APPM4605-Homework2

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1. (a) Show $(1+x)^n = 1 + nx + o(x)$ as $x \to 0$

   $\lim_{x\to 0} \frac{(1+x)^n - 1 - nx}{x} = \frac{1-1-0}{0} = \frac{0}{0}$

   Using L'Hopitals Rule $\lim_{x\to 0} \frac{n(1+x)^{n-1} - n}{1} = \frac{n-n}{1} = 0$

   (b) Show that $xsin(\sqrt{x}) = O(x^{3/2})$ as $x \to 0$

   $\lim_{x\to 0} \frac{xsin(\sqrt{x})}{x^{3/2}} = \frac{0}{0}$

   Using L'Hoptials Rule $\lim_{x\to 0} \frac{\frac{cos(\sqrt{(x)}}{2\sqrt{(x)}}}{\frac{1}{2\sqrt{(x)}}} = \lim_{x\to 0} cos(\sqrt{(x)}) = 1$ which is a constant

   (c) Show that $e^{-t} = o(\frac{1}{t^2})$ as $t \to \infty$

   $\lim_{x\to\infty} \frac{e^{-t}}{\frac{1}{t^2}} = \lim_{x\to\infty} \frac{t^2}{e^t} = \frac{\infty}{\infty}$ Using L'Hopital's Rule $\lim_{x\to\infty} \frac{2t}{e^t} = \frac{\infty}{\infty}$

   Using L'Hopital's Rule $\lim_{x\to\infty} \frac{2}{e^t} = \frac{2}{\infty} = 0$

   (d) Show that $\int_0^\epsilon e^{-x^2} = O(\epsilon)$ as $\epsilon \to 0$

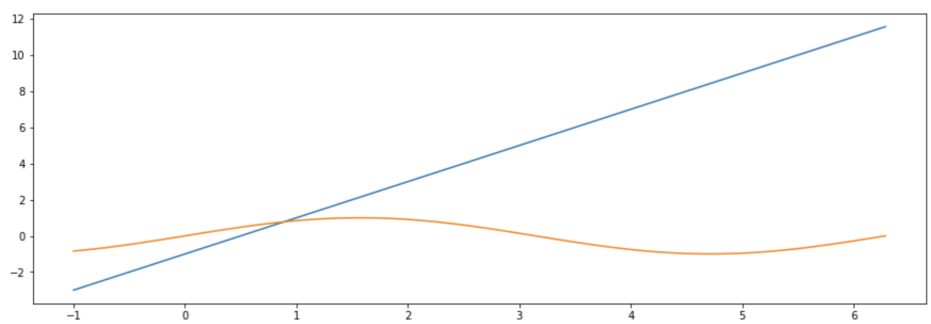
   $\lim_{\epsilon\to 0} \frac{\int_0^\epsilon e^{-x^2}}{\epsilon} = \frac{0}{0}$ Using L'Hopital's Rule $\lim_{\epsilon\to 0} \frac{e^{-x^2}}{1}$ which is a constant.

2.
```
def bisect(f,a,b,tol):
    iterations=0
    if f(a)*f(b)>=0:
        return [None,1];
    if abs(f(a))==0:
        return [a, 0]
    elif abs(f(b))==0:
        return [b, 0]
    fa=f(a)
    for i in range(1000000):
        c=(b+a)/2
        fc=f(c)
```

```
        if abs(fc)==0 or (b-a)/2<tol:
            print(iterations)
            return [c, 0]
        if (fc) * (fa)>0:
            a=c
            fa=fc
        else:
            b=c
        iterations=iterations+1
    if (i==1000000):
        return [None,1]
```

3. $2x - 1 = sin(x)$

   (a) Let $f(x) = 2x - 1$ and $g(x) = sin(x)$ and $F(x) = f(x) - g(x) = 2x - 1 - sin(x)$ to find the zero of the above equation.
   Graph of $f(x)$ and $g(x)$



   Using the above graph, we can guess an interval of $[0, \pi]$
   $F(0) = 2(0) - 1 - sin(0) = -1 < 0$
   $F(\pi) = 2(\pi) - 1 - sin(\pi) = 2\pi - 1 > 0$ $F(x)$ is continuous on $[0, \pi]$.
   Therefore, by the IVT, a root (r) exists on the closed interval $[0, \pi]$.

   (b) Say there are two roots to F(x) such that $f(a) = f(b) = 0$. Since F(x) is continuous and differential, we can apply Rolle's Thm.
   $f^{'}(c) = 0$ between a and b
   $F^{'}(x) = 2 - cos(x) = 0$ This value does not exist, so there cannot be more than one real root. Part a) proves there is at least one, so there is only one real root of the function.

   (c)
```
f = lambda x: (2*x-1-np.sin(x))
print(bisect(f,0,math.pi,1e-16))

# of iterations: 54
[0.887862211570866, 0]
```

4. $(x-5)^9$
   (a) Bisect function gives 5.000073242187501 as the root.
   (b) Bisect function gives 5.12875.
   (c) This is because the expanded version gives more error with more calculations, and represents the actual root poorly.

5. (a) Using Thm 2.1, $10^{-3} \leq \frac{3}{2^n}$
   $2^n \leq 3 * 10^3$, $log_2(2^n) \leq log_2 3000$, $n \leq 11.55074$
   Since n is a integer, $n \leq 12$, so upper bound on the number of iterations is 12.
   (b) Running the sub problem from 2 gives 11 iterations, which is one less than what was found in part a).