Discrete univariate distributions

Distribution	Probability mass function (p.m.f.) $p(x)$	Range	Parameters	$\mathbb{E}(X)$	$\mathrm{Var}(X)$	Moment-generating function $M(t)$	Comments
Binomial[1,2] $\mathsf{Bi}(n,\theta)$	$\binom{n}{x}\theta^x(1-\theta)^{n-x}$	$x\in\{0,1,\dots,n\}$	$n\in\mathbb{N}$ $0<\theta<1$	$n\theta$	$n\theta(1-\theta)$	$(1-\theta+\theta\exp(t))^n$	No. of successes in n trials θ – probability of success
Geometric $\mathsf{Geo}(\theta)$	$\theta^{x-1}(1-\theta)$	$x\in\mathbb{N}$	$0<\theta<1$	$\frac{1}{1-\theta}$	$\frac{\theta}{(1-\theta)^2}$	$\frac{(1-\theta)\exp(t)}{1-\theta\exp(t)}$	No. of trials until (and including) first failure θ – probability of success
Hypergeometric $\mathsf{HyGe}(n,N,M)$	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$x\in\{\max\{0,n-(N-M)\},\dots,\min\{n,M\}\}$	$N,n\in\mathbb{N}$ $M\in\{0,\dots,N\}$	$n\theta$	$n\theta(1-\theta)\cdot\frac{N-n}{N-1}$ (with $\theta=\frac{M}{N}$)	—[3]	No. of type I objects in a sample of size n, drawn *without* replacement from a population of size N, containing M type I objects.
Negative Binomial $\mathsf{NeBi}(k,\theta)$	$\binom{x-1}{k-1}\theta^{x-k}(1-\theta)^k$	$x\in\{k,k+1,\dots\}$	$k\in\mathbb{N}$ $0<\theta<1$	$\frac{k}{1-\theta}$	$\frac{k\theta}{(1-\theta)^2}$	$\left(\frac{(1-\theta)\exp(t)}{1-\theta\exp(t)}\right)^k$	No. of trials until (and including) k^{th} failure θ – probability of success $\mathsf{NeBi}(1,\theta)\equiv\mathsf{Geo}(\theta)$
Poisson $\mathsf{Poi}(\lambda)$	$\exp(-\lambda)\frac{\lambda^x}{x!}$	$x\in\mathbb{N}_0$	$\lambda>0$	λ	λ	$\exp(\lambda(\exp(t)-1))$	

[1] $\mathsf{Bi}(n,\theta)$ can be approximated by $\mathsf{Poi}(n\theta)$, if n large, θ small and $n\theta$ moderate.

[2] $\mathsf{Bi}(n,\theta)$ can be approximated by $\mathsf{N}(n\theta,n\theta(1-\theta))$, if n large and θ not too close to 0 or 1.

[3] No simple closed form expression exists.

Continuous univariate distributions

Distribution	Probability density function (p.d.f.) $f(x)$	Range	Parameters	$\mathbb{E}(X)$	$\mathrm{Var}(X)$	Moment-generating function $M(t)$	Comments
Beta $\mathsf{Be}(\alpha_1,\alpha_2)$	$\frac{x^{\alpha_1-1}(1-x)^{\alpha_2-1}}{\mathrm{B}(\alpha_1,\alpha_2)}$	$0\le x\le 1$	$\alpha_1>0$ $\alpha_2>0$	$\frac{\alpha_1}{\alpha_1+\alpha_2}$	$\frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_2)^2(\alpha_1+\alpha_2+1)}$	—[3]	$X_1\sim\mathsf{Ga}(\alpha_1,\theta)$ $X_2\sim\mathsf{Ga}(\alpha_2,\theta)$ independent $\Rightarrow\frac{X_1}{X_1+X_2}\sim\mathsf{Be}(\alpha_1,\alpha_2)$
Cauchy $\mathsf{Ca}(\eta,\gamma)$	$\frac{1}{\pi\gamma\left(1+\frac{(x-\eta)^2}{\gamma^2}\right)}$	$x\in\mathbb{R}$	$\eta\in\mathbb{R}$ $\gamma>0$	—[4]	—[4]	—[4]	$\mathsf{Ca}(0,1)\equiv\mathsf{t}(1)$
Chi-Squared $\chi^2(\nu)$	$\frac{x^{\frac{\nu}{2}-1}\exp\left(-\frac{x}{2}\right)}{2^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}$	$x>0$	$\nu\in\mathbb{N}$	ν	2ν	$\frac{1}{(1-2t)^{\frac{\nu}{2}}}$	$X_i\sim\mathsf{N}(0,1)$ independent $\Rightarrow\sum_{i=1}^{\nu}X_i^2\sim\chi^2(\nu)$
Exponential $\mathsf{Expo}(\theta)$	$\theta\exp(-\theta x)$	$x>0$	$\theta>0$	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$\frac{1}{1-\frac{t}{\theta}}$	
F $\mathsf{F}(\nu_1,\nu_2)$	$\frac{\nu_1^{\frac{\nu_1}{2}}\nu_2^{\frac{\nu_2}{2}}}{\mathrm{B}\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)}\frac{x^{\frac{\nu_1}{2}-1}}{(\nu_1x+\nu_2)^{\frac{\nu_1+\nu_2}{2}}}$	$x>0$	$\nu_1,\nu_2\in\mathbb{N}$	$\frac{\nu_2}{\nu_2-2}$ (for $\nu_2>2$)	$\frac{2\,\nu_2^2\,(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$ (for $\nu_2>4$)	—[4]	$X_1\sim\chi^2(\nu_1)$ $X_2\sim\chi^2(\nu_2)$ independent $\Rightarrow\frac{X_1/\nu_1}{X_2/\nu_2}\sim\mathsf{F}(\nu_1,\nu_2)$

Distribution	Probability density function (p.d.f.) $f(x)$	Range	Parameters	$\mathbb{E}(X)$	$\mathrm{Var}(X)$	Moment-generating function $M(t)$	Comments
Gamma $\mathsf{Ga}(\alpha, \theta)$	$\dfrac{\theta^\alpha x^{\alpha-1}\exp(-\theta x)}{\Gamma(\alpha)}$	$x > 0$	$\theta > 0$ $\alpha > 0$	$\dfrac{\alpha}{\theta}$	$\dfrac{\alpha}{\theta^2}$	$\dfrac{1}{\left(1-\frac{t}{\theta}\right)^\alpha}$	$\mathsf{Ga}(1,\theta) \equiv \mathsf{Expo}(\theta)$ $\mathsf{Ga}\left(\frac{\nu}{2}, \frac{1}{2}\right) \equiv \chi^2(\nu)$
Normal $\mathsf{N}(\mu, \sigma^2)$	$\dfrac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$	$x \in \mathbb{R}$	$\mu \in \mathbb{R}$ $\sigma^2 > 0$	μ	σ^2	$\exp\left(\mu t + \dfrac{\sigma^2 t^2}{2}\right)$	$\mathsf{N}(0,1)$ – standard normal $\frac{X-\mu}{\sigma} \sim \mathsf{N}(0,1)$
Pareto $\mathsf{Pa}(k, \theta)$	$\dfrac{\theta k^\theta}{x^{\theta+1}}$	$x > k$	$k > 0$ $\theta > 0$	$\dfrac{\theta k}{\theta - 1}$ (for $\theta > 1$)	$\dfrac{\theta k^2}{(\theta-1)^2(\theta-2)}$ (for $\theta > 2$)	—[3]	
Student's t $\mathsf{t}(\nu)$	$\dfrac{1}{\sqrt{\nu}\mathrm{B}\left(\frac{\nu}{2},\frac{1}{2}\right)\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}}$	$x \in \mathbb{R}$	$\nu \in \mathbb{N}$	0 (for $\nu > 1$)	$\dfrac{\nu}{\nu-2}$ (for $\nu > 2$)	—[4]	$X_1 \sim \mathsf{N}(0,1)$ $X_2 \sim \chi^2(\nu)$ independent $\Rightarrow \frac{X_1}{\sqrt{\frac{X_2}{\nu}}} \sim \mathsf{t}(\nu)$
Uniform $\mathsf{U}(a, b)$ c.d.f. for $a < x < b$:	$\dfrac{1}{b-a}$	$a \leq x \leq b$	$a, b \in \mathbb{R}$ $a < b$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\dfrac{\exp(bt)-\exp(at)}{t(b-a)}$	$\mathsf{U}(0,1) \equiv \mathsf{Be}(1,1)$
Weibull $\mathsf{We}(\alpha, \theta)$	$\alpha\theta x^{\alpha-1}\exp(-\theta x^\alpha)$	$x > 0$	$\alpha > 0$ $\theta > 0$	$\dfrac{\Gamma\left(1+\frac{1}{\alpha}\right)}{\theta^{\frac{1}{\alpha}}}$	$\dfrac{\Gamma\left(1+\frac{2}{\alpha}\right)}{\theta^{\frac{2}{\alpha}}} - (\mathbb{E}(X))^2$	—[3]	$\mathsf{We}(1,\theta) \equiv \mathsf{Expo}(\theta)$

[3] No simple closed form expression exists.

[4] Does not exist.

Multivariate distributions

Distribution	Probability mass / density function $p(\mathbf{x}) = p(x_1, \ldots, x_k)$ or $f(\mathbf{x}) = f(x_1, \ldots, x_k)$	Range	Parameters	$\mathbb{E}(X_j)$	$\mathrm{Var}(X_j)$	$\mathrm{Cov}(X_i, X_j)$	Moment-generating function $M(\mathbf{t}) = M(t_1, \ldots, t_k)$
Multinomial $\mathsf{Mu}(n, \theta_1, \ldots \theta_k)$	$\dfrac{n!}{x_1! \cdots x_k!}\theta_1^{x_1}\cdots\theta_k^{x_k}$	$x \in \mathbb{N}_0$ $\sum_{j=1}^k x_j = n$	$n \in \mathbb{N}$ $0 < \theta_j < 1$ $\sum_{j=1}^k \theta_j = 1$	$n\theta_j$	$n\theta_j(1-\theta_j)$	$-n\theta_i\theta_j$	$\left(\displaystyle\sum_{j=1}^k \theta_j \exp(t_j)\right)^n$
Normal $\mathsf{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	$\dfrac{1}{(2\pi)^{\frac{k}{2}}\lvert\boldsymbol{\Sigma}\rvert^{1/2}}\exp\left(-\dfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$	$x \in \mathbb{R}^k$	$\boldsymbol{\mu} \in \mathbb{R}^k$ $\boldsymbol{\Sigma}$ symm., pos. def.	μ_j	Σ_{jj}	Σ_{ij}	$\exp\left(\boldsymbol{\mu}^\top\mathbf{t} + \dfrac{1}{2}\mathbf{t}^\top\boldsymbol{\Sigma}\mathbf{t}\right)$
Special case: bivariate normal distribution with $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$:							
$\dfrac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2(1-\rho^2)}}\exp\left(-\dfrac{\sigma_2^2(x_1-\mu_1)^2 - 2\rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) + \sigma_1^2(x_2-\mu_2)^2}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right)$			$\mu_1, \mu_2 \in \mathbb{R}$ $\sigma_1^2, \sigma_2^2 > 0$ $-1 < \rho < 1$	μ_j	σ_j^2	$\rho\sigma_1\sigma_2$	