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Newtons Method

- **Ruggiero, Example 5, p. 199**

$$\underline{\mathbf{F}}(\underline{x}) = \begin{pmatrix} x_0 + x_1 - 3 \\ x_0^2 + x_1^2 - 9 \end{pmatrix}; \quad \underline{\mathbf{x}}_0 = \begin{pmatrix} 1.000000 \\ 5.000000 \end{pmatrix}$$

$$\underline{\underline{\mathbf{J}}}(\underline{x}) = \begin{pmatrix} 1 & 1 \\ 2x_0 & 2x_1 \end{pmatrix}$$

$\varepsilon_1 = 1.00e-06, \varepsilon_2 = 1.00e-06.$

k	$\underline{\mathbf{s}}_k$	$\underline{\mathbf{x}}_k$	$\underline{\underline{\mathbf{J}}}(\underline{\mathbf{x}}_k)$	$\underline{\mathbf{F}}(\underline{\mathbf{x}}_k)$
1	$\begin{pmatrix} 1.6250000000 \\ 1.3750000000 \end{pmatrix}$	$\begin{pmatrix} -0.6250000000 \\ 3.6250000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -1.2500000000 & 7.2500000000 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 4.53e+00 \end{pmatrix}$
2	$\begin{pmatrix} 0.5330882353 \\ -0.5330882353 \end{pmatrix}$	$\begin{pmatrix} -0.0919117647 \\ 3.0919117647 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -0.1838235294 & 6.1838235294 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 5.68e-01 \end{pmatrix}$
3	$\begin{pmatrix} 0.0892584228 \\ -0.0892584228 \end{pmatrix}$	$\begin{pmatrix} -0.0026533419 \\ 3.0026533419 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -0.0053066839 & 6.0053066839 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 1.59e-02 \end{pmatrix}$
4	$\begin{pmatrix} 0.0026509993 \\ -0.0026509993 \end{pmatrix}$	$\begin{pmatrix} -0.0000023426 \\ 3.0000023426 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -0.0000046852 & 6.0000046852 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 1.41e-05 \end{pmatrix}$
5	$\begin{pmatrix} 0.0000023426 \\ -0.0000023426 \end{pmatrix}$	$\begin{pmatrix} -0.0000000000 \\ 3.0000000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -0.0000000000 & 6.0000000000 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 1.10e-11 \end{pmatrix}$
6	$\begin{pmatrix} 0.0000000000 \\ 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} -0.0000000000 \\ 3.0000000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 & 1.0000000000 \\ -0.0000000000 & 6.0000000000 \end{pmatrix}$	$\begin{pmatrix} 0.00e+00 \\ 1.10e-11 \end{pmatrix}$

Convergence.

$$\underline{\mathbf{x}}^* = \begin{pmatrix} -0.000000 \\ 3.000000 \end{pmatrix}; \quad ||\underline{\mathbf{s}}^*|| = 0.00e+00; \quad ||\underline{\mathbf{F}}(\underline{\mathbf{x}}^*)|| = 1.10e-11$$

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Newtons Method

- **Ruggiero, Exercise 2.a, p. 205**

$$\underline{\mathbf{F}}(\underline{x}) = \begin{pmatrix} x_0^2 + x_1^2 - 2 \\ e^{x_0-1} + x_1^3 - 2 \end{pmatrix}; \quad \underline{\mathbf{x}}_0 = \begin{pmatrix} 1.500000 \\ 2.000000 \end{pmatrix}$$

$$\underline{\underline{\mathbf{J}}}(\underline{x}) = \begin{pmatrix} 2x_0 & 2x_1 \\ e^{x_0-1} & 3x_1^2 \end{pmatrix}$$

$\varepsilon_1 = 1.00e-06, \varepsilon_2 = 1.00e-06.$

k	$\underline{\mathbf{s}}_k$	$\underline{\mathbf{x}}_k$	$\underline{\underline{\mathbf{J}}}(\underline{\mathbf{x}}_k)$	$\underline{\mathbf{F}}(\underline{\mathbf{x}}_k)$
1	$\begin{pmatrix} 0.6939308000 \\ 0.5420519000 \end{pmatrix}$	$\begin{pmatrix} 0.8060692000 \\ 1.4579481000 \end{pmatrix}$	$\begin{pmatrix} 1.6121384001 & 2.9158961999 \\ 0.8237149034 & 6.3768379866 \end{pmatrix}$	$\begin{pmatrix} 7.75e-01 \\ 1.92e+00 \end{pmatrix}$
2	$\begin{pmatrix} 0.0840500702 \\ -0.3123775679 \end{pmatrix}$	$\begin{pmatrix} 0.8901192702 \\ 1.1455705321 \end{pmatrix}$	$\begin{pmatrix} 1.7802385404 & 2.2911410642 \\ 0.8959409880 & 3.9369955320 \end{pmatrix}$	$\begin{pmatrix} 1.05e-01 \\ 3.99e-01 \end{pmatrix}$
3	$\begin{pmatrix} 0.1014698784 \\ -0.1245164481 \end{pmatrix}$	$\begin{pmatrix} 0.9915891486 \\ 1.0210540840 \end{pmatrix}$	$\begin{pmatrix} 1.9831782973 & 2.0421081679 \\ 0.9916244209 & 3.1276543272 \end{pmatrix}$	$\begin{pmatrix} 2.58e-02 \\ 5.61e-02 \end{pmatrix}$
4	$\begin{pmatrix} 0.0081193217 \\ -0.0205192582 \end{pmatrix}$	$\begin{pmatrix} 0.9997084703 \\ 1.0005348258 \end{pmatrix}$	$\begin{pmatrix} 1.9994169407 & 2.0010696516 \\ 0.9997085128 & 3.0032098130 \end{pmatrix}$	$\begin{pmatrix} 4.87e-04 \\ 1.31e-03 \end{pmatrix}$
5	$\begin{pmatrix} 0.0002913577 \\ -0.0005344686 \end{pmatrix}$	$\begin{pmatrix} 0.9999998281 \\ 1.0000003572 \end{pmatrix}$	$\begin{pmatrix} 1.9999996561 & 2.0000007144 \\ 0.9999998281 & 3.0000021433 \end{pmatrix}$	$\begin{pmatrix} 3.71e-07 \\ 9.00e-07 \end{pmatrix}$
6	$\begin{pmatrix} 0.0000000000 \\ 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} 0.9999998281 \\ 1.0000003572 \end{pmatrix}$	$\begin{pmatrix} 1.9999996561 & 2.0000007144 \\ 0.9999998281 & 3.0000021433 \end{pmatrix}$	$\begin{pmatrix} 3.71e-07 \\ 9.00e-07 \end{pmatrix}$

Convergence.

$$\underline{\mathbf{x}}^* = \begin{pmatrix} 1.000000 \\ 1.000000 \end{pmatrix}; \quad ||\underline{\mathbf{s}}^*|| = 0.00e+00; \quad ||\underline{\mathbf{F}}(\underline{\mathbf{x}}^*)|| = 9.00e-07$$

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Newtons Method

- **Ruggiero, Exercise 2.b, p. 205**

$$\underline{\mathbf{F}}(\underline{x}) = \begin{pmatrix} 4x_0 - x_0^3 + x_1 \\ -\frac{1}{9}x_0^2 + x_1 - \frac{1}{4}x_1^2 + 1 \end{pmatrix}; \quad \underline{\mathbf{x}}_0 = \begin{pmatrix} -1.000000 \\ -2.000000 \end{pmatrix}$$

$$\underline{\underline{\mathbf{J}}}(\underline{x}) = \begin{pmatrix} 4 - 3x_0^2 & 1 \\ -\frac{2}{9}x_0 & 1 - \frac{1}{2}x_1 \end{pmatrix}$$

$\varepsilon_1 = 1.00e-06, \varepsilon_2 = 1.00e-06.$

k	$\underline{\mathbf{s}}_k$	$\underline{\mathbf{x}}_k$	$\underline{\underline{\mathbf{J}}}(\underline{\mathbf{x}}_k)$	$\underline{\mathbf{F}}(\underline{\mathbf{x}}_k)$
1	$\begin{pmatrix} -4.4375000000 \\ -0.5625000000 \end{pmatrix}$	$\begin{pmatrix} 3.4375000000 \\ -1.4375000000 \end{pmatrix}$	$\begin{pmatrix} -31.4492187500 & 1.0000000000 \\ -0.7638888889 & 1.7187500000 \end{pmatrix}$	$\begin{pmatrix} -2.83e+01 \\ -2.27e+00 \end{pmatrix}$
2	$\begin{pmatrix} -0.8704270411 \\ 0.9321460625 \end{pmatrix}$	$\begin{pmatrix} 2.5670729589 \\ -0.5053539375 \end{pmatrix}$	$\begin{pmatrix} -15.7695907288 & 1.0000000000 \\ -0.5704606575 & 1.2526769687 \end{pmatrix}$	$\begin{pmatrix} -7.15e+00 \\ -3.01e-01 \end{pmatrix}$
3	$\begin{pmatrix} -0.4514185397 \\ 0.0350370735 \end{pmatrix}$	$\begin{pmatrix} 2.1156544192 \\ -0.4703168639 \end{pmatrix}$	$\begin{pmatrix} -9.4279808649 & 1.0000000000 \\ -0.4701454265 & 1.2351584320 \end{pmatrix}$	$\begin{pmatrix} -1.48e+00 \\ -2.29e-02 \end{pmatrix}$
4	$\begin{pmatrix} -0.1612379175 \\ -0.0427931279 \end{pmatrix}$	$\begin{pmatrix} 1.9544165017 \\ -0.5131099919 \end{pmatrix}$	$\begin{pmatrix} -7.4592315866 & 1.0000000000 \\ -0.4343147782 & 1.2565549959 \end{pmatrix}$	$\begin{pmatrix} -1.61e-01 \\ -3.35e-03 \end{pmatrix}$
5	$\begin{pmatrix} -0.0222322600 \\ -0.0050211543 \end{pmatrix}$	$\begin{pmatrix} 1.9321842417 \\ -0.5181311462 \end{pmatrix}$	$\begin{pmatrix} -7.2000078320 & 1.0000000000 \\ -0.4293742759 & 1.2590655731 \end{pmatrix}$	$\begin{pmatrix} -2.89e-03 \\ -6.12e-05 \end{pmatrix}$
6	$\begin{pmatrix} -0.0004138273 \\ -0.0000925008 \end{pmatrix}$	$\begin{pmatrix} 1.9317704144 \\ -0.5182236470 \end{pmatrix}$	$\begin{pmatrix} -7.1952108017 & 1.0000000000 \\ -0.4292823143 & 1.2591118235 \end{pmatrix}$	$\begin{pmatrix} -9.93e-07 \\ -2.12e-08 \end{pmatrix}$
7	$\begin{pmatrix} 0.0000000000 \\ 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} 1.9317704144 \\ -0.5182236470 \end{pmatrix}$	$\begin{pmatrix} -7.1952108017 & 1.0000000000 \\ -0.4292823143 & 1.2591118235 \end{pmatrix}$	$\begin{pmatrix} -9.93e-07 \\ -2.12e-08 \end{pmatrix}$

Convergence.

$$\underline{\mathbf{x}}^* = \begin{pmatrix} 1.931770 \\ -0.518224 \end{pmatrix}; \quad ||\underline{\mathbf{s}}^*|| = 0.00e+00; \quad ||\underline{\mathbf{F}}(\underline{\mathbf{x}}^*)|| = 9.93e-07$$

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Newtons Method

- **Ruggiero, Exercise 2.c, p. 205**

$$\underline{\mathbf{F}}(\underline{x})=\begin{pmatrix}\frac{1}{9}(2x_0-x_0^2+8)+\frac{1}{4}(4x_1-x_1^2)\\ 8x_0-4x_0^2+x_1^2+1\end{pmatrix};\quad \underline{\mathbf{x}}_0=\begin{pmatrix}-1.000000\\-1.000000\end{pmatrix}$$

$$\underline{\underline{\mathbf{J}}}(\underline{x})=\begin{pmatrix}\frac{1}{9}(2-2x_0) & 1-\frac{1}{2}x_1\\ 8-8x_0 & 2x_1\end{pmatrix}$$

$\varepsilon_1=1.00e-06, \varepsilon_2=1.00e-06.$

k	$\underline{\mathbf{s}}_k$	$\underline{\mathbf{x}}_k$	$\underline{\underline{\mathbf{J}}}(\underline{\mathbf{x}}_k)$	$\underline{\mathbf{F}}(\underline{\mathbf{x}}_k)$
1	$\begin{pmatrix}-0.5982142857\\-0.2857142857\end{pmatrix}$	$\begin{pmatrix}-0.4017857143\\-0.7142857143\end{pmatrix}$	$\begin{pmatrix}0.3115079365 & 1.3571428571\\11.2142857143 & -1.4285714286\end{pmatrix}$	$\begin{pmatrix}-6.02e-02\\-5.54e+00\end{pmatrix}$
2	$\begin{pmatrix}0.4856953264\\-0.0671465985\end{pmatrix}$	$\begin{pmatrix}0.0839096121\\-0.7814323128\end{pmatrix}$	$\begin{pmatrix}0.2035756418 & 1.3907161564\\7.3287231031 & -1.5628646256\end{pmatrix}$	$\begin{pmatrix}-2.73e-02\\-1.71e+00\end{pmatrix}$
3	$\begin{pmatrix}0.2304688797\\-0.0140787737\end{pmatrix}$	$\begin{pmatrix}0.3143784918\\-0.7955110865\end{pmatrix}$	$\begin{pmatrix}0.1523603352 & 1.3977555433\\5.4849720656 & -1.5910221731\end{pmatrix}$	$\begin{pmatrix}-5.95e-03\\2.47e-01\end{pmatrix}$
4	$\begin{pmatrix}-0.0424016490\\0.0088796998\end{pmatrix}$	$\begin{pmatrix}0.2719768428\\-0.7866313868\end{pmatrix}$	$\begin{pmatrix}0.1617829238 & 1.3933156934\\5.8241852573 & -1.5732627735\end{pmatrix}$	$\begin{pmatrix}-2.19e-04\\-1.31e-01\end{pmatrix}$
5	$\begin{pmatrix}0.0219203853\\-0.0023877325\end{pmatrix}$	$\begin{pmatrix}0.2938972282\\-0.7890191192\end{pmatrix}$	$\begin{pmatrix}0.1569117271 & 1.3945095596\\5.6488221747 & -1.5780382385\end{pmatrix}$	$\begin{pmatrix}-5.48e-05\\6.01e-02\end{pmatrix}$
6	$\begin{pmatrix}-0.0103052837\\0.0011988691\end{pmatrix}$	$\begin{pmatrix}0.2835919444\\-0.7878202501\end{pmatrix}$	$\begin{pmatrix}0.1592017901 & 1.3939101251\\5.7312644445 & -1.5756405002\end{pmatrix}$	$\begin{pmatrix}-1.22e-05\\-3.02e-02\end{pmatrix}$
7	$\begin{pmatrix}0.0051077512\\-0.0005746453\end{pmatrix}$	$\begin{pmatrix}0.2886996956\\-0.7883948954\end{pmatrix}$	$\begin{pmatrix}0.1580667343 & 1.3941974477\\5.6904024352 & -1.5767897909\end{pmatrix}$	$\begin{pmatrix}-2.98e-06\\1.45e-02\end{pmatrix}$
8	$\begin{pmatrix}-0.0024719368\\0.0002823935\end{pmatrix}$	$\begin{pmatrix}0.2862277588\\-0.7881125019\end{pmatrix}$	$\begin{pmatrix}0.1586160536 & 1.3940562510\\5.7101779293 & -1.5762250039\end{pmatrix}$	$\begin{pmatrix}-6.99e-07\\-7.13e-03\end{pmatrix}$
9	$\begin{pmatrix}0.0012108071\\-0.0001372646\end{pmatrix}$	$\begin{pmatrix}0.2874385659\\-0.7882497665\end{pmatrix}$	$\begin{pmatrix}0.1583469854 & 1.3941248833\\5.7004914728 & -1.5764995330\end{pmatrix}$	$\begin{pmatrix}-1.68e-07\\3.47e-03\end{pmatrix}$
10	$\begin{pmatrix}-0.0005896641\\0.0000670952\end{pmatrix}$	$\begin{pmatrix}0.2868489018\\-0.7881826713\end{pmatrix}$	$\begin{pmatrix}0.1584780218 & 1.3940913356\\5.7052087856 & -1.5763653426\end{pmatrix}$	$\begin{pmatrix}-3.98e-08\\-1.69e-03\end{pmatrix}$
11	$\begin{pmatrix}0.0002879839\\-0.0000327090\end{pmatrix}$	$\begin{pmatrix}0.2871368857\\-0.7882153803\end{pmatrix}$	$\begin{pmatrix}0.1584140254 & 1.3941076902\\5.7029049141 & -1.5764307606\end{pmatrix}$	$\begin{pmatrix}-9.48e-09\\8.26e-04\end{pmatrix}$
12	$\begin{pmatrix}-0.0001404534\\0.0000159667\end{pmatrix}$	$\begin{pmatrix}0.2869964323\\-0.7881994136\end{pmatrix}$	$\begin{pmatrix}0.1584452373 & 1.3940997068\\5.7040285413 & -1.5763988272\end{pmatrix}$	$\begin{pmatrix}-2.26e-09\\-4.03e-04\end{pmatrix}$
13	$\begin{pmatrix}0.0000685471\\-0.0000077891\end{pmatrix}$	$\begin{pmatrix}0.2870649795\\-0.7882072027\end{pmatrix}$	$\begin{pmatrix}0.1584300046 & 1.3941036013\\5.7034801642 & -1.5764144053\end{pmatrix}$	$\begin{pmatrix}-5.37e-10\\1.97e-04\end{pmatrix}$
14	$\begin{pmatrix}-0.0000334429\\0.0000038009\end{pmatrix}$	$\begin{pmatrix}0.2870315366\\-0.7882034017\end{pmatrix}$	$\begin{pmatrix}0.1584374363 & 1.3941017009\\5.7037477072 & -1.5764068035\end{pmatrix}$	$\begin{pmatrix}-1.28e-10\\-9.60e-05\end{pmatrix}$
15	$\begin{pmatrix}0.0000163188\\-0.0000018545\end{pmatrix}$	$\begin{pmatrix}0.2870478554\\-0.7882052563\end{pmatrix}$	$\begin{pmatrix}0.1584338099 & 1.3941026281\\5.7036171570 & -1.5764105125\end{pmatrix}$	$\begin{pmatrix}-3.04e-11\\4.68e-05\end{pmatrix}$
16	$\begin{pmatrix}-0.0000079623\\0.0000009049\end{pmatrix}$	$\begin{pmatrix}0.2870398931\\-0.7882043514\end{pmatrix}$	$\begin{pmatrix}0.1584355793 & 1.3941021757\\5.7036808552 & -1.5764087027\end{pmatrix}$	$\begin{pmatrix}-7.25e-12\\-2.29e-05\end{pmatrix}$
17	$\begin{pmatrix}0.0000038851\\-0.0000004415\end{pmatrix}$	$\begin{pmatrix}0.2870437782\\-0.7882047929\end{pmatrix}$	$\begin{pmatrix}0.1584347160 & 1.3941023964\\5.7036497743 & -1.5764095858\end{pmatrix}$	$\begin{pmatrix}-1.73e-12\\1.12e-05\end{pmatrix}$
18	$\begin{pmatrix}-0.0000018957\\0.0000002154\end{pmatrix}$	$\begin{pmatrix}0.2870418825\\-0.7882045774\end{pmatrix}$	$\begin{pmatrix}0.1584351372 & 1.3941022887\\5.7036649396 & -1.5764091549\end{pmatrix}$	$\begin{pmatrix}-4.11e-13\\-5.44e-06\end{pmatrix}$
19	$\begin{pmatrix}0.0000009250\\-0.0000001051\end{pmatrix}$	$\begin{pmatrix}0.2870428075\\-0.7882046826\end{pmatrix}$	$\begin{pmatrix}0.1584349317 & 1.3941023413\\5.7036575399 & -1.5764093651\end{pmatrix}$	$\begin{pmatrix}-9.78e-14\\2.66e-06\end{pmatrix}$

No further progress.

$$\underline{\mathbf{x}}^*=\begin{pmatrix}0.287043\\-0.788205\end{pmatrix};\quad ||\underline{\mathbf{s}}^*||=0.00e+00;\quad ||\underline{\mathbf{F}}(\underline{\mathbf{x}}^*)||=2.66e-06$$

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Newtons Method

- **Ruggiero, Exercise 2.d (Rosenbrock), p. 205**

$$\underline{\mathbf{F}}(\underline{x}) = \begin{pmatrix} 10(x_1 - x_0^2) \\ 1 - x_0 \end{pmatrix}; \quad \underline{\mathbf{x}}_0 = \begin{pmatrix} -1.200000 \\ 1.000000 \end{pmatrix}$$

$$\underline{\underline{\mathbf{J}}}(\underline{x}) = \begin{pmatrix} -20x_0 & 10 \\ -1 & 0 \end{pmatrix}$$

$\varepsilon_1 = 1.00e-06, \varepsilon_2 = 1.00e-06.$

k	$\underline{\mathbf{s}}_k$	$\underline{\mathbf{x}}_k$	$\underline{\underline{\mathbf{J}}}(\underline{\mathbf{x}}_k)$	$\underline{\mathbf{F}}(\underline{\mathbf{x}}_k)$
1	$\begin{pmatrix} -2.2000000000 \\ 4.8400000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 \\ -3.8400000000 \end{pmatrix}$	$\begin{pmatrix} -20.0000000000 & 10.0000000000 \\ -1.0000000000 & 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} -4.84e+01 \\ -2.22e-16 \end{pmatrix}$
2	$\begin{pmatrix} -0.0000000000 \\ 4.8400000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 \\ 1.0000000000 \end{pmatrix}$	$\begin{pmatrix} -20.0000000000 & 10.0000000000 \\ -1.0000000000 & 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} -4.44e-15 \\ 2.22e-16 \end{pmatrix}$
3	$\begin{pmatrix} 0.0000000000 \\ 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} 1.0000000000 \\ 1.0000000000 \end{pmatrix}$	$\begin{pmatrix} -20.0000000000 & 10.0000000000 \\ -1.0000000000 & 0.0000000000 \end{pmatrix}$	$\begin{pmatrix} -4.44e-15 \\ 2.22e-16 \end{pmatrix}$

Convergence.

$$\underline{\mathbf{x}}^* = \begin{pmatrix} 1.000000 \\ 1.000000 \end{pmatrix}; \quad ||\underline{\mathbf{s}}^*|| = 0.00e+00; \quad ||\underline{\mathbf{F}}(\underline{\mathbf{x}}^*)|| = 4.44e-15$$

```
#!/usr/bin/python3

from math import *

from Vector import *
from Matrix import *
from Gauss_Pivotation import *

##!
##! Implement Newtons method for nonlinear systems.
##!

##!
##! F: Function (def) returning the list of functions to be zeroed.
##! J: Function (def) returning the Jacobian.
##! x: Starting x, list.
##!

def Newton(F,J,x,eps1=1.0E-6,eps2=1.0E-6,max_iter=100):
    xs=[]
    ss=[]
    res=0

    k=1
    while (res==0 and k<=max_iter):
        res,x=Newton_Iteration(F,J,x,eps1,eps2)
        xs.append(x)

        k+=1
    return res,xs

##!
##! Do one Newton iteration
##!

def Newton_Iteration(F,J,x,eps1,eps2):
    Fx=F(x)

    res=0
    if (Vector_Norm_Inf(Fx)<eps1):
        res=1
    else:
        Jx=J(x)
        s=Vector_Mul(Fx,-1.0)
        Gauss_Pivotation(Jx,s)

        x=Vectors_Add(x,s)
        if (Vector_Norm_Inf(s)<eps2):
            res=2

    return res,x


##!
##! Gather Latex for Newton iterations
##!

def Newton_Latex(info,F_Latex,J_Latex,F,J,x,eps1,eps2):
    res,xs=Newton(F,J,x,eps1,eps2)

    latex=[
        Latex_Title("Newtons Method"),
        [
            "$\\bullet$",
            "\\textbf{"+info+"}",
        ],
        Latex_Math([
            "\\Vector{F}(\\underline{x})=",
            Vector_Latex(
                F_Latex(),options=""
            ),
            ";\\quad",
            "\\Vector{x}_0=",
            Vector_Latex(x),
        ]),
        Latex_Math([
            "\\Matrix{J}(\\underline{x})=",
            Matrix_Latex(
                J_Latex(),options=""
            )
        ]),
        Latex_Inline([
            "\\varepsilon_1="+("%.2e" % eps1)+",",
            "\\varepsilon_2="+("%.2e" % eps2)+".",
        ]),
    ]
```

```
    table=[
        [
            "$k$",
            "$\\Vector{s}_k$",
            "$\\Vector{x}_k$",
            "$\\Matrix{J}(\\Vector{x}_k)$",
            "$\\Vector{F}(\\Vector{x}_k)$",
        ]
    ]

    for k in range(len(xs)):
        table.append(
            Newton_Latex_Iteration(F,J,x,xs,k)
        )

    latex=latex+Latex_Table(table)

    niterations=len(xs)
    eps1_val=Vector_Norm_Inf(
        Vectors_Sub(xs[niterations -1],xs[niterations -1])
    )
    eps2_val=Vector_Norm_Inf(
        F(xs[niterations -1])
    )

    if (eps2_val<=eps2):
        latex.append("Convergence.")
    elif (eps1_val<=eps1):
        latex.append("No further progress.")
    else:
        latex.append("Divergence.")

    latex=latex+[
        Latex_Math([
            "\\Vector{x}^*=",
            Vector_Latex(xs[niterations -1]),
            ";\\quad",
            "||\\Vector{s}^*||=",
            ("%.2e" % eps1_val),
            ";\\quad",
            "||\\Vector{F}(\\Vector{x}^*)||=",
            ("%.2e" % eps2_val)
        ])
    ]

    return latex+["\\clearpage\n\n"]

##!
##! Gather Latex row for one Newton iteration
##!

def Newton_Latex_Iteration(F,J,x,xs,k):
    s=[]
    if (k>0):
        s=Vectors_Sub(xs[k],xs[k-1])
    else:
        s=Vectors_Sub(x,xs[k])


    return [
        str(k+1),
        Latex_Inline([
            Vector_Latex(s,frmt="%.10f"),
        ]),
        Latex_Inline([
            Vector_Latex(xs[k],frmt="%.10f"),
        ]),
        Latex_Inline([
            Matrix_Latex(J(xs[k]),frmt="%.10f"),
        ]),
        Latex_Inline([
            Vector_Latex(F(xs[k]),frmt="%.2e"),
        ]),
    ]


#Ruggiero, Example 5, p. 199.
def F1(x):
    return [
        x[0]+x[1]-3.0,
        x[0]**2+x[1]**2-9.0,
    ]


def J1(x):
```

```
    return [
        [
            1.0,1.0
        ],
        [
            2.0*x[0],2.0*x[1]
        ],
    ]

def F1_Latex():
    return [
        "x_0+x_1-3",
        "x_0^2+x_1^2-9",
    ]

def J1_Latex():
    return [
        ["1","1"],
        ["2x_0","2x_1"]
    ]


#Ruggiero, Exercise 2.a, p. 205.
def F2(x):
    return [
        x[0]**2+x[1]**2-2.0,
        e**(x[0]-1.0)+x[1]**3-2.0,
    ]


def J2(x):
    return [
        [
            2.0*x[0],2.0*x[1]
        ],
        [
            e**(x[0]-1.0),3.0*x[1]**2
        ],
    ]

def F2_Latex():
    return [
        "x_0^2+x_1^2-2",
        "e^{x_0-1}+x_1^3-2",
    ]

def J2_Latex():
    return [
        ["2x_0","2x_1"],
        ["e^{x_0-1}","3x_1^2"],
    ]

#Ruggiero, Exercise 2.b, p. 205.
def F3(x):
    return [
        4.0*x[0]-x[0]**3+x[1],
        -x[0]**2/9.0+x[1]-0.25*x[1]**2+1.0
    ]


def J3(x):
    return [
        [
            4.0-3.0*x[0]**2,1.0
        ],
        [
            -2.0/9.0*x[0],1.0-0.5*x[1]
        ],
    ]

def F3_Latex():
    return [
        "4x_0-x_0^3+x_1",
        "-\\frac{1}{9}x_0^2+x_1-\\frac{1}{4} x_1^2+1",
    ]

def J3_Latex():
    return [
        ["4-3x_0^2","1"],
        ["-\\frac{2}{9}x_0","1-\\frac{1}{2}x_1"],
    ]

#Ruggiero, Exercise 2.c, p. 205.
def F4(x):
    return [
        (2.0*x[0]-x[0]**2+8)/9.0    +   x[1]-0.25*x[1]**2,
```

```
        8.0*x[0]-4.0+x[0]**2    +    x[1]**2+1.0
    ]


def J4(x):
    return [
        [
            (2.0-2.0*x[0])/9.0,
            1-0.5*x[1]
        ],
        [
            8.0-8.0*x[0],
            2.0*x[1]
        ],
    ]

def F4_Latex():
    return [
        "\\frac{1}{9}(2x_0-x_0^2+8)+\\frac{1}{4}(4x_1-x_1^2)",
        "8x_0-4x_0^2+x_1^2+1",
    ]

def J4_Latex():
    return [
        [
            "\\frac{1}{9}(2-2x_0)",
            "1-\\frac{1}{2}x_1"
        ],
        [
            "8-8x_0",
            "2x_1"
        ],
    ]


#Ruggiero, Exercise 2.d, p. 205 (Rosenbrock).
def F5(x):
    return [
        10.0*(x[1]-x[0]**2),
        1.0-x[0]
    ]


def J5(x):
    return [
        [
            -20.0*x[0],
            10.0
        ],
        [
            -1.0,
            0.0
        ],
    ]

def F5_Latex():
    return [
        "10(x_1-x_0^2)",
        "1-x_0",
    ]

def J5_Latex():
    return [
        [
            "-20x_0",
            "10",
        ],
        [
            "-1",
            "0",
        ],
    ]


##!
##! To keep SmtC happy.
##!

def dummy():
    return 0

###!
###! Testing ###
###!
###!
```

```
if (__name__=='__main__'):

    eps1=eps2=1.0E-6

    latex=[]

    x1=[1.0,5.0]
    latex=latex+Newton_Latex(
            "Ruggiero, Example 5, p. 199",
            F1_Latex,J1_Latex,
            F1,J1,x1,
            eps1,eps2
    )

    x2=[1.5,2.0]
    latex=latex+Newton_Latex(
            "Ruggiero, Exercise 2.a, p. 205",
            F2_Latex,J2_Latex,
            F2,J2,x2,
            eps1,eps2
    )

    x3=[-1,-2.0]
    latex=latex+Newton_Latex(
            "Ruggiero, Exercise 2.b, p. 205",
            F3_Latex,J3_Latex,
            F3,J3,x3,
            eps1,eps2
    )

    x4=[-1,-1.0]
    latex=latex+Newton_Latex(
            "Ruggiero, Exercise 2.c, p. 205",
            F4_Latex,J4_Latex,
            F4,J4,x4,
            eps1,eps2
    )

    x5=[-1.2,1.0]
    latex=latex+Newton_Latex(
            "Ruggiero, Exercise 2.d (Rosenbrock), p. 205",
            F5_Latex,J5_Latex,
            F5,J5,x5,
            eps1,eps2
    )

    latex=latex+["\\lstinputlisting{Newton.py}"]

    latex=Latex_Print(latex)

    Latex_Save("Newton.tex",latex)
```