Dr. Ole Peter Smith
Instituto de Matemática e Estatística
Universidade Federal de Goiás
ole@ufg.br - http://www.olesmith.com.br

Gauss Jacobi:

$$\underline{\underline{A}}\,\underline{x} = \begin{pmatrix} 10.000 & 2.000 & 1.000 \\ 1.000 & 5.000 & 1.000 \\ 2.000 & 3.000 & 10.000 \end{pmatrix} \underline{x} = \begin{pmatrix} 7.000000 \\ -8.000000 \\ 6.000000 \end{pmatrix} = \underline{b}$$

Iterations:

$$\underline{x}_{i+1} = \underline{\underline{C}}\,\underline{x}_i + \underline{g},$$

Where:

$$\underline{\underline{C}} = \begin{pmatrix} 0.000 & -0.200 & -0.100 \\ -0.200 & 0.000 & -0.200 \\ -0.200 & -0.300 & 0.000 \end{pmatrix}; \qquad \underline{g} = \begin{pmatrix} 0.700000 \\ -1.600000 \\ 0.600000 \end{pmatrix}; \qquad \underline{x_0} = \begin{pmatrix} 0.700000 \\ -1.600000 \\ 0.600000 \end{pmatrix}$$

Diagonally dominant: Yes

$$\underline{\alpha} = \begin{pmatrix} 0.300000 \\ 0.400000 \\ 0.500000 \end{pmatrix}; \qquad ||\underline{\alpha}|| = 0.500000$$

Iteration 0:

$$\underline{x}_0 = \begin{pmatrix} 0.700000 \\ -1.600000 \\ 0.600000 \end{pmatrix}; \quad \underline{r}_0 = \underline{\underline{A}}\,\underline{x}_0 - \underline{b} = \begin{pmatrix} -2.60E+00 \\ 1.30E+00 \\ -3.40E+00 \end{pmatrix}; \quad ||\underline{r}_0|| = 4.25E-01;$$

Iteration 1:

$$\underline{x}_1 = \begin{pmatrix} 0.960000 \\ -1.860000 \\ 0.940000 \end{pmatrix}; \quad \underline{r}_1 = \underline{\underline{A}}\,\underline{x}_1 - \underline{b} = \begin{pmatrix} -1.80E-01 \\ 6.00E-01 \\ -2.60E-01 \end{pmatrix}; \quad ||\underline{r}_1|| = 7.50E-02;$$

$$\underline{d}_1 = \underline{x}_1 - \underline{x}_0 = \begin{pmatrix} 2.60E-01 \\ -2.60E-01 \\ 3.40E-01 \end{pmatrix}; \quad ||\underline{d}_1|| = 2.12E-01$$

Iteration 2:

$$\underline{x}_2 = \begin{pmatrix} 0.978000 \\ -1.980000 \\ 0.966000 \end{pmatrix}; \quad \underline{r}_2 = \underline{\underline{A}}\,\underline{x}_2 - \underline{b} = \begin{pmatrix} -2.14E-01 \\ 4.40E-02 \\ -3.24E-01 \end{pmatrix}; \quad ||\underline{r}_2|| = 4.05E-02;$$

$$\underline{d}_2 = \underline{x}_2 - \underline{x}_1 = \begin{pmatrix} 1.80E-02 \\ -1.20E-01 \\ 2.60E-02 \end{pmatrix}; \quad ||\underline{d}_2|| = 6.45E-02$$

Iteration 3:

$$\underline{x}_3 = \begin{pmatrix} 0.999400 \\ -1.988800 \\ 0.998400 \end{pmatrix}; \quad \underline{r}_3 = \underline{\underline{A}}\,\underline{x}_3 - \underline{b} = \begin{pmatrix} 1.48E-02 \\ 5.38E-02 \\ 1.64E-02 \end{pmatrix}; \quad ||\underline{r}_3|| = 6.72E-03;$$

$$\underline{d}_3 = \underline{x}_3 - \underline{x}_2 = \begin{pmatrix} 2.14E-02 \\ -8.80E-03 \\ 3.24E-02 \end{pmatrix}; \quad ||\underline{d}_3|| = 1.64E-02$$

Iteration 4:

$$\underline{x}_4 = \begin{pmatrix} 0.997920 \\ -1.999560 \\ 0.996760 \end{pmatrix}; \quad \underline{r}_4 = \underline{\underline{A}}\,\underline{x}_4 - \underline{b} = \begin{pmatrix} -2.32E-02 \\ -3.12E-03 \\ -3.52E-02 \end{pmatrix}; \quad ||\underline{r}_4|| = 4.41E-03;$$

$$\underline{d}_4 = \underline{x}_4 - \underline{x}_3 = \begin{pmatrix} -1.48E-03 \\ -1.08E-02 \\ -1.64E-03 \end{pmatrix}; \quad ||\underline{d}_4|| = 5.41E-03$$

Iteration 5:

$$\underline{x}_5 = \begin{pmatrix} 1.000236 \\ -1.998936 \\ 1.000284 \end{pmatrix}; \quad \underline{r}_5 = \underline{\underline{A}}\,\underline{x}_5 - \underline{b} = \begin{pmatrix} 4.77E-03 \\ 5.84E-03 \\ 6.50E-03 \end{pmatrix}; \quad ||\underline{r}_5|| = 8.13E-04;$$

$$\underline{d}_5 = \underline{x}_5 - \underline{x}_4 = \begin{pmatrix} 2.32E-03 \\ 6.24E-04 \\ 3.52E-03 \end{pmatrix}; \quad ||\underline{d}_5|| = 1.76E-03$$

Iteration 6:

$$\underline{x}_6 = \begin{pmatrix} 0.999759 \\ -2.000104 \\ 0.999634 \end{pmatrix}; \quad \underline{r}_6 = \underline{\underline{A}}\,\underline{x}_6 - \underline{b} = \begin{pmatrix} -2.99E-03 \\ -1.13E-03 \\ -4.46E-03 \end{pmatrix}; \quad ||\underline{r}_6|| = 5.57E-04;$$

$$\underline{d}_6 = \underline{x}_6 - \underline{x}_5 = \begin{pmatrix} -4.77E-04 \\ -1.17E-03 \\ -6.50E-04 \end{pmatrix}; \quad ||\underline{d}_6|| = 5.84E-04$$

Iteration 7:

$$\underline{x}_7 = \begin{pmatrix} 1.000057 \\ -1.999878 \\ 1.000079 \end{pmatrix}; \quad \underline{r}_7 = \underline{\underline{A}}\,\underline{x}_7 - \underline{b} = \begin{pmatrix} 8.97E-04 \\ 7.44E-04 \\ 1.27E-03 \end{pmatrix}; \quad ||\underline{r}_7|| = 1.59E-04;$$

$$\underline{d}_7 = \underline{x}_7 - \underline{x}_6 = \begin{pmatrix} 2.99E-04 \\ 2.26E-04 \\ 4.46E-04 \end{pmatrix}; \quad ||\underline{d}_7|| = 2.23E-04$$

Iteration 8:

$$\underline{x}_8 = \begin{pmatrix} 0.999968 \\ -2.000027 \\ 0.999952 \end{pmatrix}; \quad \underline{r}_8 = \underline{\underline{A}}\,\underline{x}_8 - \underline{b} = \begin{pmatrix} -4.25E-04 \\ -2.17E-04 \\ -6.26E-04 \end{pmatrix}; \quad ||\underline{r}_8|| = 7.83E-05;$$

$$\underline{d}_8 = \underline{x}_8 - \underline{x}_7 = \begin{pmatrix} -8.97E-05 \\ -1.49E-04 \\ -1.27E-04 \end{pmatrix}\ ;\ ||\underline{d}_8|| = 7.45E-05$$

Iteration 9:

$$\underline{x}_9 = \begin{pmatrix} 1.000010 \\ -1.999984 \\ 1.000015 \end{pmatrix}\ ;\ \underline{r}_9 = \underline{\underline{A}}\,\underline{x}_9 - \underline{b} = \begin{pmatrix} 1.49E-04 \\ 1.05E-04 \\ 2.15E-04 \end{pmatrix}\ ;\ ||\underline{r}_9|| = 2.69E-05;$$

$$\underline{d}_9 = \underline{x}_9 - \underline{x}_8 = \begin{pmatrix} 4.25E-05 \\ 4.34E-05 \\ 6.26E-05 \end{pmatrix}\ ;\ ||\underline{d}_9|| = 3.13E-05$$

Iteration 10:

$$\underline{x}_{10} = \begin{pmatrix} 0.999995 \\ -2.000005 \\ 0.999993 \end{pmatrix}\ ;\ \underline{r}_{10} = \underline{\underline{A}}\,\underline{x}_{10} - \underline{b} = \begin{pmatrix} -6.36E-05 \\ -3.65E-05 \\ -9.30E-05 \end{pmatrix}\ ;\ ||\underline{r}_{10}|| = 1.16E-05;$$

$$\underline{d}_{10} = \underline{x}_{10} - \underline{x}_9 = \begin{pmatrix} -1.49E-05 \\ -2.10E-05 \\ -2.15E-05 \end{pmatrix}\ ;\ ||\underline{d}_{10}|| = 1.08E-05$$

Iteration 11:

$$\underline{x}_{11} = \begin{pmatrix} 1.000002 \\ -1.999998 \\ 1.000002 \end{pmatrix}\ ;\ \underline{r}_{11} = \underline{\underline{A}}\,\underline{x}_{11} - \underline{b} = \begin{pmatrix} 2.39E-05 \\ 1.57E-05 \\ 3.46E-05 \end{pmatrix}\ ;\ ||\underline{r}_{11}|| = 4.32E-06;$$

$$\underline{d}_{11} = \underline{x}_{11} - \underline{x}_{10} = \begin{pmatrix} 6.36E-06 \\ 7.29E-06 \\ 9.30E-06 \end{pmatrix}\ ;\ ||\underline{d}_{11}|| = 4.65E-06$$

Iteration 12:

$$\underline{x}_{12} = \begin{pmatrix} 0.999999 \\ -2.000001 \\ 0.999999 \end{pmatrix}\ ;\ \underline{r}_{12} = \underline{\underline{A}}\,\underline{x}_{12} - \underline{b} = \begin{pmatrix} -9.72E-06 \\ -5.85E-06 \\ -1.42E-05 \end{pmatrix}\ ;\ ||\underline{r}_{12}|| = 1.77E-06;$$

$$\underline{d}_{12} = \underline{x}_{12} - \underline{x}_{11} = \begin{pmatrix} -2.39E-06 \\ -3.13E-06 \\ -3.46E-06 \end{pmatrix}\ ;\ ||\underline{d}_{12}|| = 1.73E-06$$

Iteration 13:

$$\underline{x}_{13} = \begin{pmatrix} 1.000000 \\ -2.000000 \\ 1.000000 \end{pmatrix}\ ;\ \underline{r}_{13} = \underline{\underline{A}}\,\underline{x}_{13} - \underline{b} = \begin{pmatrix} 3.76E-06 \\ 2.39E-06 \\ 5.45E-06 \end{pmatrix}\ ;\ ||\underline{r}_{13}|| = 6.82E-07;$$

$$\underline{d}_{13} = \underline{x}_{13} - \underline{x}_{12} = \begin{pmatrix} 9.72E-07 \\ 1.17E-06 \\ 1.42E-06 \end{pmatrix}\ ;\ ||\underline{d}_{13}|| = 7.08E-07$$

Convergence, 13 iterations: 7.1E-07<1.0E-06

$$\underline{\underline{A}}\,\underline{x}^* - \underline{b} = \begin{pmatrix} 10.000 & 2.000 & 1.000 \\ 1.000 & 5.000 & 1.000 \\ 2.000 & 3.000 & 10.000 \end{pmatrix} \begin{pmatrix} 1.000000 \\ -2.000000 \\ 1.000000 \end{pmatrix} - \begin{pmatrix} 7.000000 \\ -8.000000 \\ 6.000000 \end{pmatrix} = \begin{pmatrix} 0.000004 \\ 0.000002 \\ 0.000005 \end{pmatrix}$$

$$\underline{x}^* = \begin{pmatrix} 1.0E+00 \\ -2.0E+00 \\ 1.0E+00 \end{pmatrix};\qquad \underline{r} = \begin{pmatrix} 3.8E-06 \\ 2.4E-06 \\ 5.5E-06 \end{pmatrix};\qquad ||r|| = 6.8E-07$$