HST code
Documentation

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1 Initialization

1.1 Wave numbers

Setup horizontal, vertical and spanwise wave numbers as

$$k_x(x) = x, \qquad k_y(y) = y - \frac{y+1}{N_y(\frac{N_y}{2}+2)}, \qquad k_z(z) = z - \frac{z+1}{N_z(\frac{N_z}{2}+2)}.$$

1.2 Mask

$$k^2(x,y,z) = \left(\frac{k_x(x)}{N_x}\right)^2 + \left(\frac{k_y(y)}{N_y}\right)^2 + \left(\frac{k_z(z)}{N_z}\right)^2$$

If $k^2 > \dfrac{2}{9}$ then $mask = 1$.

For all $y = N_y/2$ and for all $z = N_z/2$: $mask = 0$.

1.3 Energy

$$k_0 = 1$$

If the case is $128 \times 256 \times 512$ cube than

$$k_p = 6.68 \qquad \gamma = 7.5 \cdot 10^{-5}.$$

If the case is $64 \times 64 \times 64$ cube than

$$k_p = 8.08 \qquad \gamma = 7.888 \cdot 10^{-4}.$$

It seems that other cases we can't calculate.

For wavenumber $k_0 \leqslant k \leqslant k_p$:

$$e(k) = \gamma k^2$$

For wavenumber $k_p < k \leqslant k_{max}$:

$$e(k) = (\gamma k_p)^{11/3} k^{-5/3}$$

1.4 Initial velocity field

$$k_{max} = \frac{\sqrt{2}}{3} N_x;$$

$$k(x,y,z) = \sqrt{\left(\frac{k_x(x)}{N_x}\right)^2 + \left(\frac{k_y(y)}{N_y}\right)^2 + \left(\frac{k_z(z)}{N_z}\right)^2}$$

$$e_f(x,y,z) = mask * \frac{\sqrt{e(k)}}{\sqrt{2\pi} k}$$

Using that $\mathrm{Im}(e^{i\theta}) = i\sin\theta$ we get

$$\alpha = e_f e^{i\theta_1}\cos\phi = e_f(\cos\theta_1 + i\sin\theta_1)\cos\phi,$$
$$\beta = e_f e^{i\theta_2}\sin\phi = e_f(\cos\theta_2 + i\sin\theta_2)\sin\phi,$$
$$\delta = e_f e^{i\theta_3} = e_f(\cos\theta_3 + i\sin\theta_3),$$

where $\theta_1,\ \theta_2,\ \theta_3$ and ϕ are random angles.

If $k_x(x)^2 + k_y(y)^2 = 0$ then velocity field

$$U_x(x,y,z) = \alpha, \qquad U_y(x,y,z) = \beta, \qquad U_z(x,y,z) = 0;$$

else

$$U_x(x,y,z) = \frac{\alpha k k_y(y) + \beta k_z(z) k_x(x)}{k\sqrt{k_x(x)^2 + k_y(y)^2}},$$

$$U_y(x,y,z) = \frac{\alpha k k_x(x) + \beta k_z(z) k_y(y)}{k\sqrt{k_x(x)^2 + k_y(y)^2}},$$

$$U_z(x,y,z) = -\frac{\beta\sqrt{k_x(x)^2 + k_y(y)^2}}{k}.$$

If $e_f \leqslant 0$ (here included $mask = 0$) than all components of velocity field equal zero $U_i(x,y,z) = 0$. For point $(0,0,0)$ all components of velocity field equal zero $U_i(x,y,z) = 0$.