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Ellipse, Natural Parametrization

Equation:

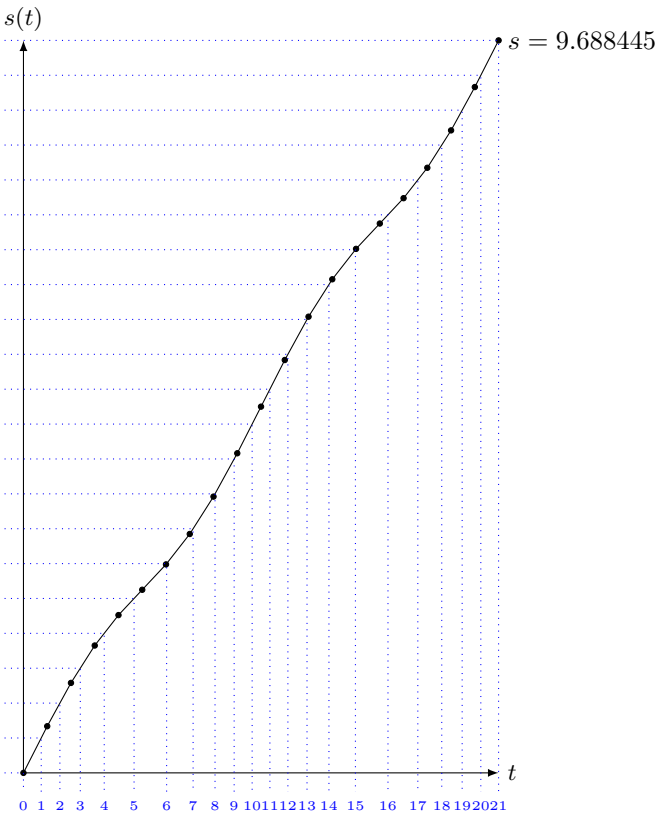$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Parametrization:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = a \begin{pmatrix} \cos t \\ \epsilon \sin t \end{pmatrix},$$

$\epsilon = \frac{b}{a}$. Take: $a = 1.0$, $b = 2.0$.

Arc Length by Numerical Integration: $N = 20$ Intervals

n	t_n	$ds(t_n)$	$s(t_n)$
0	0.0	0.616859725942879	0.0
1	0.3141592653589793	0.5731113712100473	0.616859725942879
2	0.6283185307179586	0.49457155487030247	1.1899710971529263
3	0.9424777960769379	0.4023248555098071	1.6845426520232287
4	1.2566370614359172	0.33524384394514195	2.086867507533036
5	1.5707963267948966	0.33524384394514195	2.422111351478178
6	1.8849555921538759	0.4023248555098071	2.7573551954233197
7	2.199114857512855	0.49457155487030224	3.159680050933127
8	2.5132741228718345	0.5731113712100475	3.654251605803429
9	2.827433388230814	0.616859725942879	4.227362977013477
10	3.141592653589793	0.616859725942879	4.844222702956356
11	3.4557519189487724	0.5731113712100475	5.461082428899235
12	3.7699111843077517	0.49457155487030224	6.034193800109282
13	4.084070449666731	0.4023248555098071	6.528765354979584
14	4.39822971502571	0.3352438439451424	6.9310902104893914
15	4.71238898038469	0.3352438439451415	7.266334054434534
16	5.026548245743669	0.4023248555098071	7.601577898379675
17	5.340707511102648	0.49457155487030313	8.003902753889482
18	5.654866776461628	0.5731113712100466	8.498474308759786
19	5.969026041820607	0.616859725942879	9.071585679969832
20	6.283185307179586	-	9.688445405912711

$s(t)$

$s = 9.688445$

t

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

$\mu\alpha\tau\epsilon\mu\alpha\tau\iota\kappa\alpha$
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Natural Parametrization, Equidistante $s(t_i)$

n	s_n	$s_n - s_{n-1}$	t_n
0	0.0	-	0.0
1	0.46135454313870056	0.46135454313870056	0.23496233948640535
2	0.9227090862774011	0.46135454313870056	0.48181500415602296
3	1.3840636294161017	0.4613545431387006	0.7516090166891012
4	1.8454181725548022	0.4613545431387005	1.0680990065298688
5	2.3067727156935027	0.4613545431387005	1.462711713937547
6	2.7681272588322035	0.4613545431387007	1.8933670623391992
7	3.229481801970904	0.4613545431387007	2.2434539761112737
8	3.690836345109605	0.4613545431387007	2.533328575463677
9	4.152190888248305	0.4613545431387003	2.7862267234038636
10	4.6135454313870055	0.4613545431387003	3.0241114838465903
11	5.074899974525706	0.4613545431387003	3.259073823332996
12	5.536254517664406	0.4613545431387003	3.4969585837757227
13	5.997609060803106	0.4613545431387003	3.7498567317159086
14	6.458963603941807	0.4613545431387003	4.039731331068312
15	6.920318147080507	0.4613545431387003	4.389818244840386
16	7.381672690219207	0.4613545431387003	4.8204735932420375
17	7.843027233357907	0.4613545431387003	5.215086300649716
18	8.304381776496609	0.46135454313870117	5.531576290490484
19	8.765736319635309	0.4613545431387003	5.801370303023562
20	9.22709086277401	0.4613545431387003	6.04822296769318
21	9.688445405912711	0.46135454313870206	6.283185307179586

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