

Awareness Logic: an Epistemological Defence

Correlations between Awareness Logic and Epistemology

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Abstract: with this paper, we intend to clarify some of the central notions that are commonly used in contemporary developments of Epistemic Logic, which lack a proper theoretical foundation. We want to follow the steps of some prominent epistemologists and epistemic logicians, who advocate for a correlation between their respective fields of study. We will proceed with a first small step that will consist in adapting one contemporary version of Epistemic Internalism to the framework of Awareness Logic, such that the key concepts in this logic can be sustained by an epistemological view, which, in turn, can work as a theoretical foundation for Awareness Logic.

Keywords: Epistemology – Epistemic Internalism – Epistemic Logic – Awareness Logic – Justification.

1. Introduction and Motivations

Nowadays, when we think about Epistemology and Logic in general, we often find ourselves looking into two apparently separated areas: ana-

lytic philosophy or philosophy of science, for Epistemology, and formal sciences or computational sciences, for Logic. A quick glimpse into both areas will probably result in the conviction that they have little in common. However, first impressions should not be trusted. Hence, our main motivation is to change these impressions and find correlates between Epistemology and Epistemic Logic.¹

In the past decades, Epistemic Logic has been also concerned with the modelling of knowledge of resource-limited agents (with bounded reasoning capabilities). This task has its origins in the need of solving the problem of logical omniscience, and also the willingness of formalizing real knowledge, meaning knowledge of human beings and most computing machines.

The list of different approaches that achieve this task is very long, each of those approaches focus on different strategies for solving the problem. Most of these approaches fulfil the purpose of modelling real knowledge, but not every approach is equally intuitive (some of them resort to very complex mathematical models). The logic we will be considering here, Awareness Logic, is an extension of epistemic logic that includes the agent's awareness in its system as a syntactic operator, and distinguishes between implicit and explicit knowledge.

We want to argue in favour of this logic based on the fact that from all the possible solutions to the problem of logical omniscience, Awareness Logic is the one that corresponds to the common-sense intuitions of how the knowledge of human beings works, as well as computing machines do. For doing so, we want to consider some key epistemological concepts, such as Epistemic Internalism. These concepts will help us to unravel the theoretical notions that underlie Awareness Logic.

We will also try to establish some fine-grained definitions for these epistemological concepts, for relating them with their logical correlates. Accuracy in the use of the terms is always beneficial, even more in a philosophical debate of this magnitude. Therefore, by redefining some propositions and classifying some uses of each concept, we want to

1 Two outstanding works that have already tried to build bridges between epistemic logic and epistemology are (Hendricks and Symon, 2006) and (van Benthem, 2006).

achieve a more complete overview of our field of study, and lay down some theoretical foundations for Awareness Logic.

2. Awareness Logic: an Extension of Epistemic Logic

2.1 Origins of Epistemic Logic and Logical Omniscience

The origin of Epistemic Logic (EL) can be found in Hintikka's work (Hintikka, 1962), where the basic foundations of the modal logic of knowledge are established.² As an extension of modal logic, it employs a version of the semantics of possible worlds (also called Kripke semantics nowadays), where the truth values of the propositions depend on the accessibility relation that we establish between the possible worlds, or epistemic states, the agent can consider.

As mentioned above, standard Epistemic Logic suffers from the problem of logical omniscience, in fact, this problem is intrinsic to Kripke semantics and Hintikka himself pointed it out in (Hintikka, 1975). Thus, contemporary developments and extensions of Epistemic Logic need to confront this problem and propose a different framework that overcomes the idealization of knowledge.

The notion of “agents with limited resources” is central in a large list of nowadays developments of Epistemic Logic, since it simulates most machines and also human beings. The limitation of the reasoning abilities of the agents can be interpreted in different ways and has given rise to a variety of alternative proposals. We will only consider one of those proposals here: Awareness Logic.

2.2 Awareness Logic as one of the Possible Solutions

Fagin and Halpern presented their concept of “awareness of the agent” in (Fagin and Halpern, 1988) and later on in (Fagin et. al., 1995), where they established “awareness logic” as an extension of Epistemic Logic. They introduce the notion of the agent’s awareness as a syntactic concept. As such, they incorporate an awareness operator into the system

² Though the first development of a logic of knowledge was published some years before in (von Wright, 1951).

of Epistemic Logic that works as a filter on knowledge. They also split knowledge into implicit and explicit:

Implicit knowledge represents the standard ideal knowledge as presented in traditional epistemic logic. Being “implicit” means that it embraces the whole corpus of knowledge an agent could ideally know, meaning here, that the implicit knowledge is closed under logical consequence. This means that implicit knowledge is formed by every logical consequence of the agent’s knowledge. As such, this knowledge is unable to stand for what is commonly known as “knowledge” in relation to human beings and computing machines. Implicit knowledge has proven itself very useful in computational applications (like distributed knowledge and the wise man’s knowledge), but when it comes to represent real knowledge it can only act as an idealization.

Explicit knowledge, on the other hand, is what we can call “real knowledge”, those propositions that the agent actually knows and uses in his daily reasoning. The explicit knowledge, in this version of Awareness Logic, is limited due to the epistemic awareness. In order for knowledge to become explicit, the propositions the agent knows must also be part of her awareness. In other words, explicit knowledge is defined by the implicit knowledge the agent is aware of.

2.3 The concept of Epistemic Awareness and Explicit Knowledge

The awareness we are referring to is always epistemic, that is, it accounts for the fact of realizing one’s own information. The first entry of ‘to realize’ in Oxford Dictionaries³ says “[b]ecome fully aware of (something) as a fact; understand clearly”; which supports the choice of only considering this epistemic sense of awareness. By establishing this notion of “Epistemic Awareness” (“Awareness” for simplicity), we hope to rule out every moral- or consciousness-related interpretation the term could possibly evoke.

The agent’s awareness represents the information she entertains in a broad sense, irrespective of its condition (she can be aware of true or false information, but also aware of doubts or other types of informa-

³ Available at <https://en.oxforddictionaries.com/definition/realize> (accessed 5th July 2017).

tion). From a formal point of view, awareness is introduced in the formalism of Epistemic Logic as a syntactic operator. This means that it does not consider the truth value of the propositions it contains, and indeed the truth value is irrelevant to the operator. We say that agent i is aware of p at a given world $(A_i p)$, if and only if p belongs to the awareness set of the agent in this world. This awareness set is an arbitrary set of propositions we define for each agent.

The implicit knowledge operator works as in standard Epistemic Logic, that is, agent i implicitly knows p in world w ($K_i p$) if and only if in every other world w' the agent has access to, p is true. This means that implicit knowledge relies on the accessibility relation that we impose on each agent in the semantic structure (possible worlds). The explicit knowledge operator is defined as follows: agent i explicitly knows p ($K_{i^{\text{ex}}} p$) if and only if both agent i knows implicitly p and agent i is aware of p .

In a more theoretical sense, we could reformulate this conception in the following terms:

(Def. 1) Explicit knowledge: agent i actually (explicitly) knows proposition p , if and only if

- (i) Agent i could ideally come to know (*implicitly* knows) p ; and
- (ii) Agent i is aware of p .

In clause (i) we refer to the implicit knowledge, which includes every possible formula the agent could infer from her knowledge, since it is closed under logical consequence, and hence represents an ideal knowledge that no human being, nor any computing machine, could possess.

With clause (ii) we specify that for the knowledge to become explicit, it needs to be part of the agent's awareness.

The sense of awareness we explained above corresponds to what (Dretske, 1995) called “awareness of things”. In his paper, he pointed out an important distinction in the use of the term “awareness”: he distinguished “awareness of things” (awareness of X) from “awareness of facts” (awareness that X). In general, we could sum up the basic intuition behind this distinction (leaving out the details that underlie his studies on perception) with the following quote from the mentioned paper:

(Ref. 1) “Things are neither true nor false though, in the case of events, states of affairs, and conditions, we sometimes speak of them as what makes a statement true. Facts are what we express in making true statements about things. We describe our awareness of facts by using a factive complement, a that-clause, after the verb; we describe our awareness of things by using a (concrete) noun or noun phrase as direct object of the verb”. (Dretske, 1995, 264.)

Therefore, we could say that awareness of facts stands for the awareness of those thoughts (or believed propositions) we can form, while awareness of things would be somehow the very fact of perceiving and forming our mental content. In this sense, awareness of facts is more complex than awareness of things and represents a smaller range of all the things that could belong to our awareness. We could re-express the distinction by saying that every conceivable information is suitable of being part of our awareness of things; while only that information from which we can form a proposition and hence, to which we can add a truth value, will belong to our awareness of facts.

This being the case, the awareness involved in awareness logic corresponds by analogy to Dretske’s “awareness of things”, since, as explained above, every information is eligible for the agent’s awareness. Although in awareness logic the awareness set is formed by propositions, they are considered syntactically (as sentences) and thus their truth value plays no role; henceforth, they could be interpreted as “things”. On the other hand, awareness of facts could be equivalent, in this version of awareness logic, to explicit knowledge, since it would only include the true propositions the agent knows; and explicit knowledge does precisely this, selecting those propositions from the awareness set, that are also part of the implicit knowledge set of the agent.

This distinction is crucial for what follows. In the next sections, we will consider a definition of Epistemic Internalism that alludes to the agent’s awareness without specifying which awareness it refers to. In order for it to work as a theoretical correlate for awareness logic, we will need to clarify that we are dealing with awareness of things.

3. Epistemology and Logical Considerations

What Epistemology is and how it has developed in the last decades is a topic that goes far beyond the limits of this paper. Furthermore, as we announced at the beginning, we do not pretend to enter the heated debates in Epistemology; what we want is to select some definitions or views that are suitable of working as theoretical correlates to Awareness Logic.

In general, we could say that there are two main traditions in Epistemology: *Internalism* and *Externalism*. Both search for the foundations of justification in knowledge. While internalism finds it “inside the agent”, that is, in the reasoning; externalism advocates for an “outside justification”, meaning that it has to be related to the social and physical context of the agent. Arguments and counterarguments from both views caused rivers of ink to flow and nowadays it is still an open question. Although the traditions are opposed, there are also other proposals (mixed approaches), that argue in favour of the compatibility of both, by limiting their scopes and redefining the main concepts.

3.1 Epistemic Internalism and Awareness Requirement

At this initial stage of this research, we will only consider Epistemic Internalism, one concrete proposal to be more specific. In order for it to capture the basic intuitions that underlie Awareness Logic, we will re-write some concepts and propose a new definition that accommodates itself to the logical framework.

Nowadays, most versions of Epistemic Internalism include the agent’s awareness in some sense or another. We will consider here the proposal that B.J.C. Madison presents in (Madison, 2014).⁴ There, he establishes that Epistemic Internalism needs an “Awareness Requirement”, which, in turn, he takes from M. Bergmann in (Bergmann, 2006). Madison says the following:

(Ref.2) “A traditional way in which the epistemically internal is understood maintains that it is a necessary condition of being justified in holding a particular belief that the sub-

⁴ A first version of this can be found in (Madison, 2009).

ject is consciously aware of some reason to think that the belief is true (or that the subject could easily become aware of the reason, upon reflection). However, it is not enough that subjects are merely *aware of* the existence of their grounds; they must *appreciate* the existence and relevance of the grounds to what is believed". (Madison, 2014, 62. Original italics.)

Madison's argument continues with the following condition for epistemic internalism:

(Ref. 3) "AWARENESS: S is justified in believing that p only if:

- (i) There is something, X , that contributes to the justification of belief B ; and
- (ii) For all X that contributes, S is aware (or potentially aware) *that X contributes to the justification of belief B*". (Madison, 2014, 62. Original italics.)

We could probably have chosen any other proposal of Epistemic Internalism, but we decided to focus on this specific one, because it clarifies the important role of the Awareness Requirement. Since our main goal is to adapt the concept of Epistemic Internalism to Awareness Logic, we considered that this view would make an appropriate starting point.

Nevertheless, for our logical purposes, this formulation lacks internal coherence in the use of some terms such as " p " at the beginning, "something, X " in (i) and "believe B " in (i) and (ii). Therefore, in order to establish a common terminology that suits the logical framework, we will re-define Madison's proposal as follows:

(Def. 2) Epistemic internalism: agent a is justified in believing proposition p if and only if:

- (i) There is some proposition q , that contributes to the justification of proposition p ; and
- (ii) For all q that contributes, agent a is aware (or potentially aware) that q contributes to the justification of p .

3.2 Logical Considerations on Epistemic Internalism

We take (Def. 2) as the starting point of the argument that will lead us to a logical version of Epistemic Internalism. This argument is based on three questions we throw on the aforementioned definition of Epistemic Internalism:

- (Q1) Which awareness is (Def. 2) referring to?
- (Q2) Which knowledge is (Def. 2) defining?
- (Q3) What does the notion of “contribution to the justification” in (Def.2) mean?

For the answer of (Q1) we need to recall the two senses of awareness we are considering, namely “awareness of things” and “awareness of facts”. Above, we agreed that the version of Awareness Logic we are taking here into account, employs awareness of things. When it comes to defining how an agent knows what she knows, it seems only fair, to settle that the awareness involved is also awareness of things, since this sense of awareness includes every possible information independent of its truth or falsehood. Furthermore, as explained above, awareness of facts would be equivalent to explicit knowledge in the framework of Awareness Logic, and defining how an agent knows alluding to her knowledge would be redundant. Hence, we can answer the first question with:

- (A1) Epistemic internalism refers to awareness of things.

For an appropriate response to (Q2) we need to remember what implicit and explicit knowledge is. Broadly speaking, implicit knowledge represents the ideal knowledge of agents that are logically omniscient; and explicit knowledge is the real knowledge we employ in our daily reasoning. Epistemology studies the knowledge of the subjects, hence, of human beings (in our case we also include computing machines). Therefore, what Epistemology is defining has to be explicit knowledge. Implicit knowledge could be seen here as a logical construct, which results very useful in many computational applications, but has no direct boundaries with the human reality. The important feature of implicit knowledge is that it is formed by our ideal knowledge, thus it shows us in theory what we ideally could know. But for epistemological consider-

ations, implicit knowledge has no significance. Thus, we can establish the second answer:

(A2) Epistemic internalism defines how the subjects explicitly know.

The answer to (Q3) is slightly more challenging. We want to establish what “contributes to the justification of proposition p ” refers to. Intuitively it does not seem so complicated, but when it comes to expressing it with logical concepts, the “contribution to the justification of p ” has no concrete transcription. Hence, we need to unravel what this concept entails and specify our framework and initial assumptions. As mentioned before, we assume the system of Awareness Logic by Fagin and Halpern in (Fagin and Halpern, 1988), specifically the “Logic of General Awareness”, where no restriction is imposed on the awareness function.

The notion of “justification of p ” by itself could be re-interpreted in logic⁵ as a relation of logical consequence, where p follows from a set of propositions. Thus, the contribution to the justification of p would be a contribution to the set of propositions from which p results. From this point of view, “contribution” might be considered as the membership of some proposition q to the mentioned set. This being so, we can answer (Q3) with:

(A3) In the description of Epistemic Internalism we refer to the relation of logical consequence between a set of propositions and a concrete proposition, p .

After considering these questions and answering them, we have settled now a common ground for presenting our version of Epistemic Internalism from the perspective of Awareness Logic. What we are looking forward to achieving next, is to re-interpret Madison’s definition of Epistemic Internalism, with the help of the concepts of explicit and implicit knowledge and awareness of things, that we take from Awareness Logic.

⁵ There is a Justification Logic by Sergei Artemov, which we leave out in this research, but will be considered in future works.

3. Epistemic Internalism from the Perspective of Awareness Logic

We will now express Epistemic Internalism in the terms of Awareness Logic and name the set of propositions, from which the proposition p will be a logical consequence, the *knowledge basis*.

Madison refers to justified beliefs in his definition of Epistemic Internalism, but we will focus on knowledge, in order to adapt it to the framework of Awareness Logic (following here Plato's old thesis of knowledge as justified true belief). As such, instead of referring to beliefs that justify other beliefs, we will speak of a knowledge basis that sustains our explicit knowledge. Furthermore we will allude to the agent's awareness of things and its relation to explicit knowledge.

Henceforth, the knowledge basis (KB hereafter) is formed by propositions the agent could implicitly come to know, that is, KB is a subset of the implicit knowledge set of the agent.⁶ Thus, KB constitutes a collection of propositions from which p will be its logical consequence. The way in which the agent is aware of all propositions in the KB, or only of some propositions, will determine the strength of our version of Epistemic Internalism.

In re-defining (Def. 2), and, again, in order for the terminology to suit the logical framework, we will remove clause (i) and turn it into a preliminary definition. Regarding clause (ii) we will re-interpret it in two different senses: (a) universal consideration, and (b) existential consideration. In (a) we will make the agent aware of the whole KB, whereas in (b) the agent will only need to be aware of some relevant proposition included in the KB. This being so, we will have defined two interpretations⁷ of Epistemic Internalism, that differ in the amount of implicit knowledge that is shared with the agent's awareness. We could say that the universal version is the weak one, and the existential version, the strong one.⁸ Now, we can

⁶ We defined "implicit knowledge" as a logical construct that entails all the possible logical consequences of the knowledge of the agent, such that the explicit (real) knowledge is selected from it. Therefore the set of propositions that supports the explicit knowledge of the agent must be a subset of her implicit knowledge set.

⁷ According to the system of Awareness Logic.

⁸ Inspired by the classification in (Pappas, 2014).

present our version of Epistemic Internalism from the perspective of Awareness Logic:

(Def. 3) Awareness logic epistemic internalism: Let p be any proposition and let KB be the knowledge basis, such that p is a logical consequence of KB ($KB \models p$). Then, agent a knows explicitly p ,

- (a) if and only if, for all q that belongs to KB , agent a is aware of q (Universal); or
- (b) if and only if, for some q that belongs to KB , agent a is aware of q (Existential).

If we compare (Def.3) with (Def.1) of explicit knowledge, we will find some interesting parallels. In (Def.1) we expressed in a theoretical sense how explicit knowledge is defined in the system of Awareness Logic. We said that for knowledge to be explicit, it needs to be implicit knowledge and also be part of the agent's awareness. (Def.3) has been constructed with (Def.1) in mind, but is inspired in Madison's definition of Epistemic Internalism. What we wanted to capture is the background intuition, present in Madison's proposal, that for an information to become knowledge, there is a relevant set from which this knowledge results, and the agent needs to be aware of this set (or, at least, of part of it). Expressing this in the terms of Awareness Logic has brought us to (Def.3), where we establish that there is a knowledge basis from which the knowledge of p results, and that the agent needs to be aware of this basis.

One interesting fact is that when it comes to express these notions in a logical sense, the awareness we allude to is always awareness of facts (subset of awareness of things), since it is concerned with true propositions. This being so, what we have achieved with our (Def.3) is a restriction of the explicit knowledge. In (Def. 1) we established the sense of explicit knowledge that corresponds to Awareness Logic, but with (Def. 3) we selected a part of these explicitly known propositions and created a new subset, formed only by those propositions the agent explicitly knows *and* is able to justify, meaning here, that she is able to deduce them from other propositions.

3. Conclusions and Future Work

With (Def. 3) we have achieved the first small step of our main goal, namely, widen the theoretical foundations of Awareness Logic, while re-defining some central epistemological concepts that work as correlates to the main topics in epistemic logic.

The next natural step would be, first, to analyse other versions of internalism, secondly, to take a look at Epistemic Externalism, and finally, to consider the views that argue in favour of the compatibility of both Internalism and Externalism (such as Epistemic Disjunctivism). We think that these views might be of great interest to Awareness Logic, and Epistemic Logic in general.

Parallel to that, we believe that the recent developments in Epistemic Logic, that include the dynamics of information, will need to be considered in detail. The concept of epistemic actions in these systems and the notion of justification (or “evidence”) are also suitable for theoretical considerations.

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