

Inference

Christos Dimitrakakis

March 27, 2025

Outline

Logical inference

- Set theory and logic
- Logical inference

Probability background

- Probability facts
- Conditional probability and independence
- Posterior distributions and model estimation
- Random variables, expectation and variance

Graphical models

- Graphical model
- Exercises

Logical inference

- Set theory and logic

- Logical inference

Probability background

- Probability facts

- Conditional probability and independence

- Posterior distributions and model estimation

- Random variables, expectation and variance

Graphical models

- Graphical model

- Exercises

Logical inference

- Set theory and logic

- Logical inference

Probability background

- Probability facts

- Conditional probability and independence

- Posterior distributions and model estimation

- Random variables, expectation and variance

Graphical models

- Graphical model

- Exercises

Set theory

- ▶ First, consider some universal set Ω .
- ▶ A set A is a collection of points x in Ω .
- ▶ $\{x \in \Omega : f(x)\}$: the set of points in Ω with the property that $f(x)$ is true.

Unary operators

- ▶ $\neg A = \{x \in \Omega : x \notin A\}$.

Binary operators

- ▶ $A \cup B$ if $\{x \in \Omega : x \in A \vee x \in B\}$ - (c.f. $A \vee B$)
- ▶ $A \cap B$ if $\{x \in \Omega : x \in A \wedge x \in B\}$ - (c.f. $A \wedge B$)

Binary relations

- ▶ $A \subset B$ if $x \in A \Rightarrow x \in B$ - (c.f. $A \Rightarrow B$)
- ▶ $A = B$ if $x \in A \Leftrightarrow x \in B$ - (c.f. $A \Leftrightarrow B$)

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

The inference problem

- ▶ Given statements A_1, \dots, A_n we know to be true (i.e. a knowledge base), is another statement B true?

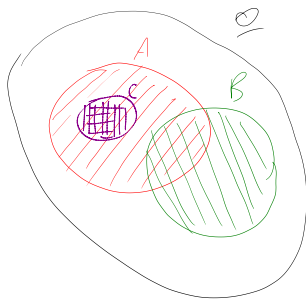
The following statements are equivalent:

- ▶ $A \implies B$ iff $(A \cap \neg B) = \emptyset$.
- ▶ $A \implies B$ iff $A \subset B$.

In addition

- ▶ If $(A \implies B) \wedge A$ then B .
- ▶ If $(A \wedge B)$ then A .

Illustration



$$(A|C) =$$

inferred known

$$(B|C) =$$

$$(C|A) =$$

$$(A \cap B|C)$$

Logical inference

- Set theory and logic

- Logical inference

Probability background

- Probability facts

- Conditional probability and independence

- Posterior distributions and model estimation

- Random variables, expectation and variance

Graphical models

- Graphical model

- Exercises

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

Events as sets

The universe and random outcomes

- ▶ The Ω contains all events that can happen.
- ▶ When something happens, we observe an element $\omega \in \Omega$.

Events in the universe

- ▶ An event is true if $\omega \in A$, and false if $\omega \notin A$.
- ▶ The negative event $\neg A = \Omega \setminus A$ is the set
- ▶ The possible events are a collection of subsets Σ of Ω so that

(i) $\Omega \in \Sigma$, (ii) $A, B \in \Sigma \Rightarrow A \cup B \in \Sigma$ (iii) $A \in \Sigma \Rightarrow \neg A \in \Sigma$

Example: Traffic violation

- ▶ A car is moving with speed $\omega \in [0, \infty)$ in front of the speed camera.
- ▶ $A_0 = [0, 50]$: below the speed limit
- ▶ $A_1 = (50, 60]$: low fine
- ▶ $A_2 = (60, \infty]$: high fine
- ▶ $A_3 = (100, \infty)$: Suspension of license
- ▶ All combinations of the above events are interesting.

Probability fundamentals

Probability measure P

Probability can be seen as an area-like function assigning a likelihood to sets.

- ▶ $P : \Sigma \rightarrow [0, 1]$ gives the likelihood $P(A)$ of an event $A \in \Sigma$.
- ▶ $P(\Omega) = 1$
- ▶ For $A, B \subset \Omega$, if $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$.

Marginalisation

Partition

If A_1, \dots, A_n are a partition of B then:

- ▶ $A_j \cap A_i = \emptyset$ for $i \neq j$
- ▶ $\bigcup_{i=1}^n A_i = B$.

Marginalisation

If $A_1, \dots, A_n \subset \Omega$ are a partition of Ω

$$P(B) = \sum_{i=1}^n P(B \cap A_i).$$

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

Conditional probability

Definition (Conditional probability)

The conditional probability of an event A given an event B is defined as

$$P(A|B) \triangleq \frac{P(A \cap B)}{P(B)}$$

The above definition requires $P(B)$ to exist and be positive.

Conditional probabilities as a collection of probabilities

More generally, we can define conditional probabilities as simply a collection of probability distributions:

$$\{P_\theta : \theta \in \Theta\},$$

where Θ is indexing possible values of θ .

- θ is sometimes called the **model** or **parameter**

The theorem of Bayes

Theorem (Bayes's theorem)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The theorem of Bayes

Theorem (Bayes's theorem)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The general case

If A_1, \dots, A_n are a partition of Ω , meaning that they are mutually exclusive events (i.e. $A_i \cap A_j = \emptyset$ for $i \neq j$) such that one of them must be true (i.e. $\bigcup_{i=1}^n A_i = \Omega$), then

$$P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$$

and

$$P(A_j|B) = \frac{P(B|A_j)}{\sum_{i=1}^n P(B|A_i)P(A_i)}$$

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

Bayes's theorem

As a conditional measure

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)} = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | \neg A)P(\neg A)}$$

Bayes's theorem

As a conditional measure

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)} = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | \neg A)P(\neg A)}$$

As a causal explanation

$$\mathbb{P}(\text{cause} | \text{effect}) = \frac{\mathbb{P}(\text{effect} | \text{cause}) \mathbb{P}(\text{cause})}{\mathbb{P}(\text{effect})}$$

Bayes's theorem

As a conditional measure

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)} = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | \neg A)P(\neg A)}$$

As a causal explanation

$$\mathbb{P}(\text{cause} | \text{effect}) = \frac{\mathbb{P}(\text{effect} | \text{cause}) \mathbb{P}(\text{cause})}{\mathbb{P}(\text{effect})}$$

As model inference

- ▶ Prior $\beta(\theta)$
- ▶ Model class $\{P_{\theta}(\beta) : \theta \in \Theta\}$
- ▶ Data x

$$\beta(\theta | x) = \frac{P_{\theta}(x)\beta(\theta)}{\mathbb{P}_{\beta}(x)} = \frac{P_{\theta}(x)\beta(\theta)}{\sum_{\theta' \in \Theta} P_{\theta'}(x)\beta(\theta')}$$

Example: COVID symptoms

Activity (with playing cards or dice)

- ▶ Pick two (x, y) from 1 to 10.
- ▶ If $(x = 1 \text{ and } y < 9)$, **or** $(x \text{ is even and } y \geq 9)$, you have **symptoms**.
- ▶ Do you have COVID?

Example: COVID symptoms

Activity (with playing cards or dice)

- ▶ Pick two (x, y) from 1 to 10.
- ▶ If $(x = 1 \text{ and } y < 9)$, **or** $(x \text{ is even and } y \geq 9)$, you have **symptoms**.
- ▶ Do you have COVID?

Information

- ▶ 20% of people have COVID
- ▶ 50% of people **with** COVID have symptoms.
- ▶ 10% of people with **no** COVID have symptoms.
- ▶ If you **do** have symptoms, what are the chances you have COVID?

Example: COVID symptoms

Activity (with playing cards or dice)

- ▶ Pick two (x, y) from 1 to 10.
- ▶ If $(x = 1 \text{ and } y < 9)$, **or** $(x \text{ is even and } y \geq 9)$, you have **symptoms**.
- ▶ Do you have COVID?

Information

- ▶ 20% of people have COVID
- ▶ 50% of people **with** COVID have symptoms.
- ▶ 10% of people with **no** COVID have symptoms.
- ▶ If you **do** have symptoms, what are the chances you have COVID?

Formalisation

- ▶ Prior $P(C) = 0.1$:
- ▶ Likelihood: $P(S|C) = 0.5$, $P(S|\neg C) = 0.1$
- ▶ Posterior:

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|\neg C)P(\neg C)}$$

Example: The k-meteorologists problem (set notation)

- ▶ R_t : The **event** that it rains at time t .
- ▶ A set of stations Θ , with $\theta \in \Theta$ making weather predictions:

$$P(R_{t+1} \mid R_1, \dots, R_t, \theta),$$

- ▶ A **prior probability** $P(\theta)$ on the stations.
- ▶ The **marginal** probability

$$P(R_1, \dots, R_t) = \sum_{\theta \in \Theta} P(R_1, \dots, R_t \mid \theta) P(\theta)$$

- ▶ The **posterior** probability

$$\begin{aligned} P(\theta \mid R_1, \dots, R_t) &= \frac{P(R_1, \dots, R_t \mid \theta) P(\theta)}{P(R_1, \dots, R_t)} = \frac{\prod_{i=1}^t P(R_i \mid R_1, \dots, R_{i-1}, \theta) P(\theta)}{P(R_1, \dots, R_t)} \\ &= \frac{P(R_t \mid R_1, \dots, R_{t-1} \mid \theta) P(\theta \mid R_1, \dots, R_{t-1})}{P(R_t \mid R_1, \dots, R_{t-1})} \end{aligned}$$

- ▶ The **marginal posterior** probability

$$P(R_{t+1} \mid R_1, \dots, R_t) = \sum_{\theta \in \Theta} P(R_{t+1} \mid R_1, \dots, R_t, \theta) P(\theta \mid R_1, \dots, R_t)$$

Example: The k-meteorologists problem (stat notation)

- ▶ $x_t \in \{0, 1\}$: A **random variable**, telling us whether it rains at time t .
- ▶ A set of stations Θ , with $\theta \in \Theta$ making weather predictions:

$$P_{\theta}(x_{t+1} \mid x_1, \dots, x_t)$$

- ▶ A **prior probability** $\beta(\theta)$ on the stations.
- ▶ The **marginal** probability

$$\mathbb{P}_{\beta}(x_1, \dots, x_t) = \sum_{\theta \in \Theta} P_{\theta}(x_1, \dots, x_t) \beta(\theta)$$

- ▶ The **posterior** probability

$$\begin{aligned} \beta(\theta \mid x_1, \dots, x_t) &= \frac{P_{\theta}(x_1, \dots, x_t) \beta(\theta)}{\mathbb{P}_{\beta}(x_1, \dots, x_t)} = \frac{\prod_{i=1}^t P_{\theta}(x_i \mid x_1, \dots, x_{i-1}) \beta(\theta)}{\mathbb{P}_{\beta}(x_1, \dots, x_t)} \\ &= \frac{P_{\theta}(x_t \mid x_1, \dots, x_{t-1}) \beta(\theta \mid x_1, \dots, x_{t-1})}{\mathbb{P}_{\beta}(x_t \mid x_1, \dots, x_{t-1})} \end{aligned}$$

- ▶ The **marginal posterior** probability

$$\mathbb{P}_{\beta}(x_{t+1} \mid x_1, \dots, x_t) = \sum_{\theta \in \Theta} P_{\theta}(x_{t+1} \mid x_1, \dots, x_t) \beta(\theta \mid x_1, \dots, x_t)$$

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

Random variables

A random variable $f : \Omega \rightarrow \mathbb{R}$ is a real-valued **function**, with $\omega \sim P$.

The distribution of f

The probability that f lies in some subset $A \subset \mathbb{R}$ is

$$P_f(A) \triangleq P(\{\omega \in \Omega : f(\omega) \in A\}),$$

and we write $f \sim P_f$.

Shorthands for RV

- ▶ For RVs $f : \Omega \rightarrow \mathbb{R}$, we write $P(f \in A)$ to mean $P_f(A)$.
- ▶ For RVs $f : \Omega \rightarrow X$, where X is a finite set e.g. $\{1, 2, \dots, n\}$, we write $P(f = x) = P_f(\{x\})$ for any $x \in X$.

Independence of random variables

Two RVs f, g are independent in the same way that events are independent.

$$P(f \in A \wedge g \in B) = P(f \in A)P(g \in B) = P_f(A)P_g(B).$$

In that sense, $f \sim P_f$ and $g \sim P_g$.

Formal definition

More specifically, we are measuring the set of ω values for which $f(\omega) \in A$ and $g(\omega) \in B$:

$$P(\{\omega : f(\omega) \in A, g(\omega) \in B\}) = P_f(A)P_g(B).$$

Shorthand notation

Since the above is very cumbersome, we usually just write that

$$P(f, g) = P(f)P(g)$$

for any two independent random variables f, g .

Expectation

For any real-valued random variable $f : \Omega \rightarrow \mathbb{R}$, the expectation with respect to a probability measure P is

$$\mathbb{E}_P(f) = \sum_{\omega \in \Omega} f(\omega)P(\omega).$$

When Ω is continuous, we can use a density p

$$\mathbb{E}_P(f) = \int_{\Omega} f(\omega)p(\omega)d\omega.$$

Linearity of expectations

For any RVs x, y :

$$\mathbb{E}_P(x + y) = \mathbb{E}_P(x) + \mathbb{E}_P(y)$$

Multiple variables

The joint distribution $P(x, y)$

For two (or more) RVs $x : \Omega \rightarrow \mathbb{R}$, and $y : \Omega \rightarrow \mathbb{R}$, this is a **shorthand** for the distribution of $(x(\omega), y(\omega))$ when $\omega \sim P$. We can also use $P(x = i, y = j)$ for the probability that the two variables assume the values i, j respectively.

Independence

If x, y are independent RVs then $P(x, y) = P_x(x)P_y(y)$.

Correlation

If x, y are **not** correlated then $\mathbb{E}_P(xy) = \mathbb{E}(x) \mathbb{E}(y)$.

IID (Independent and Identically Distributed) random variables

A sequence x_t of r.v.s is IID if $x_t \sim P$ so that

$$(x_1, \dots, x_t, \dots, x_T) \sim P^T$$

i.e. a T -length sample is drawn from the product distribution
 $P^T = P \times P \times \dots \times P$.

Conditional expectation

The conditional expectation of a random variable $f : \Omega \rightarrow \mathbb{R}$, with respect to a probability measure P conditioned on some event B is simply

$$\mathbb{E}_P(f|B) = \sum_{\omega \in \Omega} f(\omega)P(\omega|B).$$

Conditional expectations are similar to conditional probabilities.

Conditional probabilities of RVs

Similarly to the notation over sets,

$$P(A \cap B) = P(A | B)P(B),$$

when dealing with RVs, it is common to use the notation

$$P(x, y) = P(x|y)P(y)$$

This equation works for all possible values of x, y e.g.

$$P(x = 1, y = 0) = P(x = 1|y = 0)P(y = 0)$$

which then denotes the probability masses of each

Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

Random variables, expectation and variance

Graphical models

Graphical model

Exercises

Independence

Independent events $A \perp\!\!\!\perp B$

- ▶ A, B are **independent** iff $P(A \cap B) = P(A)P(B)$.
- ▶ Knowing if A happened, does not tell us anything about whether B happened

Conditional independence $A \perp\!\!\!\perp B \mid C$

- ▶ A, B are **conditionally independent** given C iff $P(A \cap B \mid C) = P(A \mid C)P(B \mid C)$.
- ▶ Knowing if C happened tells us all we need to know about A and B .

For random variables

- ▶ Independence: $P(x, y) = P(x)P(y)$.
- ▶ Conditional independence: $P(x, y \mid z) = P(x \mid z)P(y \mid z)$.

Model specification: Independent

x_1

x_2

$f = \text{Bernoulli}(1/2)$

$g = \text{Bernoulli}(0.8)$

$x_1 \sim f$

$x_2 \sim g$

```
def f():
```

```
    return np.random.choice(2)
```

```
def g:
```

```
    return np.random.choice(2, [0.2, 0.8])
```

```
x1 = f()
```

```
x2 = g()
```

Model specification: Gaussian Dependent variables



$f = \text{Normal}(0, 1)$

$g(a) = \text{Normal}(a, 1)$

$x_1 \sim f$

$x_2 | x_1 = a \sim g(a)$

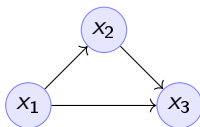
```
def f():  
    return np.random.normal(0, 1)  
def g(a):  
    return np.random.normal(a)  
x1 = f()  
x2 = g(x1)
```

Model specification: Bernoulli Dependent variables



$f = \text{Bernoulli}(1/2)$	<code>def f():</code>
$g(a) = \text{Bernoulli}(\theta_a)$	<code> return np.random.choice(2)</code>
$x_1 \sim f$	<code>def g(a):</code>
$x_2 x_1 = a \sim g(a)$	<code> theta = [0.6, 0.5]</code>
$\theta = (0.6, 0.5)$	<code> return np.random.choice(2,</code>
	<code> [1 - theta[a], theta[a]])</code>
	<code> x1 = f()</code>
	<code> x2 = g(x1)</code>

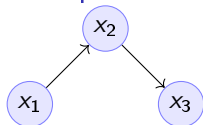
Graphical models



- ▶ Variables: x_1, x_2, x_3
- ▶ Arrows denote dependencies between variables.

Conditional independence

Example



Graphical model for the factorisation

$$\mathbb{P}(x_3 \mid x_2) \mathbb{P}(x_2 \mid x_1) \mathbb{P}(x_1).$$

Definition

- ▶ Consider variables x_1, \dots, x_n .
- ▶ Let B, D be subsets of $[n]$.

We say x_i is **conditionally independent** of x_B given x_D and write

$$x_i \perp\!\!\!\perp x_B \mid x_D$$

if and only if:

$$\mathbb{P}(x_i, x_B \mid x_D) = \mathbb{P}(x_i \mid x_D) \mathbb{P}(x_B \mid x_D).$$

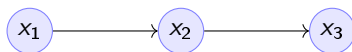
Directed graphical model

A collection of n random variables $x_i : \Omega \rightarrow X_i$, and let $X \triangleq \prod_i X_i$, with underlying probability measure P on Ω . Let $\mathbf{x} = (x_i)_{i=1}^n$ and for any subset $B \subset [n]$ let

$$\mathbf{x}_B \triangleq (x_i)_{i \in B} \tag{1}$$

$$\mathbf{x}_{-j} \triangleq (x_i)_{i \neq j} \tag{2}$$

Model specification: Chain



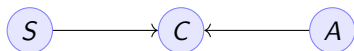
$$x_1 \sim f \quad (3)$$

$$x_2 \mid x_1 = a \sim g(a) \quad (4)$$

$$x_3 \mid x_2 = b \sim h(b), \quad (5)$$

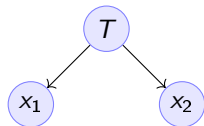
```
def f():  
    return np.random.uniform()  
def g(a):  
    return np.random.uniform() + a  
def h(b):  
    return np.random.uniform * b  
x1 = f()  
x2 = g(x1)  
x3 = h(x2)
```

Smoking and lung cancer



Smoking and lung cancer graphical model, where S : Smoking, C : cancer, A : asbestos exposure.

Time of arrival at work



Time of arrival at work graphical model where T is a traffic jam and x_1 is the time John arrives at the office and x_2 is the time Jane arrives at the office.

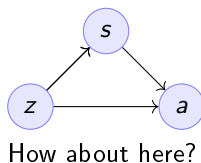
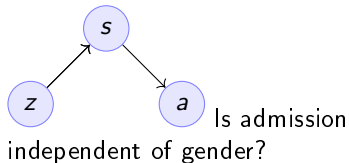
*Conditional independence:

- ▶ Even though x_1, x_2 are **not independent**, they become independent once you know T .

School admission

School	Male	Female
A	62	82
B	63	68
C	37	34
D	33	35
E	28	24
F	6	7

- ▶ z : gender
- ▶ s : school applied to
- ▶ a : admission



Logical inference

Set theory and logic

Logical inference

Probability background

Probability facts

Conditional probability and independence

Posterior distributions and model estimation

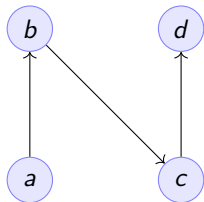
Random variables, expectation and variance

Graphical models

Graphical model

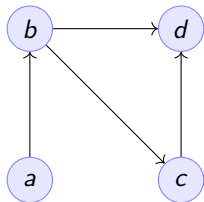
Exercises

What is the model for this graph?



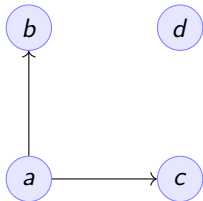
$$P(a, b, c, d) = \dots$$

What is the model for this graph?



$$P(a, b, c, d) =$$

What is the model for this graph?



$$P(a, b, c, d) =$$

Draw the graph for this model

b

d

a

c

$$P(a, b, c, d) = P(a)P(b|a)P(c|b)P(d|b)$$

Draw the graph for this model

b

d

a

c

$$P(a, b, c, d) = P(a)P(b|a)P(d|c)P(c)$$

Draw the graph for this model

b

d

a

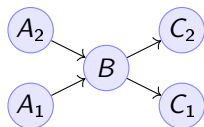
c

$$P(a, b, c, d) = P(a)P(b|a)P(c|a)P(d|b, c)$$

Conditional independence

For any set of events A_1, A_2, A_3, \dots , we can write their co-occurrence probability as $\prod_i P(A_i | \cap A_1 \cap A_2 \cap \dots \cap A_{i-1})$. However, we can use a **Bayesian network** to define conditional independence structures.

If A is a parent of B and C is a child of B , and there are **no other paths** from A to C then the following conditional independence holds:



$$P(C | B, A) = P(C | B)$$

i.e. C is conditionally independent of A given B .

Conditional probability tables

We can now write the distribution of the above example as

$$P(B, C_1, C_2) = P(A_1)P(A_2)P(B|A_1 \cap A_2)P(C_1|B)P(C_2|B).$$

Example: COVID test

Information

- ▶ 10% of people have COVID
- ▶ 50% of people with COVID have a positive **test**
- ▶ 50% of people with COVID have **symptoms**
- ▶ 10% of people without COVID have a positive **test**
- ▶ 20% of people without COVID have **symptoms**

Example: COVID test

Information

- ▶ 10% of people have COVID
- ▶ 50% of people with COVID have a positive **test**
- ▶ 50% of people with COVID have **symptoms**
- ▶ 10% of people without COVID have a positive **test**
- ▶ 20% of people without COVID have **symptoms**

Formalisation

- ▶ Prior: $P(C = 1) = 0.1$
- ▶ Likelihood: $P(T, S|C) = P(T|C)P(S|C)$, $P(T, S|\neg C)$ for all values of T, S, C .
- ▶ Posterior:

$$P(C|T, S) = \frac{P(S|C)P(T|C)P(C)}{\sum_{i=0}^1 P(S|C=i)P(T|C=i)P(C=i)}$$

Example: Naive Bayes models

Sometimes we observe multiple effects that have a common cause, but which are otherwise independent:

$$\mathbb{P}(\text{effect}_1, \dots, \text{effect}_n \mid \text{cause}) = \prod_{i=1}^n \mathbb{P}(\text{effect}_i \mid \text{cause})$$

Naive Bayes model

- ▶ Observations $(\mathbf{x}_t, y_t)_{t=1}^T$ with $\mathbf{x}_t = (x_{t,1}, \dots, x_{t,n})$.
- ▶ Probability **models** $P_\theta(y \mid \mathbf{x}) = \prod_{i=1}^n P_\theta(y \mid x_i)$.

Example: Wumpus world

	⦿	

	O	
	⦿	

	⦿	O

	O	
	⦿	O

Details

- ▶ Probability of each world A_i being true: $1/4$
- ▶ Probability of each hole generating a breeze:
 $P(B_1|A_2 \cup A_4) = P(B_2|A_3 \cup A_4)$ with B_1, B_2 conditionally independent given A .

Questions

- ▶ What is the probability of feeling a breeze $B = B_1 \cup B_2$ in each world?
- ▶ What is the probability of a hole above if you **feel** a breeze?
- ▶ What is the probability of a hole above if you **don't** feel a breeze?