# APPM4605 Homework1

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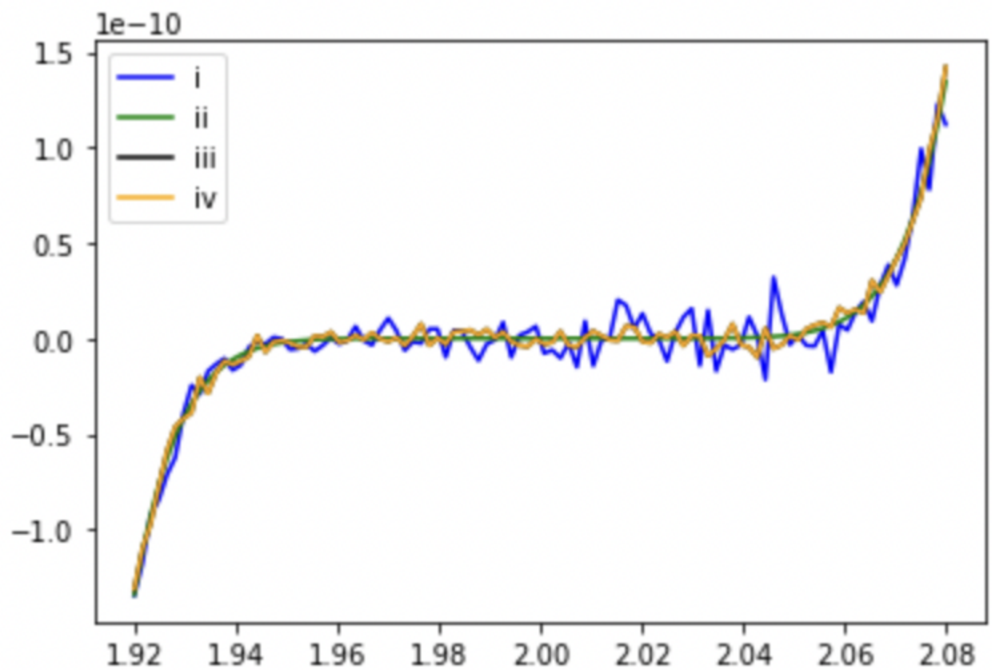
1. $p(x) = (x-2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4680x^2 + 2304x - 512$




   (a) (i) produces the most erratic plot, (iv) and (iii) are similar in that they are still somewhat erratic but less so than (i), and (ii) produces a smooth plot. I think (ii) is the most correct algorithm, since it will have the smallest absolute error as it does the fewest calculations.

2. (a) Evaluate $f(x) = (\sqrt{x+1}) - 1$ for x=0
   $f(x) = (\sqrt{x+1}) - 1 = (\sqrt{x+1} - 1) * \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \frac{x+1-1}{\sqrt{x+1}+1} = \frac{x}{\sqrt{x+1}+1}$

   (b) Evaluate $f(x) = sin(2(x+a)) - sin(2a)$ for x=0

Use trig identity $sin(a) - sin(b) = 2cos\frac{a+b}{2}sin\frac{a-b}{2}$
Let $a = 2(x + a)$, $b = 2a$
Rewrite as $2cos\frac{(2x+2a+2a)}{2}sin\frac{(2x+2a-2a)}{2} = 2cos(x + 2a)sinx$

3. Evaluate the quadratic equation $x^2 - 56x + 1$

   (a) $\frac{56\pm\sqrt{56^2-4(1)(1)}}{2(1)} = \frac{56\pm\sqrt{3132}}{2} = \frac{56\pm2\sqrt{783}}{2} = 28 \pm 3\sqrt{87}$
   $\frac{56\pm\sqrt{56^2-4(1)(1)}}{2(1)} = \frac{56\pm\sqrt{3120}}{2} = \frac{56\pm55.8}{2} = 28 \pm 27.9$
   $r_1 \approx 55.9, r_2 \approx .1$
   $\text{RError}_1 = \frac{|28+3\sqrt{87}-(55.9)|}{|28+3\sqrt{87})|} \approx .001467203$
   $\text{RError}_2 = \frac{|28-3\sqrt{87}-(.1)|}{|28-3\sqrt{87}|} \approx 4.5982137$

   (b) $(x - r_1)(x - r_2) = 0$
   $(x - r_1)(x - r_2) = x^2 - r_2x - r_1x + r_1r2 = x^2 - (r_2 + r_1)x + r_1r2 = ax^2 + bx + c = x^2 + \frac{b}{a}x + \frac{c}{a}$
   $(r_2 + r_1) = \frac{-b}{a}, r_1r_2 = \frac{c}{a}$
   The equation $r_1r_2 = \frac{c}{a}$ can be used to find a better approximation of the "bad" root since multiplication does not have loss of precision.

4. $f(x) = e^x - 1$

   (a) This algorithm is conditionally stable. For numbers close to zero, the function experiences loss of precision and is unstable (subtracting 2 similar numbers of same order magnitude). Otherwise, the function is stable.

   (b) The algorithm produces 8 correct digits, which is expected since the algorithm is unstable for numbers near zero.

   (c) $f(x) = e^x - 1 \approx x + \frac{x^2}{2} + \frac{x^3}{6}$
   Using the Remainder Estimation Thm, $|R_n(x)| \leq |\frac{f^{n+1}}{(n+1)!} * x^{n+1}|$ on the interval $[-10^{-9}, 10^{-9}]$
   $|f^{3+1}(t)|$ for $t \in [-10^{-9}, 10^{-9}]$ will be maxed at $10^{-9}$
   $|e^{10^{-9}}| \approx 1$
   $|R_3(x)| \leq |\frac{1}{(4)!} * (10^{-9})^4| = \frac{10^{-36}}{24}$
   Relative Error $= \frac{\frac{10^{-36}}{24}}{10^{-9}} = \frac{10^{-27}}{24} < 10^{-16}$
   Therefore, by Taylor's Remainder Thm, the polynomial is accurate on the interval $[-10^{-9}, 10^{-9}]$ up to 16 digits

   (d) $x + \frac{x^2}{2} + \frac{x^3}{6}$
   where $x = 9.99999999500000 * 10^{-10}$ gives 16 digits of precision