

# Compressed Sensing

## └ Problem Statement

## └ Problem Statement

### Problem Statement

**Goal:** Recover a (sparse) signal from its reduced representation (measure).

Fundamental theorem of linear algebra: in order to effectively reconstruct a signal we need a sequence of measurements that is at least as long as the original signal.

Assume sparsity -> recover signal from "incomplete" measures!

This applies primarily in situations when repeated measures are costly or harmful (like in the case of magnetic resonance imaging) or when measures are naturally sparse (like radar scans).

1. A question of interest is whether there is a good way of obtaining the compressed version of the signal directly, without taking many measurements of it. Compressed sensing is relying on taking a small amount of linear and non-adaptive measurements. Interestingly enough, all provably good measurement matrices happen to be random matrices. These include the Gaussian, Bernoulli and partial random Fourier matrices.

# Compressed Sensing

- Problem Statement
- Applications

## Applications on naturally sparse data

- Magnetic Resonance Imaging (MRI)
- Radar data
- Wireless communication
- Astronomical signal and image processing
- Camera design and imaging
- Collaborative filtering
- Matrix completion

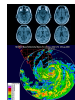


Image sources: [1] and [2]

1. In radar imaging, only a small amount of targets is monitored at the same time, so sparsity becomes a very realistic assumption. Standard methods for radar imaging actually use the sparsity assumption as well, but only at the very end of the signal processing procedure in order to clean up the noise in the resulting image. Using sparsity from the very beginning by using compressive sensing methods is therefore a natural approach.
2. Collaborative filtering and matrix completion are actively used in recommender systems

## Compressed Sensing

## └ Extensions of Compressed Sensing

## └ Extensions of Compressed Sensing

## Extensions of Compressed Sensing

**Matrix input.** We could consider restoring not only vector input, but extend it to matrices of minimal rank consistent with a given underdetermined linear system of equations.

**Matrix completion.** In the matrix completion setup the measurements are the pointwise observations of entries of the matrix. The RIP fails completely in this setting, and 'localized' low-rank matrices in the null space of  $S$  cannot be recovered by any method whatsoever. However, if certain conditions on the left and right singular vectors of the underlying low-rank matrix are imposed, essentially requiring that such vectors are uncorrelated with the canonical basis, then it was shown that such incoherent matrices of rank at most  $k$  can be recovered from  $m$  randomly chosen entries with high probability provided

$$m \geq Ck \max\{n, p\} \log^2(\max\{n, p\}).$$

1. Another generalization considers nonlinear nonadaptive measurements. The simplest nonlinear example is the quadratic measurements. The associated recovery task is called the phase retrieval problem. It appears in mostly physical situations where only intensity values can be observed. Alternatively, we can consider recovery from higher order measurements. Polynomial-type measurements can actually be recast into an affine low-rank minimization problem discussed before.