



Gauss Jacobi:

$$\underline{\underline{A}} \underline{x} = \begin{pmatrix} 5.000 & 1.000 & 1.000 \\ 3.000 & 4.000 & 1.000 \\ 3.000 & 3.000 & 6.000 \end{pmatrix} \underline{x} = \begin{pmatrix} 5.000000 \\ 6.000000 \\ 0.000000 \end{pmatrix} = \underline{b}$$

Iterations:

$$\underline{x}_{i+1} = \underline{\underline{C}} \underline{x}_i + \underline{g},$$

Where:

$$\underline{\underline{C}} = \begin{pmatrix} 0.000 & -0.200 & -0.200 \\ -0.750 & 0.000 & -0.250 \\ -0.500 & -0.500 & 0.000 \end{pmatrix}; \quad \underline{g} = \begin{pmatrix} 1.000000 \\ 1.500000 \\ 0.000000 \end{pmatrix}; \quad \underline{x}_0 = \begin{pmatrix} 0.000000 \\ 0.000000 \\ 0.000000 \end{pmatrix}$$

Diagonally dominant: No

$$\underline{\alpha} = \begin{pmatrix} 0.400000 \\ 1.000000 \\ 1.000000 \end{pmatrix}; \quad ||\underline{\alpha}|| = 1.000000$$

Iteration 0:

$$\underline{x}_0 = \begin{pmatrix} 0.000000 \\ 0.000000 \\ 0.000000 \end{pmatrix}; \quad \underline{r}_0 = \underline{\underline{A}} \underline{x}_0 - \underline{b} = \begin{pmatrix} -5.00E+00 \\ -6.00E+00 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_0|| = 1.00E+00;$$

Iteration 1:

$$\underline{x}_1 = \begin{pmatrix} 1.000000 \\ 0.750000 \\ -0.875000 \end{pmatrix}; \quad \underline{r}_1 = \underline{\underline{A}} \underline{x}_1 - \underline{b} = \begin{pmatrix} -1.25E-01 \\ -8.75E-01 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_1|| = 1.46E-01;$$

$$\underline{d}_1 = \underline{x}_1 - \underline{x}_0 = \begin{pmatrix} 1.00E+00 \\ 7.50E-01 \\ -8.75E-01 \end{pmatrix}; \quad ||\underline{d}_1|| = 0.00E+00$$

Iteration 2:

$$\underline{x}_2 = \begin{pmatrix} 1.025000 \\ 0.950000 \\ -0.987500 \end{pmatrix}; \quad \underline{r}_2 = \underline{\underline{A}} \underline{x}_2 - \underline{b} = \begin{pmatrix} 8.75E-02 \\ -1.12E-01 \\ -8.88E-16 \end{pmatrix}; \quad ||\underline{r}_2|| = 1.87E-02;$$

$$\underline{d}_2 = \underline{x}_2 - \underline{x}_1 = \begin{pmatrix} 2.50E-02 \\ 2.00E-01 \\ -1.13E-01 \end{pmatrix}; \quad ||\underline{d}_2|| = 2.00E-01$$

Iteration 3:

$$\underline{x}_3 = \begin{pmatrix} 1.007500 \\ 0.991250 \\ -0.999375 \end{pmatrix}; \quad \underline{r}_3 = \underline{\underline{A}} \underline{x}_3 - \underline{b} = \begin{pmatrix} 2.94E-02 \\ -1.19E-02 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_3|| = 4.90E-03;$$

$$\underline{d}_3 = \underline{x}_3 - \underline{x}_2 = \begin{pmatrix} -1.75E-02 \\ 4.13E-02 \\ -1.19E-02 \end{pmatrix}; \quad ||\underline{d}_3|| = 4.02E-02$$

Iteration 4:

$$\underline{x}_4 = \begin{pmatrix} 1.001625 \\ 0.998625 \\ -1.000125 \end{pmatrix}; \quad \underline{r}_4 = \underline{\underline{A}} \underline{x}_4 - \underline{b} = \begin{pmatrix} 6.62E-03 \\ -7.50E-04 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_4|| = 1.10E-03;$$

$$\underline{d}_4 = \underline{x}_4 - \underline{x}_3 = \begin{pmatrix} -5.87E-03 \\ 7.37E-03 \\ -7.50E-04 \end{pmatrix}; \quad ||\underline{d}_4|| = 7.32E-03$$

Iteration 5:

$$\underline{x}_5 = \begin{pmatrix} 1.000300 \\ 0.999806 \\ -1.000053 \end{pmatrix}; \quad \underline{r}_5 = \underline{\underline{A}} \underline{x}_5 - \underline{b} = \begin{pmatrix} 1.25E-03 \\ 7.19E-05 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_5|| = 2.09E-04;$$

$$\underline{d}_5 = \underline{x}_5 - \underline{x}_4 = \begin{pmatrix} -1.33E-03 \\ 1.18E-03 \\ 7.19E-05 \end{pmatrix}; \quad ||\underline{d}_5|| = 1.32E-03$$

Iteration 6:

$$\underline{x}_6 = \begin{pmatrix} 1.000049 \\ 0.999976 \\ -1.000013 \end{pmatrix}; \quad \underline{r}_6 = \underline{\underline{A}} \underline{x}_6 - \underline{b} = \begin{pmatrix} 2.10E-04 \\ 4.03E-05 \\ -8.88E-16 \end{pmatrix}; \quad ||\underline{r}_6|| = 3.51E-05;$$

$$\underline{d}_6 = \underline{x}_6 - \underline{x}_5 = \begin{pmatrix} -2.51E-04 \\ 1.70E-04 \\ 4.03E-05 \end{pmatrix}; \quad ||\underline{d}_6|| = 2.51E-04$$

Iteration 7:

$$\underline{x}_7 = \begin{pmatrix} 1.000007 \\ 0.999998 \\ -1.000003 \end{pmatrix}; \quad \underline{r}_7 = \underline{\underline{A}} \underline{x}_7 - \underline{b} = \begin{pmatrix} 3.18E-05 \\ 1.03E-05 \\ -8.88E-16 \end{pmatrix}; \quad ||\underline{r}_7|| = 5.29E-06;$$

$$\underline{d}_7 = \underline{x}_7 - \underline{x}_6 = \begin{pmatrix} -4.21E-05 \\ 2.15E-05 \\ 1.03E-05 \end{pmatrix}; \quad ||\underline{d}_7|| = 4.21E-05$$

Iteration 8:

$$\underline{x}_8 = \begin{pmatrix} 1.000001 \\ 1.000000 \\ -1.000000 \end{pmatrix}; \quad \underline{r}_8 = \underline{\underline{A}} \underline{x}_8 - \underline{b} = \begin{pmatrix} 4.27E-06 \\ 2.08E-06 \\ 0.00E+00 \end{pmatrix}; \quad ||\underline{r}_8|| = 7.12E-07;$$



$$\underline{d}_8 = \underline{x}_8 - \underline{x}_7 = \begin{pmatrix} -6.35E-06 \\ 2.19E-06 \\ 2.08E-06 \end{pmatrix}; \quad ||\underline{d}_8|| = 6.35E-06$$

Iteration 9:

$$\underline{x}_9 = \begin{pmatrix} 1.000000 \\ 1.000000 \\ -1.000000 \end{pmatrix}; \quad \underline{r}_9 = \underline{A} \underline{x}_9 - \underline{b} = \begin{pmatrix} 4.87E-07 \\ 3.67E-07 \\ 8.88E-16 \end{pmatrix}; \quad ||\underline{r}_9|| = 8.12E-08;$$

$$\underline{d}_9 = \underline{x}_9 - \underline{x}_8 = \begin{pmatrix} -8.54E-07 \\ 1.20E-07 \\ 3.67E-07 \end{pmatrix}; \quad ||\underline{d}_9|| = 8.54E-07$$

Convergence, 9 iterations: 8.5E-07<1.0E-06

$$\underline{A} \underline{x}^* - \underline{b} = \begin{pmatrix} 5.000 & 1.000 & 1.000 \\ 3.000 & 4.000 & 1.000 \\ 3.000 & 3.000 & 6.000 \end{pmatrix} \begin{pmatrix} 1.000000 \\ 1.000000 \\ -1.000000 \end{pmatrix} - \begin{pmatrix} 5.000000 \\ 6.000000 \\ 0.000000 \end{pmatrix} = \begin{pmatrix} 0.000000 \\ 0.000000 \\ 0.000000 \end{pmatrix}$$

$$\underline{x}^* = \begin{pmatrix} 1.0E+00 \\ 1.0E+00 \\ -1.0E+00 \end{pmatrix}; \quad \underline{r} = \begin{pmatrix} 4.9E-07 \\ 3.7E-07 \\ 8.9E-16 \end{pmatrix}; \quad ||\underline{r}|| = 8.1E-08$$