APPM4605 Homework4

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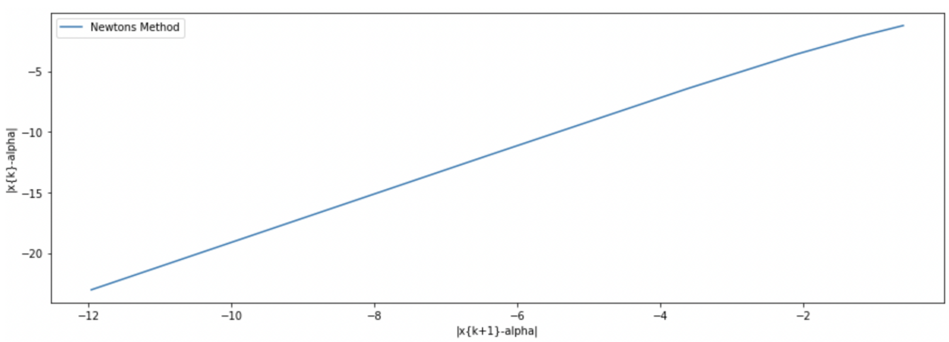
1. (a) Newton's Method

```
|-n-|---------error---------|
| 1 | 0.31937172774869116 |
| 2 | 0.24988928401224642 |
| 3 | 0.175768032129626 |
| 4 | 0.09343252333612329 |
| 5 | 0.02518515860280779 |
| 6 | 0.0016227458268762707 |
| 7 | 6.389843407950124e-06 |
| 8 | 9.870171346904044e-11 |
1.1347241384015194
```

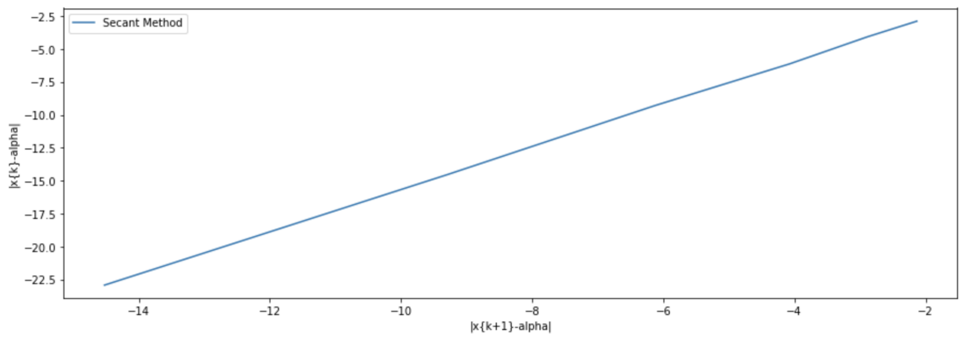
Secant Method

```
|-n-|---------error---------|
| 1 | 0.016129032258064502 |
| 2 | 0.17444873641857295 |
| 3 | 0.07292193773508582 |
| 4 | 0.014875719274581423 |
| 5 | 0.0022852577887197967 |
| 6 | 9.316205614795514e-05 |
| 7 | 4.92342510982624e-07 |
| 8 | 1.1030376612097825e-10 |
| 9 | 2.220446049250313e-16 |
```

Yes, the error of the secant method decreases slower than that of Newton's method.



(b)



The slope of the Newton's Method line is about 1.8664962708109956 and the slope of the Secant Method line is about 1.5417058924772813. This is the order of convergence (superlinear).

2. (a) The system converges to $(.5, 0.8660254037844386)$ after 56 iterations.

   (b) The numerical matrix is the inverse of the Jacobian matrix evaluated at (1,1).

   (c) After 20 iterations, Newton's Method converges to $[0.4999999972252189, 0.8660253989783768]$

   (d) The exact solution is $(1/2, \frac{\sqrt{3}}{2})$.
   $f(x, y) = 3(1/2)^2 - (\frac{\sqrt{3}}{2})^2 = 0.0$
   $g(x, y) = 3(1/2)(\frac{\sqrt{3}}{2})^2 - (1/2)^3 - 1 = 0.0$

3.
$$G = \begin{cases} x = \frac{1}{\sqrt(2)}\sqrt(1 + (x + y)^2) - \frac{2}{3} \\ y = \frac{1}{\sqrt(2)}\sqrt(1 + (x - y)^2) - \frac{2}{3} \end{cases}$$

Let $D = \{(x_1, x_2)^t | -.5 \leq x_i \leq .5, \text{ for each } i = 1, 2\}$
$g_1(x_1, x_2) = \frac{1}{\sqrt(2)}\sqrt(1 + (x_1 + x_2)^2) - \frac{2}{3}$
$g_2(x_1, x_2) = \frac{1}{\sqrt(2)}\sqrt(1 + (x_1 - x_2)^2) - \frac{2}{3}$

$|g_1(x_1, x_2)| = |\frac{1}{\sqrt(2)}\sqrt(1 + (x_1 + x_2)^2) - \frac{2}{3} = |\frac{1}{\sqrt(2)}\sqrt(1 + (1)^2) - \frac{2}{3}| \leq \frac{1}{3}$
$|g_2(x_1, x_2)| = |\frac{1}{\sqrt(2)}\sqrt(1 + (x_1 - x_2)^2) - \frac{2}{3}| = |\frac{1}{\sqrt(2)}\sqrt(1 + (1)^2) - \frac{2}{3}| \leq \frac{1}{3}$
$-\frac{1}{2} \leq g_i(x_1, x_2) \leq \frac{1}{2}$
Therefore, $G \in D$ whenever $x \in D$
$|\frac{\partial g_1}{\partial x_1}| = |\frac{\partial g_1}{\partial x_2}| = |\frac{x_1+x_2}{\sqrt(2)((x_1+x_2)^2+1)^{1/2}}| = |\frac{2}{\sqrt(1)((1)^2+1)^{1/2}}| \leq 0\frac{1}{2}$
$|\frac{\partial g_2}{\partial x_1}| = |\frac{x_1-x_2}{\sqrt(2)((x_1-x_2)^2+1)^{1/2}}| = |\frac{2}{\sqrt(1)((1)^2+1)^{1/2}}| \leq \frac{1}{2}$
$|\frac{\partial g_2}{\partial x_2}| = |\frac{x_2-x_1}{\sqrt(2)((x_1-x_2)^2+1)^{1/2}}| = |\frac{2}{\sqrt(1)((1)^2+1)^{1/2}}| \leq \frac{1}{2}$
Therefore, $|\frac{\partial g_i}{\partial x_j}| \leq \frac{1}{2}$ for $K = 1$ for $i = 1, 2$ and $j = 1, 2$
By Thm. 10.6, $D = \{(x_1, x_2)^t | -.5 \leq x_i \leq .5, \text{ for each } i = 1, 2\}$ is a region

where G is guaranteed to converge to a unique solution for any starting point $(x_0, y_0) \in D$.

4. (a) direction $= \pm \frac{\nabla f}{||\nabla f||}$

   magnitude $= \frac{f(x,y)}{\nabla f \cdot \pm \frac{\nabla f}{||\nabla f||}} = \pm \frac{f(x,y)||\nabla f||}{||\nabla f||^2}$

   (direction)(magnitude)$= -\frac{f(x,y)\nabla f}{f_x^2 + f_y^2}$

   This will always result in a negative since the sign of f(x,y) and ∇f will be different.

   $x_{n+1} = x_n + \frac{-f_x(f)}{f_x^2 + f_y^2}$

   $x_{n+1} = x_n + \frac{-f_y(f)}{f_x^2 + f_y^2}$

   (b)

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[error_x,error_y,error_z]
[0.10606060606060597, 0.4242424242424243, 0.4242424242424243]
[0.012134450190934398, 0.06250072975056575, 0.06250072975056575]
[0.0002836932607810372, 0.001412588736741105, 0.001412588736741105]
[1.452206574992232e-07, 7.225318838788297e-07, 7.225318838788297e-07]
[3.7969627442180354e-14, 1.8918200339612667e-13, 1.8918200339612667e-13]
[0.0, 2.220446049250313e-16, 2.220446049250313e-16]

]: [1.093642317388195, 1.3603283832230446, 1.3603283832230446]
```