

## Minor corrections to the thesis

### Abstract

1. “We first present” (was in past tense)
2. “we also adapt the criteria for use in blocked experiments” (was in passive voice)

### Introduction

1. Page 2. “.. all responses are put in an  $n$ -dimensional vector  $\mathbf{Y}$ ”
2. Page 3. Extra “further” removed

### Chapter 2. Background and Related Work

1. Page 9. “.. and derived the  $DP$ -criterion: minimising  $(F_{p,d;1-\alpha_{DP}})^p |(\mathbf{X}'\mathbf{X})^{-1}|$ ” (was  $\nu$  instead of  $d$ )
2. Page 10. Expanding the explanation of where L-optimality comes from: “If we are interested in the functions of parameters given by  $\mathbf{L}'\boldsymbol{\beta}$ , then the variance-covariance matrix of  $\mathbf{L}'\hat{\boldsymbol{\beta}}$  is...”
3. Page 10. Explaining why Sidak’s corrections were used: “Here we shall use Sidak’s correction (Šidák, 1967), which is...”
4. Page 10. “.. for example,  $I$ -optimality, that is minimising **average** prediction variance”
5. Page 11. Rephrasing in order to make a sentence more readable: "The idea of compound criteria  $\langle \dots \rangle$  allows finding a compromise between two or more desirable properties while searching for the design, and explore how the changes in the allocation of weights between the individual criteria affect the optimal designs’ performances."
6. Page 11. “.. prior knowledge of the **experimenter**” (misprint)
7. Page 12. “.. it is sometimes **necessary** to allocate units in blocks”
8. Page 12. “.. its  $(i,j)^{th}$  element”
9. Page 12. “.. and  $\boldsymbol{\beta}_B$  is the vector of block effects”
10. Page 13. “.. matrix whose elements indicate **the** treatments”
11. Page 13. “.. the number of replications after **subtracting the number of replicates** taken for the estimation of block contrasts.”
12. Page 15. “.. Goos and Vandebroek (2001) considered the three cases when D-optimal designs for split-plot experiments do not depend on the value of  $\eta$ ; in other practical cases an estimate of the variance ratio is to be provided.”

13. Page 16. “Arnouts and Goos (2012) presented a coordinate-exchange algorithm ... examples of D- and I-optimal designs.”
14. Page 16. “.. assumed possible values of  $\eta_i$ ” (misprint)
15. Page 16. “.. in the presence” (misprint)
16. Page 17. “In this thesis, due to the primary interest being the quality of inference,..”
17. Page 18. Removed “Therefore” in the 4th paragraph.

### Chapter 3. Compound criteria. Some amendments

1. Page 20. Adding significance levels where they were missed:  $F_{n-p-d,d;1-\alpha_{LoF}}$  instead of  $F_{n-p-d,d}$ , and on Figure 3.1 on page 21.
2. Page 21. Expanding interpretation of the plot in Figure 3.1: “Larger numbers of available residual degrees of freedom ...”
3. Page 21. “The third component in both criteria corresponds to the DF-efficiency defined in (2.7).”
4. Pages 22 and 24. Tables 3.1 and 3.4: added columns with DF-efficiency values
5. Page 23. “..therefore, their  $DF$ -efficiency values  $\frac{n-d}{n}$  are below 75% (i.e. when  $d > 10$ ).”
6. Page 24. “..the design optimal  $\langle \dots \rangle$  (#12) is also optimal with respect to the criterion with equal weights put on the  $DF$ - and  $LoF$ -components”
7. Page 25. “..except for the point number 18 – 19 and the point number 36 – 37”
8. Page 25. “..the resulting efficiency values. It also would be sensible..” (breaking the sentence)
9. Page 27. Adding extra comment: “This particular parametrisation  $\langle \dots \rangle$  designs which are more than 50% efficient.”
10. Page 27. Adding subsection 3.2.2 Optimal designs (no text amendments)
11. Pages 28 and 29. Tables 3.6 and 3.7: added columns with DF-efficiency values
12. Page 29. Paraphrasing the last paragraph: “It was desired to alter the form of an efficiency’s contribution..”

### Chapter 4. Generalised Compound Criteria

1. Page 31. Adding bullet points in the chapter introduction
2. Page 32 - 33. Adding subsections 4.1.1 and 4.1.2
3. Page 32. Equation (4.3) –  $\varepsilon$  in the subscripts
4. Page 33. “..how large the magnitude of each of the potential terms is assumed to **possibly** be..”
5. Page 34. “..we incorporate  $L$ -optimality for the primary terms..”
6. Pages 35 and 36. “..and (4.11)”
7. Page 35. Adding the reference to Section 3.1.1
8. Page 37. Adding the reference to DuMouchel and Jones (1994))
9. !!!Page 37. Amending the derivation of the formula for the confidence region for  $\beta_2$  over the posterior distribution (introducing the corresponding F-quantile).
10. Page 39. “Henceforth” instead of “Here and further on”
11. Page 40. “The results should” instead of “The results would”
12. Page 43. Table 4.6, 2nd part, column names: **L**, **LP**, **LoF(LP)** and **Bias(L)**
13. Page 44. Rephrasing: “similar tendency seems to be true”
14. Page 49. Paragraph 3. Adding an explanation regarding large *LoF-efficiencies* in the case of smaller  $\tau^2$ : “Such consistently high performance...”
15. Page 53. “..each factor can take five levels..”
16. Page 53. “all third and **fourth** order terms are taken as potentially missed terms”
17. Page 53. Adding: “ Most of the replicates are at corner points...”
18. Page 58. Changing the name of Section 4.3 (to “Blocked Experiments”) and adding subsection 4.3.1 “Generalised criteria”
19. Page 58. Adding “ $\beta_B$  is the vector of **fixed block effects**,...”
20. Page 59.  $\beta_B$  instead of  $\beta_0$
21. Page 59. “Therefore, ...”
22. Page 60. “**the**  $D$ -optimal design”
23. !!Page 64. Adding a comment regarding ignoring the orthonormalisation: “Ignoring such a requirement would amend the formulae of the

bias components, but in general it should not be expected to considerably influence the resulting optimal designs and their performances; though the orthonormalisation step should not be blindly omitted, especially if the prediction bias is of a particular interest.”

## Chapter 5. MSE-based Criteria

1. Page 66. Adding an explanation of ‘pseudo-Bayesian’: “..(hence we shall refer to such an approach as “pseudo-Bayesian”)..”
2. Page 67. “.. and  $\mathbf{u}, \mathbf{v}$  are column vectors”
3. Page 68. Adding a definition of  $\tilde{\beta}_{2i}$  after the equation (5.5): “Here  $\tilde{\beta}_{2i}$ ,  $i = 1..N$  – independent random vectors sampled from  $\mathcal{N}(\mathbf{0}, \tau^2 \sigma^2 \mathbf{I}_q)$ .”
4. Page 70. Emphasising the computational advantage of the trace-based criterion: “The main advantage is the absence of the necessity of any additional numerical evaluations, and...”
5. Page 74. Amending the interpretation: “..the decreased scale of the potentially missed contamination results in **an easier achievable compromise** between...”
6. Page 74. “.. in the corresponding tables”
7. Page 74. Rephrasing: “..in the case of  $MSE(D)$ -efficient designs with smaller  $\tau^2$ .”

## Chapter 6. MSE-based Criteria for Blocked Experiments

1. Page 78. Equation (6.2):  $\beta_B$  and  $\beta_1$  instead of  $\tilde{\beta}_b$  and  $\tilde{\beta}_p$
2. Page 82. “in the design #8”
3. Page 84. Adding a comment regarding relative efficiencies between the designs optimal w.r.t. the continuous and point-prior-based criteria: “As for the efficiency losses...”
4. Page 85. Reason for fixing two centre points per block in the case-study: “... the experimenters wanted to have at least two centre points in each block to ensure representation of the conditions thought a priori most likely to be best (with dosages of 95% for each supplement)”
5. Page 87. “ $LoF(DP)$ –”
6. Page 88. Amending the explanation of the “Relative efficiency”: “The last column of this table, “Relative Efficiency,” is..”
7. Page 91. Misprint: “experimenters”

## Chapter 7. MSE-based Criteria for Multistratum Experiments

1. Page 94. Changing **REML Methodology** from a subsection to section
2. Page 96. Changing the title of Section 7.2.2: “Yates’ procedure”
3. Page 97. “ $m_j = \dots$  **units**”
4. Page 98. Adding the **Construction procedure** subsection title (no text amendments)
5. Page 103. Expanding the sentence: “Now we need to evaluate the number of degrees of freedom for..”
6. Page 103. “.. the factors applied at the current and **all** higher strata”
7. Page 104. Explaining why  $\tau^2 = \sqrt{1/q}$  was used: “As this number tends to become..”
8. Page 104. “Regarding the **third-order** potential terms..”
9. Page 106. “(designs #3)”
10. Page 106. “.. designs #5 and #7 provide the worst performance with respect to the  $MSE(L)$ – and  $LP$ -components respectively”
11. Page 109. “i.e. designs #6 in Table 7.6”
12. Page 111. Adding the **Discussion** subsection title (no text amendments)

## Chapter 8. Conclusions and Future Work

1. Page 114. Adding a comment regarding advantages and disadvantages of the Pareto frontier approach: “On one hand, such an algorithm would result in a set of designs...”

## References

- Arnouts, H. and Goos, P. (2012) Staggered-level designs for experiments with more than one hard-to-change factor. *Technometrics*, **54**, 355–366.
- DuMouchel, W. and Jones, B. (1994) A simple Bayesian modification of D-optimal designs to reduce dependence on an assumed model. *Technometrics*, **36**, 37–47.
- Goos, P. and Vandebroek, M. (2001) D-optimal response surface designs in the presence of random block effects. *Computational statistics & data analysis*, **37**, 433–453.

Šidák, Z. (1967) Rectangular confidence regions for the means of multivariate normal distributions. *Journal of the American Statistical Association*, **62**, 626–633.