

Correctness of a program

Let $P = \{p_b \mid p_b \text{ is a block of a program}\}$

Let $TS = \{ts_i \mid ts_i \text{ is a set of tests for each } p_b\}$

Let $ts_i = \{t_j \mid t_j \text{ is a test}\}$

$t_j = (f_k, inputs_{f_k}, output_{f_k})$

where, $output_{f_k} = f_k(inputs_{f_k})$

and, $inputs_{f_k} = \{v_s \mid v_s \text{ is a value}\}$

Moreover,

$p_b = (f_c, inputs_{f_c})$

where, $f_c(inputs_{f_c}) \Rightarrow output_{f_c}$

and, $inputs_{f_c} = \{v_s \mid v_s \text{ is a value}\}$

Let $correctness(P, TS) \Leftrightarrow \forall p_b \in P, \forall ts_i \in TS \mid correct(ts_i, p_b)$

$correct(ts_i, p_b) \Leftrightarrow \forall t_j \in ts_i \mid success(t_j, p_b)$

$success(t_j, p_b) \models \varphi$

Where $\varphi = x \wedge \neg x$

$success(t_j, p_b) \rightarrow \begin{cases} 1 & \text{if } [t_j]_{output_{f_k}} = [p_b]_{output_{f_c}} \\ 0, & \text{otherwise} \end{cases}$

Satisfaction of requirements

Let $R = \{RS \mid RS \text{ is a set of requirements}\}$

$RS = \{r_s \mid r_s \text{ is a requirement}\}$

$satisfyPR(P, TS, R) \Leftrightarrow \forall ts_i \in TS, \forall p_b \in P, \forall RS \in R \mid satisfaction(p_b, ts_i, RS)$

$satisfaction(p_b, ts_i, RS) \Leftrightarrow \forall t_j \in ts_i, \forall r_s \in RS \mid satisfy(success(t_j, p_b), r_s)$

$satisfy(success(t_j, p_b), r_s) \models \varphi$

Where $\varphi = x \wedge \neg x$

Let $r_s = (\text{pre-condition and post-condition})$

$conditionOutput = success(t_j, p_b) = 1 \text{ and } [t_j]_{output_{f_k}} = [p_b]_{output_{f_c}} = r_s(\text{post-condition})$

$conditionInput = [t_j]_{input_{f_k}} = [p_b]_{input_{f_c}} = r_s(\text{pre-condition})$

$$satisfy(success(t_j, p_b), r_s) \rightarrow \begin{cases} 1, & \text{if conditionOutput and conditionInput} \\ 0, & \text{otherwise} \end{cases}$$