Functional Forkshop: Part 1

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About us

Valik

Server guild manager in Kyiv. Formerly forced people to use functional programming style in the Domains (Premium) team. Now works on Tagless Infra to provide you with the best tools for your daily needs. Which are all functional, of course.

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Boris

Developer at Payments by Wix team. Jumps between two extremes - Emacs Lisp and Haskell. Wants to force people around to use both languages, but can't explain why.

About the Forkshop

- Basic forkshop is split into several parts:
  1. Type classes, Semigroups and Monoids.
  2. Functors and Applicative Functors.
  3. Monads.
  4. Readers.
  5. Comonads.
- Theory and practice. Make sure that you are ready to write the code.
- Target audience is Scala developers learning FP.
- Forkshop is duplicated in Haskell.

Whys

- Functional programming roams (a bit).
  - More projects are using functional programming techniques and idioms (at different scale).
- Some people are still confused by all these functional talks (`OptionT`, type lambdas etc).
- Having a common language and understanding of some fundamental stuff is important.

Agenda

- Type classes
- Semigroups
- Monoids
- 3 interesting™ tasks

Type classes

- Problem:
  - Application design
  - Data and logic separation
- Plan:
  - Write a simple application
  - Evolve it
  - Check different approaches
  - Introduce type classes

Application definition

- We are writing a game.
- With multiple different creatures.
- Everyone introduces themselves.
- Introduction consists of animations and text showing in a bubble.

Meet the hero

```
case class Hero(name: String, job: String, level: Int) {
  def introduce(): String = s"Hi! My name is $name. I am $level level $job."
}

object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)

  someRealShitSounds()
  drawBubble(player.introduce())
  someRealShitAnimations()
}

// Hi! My name is Valik. I am 20 level Black Mage.
```

Every hero needs a monster

```scala
case class Orc(name: String, level: Int) {
  def introduce(): String =
    s"Lok-tar ogar! Me be $name. Me be strong. Level $level strong!"
}

case class Ooze(level: Int) {
  def introduce(): String = 1.to(level).map(_=>"brlup").mkString("-")
}
```

Game

```
object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
  val orc = Orc("Garrosh", 105)
  val ooze = Ooze(2)

  // Introduce player
  someRealShitSounds()
  drawBubble(player.introduce())
  someRealShitAnimations()

  // Introduce orc
  someRealShitSounds()
  drawBubble(orc.introduce())
  someRealShitAnimations()

  // Introduce ooze
  someRealShitSounds()
  drawBubble(ooze.introduce())
  someRealShitAnimations()
}

// Hi! My name is Valik. I am 20 level Black Mage.
// Lok-tar ogar! Me be Garrosh. Me be strong. Level 105 strong!
// brlup-brlup
```

Game

```
object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
  val orc = Orc("Garrosh", 105)
  val ooze = Ooze(2)

  // Introduce player
  someRealShitSounds()
  drawBubble(player.introduce())
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  // Introduce orc
  someRealShitSounds()
  drawBubble(orc.introduce())
  someRealShitAnimations()

  // Introduce ooze
  someRealShitSounds()
  drawBubble(ooze.introduce())
  someRealShitAnimations()
}

// Hi! My name is Valik. I am 20 level Black Mage.
// Lok-tar ogar! Me be Garrosh. Me be strong. Level 105 strong!
// brlup-brlup
```

Issues with this code:

1. Repetition
2. Noise
3. Refactoring (all these `introduce` are not related to each other).

Be DRY, leap for disaster

```
object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
  val orc = Orc("Garrosh", 105)
  val ooze = Ooze(2)

  // Introduce player
  someRealShitSounds()
  drawBubble(player.introduce())
  someRealShitAnimations()

  // Introduce orc
  someRealShitSounds()
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  // Introduce ooze
  someRealShitSounds()
  drawBubble(ooze.introduce())
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}
```

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  drawBubble(player.introduce())
  someRealShitAnimations()

  // Introduce orc
  someRealShitSounds()
  drawBubble(orc.introduce())
  someRealShitAnimations()

  // Introduce ooze
  someRealShitSounds()
  drawBubble(ooze.introduce())
  someRealShitAnimations()
}
```

```
def introduce(phrase: String): Unit = {
  someRealShitSounds()
  drawBubble(phrase)
  someRealShitAnimations()
}
```

Be DRY, leap for disaster

```
object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
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  // Introduce player
  someRealShitSounds()
  drawBubble(player.introduce())
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  someRealShitSounds()
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```
def introduce(phrase: String): Unit = {
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object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
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  introduce(player.introduce())
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```

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object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
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```

```
def introduce(phrase: String): Unit = {
  someRealShitSounds()
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}
object Game extends App {
  val player = Hero("Valik", "Black Mage", 20)
  val orc = Orc("Garrosh", 105)
  val ooze = Ooze(2)

  introduce(player.introduce())
  introduce(orc.introduce())
  introduce(ooze.introduce())
}
```

Cool, but can we `introduce(player)`?

Introducing abstractions

```
//


case class Hero(...) {
  def introduce(): String = s"..."
}

case class Orc(...) {
  def introduce(): String = s"..."
}

case class Ooze(...) {
  def introduce(): String = s"..."
}
```

Introducing abstractions

```
//


case class Hero(...) {
  def introduce(): String = s"..."
}

case class Orc(...) {
  def introduce(): String = s"..."
}

case class Ooze(...) {
  def introduce(): String = s"..."
}
```

```
trait Introducible {
  def introduce(): String
}

case class Hero(...) extends Introducible {
  override def introduce(): String = s"..."
}

case class Orc(...) extends Introducible {
  override def introduce(): String = s"..."
}

case class Ooze(...) extends Introducible {
  override def introduce(): String = s"..."
}
```

Game with trait

```
def introduce(phrase: String): Unit = {
  someRealShitSounds()
  drawBubble(phrase)
  someRealShitAnimations()
}

object Game extends App {
  /* ... */

  introduce(player.introduce())
  introduce(orc.introduce())
  introduce(ooze.introduce())
}
```

Game with `trait`

```
def introduce(phrase: String): Unit = {
  someRealShitSounds()
  drawBubble(phrase)
  someRealShitAnimations()
}

object Game extends App {
  /* ... */

  introduce(player.introduce())
  introduce(orc.introduce())
  introduce(ooze.introduce())
}
```

```
def introduce(creature: Introducible): Unit = {
  someRealShitSounds()
  drawBubble(creature.introduce())
  someRealShitAnimations()
}

object Game extends App {
  /* ... */

  introduce(player)
  introduce(orc)
  introduce(ooze)
}
```

Game with trait

- No more `introduce(_.introduce())`.
- We are adaptive. Less code needs to be changed if we need something new in the `introduce` function (e.g. sound name) - just add new 'method' to the trait.
- Refactoring becomes easier.

```
def introduce(phrase: String): Unit = {
  someRealShitSounds()
  drawBubble(phrase)
  someRealShitAnimations()
}

object Game extends App {
  /* ... */

  introduce(player.introduce())
  introduce(orc.introduce())
  introduce(ooze.introduce())
}
```

```
def introduce(creature: Introducible): Unit = {
  someRealShitSounds()
  drawBubble(creature.introduce())
  someRealShitAnimations()
}

object Game extends App {
  /* ... */

  introduce(player)
  introduce(orc)
  introduce(ooze)
}
```

Here comes the cockatrice

```
import io.proprietary.monsters.cockatrice._

/* ... */

object Game extends App {
  /* ... */

  val cockatrice = Cockatrice(level = 666, element = Element.Fire)

  introduce(cockatrice) // ???
                        // ain't gonna work
}
```

Shawarma to the rescue

```scala
import io.proprietary.monsters.cockatrice._

/* ... */

case class CockatriceWrapper(cockatrice: Cockatrice) extends Introducible {
  override def introduce(): String = {
    import cockatrice._
    s"Haha. I am a ${element.shortName} cockatrice of level ${level}."
  }
}

object Game extends App {
  /* ... */

  val cockatrice = Cockatrice(level = 666, element = Element.Fire)
  val cockatriceW = CockatriceWrapper(cockatrice)

  introduce(cockatriceW)

  /* ... */
}


// Haha. I am a fire cockatrice of level 666.
```

Calm down and reevaluate our goal

- Abstraction - caring about what you can do and not what you are. E.g. separation of data and behaviour.

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- Abstraction - caring about what you can do and not what you are. E.g. separation of data and behaviour.
- Composition - having a way to express something that can do several things at once.
- Extensibility - extending all kind of types:
  - types we own
  - types we don't own
  - even built-in types

trait + wrapper: abstraction

Abstraction holds. Proof is the `introduce` function itself.

```
def introduce(creature: Introducible): Unit = {
  someRealShitSounds()
  drawBubble(creature.introduce())
  someRealShitAnimations()
}
```

`trait` + wrapper: composition

Composition holds thanks to `with` keyword.

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```
trait CanAttack {
  def attack(): Unit
}

def patheticAttack[A <: Introducible with CanAttack](creature: A): Unit
```

trait + wrapper: composition

Composition holds thanks to `with` keyword.

```
trait CanAttack {
  def attack(): Unit
}

def patheticAttack[A <: Introducible with CanAttack](creature: A): Unit
```

`with` keyword is not commutative

`Introducible with CanAttack != CanAttack with Introducible`.

`trait` + wrapper: extensibility

Extensibility holds, but with several caveats:

1. No consistency - we wrap only types we don't own.
2. Wrappers don't compose very well. You might even wrap your wrappers.
3. Bad usability - you can't interchangeably use wrapper and the underlying value.

You know where it's going to, right?

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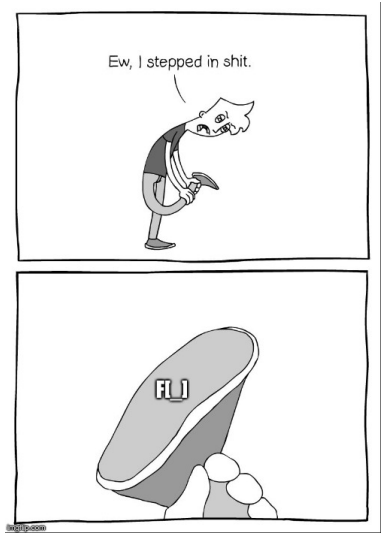

Dividing data and behaviour

```
trait Introducible {
  def introduce(): String
}

def introduce(creatute: Introducible): Unit = {

  /* ... */
  drawBubble(creatute.introduce())
  /* ... */
}
```

Dividing data and behaviour

```
trait Introducible {
  def introduce(): String
}

def introduce(creatute: Introducible): Unit = {

  /* ... */
  drawBubble(creatute.introduce())
  /* ... */
}
```

```
trait Introducible[A] {
  def introduce(a: A): String
}

def introduce[A](creature: A,
                 impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}
```

Usage

```
// Define new trait
trait Introducible[A] {
  def introduce(a: A): String
}
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// Define new trait
trait Introducible[A] {
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// Remove behaviour from data
case class Hero(name: String, job: String, level: Int)
```

Usage

```scala
// Define new trait
trait Introducible[A] {
  def introduce(a: A): String
}


// Remove behaviour from data
case class Hero(name: String, job: String, level: Int)


// Implement behaviour as a value in companion object
object Hero {
  val introducibleHero: Introducible[Hero] = new Introducible[Hero] {
    override def introduce(a: Hero): String =
      s"..."
  }
}
```

Usage

```scala
// Define new trait
trait Introducible[A] {
  def introduce(a: A): String
}


// Remove behaviour from data
case class Hero(name: String, job: String, level: Int)


// Implement behaviour as a value in companion object
object Hero {
  val introducibleHero: Introducible[Hero] = new Introducible[Hero] {
    override def introduce(a: Hero): String =
      s"..."
  }
}


// Pass data and behaviour separately
object Game extends App {
  /* ... */
  introduce(
    creature = hero,
    impl = Hero.introducibleHero
  )
}
```

External types? Pff...

```scala
import io.proprietary.monsters.cockatrice._

// Implement behaviour as a value in companion object
object CockatriceInstances {
  val introducibleCockatrice: Introducible[Cockatrice] = new Introducible[Cockatrice] {
    override def introduce(a: Cockatrice): String =
      s"..."
  }
}
```

External types? Pff...

```scala
import io.proprietary.monsters.cockatrice._

// Implement behaviour as a value in companion object
object CockatriceInstances {
  val introducibleCockatrice: Introducible[Cockatrice] = new Introducible[Cockatrice] {
    override def introduce(a: Cockatrice): String =
      s"..."
  }
}


// Pass data and behaviour separately
object Game extends App {
  /* ... */
  introduce(
    creature = cockatrice,
    impl = CockatriceInstances.introducibleCockatrice
  )
}
```

But passing implementation around is. . .

Cucumbersome

So implicits :(

```
object Hero {
  val introducibleHero:
      Introducible[Hero] = ???
}

object CockatriceInstances {
  val introducibleCockatrice:
      Introducible[Cockatrice] = ???
}

def introduce[A](creature: A,
                 impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}

object Game extends App {
  /* ... */
  introduce(hero, introducibleHero)
  introduce(cockatrice, introducibleCockatrice)
}
```

So implicits :(

```
object Hero {
  val introducibleHero:
      Introducible[Hero] = ???
}

object CockatriceInstances {
  val introducibleCockatrice:
      Introducible[Cockatrice] = ???
}

def introduce[A](creature: A,
                 impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}

object Game extends App {
  /* ... */
  introduce(hero, introducibleHero)
  introduce(cockatrice, introducibleCockatrice)
}
```

```
object Hero {
  implicit val introducibleHero:
      Introducible[Hero] = ???
}

object CockatriceInstances {
  implicit val introducibleCockatrice:
      Introducible[Cockatrice] = ???
}

def introduce[A](creature: A)
            (implicit impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}

object Game extends App {
  /* ... */
  introduce(hero)
  introduce(cockatrice)
}
```

Summoning the summoner

```scala
trait Introducible[A] {
  def introduce(a: A): String
}
```

```scala
def introduce[A](creature: A)
                (implicit impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}
```

Summoning the summoner

```scala
trait Introducible[A] {
  def introduce(a: A): String
}
```

```scala
def introduce[A](creature: A)
                (implicit impl: Introducible[A]): Unit = {
  /* ... */
  drawBubble(impl.introduce(creature))
  /* ... */
}
```

```scala
trait Introducible[A] {
  def introduce(a: A): String
}

object Introducible {
  def apply[A: Introducible]: Introducible[A] =
    implicitly[Introducible[A]]
}
```

```scala
def introduce[A: Introducible](creature: A): Unit = {

  /* ... */
  drawBubble(Introducible[A].introduce(creature))
  /* ... */
}
```

What have we done?

Type class is just a construct that supports ad hoc polymorphism. E.g. allows one to define polymorphic functions that can be applied to arguments of different types and behave differently based the type of the arguments.

In other words, type classes are solution for supporting function overloading.

What have we done?

Type class is just a construct that supports ad hoc polymorphism. E.g. allows one to define polymorphic functions that can be applied to arguments of different types and behave differently based the type of the arguments.

In other words, type classes are solution for supporting function overloading.

In Scala this can be achieved in several ways:

- Class inheritance or traits.
- Type classes (traits + implicits).

Type classes: abstraction

Abstraction holds. Proof is the `introduce` function itself.

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Abstraction holds. Proof is the `introduce` function itself.

```
def introduce(creature: Introducible): Unit = {
  /* ... */
  drawBubble(creature.introduce())
  /* ... */
}
```

```
def introduce[A: Introducible](creature: A): Unit = {
  /* ... */
  drawBubble(Introducible[A].introduce(creature))
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Type classes: abstraction

Abstraction holds. Proof is the `introduce` function itself.

```
def introduce(creature: Introducible): Unit = {
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  /* ... */
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def introduce[A: Introducible](creature: A): Unit = {
  /* ... */
  drawBubble(Introducible[A].introduce(creature))
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```

We gain literal data and behaviour separation.

Type classes: composition

Composition holds. We just pass two different behaviours.

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```
def patheticAttack[A <: Introducible with CanAttack](creature: A): Unit
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Type classes: composition

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```
def patheticAttack[A <: Introducible with CanAttack](creature: A): Unit
```

```
def patheticAttack[A : Introducible : CanAttack](creature: A): Unit
def patheticAttack[A](creature: A)
                     (implicit introducibleImpl: Introducible[A],
                      implicit canAttackImpl: CanAttack[A]): Unit
```

Type classes: composition

Composition holds. We just pass two different behaviours.

```
def patheticAttack[A <: Introducible with CanAttack](creature: A): Unit
```

```
def patheticAttack[A : Introducible : CanAttack](creature: A): Unit
def patheticAttack[A](creature: A)
                     (implicit introducibleImpl: Introducible[A],
                      implicit canAttackImpl: CanAttack[A]): Unit
```

But with type classes we don't care about the order.

Type classes: extensibility

Extensibility holds with some gains:

1. Consistency - we treat our own type the same way we treat external types.
2. Usability - no wrappers, no interchangeability problem.

Type classes: final thoughts

1. Simple idea giving us good properties.
2. Found a good use for controversial implicits feature.
3. Literal separation of data and values.
4. Good for overloading.
5. As with any abstraction, not so good for performance.

Warning

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2. Laws and properties.

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Please keep calm and remember that our focus is practical importance :)

Semigroup

Semigroup is a set S with binary closed operation $\cdot : S \times S \to S$ that satisfies associativity property:

$$\forall a, b, c \in S : (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Operation is closed when $\forall a, b \in S : a \cdot b \in S$.

But it's not that scary

```scala
package object typeclass {

  //
  // Laws:
  //   1. ∀a, b, c ∈ A : (a · b) · c = a · (b · c)
  //
  trait Semigroup[A] {
    def combine(x: A, y: A): A
  }

  object Semigroup {
    def apply[A: Semigroup]: Semigroup[A] =
      implicitly[Semigroup[A]]
  }

}
```

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package object typeclass {

  //
  // Laws:
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    def apply[A: Semigroup]: Semigroup[A] =
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}
```

In simple words, semigroup is a set with means of combining elements of that set.

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```
package object typeclass {

  //
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  trait Semigroup[A] {
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    def apply[A: Semigroup]: Semigroup[A] =
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  }

}
```

In simple words, semigroup is a set with means of combining elements of that set.

Important!

Semigroup is a pair of the set and the operation.

You can't say that string is a semigroup, you must provide an operation.

And in many cases there is more than one operation for a set to form a semigroup.

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- Laws describe properties of these operations and connection between operations in one type class.

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- So type classes should have some associated laws.
- Laws describe properties of these operations and connection between operations in one type class.
- Contract of the interface gives us confidence when we write generic code.

What is law?

- In programming world it's just a contract.
- Operations in the type classes are very generic.

```
def combine(x: A, y: A): A
```

- So type classes should have some associated laws.
- Laws describe properties of these operations and connection between operations in one type class.
- Contract of the interface gives us confidence when we write generic code.
- And as you will see, we really care about these laws.

Instance example

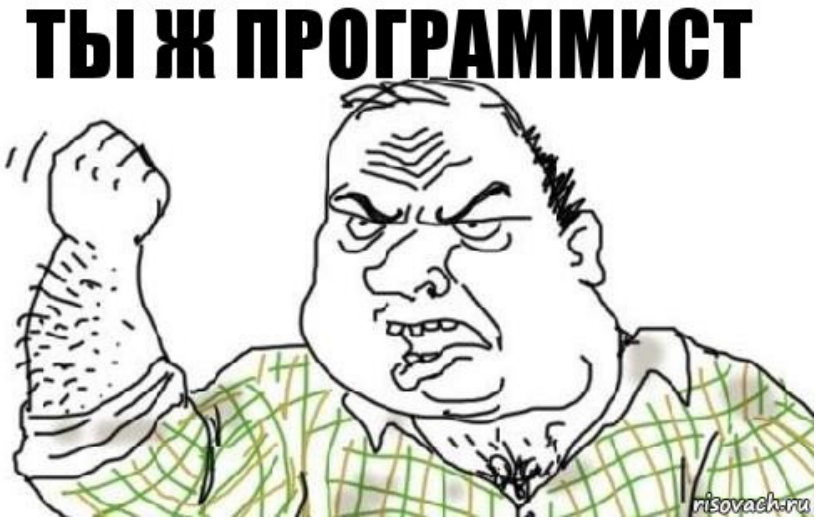
```
package object implicits {
  implicit val stringSemigroup: Semigroup[String] = new Semigroup[String] {
    override def combine(x: String, y: String): String = x + y
  }
}
```

Checking laws - ~~pen and paper~~ in comments

```
package object implicits {
  implicit val stringSemigroup: Semigroup[String] = new Semigroup[String] {
    override def combine(x: String, y: String): String = x + y
  }
}

/*
combine(a, combine(b, c))
  = combine(a, b + c)
  = a + (b + c)
  = (associativity of +)
  = (a + b) + c = combine(a + b, c)
  = combine(combine(a, b), c)
*/
```

You're programmer after all



Question on the interview: property based testing

```
object SemigroupSpecification extends Properties("Semigroup") with SemigroupSpecificationSupport {
 include(semigroup[String](stringSemigroup))
}

trait SemigroupSpecificationSupport {
 def semigroup[A](sg: Semigroup[A])(implicit ar: Arbitrary[A], tag: ClassTag[A]): Properties =
   new Properties(s"Semigroup[${tag.toString}]") {


     // ∀a, b, c ∈ A : (a · b) · c = a · (b · c)
     property("associativity") = forAll { (a: A, b: A, c: A) =>
       sg.combine(sg.combine(a, b), c) =? sg.combine(a, sg.combine(b, c))
     }


   }
}

/*
+ Semigroup.Semigroup[java.lang.String].associativity: OK, passed 100 tests
  .
*/
```

More than one valid instance

```
package object implicits {
  implicit val stringSemigroup: Semigroup[String] = new Semigroup[String] {
    override def combine(x: String, y: String): String = x
  }
}
```

More examples

- Numbers with $+$, $*$, *min*, *max*
- Booleans with conjunction, disjunction, implication etc.
- Square nonnegative matrices with multiplication.
- Lists, Strings, Maps etc. with concatenation/union
- We will see even more examples during practical part.

Contra-examples

- $\{\mathbb{N}, /\}$ is not a Semigroup, because $/$ is not associative.
- The same goes for $\{\mathbb{N}, a^b\}$.
- $\{\mathbb{N}, -\}$ is not a Semigroup, because $-$ is not a closed operation, e.g. $\exists a, b \in \mathbb{N} : a - b \notin \mathbb{N}$, for example $10 - 15 = -5 \notin \mathbb{N}$.

Lets hack

1. Clone `git@github.com:d12frosted/wax.git`
2. Import it as `sbt` project.

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1. Task 2
   1.1 Create `Semigroup` typeclass manually in `wax.typeclass.semigroup.manual.typeclass`
   1.2 Implement missing `Semigroup` instances in `wax.typeclass.semigroup.manual.implicits`
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Monoid

A monoid is a set S with binary closed operation $\cdot : S \times S \to S$ that satisfies associativity property:

$$\forall a, b, c \in S : (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

and identity element e that satisfies

$$\forall a \in S : e \cdot a = a \cdot e = a$$

Operation is closed when $\forall a, b \in S : a \cdot b \in S$.

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In other words, monoid is just a semigroup with identity element.

Again, it's not that scary

```scala
package object typeclass {

  //
  // Laws:
  //   1. ∀a, b, c ∈ S : (a · b) · c = a · (b · c)
  //   2. ∀a ∈ S : e · a = a · e = a
  //
  trait Monoid[A] extends Semigroup[A] {
    def empty: A
  }

  object Monoid {
    def apply[A: Monoid]: Monoid[A] = implicitly[Monoid[A]]
  }

}
```

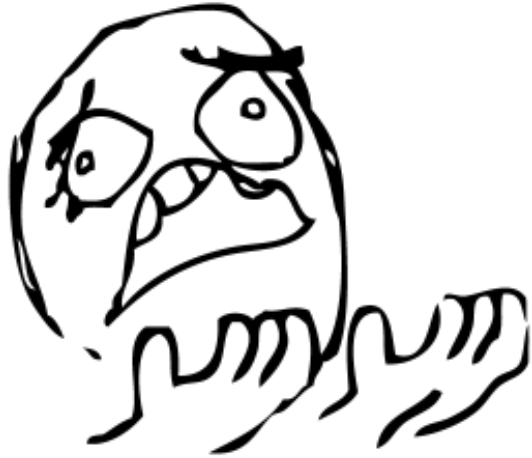
Examples

- $\{\mathbb{N}_0, +\}$, where 0 is the identity element.
- $\{\mathbb{N}, *\}$, where 1 is the identity element.
- Boolean with XOR, XNOR, OR, AND.
- String with concatenation (empty string is identity element).

But not every Semigroup forms a Monoid (we are not talking about free monoids here):

- `BigNumber` practically doesn't have identity element for `min`.

The most important question



Why did we learn this?

The Fibonacci numbers

On the interview we ask people to write a function that returns the nth Fibonacci number.

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}, \forall n > 1$$

Solution

What we expect

```
def fib(n: Int): Int = {
  def fibTail(n: Int, a: Int, b: Int): Int = n match {
    case 0 => a
    case _ => fibTail(n - 1, b, a + b)
  }

  fibTail(n, 0, 1)
}
```

Ideal solution

$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}$$
$$= \frac{\phi^n - (-\phi)^{-n}}{2\phi - 1}$$

$$\phi = \frac{1 + \sqrt{5}}{2}$$

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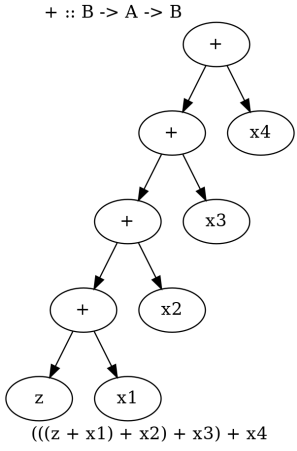
As they say, truth is somewhere in the logarithm.

Two folds

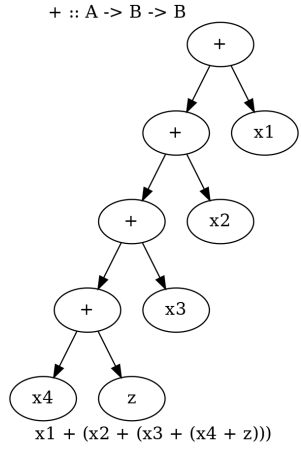
- `def foldl[A, B](xs: Seq[A])(z: B)(op: B => A => B): B`
  - Folds the structure from left to right
- `def foldr[A, B](xs: Seq[A])(z: B)(op: A => B => B): B`
  - Folds the structure from right to left
- Since combining function is asymmetrical in its types:
  - It's impossible to place parentheses in the arbitrary fashion or even just change the direction of the `fold`
  - It's impossible to implement a total `fold` without default value of type `B`

Example

foldl



foldr



What Monoid gives us

- Combining function is symmetrical (`combine : A -> A -> A`).
- Monoid provides identity element of type `A` (`empty`).
- So we can define a special `fold`
  - `def foldMonoid[A: Monoid](xs: Seq[A]): A`
- Associativity law says that we can put parentheses in an arbitrary fashion.
- Identity law says that we can place identity element anywhere - on the far left, on the far right or even in the middle.
- Laws that we checked are giving us means to abstract implementation.
- When we ask consumer of our API to provide us a Monoid we don't want only a behaviour but behaviour that follows the laws. We want these properties as much as we want the functions.

Power in terms of Monoid

- In some cases all elements of the list are the same.

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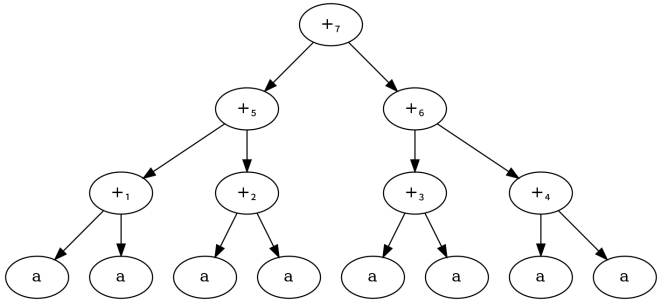
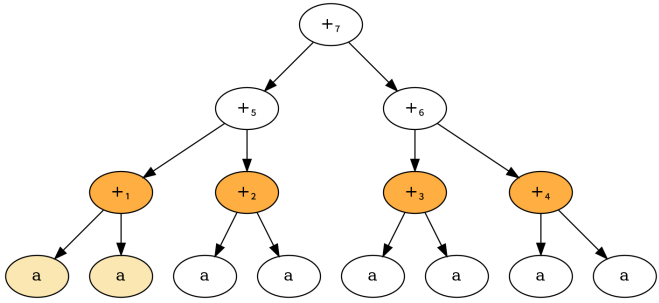
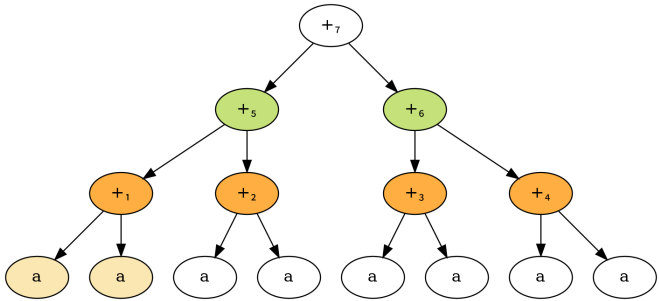


Power in terms of Monoid



Evaluating $a + a$ always yields the same result. So there is no point in repeating this calculation 4 times.

Power in terms of Monoid



The same thing with the upper level. In this particular example, we can avoid 4 operations out of 7. In general, this optimisation leads to the result in $\log n$ operations.

Power in terms of Monoid

All this means that we can define a function `power`:

```
def power[A: Monoid](a: A, n: Int): A = {
  ???
}
```

Back to Fibonacci

Fibonacci number can be defined in a different way.

$$\begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$$

Back to Fibonacci

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- The Fibonacci number can be calculated using square nonnegative matrix multiplication.
- Square nonnegative matrices form Monoid with multiplication.
- So we can put parentheses in a way we like it.

Time for work

- Open `wax.exercise.fibonacci` module.
- Task is to implement monoid for `Matrix2x2`.
- Run `MatrixMonoidSpecification` to test your instance.
- Run `Main` object and compare benchmarking results of tail recursive and matrix-based implementations.

Outcome

- Just think about it.
  - Giving any monoid we have a helper that allows us to efficiently calculate a^n.
  - This is not only because of the operations, but the laws (or so-called properties) that come with these operations.
  - Monoids are everywhere around us. We deal with them every day, without even noticing it.
- You might forget how matrix multiplication works, but now you remember, right?

Folds with Monoids

- We already know that Monoids give us an ability to place parentheses in any fashion.
- We already saw that when it comes to folding the list of the same elements we gain performance.
- But what if the elements are not equal? Do we gain anything?

Folds with Monoids

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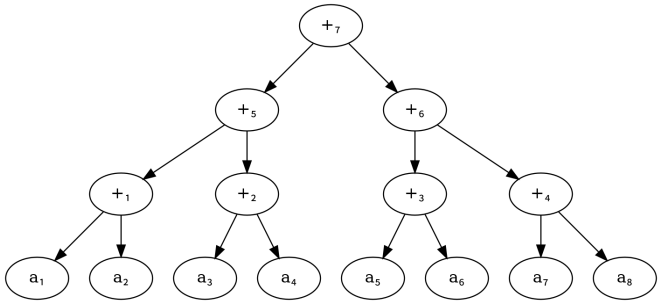
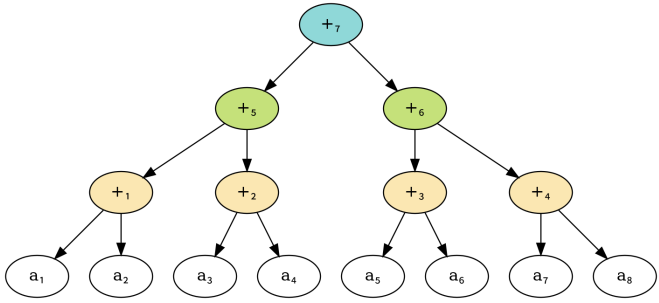


Folds with Monoids



Every expression on each level does not depend on other expressions from the same level, which means that we can evaluate them in parallel.

MapReduce

- Sometimes we have a collection of elements that don't form Monoid.
- But we can transform (going ahead, `map`) them into something that is a Monoid
  - Again, going ahead, this also can be done in parallel.
- There is a strange accent, where people pronounce 'fold' as 'reduce'.
- This is how we get the `mapReduce`.

Getting the top used words from set of books

- Open `wax.exercise.mapreduce` module.
- Task is to
  - Implement monoid for `MapReduce.Result[A]`.
  - Implement the `job` function. Let's find the most used word among all the books that is also longer than 4 symbols.
- Books are located in the `resources` directory.
- Use `allBooks` to load all available books or `authorBooks` to load all books of specific author.
  - `authorBooks("boris")` - you can use this author with small amount of text to test your `job`.
- Compare benchmark results.

Things to note

- Functional programming is not about `Monads` and `IO`.
  - Funny enough, first versions of Haskell were naked and no one dared to tell the committee that `IO` is missing.
- Functions matter.

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- Functional programming is not about `Monads` and `IO`.
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- Can a function be monoid?

Let's start with some wrappers (pun intended)

- Suppose that we have some case class `Wrapper[A](value: A)`
- Can it be a monoid?
- Well, generally speaking, not! Because we know nothing about the type `A`.
- But what if `A` is a monoid?

Let's start with some wrappers (pun intended)

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- Can it be a monoid?
- Well, generally speaking, not! Because we know nothing about the type `A`.
- But what if `A` is a monoid?

```scala
case class Wrapper[A](value: A)

object Wrapper {
  implicit def wrapperMonoid[A: Monoid]: Monoid[Wrapper[A]] = new Monoid[Wrapper[A]] {
    override def empty: Wrapper[A] = Wrapper(Monoid[A].empty)

    override def combine(x: Wrapper[A], y: Wrapper[A]): Wrapper[A] =
      Wrapper(Monoid[A].combine(x.value, y.value))
  }
}
```

Wrappers of monoids are monoids

- So IO can also be a monoid
  ```
  def ioMonoid[A: Monoid]: Monoid[IO[A]] = ???
  ```
- Which means that we can combine IO actions (in some new sense).
- Functions are wrappers (in some sense), so they also can be monoids
  ```
  def functionMonoid[A, B: Monoid]: Monoid[Function[A, B]] = ???
  ```
- Which means that we can combine functions (in some new sense).

Logger

- `Logger` is basically a function from `String` to `IO[Unit]`.
  ```
  type Logger = String => IO[Unit]
  ```
- `Unit` forms a monoid.
- So `IO[Unit]` forms a monoid.
- So `String => IO[Unit]` forms a monoid.
- So `Logger` forms a monoid.
- So we can combine loggers
  - `combine(fileLogger, consoleLogger)` - logs both into file and to console

Logger

- Open `wax.exercise.logging` module
- Task is to implement monoid for `IO[Logger]`
- Have fun!

Recap (recup?)

- Semigroup is something with means of combining these somethings.
- Monoid is semigroup that also has neutral element that doesn't affect a combination.
- Associativity is a powerful property giving us an ability to solve some tasks.
  - a^n in $\log n$
  - `mapReduce`
- Monoids are everywhere. They act like a plague, once something forms a monoid, something else also begins to form a monoid.
- We want some rest after a long session of forkshop.

Questions?

$\epsilon\rho\omega\tau\eta\sigma\eta?$

Thank you very much!

We hope you enjoyed this session.