1、

咖啡店打算要販售拿鐵、卡布奇諾、美式咖啡、奶茶四款飲料
拿鐵需要使用250毫升牛奶和16克咖啡豆A
卡布奇諾需要150毫升牛奶和14克咖啡豆A
美式咖啡需要16克的咖啡豆B
奶茶需要250毫升牛奶、1個茶包和15公克的糖

已知每天材料準備了12公升的牛奶，800克的咖啡豆A，400克的咖啡豆B、10個茶包和200公克的糖
拿鐵一杯可獲利80元，卡布奇諾一杯獲利70元，美式咖啡60元和奶茶一杯50元(不算其他成本)
假設不論如何配置飲料皆能全部售出，請問每天應配置四款飲料個多少杯能使獲利最大？
假設拿鐵a杯、卡布b杯、美式c杯和奶茶d杯，獲利e=80a+70b+60c+50d

限制條件：
250a+150b+250d<=12000
16a+16b<=800
16c<=400
15d<=200
d<=10
a, b, c, d>=1

4.

(I) Show that $A' = A + uv^T$, u and v are two n vectors.

Since A' and A are the same except the last row, the addition of uv^T (outer product of u, v^T) can be seen as performing a rank-one update

For $n=3$, $u = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $uv^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} [x\ y\ z] = \begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & cy & cz \end{bmatrix}$. If $a, b = 0$, $c \neq 0$,

$\Rightarrow \begin{bmatrix} A' \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} + uv^T = \begin{bmatrix} A \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ cx & cy & cz \end{bmatrix}$ for some c and x, y, z.

(II) Show that $A'^{-1} = (A + uv^T)^{-1} = A^{-1} - \dfrac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$

($\Rightarrow$)

First suppose that if $1 + v^TA^{-1}u \neq 0$, $A + uv^T$ is invertible given as above. Let

$A + uv^T = X$, $A^{-1} - \dfrac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u} = Y$, show that $XY = YX = I$.

$XY = (A + uv^T)\left(A^{-1} - \dfrac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}\right) = AA^{-1} + uv^TA^{-1} - \left[(AA^{-1}uv^TA^{-1} + uv^TA^{-1}uv^TA^{-1})/(1 + v^TA^{-1}u)\right]$

$= I + uv^TA^{-1} - \left[(uv^TA^{-1} + uv^TA^{-1}uv^TA^{-1})/(1 + v^TA^{-1}u)\right] = I + uv^TA^{-1} - \dfrac{u(1 + v^TA^{-1}u)v^TA^{-1}}{1 + v^TA^{-1}u}$

$= I + uv^TA^{-1} - uv^TA^{-1} = I$. Similarly, $YX = I$ can be shown. So if $1 + v^TA^{-1}u \neq 0$,

$(A + uv^T)^{-1} = A^{-1} - \dfrac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$ #

(⇒) Suppose $A+uv^T$ is invertible and $1+v^TA^{-1}u=0$. Then we have the following

$(A+uv^T)A^{-1}u = AA^{-1}u + uv^TA^{-1}u = u + u(v^TA^{-1}u) = u(1+v^TA^{-1}u) = 0$. Since $A+uv^T$ is invertible, $A^{-1}u=0$. Then since A^{-1} is also invertible, we get $u=0$. This is a contradiction ($1+v^TA^{-1}u=0$). So when $A+uv^T$ is invertible, $1+v^TA^{-1}u\neq 0$.

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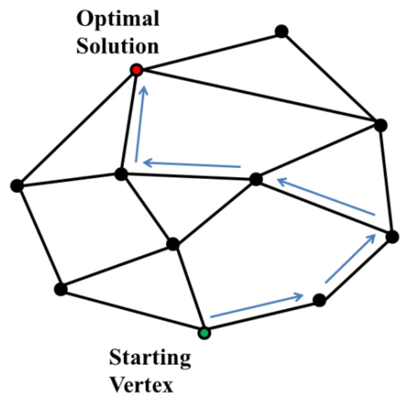
5. 單形法 (Simplex algorithm)

1. 找出可行解中其中一端點，計算目標函數值
2. 檢查相鄰端點的目標函數值，選最小的並移動到此點.
3. 重複2.直到不存在更好的相鄰端點，此即為最佳解.



代數運算上：

e.g. $z_{max.} = 2x+y$, $\begin{cases} 4x+3y \le 12 \\ 2x+y \le 4 \\ x+2y \le 4 \\ x_1, x_2 \ge 0 \end{cases}$ 改寫成矩陣 $\longrightarrow$ $\begin{cases} 4x+3y+u=12 \\ 2x+y+v=4 \quad \text{(加入惰變數)} \\ x+2y+w=4 \\ -2x-y+z=0 \end{cases}$

⇒

x	y	u	v	w	z	
4	3	1	0	0	0	12
2	1	0	1	0	0	4
1	2	0	0	1	0	4
-2	-1	0	0	0	1	0

⇒ 通過行列運算將此行變為正數，此時的 (x, y, z) 為最佳解.

Source:
https://www.youtube.com/watch?v=NXseqFK2BEo
https://ithelp.ithome.com.tw/articles/10214898
https://ccjou.wordpress.com/2013/09/27/%E7%B7%9A%E6%80%A7%E8%A6%8F%E5%8A%83-%E5%9B%9B%EF%BC%9A%E5%96%AE%E5%BD%A2%E6%B3%95/