Bayesian defect modelling

Olga Egorova

15 July 2020

Bayesian model calibration is to be adopted for making the inference on the material and/or defect parameters $\boldsymbol{\xi}$ based on the observed defective samples, and through forward modelling by defining a defect as a deformation field parameterised by $\boldsymbol{\xi}$.

The posterior derived for (calibration) parameters $\boldsymbol{\xi}$ is then used for further FE modelling in order to assess their effect on material strength or any other mai property of interest.

Defining a defect. Wrinkle. Cantilever beam.

A defect is defined as a mapping W from $(\boldsymbol{x}, \boldsymbol{\xi}) \in \Omega \times \Xi$, from the pristine state $\Omega \subset \mathrm{R}^3$ to the deformed state $W(\Omega) \subset \mathrm{R}^3$. A set of parameters ξ (let's say that $\dim(\boldsymbol{\xi}) = N_{\boldsymbol{\xi}}$) is referred to as a "defect profile", "material model parameters" – so that setting them defines uniquely the transformation between the pristine and deformed states.

Let's consider a wrinkle. Wrinkle defect is defined through the deformed field:

$$W(x, \boldsymbol{\xi}) = \sum_{i=1}^{N} a_i \psi_i(x, \boldsymbol{b})$$

In the toy example of a three-layer cantilever beam, a simple in-plane misalignment of the fibres, e.g. disturbance in the baseline angles will be treated as "defect" parameters: $\boldsymbol{\xi} = (\theta_1, \theta_2, \theta_3)^T$.

Forward Model

The forward model $F(\boldsymbol{\xi}) : \Xi \longrightarrow \mathrm{D}$ – deterministic mapping from a set of defect generating parameters to the resulting (observed) defect.

In case of a wrinkle the observed defect is presented as a set of misalignment angles: $\boldsymbol{\phi}_{obs} = (\phi_1, \dots \phi_{N_\phi})^T \in \mathrm{D} \subset \mathrm{R}^{N_\phi}$, with each of them defined as

$$\phi_j = \tan^{-1}\left(\sum_{k=1}^{N_{\boldsymbol{\xi}}} \boldsymbol{\xi}^k \frac{d\psi(\boldsymbol{x_j})}{dx_1}\right).$$

The choice of measurement points $\boldsymbol{x_j}$ $(j = 1 \dots N_\phi)$ for each of the misalignment angles to be measured is a separate matter of consideration.

The forward model for the cantilever beam maps a set of angle disturbances $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)^T$ is mapped to the displacements observed on the surface of the beam at N_F observational points: $\boldsymbol{d}_{obs} = (d_1, \dots d_{N_d})^T$.

Data. Statistical model

When n independent objects (wrinkles, beams) are observed, we denote the data measured by $D_{obs} = [\boldsymbol{d}^{(1)}, \boldsymbol{d}^{(2)}, \ldots, \boldsymbol{d}^{(n)}]$ – an $N_d \times n$ matrix of displacements, and we assume the following stochastic relationship between each of the observations and the forward model:

$$\boldsymbol{d}^{(i)} = F(\boldsymbol{\xi}) + \boldsymbol{\varepsilon_i},$$

with the vector of measurement errors $\boldsymbol{\varepsilon_i}$ following a normal ditribution with zero mean and variance-covariance matrix Σ_ϵ – which contains not just the variance of measurement erros of individual observations at points $\boldsymbol{x_j}$, but also might/should take into account the covariance structure across measurements within one object.

Covariance structure

We shall consider a few covariance structures, starting from a simple one − where we assume that all obeseravtions from all measurement points and objects (samples) are independent and identically distributed, i.e. $\boldsymbol{\varepsilon}_{ij} \sim \mathrm{N}(0, \sigma_\varepsilon)$.

Linking to calibraiton modelling

Design questions