Fairness

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What is it?

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- Proportionality and representation.

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- Non-discrimination.

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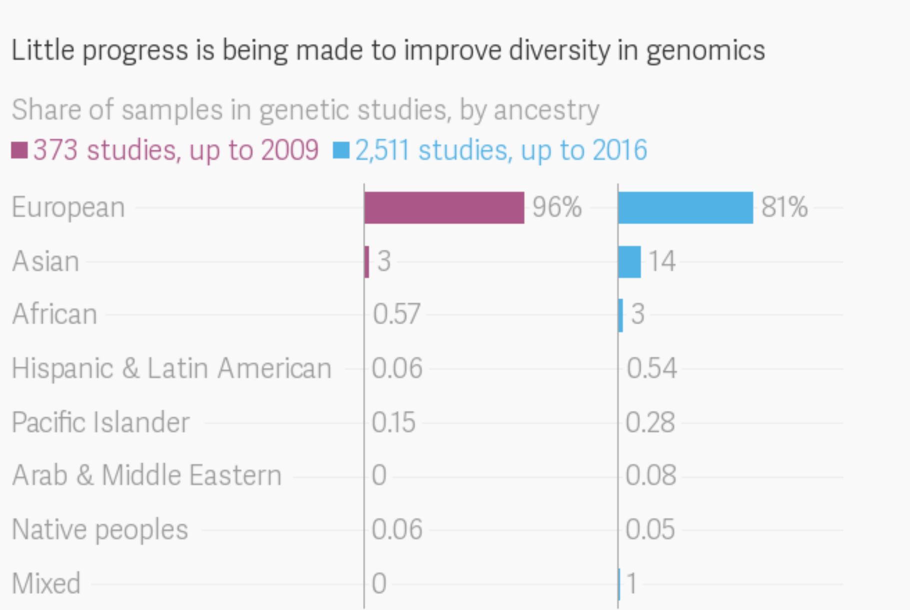
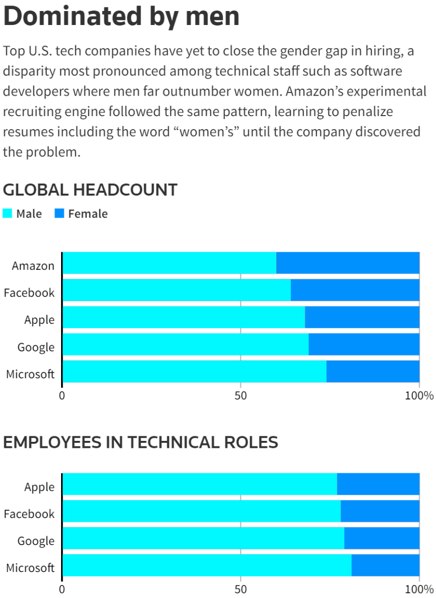
- Admit randomly?
- Use prediction of individual academic performance?

Proportional representation



Hiring decisions



Fairness and information

Example 3 (College admissions data)

School	Male	Female
A	62%	82%
B	63%	68%
C	37%	34%
D	33%	35%
E	28%	24%
F	6%	7%
Average	*45%*	*38%*

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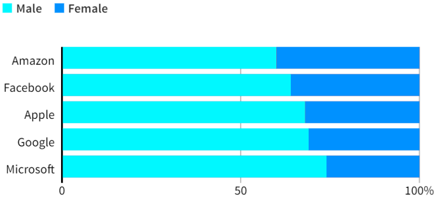
Hiring decisions

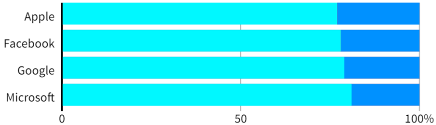
Dominated by men

Top U.S. tech companies have yet to close the gender gap in hiring, a disparity most pronounced among technical staff such as software developers where men far outnumber women. Amazon's experimental recruiting engine followed the same pattern, learning to penalize resumes including the word "women's" until the company discovered the problem.

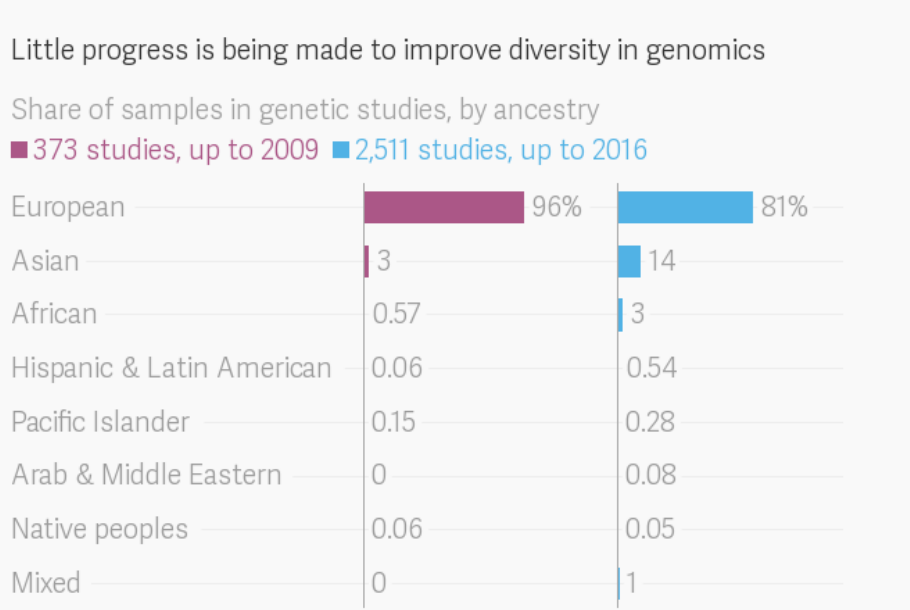
GLOBAL HEADCOUNT



EMPLOYEES IN TECHNICAL ROLES



Group fairness and proportionality



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- Admit randomly?
- Use prediction of individual academic performance?
- Should we take into account group membership or other population information?

Bail decisions

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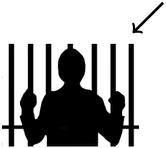

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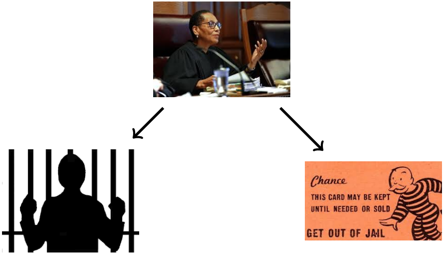

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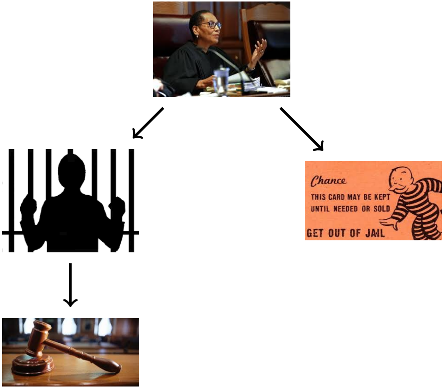

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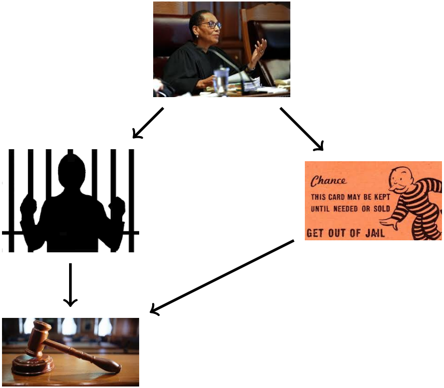

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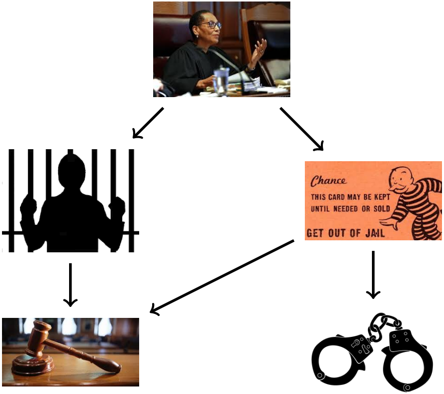

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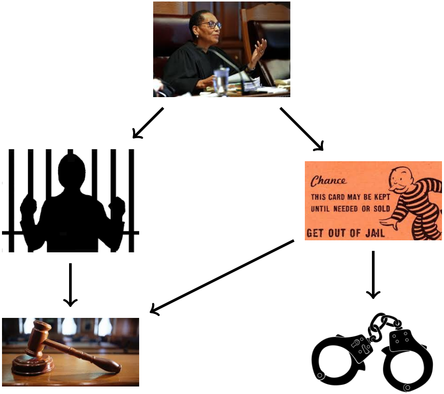
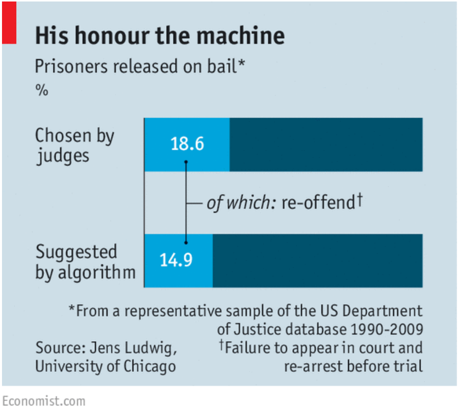




Demographic parity and equality of opportunity.

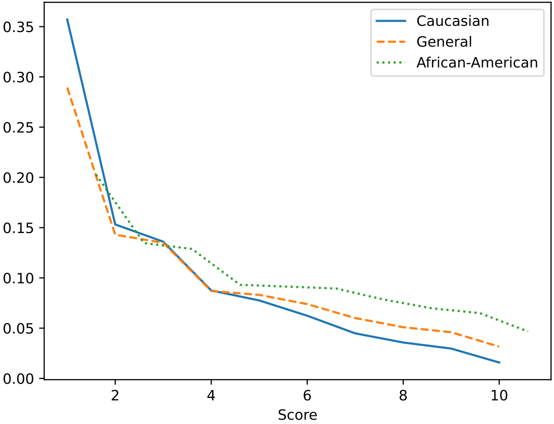


Figure: Apparent bias in risk scores towards black versus white defendants.

$$\mathbb{P}_\theta^\pi(a_t | z_t) = \mathbb{P}_\theta^\pi(a_t). \tag{3.1}$$

Calibration.

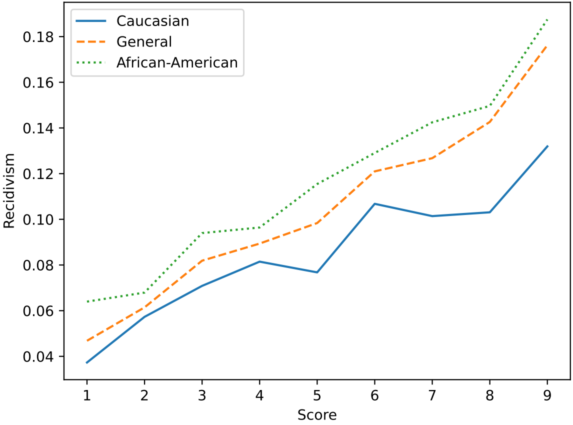
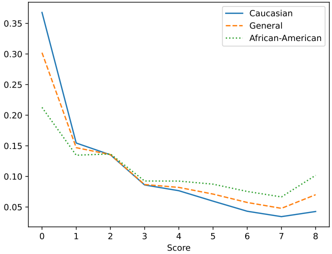


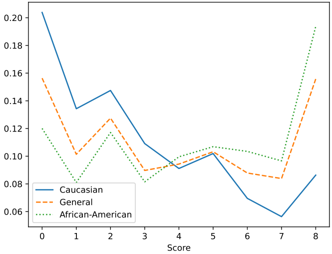
Figure: Recidivism rates by risk score.

$$\mathbb{P}^{\pi}_{\theta}(y_t | a_t, z_t) = \mathbb{P}^{\pi}_{\theta}(y_t | a_t). \tag{3.2}$$

Balance



(a) No recidivism



(b) Recidivism

Figure: Score breakdown based on recidivism rates.

$$\mathbb{P}^{\pi}_{\theta}(a_t | y_t, z_t) = \mathbb{P}^{\pi}_{\theta}(a_t | y_t). \tag{3.3}$$

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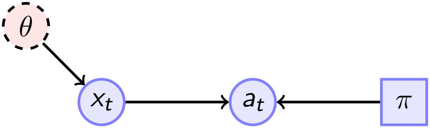

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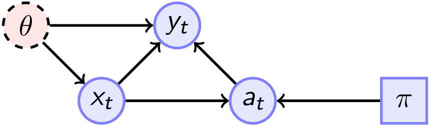

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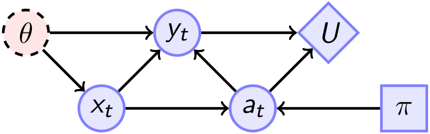

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- y_t: outcome
- U: utility

Example 6 (Classification)

- y: labels
- a: label prediction
- $U(a, y) = \mathbb{I}\{a = y\}$.
- Here the outcome is independent of the action:

$$\mathbb{P}^{\pi}_{\theta}(y_t \mid a_t, x_t, z_t) = \mathbb{P}^{\pi}_{\theta}(y_t \mid x_t, z_t).$$

Example 7 (Regression)

In this setting we can also relax our framework to work with expectations instead of probabilities. For example, calibration and balance can be written as the conditions:

$$\mathbb{E}_\theta^\pi(y_t|a_t, z_t) = \mathbb{E}_\theta^\pi(y_t|a_t), \qquad \mathbb{E}_\theta^\pi(a_t|y_t, z_t) = \mathbb{E}_\theta^\pi(a_t|y_t),$$

respectively.

Measuring and optimising fairness and utility.

Expected utility.

Let us write out expected utility, with $\mathbb{P}^\pi_\theta(\cdot) \equiv \mathbb{P}(\cdot \mid \theta, \pi)$

$$\mathbb{E}^\pi_\theta(U) = \sum_{x,z,y} \mathbb{P}^\pi_\theta(y, a, x, z) U(a, y).$$

The y_t, a_t, x_t, z_t is already sampled from $\mathbb{P}^\pi_\theta$, so we can approximate the expected utility of the historical policy with

$$\widehat{E}_n(U) = \sum_{t=1}^n U(a_t, y_t), \qquad a_t, y_t \sim \mathbb{P}^\pi_\theta .$$

Model specification

For simplicity, assume $z = x_1$, i.e. one of the features. Then

$$\mathbb{P}^\pi_\theta(y, a, x, z) = \underbrace{\mathbb{P}_\theta(y \mid a, x)}_{\text{outcome distribution}} \underbrace{\pi(a \mid x)}_{\text{policy}} \underbrace{\mathbb{P}_\theta(x)}_{\text{feature distribution}}$$

Using the model, we can estimate the expected utility of any policy.

Deviation from balance.

$$\mathbb{P}^{\pi}_{\theta}(a|y,z), \qquad \mathbb{P}^{\pi}_{\theta}(a|y).$$

for all values of y, z. Let us first look at the total variation distance:

$$\| \mathbb{P}^{\pi}_{\theta}(a|y,z) - \mathbb{P}^{\pi}_{\theta}(a|y)\|_1 = \sum_a | \mathbb{P}^{\pi}_{\theta}(a|y,z) - \mathbb{P}^{\pi}_{\theta}(a|y)|_1.$$

$$\hat{P}_n(a|y,z) = \sum_t \frac{\mathbb{I}\left\{a_t = a \wedge y_t = y \wedge z_t = i\right\}}{\mathbb{I}\left\{y_t = y \wedge z_t = i\right\}}$$

We can then plug those into our original measure:

$$\| \mathbb{P}^{\pi}_{\theta}(a|y,z) - \mathbb{P}^{\pi}_{\theta}(a|y)\|_1 \approx \sum_a |\hat{P}_n(a|y,z) - \hat{P}_n(a|y)|.$$

Formalisation of the problem

Unconstrained optimisation.

Let $\lambda \in [0, 1]$:

$$\max_{\pi} (1 - \lambda) U(\pi, \theta) - \lambda F(\pi, \theta)$$

Constrained fairness.

$$\max_{\pi \in \Pi} U(\pi, \theta)$$
$$\text{s.t.} F(\pi, \theta) \leq \epsilon.$$

Constrained utility.

Let $\epsilon \geq 0$:

$$\max_{\pi \in \Pi} F(\pi, \theta)$$
$$\text{s.t.} U(\pi, \theta) \geq \max U(\pi', \theta) - \epsilon$$

Individual fairness

Main variables for individual fairness.

- $x_i \in \mathbb{R}^d$: Individual features.
- $w_i \in \mathbb{R}$: Individual worth.
- $u_i : \mathcal{A} \to \mathbb{R}$: Individual utility.
- $a \in \mathcal{A}$: Action taken by the decision maker.
- π: The decision maker's policy for making decisions.

Meritocracy for given utilities and worths.

Definition 8

A **decision** a is **fair** if, for all i, j, we have $w_i > w_j \Rightarrow u_i(a) \geq u_j(a)$.

Example 9

w		$a = 0$	$a = 1$
1	u_1	0	1
0	u_2	1	0

Definition 10

A **policy** π is **fair** if, for all i, j it holds that: $\mathbb{E}_\pi[u_i | w] > \mathbb{E}[u_j | w] \Rightarrow w_i > w_j$.

Example 11 (Ranking and top-k cohort selection.)

- n individuals
- i-th individual receiving an evaluation w_i.
- $a_i = 1$ if we select an individual, and 0 otherwise.
- Ranked selection algorithm π: first rank the individuals so that $w_i > w_{i+1}$. We then can assign $a_i = 1$ for all $i \leq k$.

Example 12

Matching problems.

- $u_1(x) > u_1(y) > u_1(z)$,
- $u_2(z) > u_2(y) > u_2(x)$
- $u_3(z) > u_3(x) > u_3(y)$

and $w_1 > w_2 > w_3$. A stable matching is then $a_1 = x, a_2 = z, a_3 = y$.

Treating similar individuals similarly.

- $d(x_i, x_j)$: Distance between individuals i, j.
- $D[\pi(a_i|x_i), \pi(a_j|x_j)]$: Distance between policy decisions.
- Utility function $U(\pi)$.

Our goal is to find a decision rule π maximising $U(\pi)$, so that it is Lipschitz-smooth with respect to D, d:

$$D[\pi(a_i|x_i), \pi(a_j|x_j)] \leq d(x_i, x_j)$$

.

Lipschitz functions

Example 13

A real-valued function $f : \mathbb{R}^n \to \mathbb{R}$ is Lipschitz iff

$$|f(x) - f(x')| \leq \|x - x'\|, \qquad \forall x, x' \in \mathbb{R}^n$$

It is sufficient for $\left|\frac{d}{dx} f(x)\right| < 1$ for the function to be Lipschitz. One example is $\sin(x)$, whose derivative is $\cos(x)$.

Definition 14 (Lipschitz function)

A function $f : X \to Y$ is (d, D)-Lipshitz, where d, D are (semi)-metrics on X, Y, iff

$$D[f(x), f(x')] \leq d(x, x').$$

Distances for distributions

If P, Q are distributions on Ω, we define

Definition 15 (Total variation)

$$D_{TV}(P, Q) \triangleq \max_{A \subset \Omega} |P(A) - Q(A) = \frac{1}{2} \|P - Q\|_1 = \frac{1}{2} \sum_{\omega} |P(\omega) - Q(\omega)|$$

Definition 16 (KL-divergence)

Otherwise known as the relative entropy:

$$D_{KL}(P, Q) \triangleq \sum_{\omega} P(\omega) \ln \frac{P(\omega)}{Q(\omega)}$$

Connection to meritocracy.

- Individual features: x
- Policy $\pi(a \mid x)$.
- Individual decisions: $a = 1$

$$\mathbb{E}^\pi(u|x) = \pi(a = 1|x)u(x).$$

- Lipschitz utility function: $|u(x) - u(x')| \leq Ld(x, x')$
- Policy smoothness:

$$\pi(a = 1|x) \geq \pi(a = 1|x') - d(x, x') \geq \pi(a = 1|x') - |u(x) - u(x')|/L$$

- Since π is maximising, $u(x) > u(x') \Rightarrow \pi(a = 1|x) \geq \pi(a = 1|x')$.

Smoothness and parity.

Can group fairness be satisfied in this setting? To define groups, we select $S, T \subset \mathcal{X}$. We can say a policy satisfies approximate parity between the groups if:

Definition 17

ϵ-parity

$$\|\pi(a|x \in S) - \pi(a|x \in T)\|_1 \leq \epsilon.$$

We can also similarly define the worst-case bias for some action:

Definition 18

Bias for some action a:

$$\max_\pi \pi(a|x \in S) - \pi(a|x \in T) \qquad \text{s.t. } \pi \text{ is Lipschitz}$$