APPM4650 Homework11

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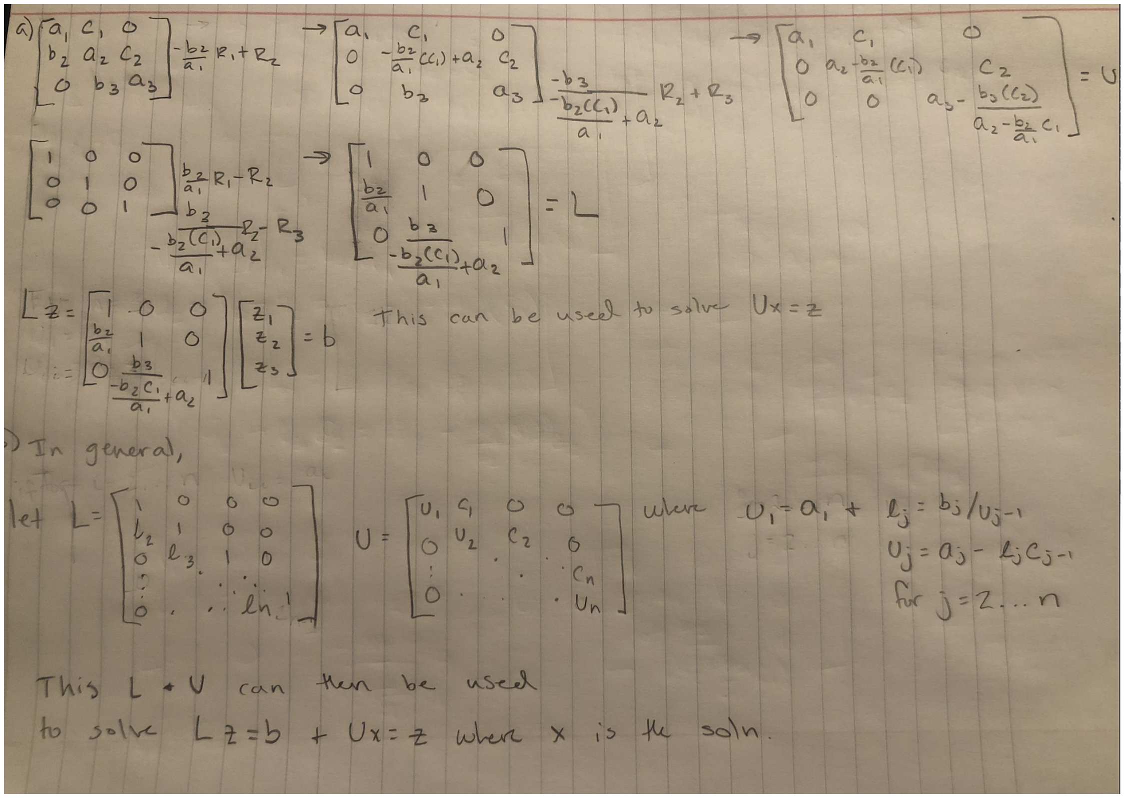
1. $2x_1 - 6\alpha x_2 = 3$
   $3\alpha x_1 - x_2 = \frac{3}{2}$
   $\begin{pmatrix} 2 & -6\alpha \\ 3\alpha & -1 \end{pmatrix}$
   The alphas that would make this equation sinuglar are given by the equation $-2 + 18\alpha^2 = 0$
   Solving for alpha gives $\alpha = \pm\frac{1}{3}$

   (a) When $\alpha = -\frac{1}{3}$ the two equations would be
   $2x_1 + 2x_2 = 3$ and $-x_1 - x_2 = \frac{3}{2}$
   Multiplying the 2nd equation by 2 and adding it to the 1st would give $0 + 0 = 6$ which is never true. Therefore, there would be no solutions.

   (b) When $\alpha = \frac{1}{3}$, both equations would be equal resulting in infinite solutions.

   (c) $\begin{pmatrix} 2 & -6a & 3 \\ 3a & -1 & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 3a & -1 & \frac{3}{2} \\ 0 & \frac{-18a^2+2}{3a} & \frac{3a-1}{a} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{3}{2(3a+1)} \\ 0 & 1 & -\frac{3}{2(3a+1)} \end{pmatrix}$
   $x_1 = \frac{3}{2(3a+1)}$
   $x_2 = -\frac{3}{2(3a+1)}$

2. (a) 

a) $\begin{bmatrix} a_1 & c_1 & 0 \\ b_2 & a_2 & c_2 \\ 0 & b_3 & a_3 \end{bmatrix} -\frac{b_2}{a_1}R_1 + R_2 \rightarrow \begin{bmatrix} a_1 & c_1 & 0 \\ 0 & -\frac{b_2}{a_1}(c_1) + a_2 & c_2 \\ 0 & b_3 & a_3 \end{bmatrix} \frac{-b_3}{-\frac{b_2(c_1)}{a_1} + a_2} R_2 + R_3 \rightarrow \begin{bmatrix} a_1 & c_1 & 0 \\ 0 & a_2 - \frac{b_2}{a_1}(c_1) & c_2 \\ 0 & 0 & a_3 - \frac{b_3(c_2)}{a_2 - \frac{b_2}{a_1}c_1} \end{bmatrix} = U$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \frac{b_2}{a_1}R_1 - R_2 \\ \frac{b_3}{-\frac{b_2(c_1)}{a_1} + a_2} R_2 - R_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{b_2}{a_1} & 1 & 0 \\ 0 & \frac{b_3}{-\frac{b_2(c_1)}{a_1} + a_2} & 1 \end{bmatrix} = L$

$Lz = \begin{bmatrix} 1 & 0 & 0 \\ \frac{b_2}{a_1} & 1 & 0 \\ 0 & \frac{b_3}{-\frac{b_2 c_1}{a_1} + a_2} & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = b$ This can be used to solve $Ux = z$

b) In general,

let $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_2 & 1 & 0 & 0 \\ 0 & l_3 & 1 & 0 \\ \vdots & & \ddots & \ddots \\ 0 & \cdots & l_n & 1 \end{bmatrix}$ $U = \begin{bmatrix} u_1 & c_1 & 0 & 0 \\ 0 & u_2 & c_2 & 0 \\ \vdots & & \ddots & c_n \\ 0 & \cdots & & u_n \end{bmatrix}$ where $u_1 = a_1$ + $l_j = b_j / u_{j-1}$
$u_j = a_j - l_j c_{j-1}$
for $j = 2 \ldots n$

This L + U can then be used
to solve $Lz = b$ + $Ux = z$ where x is the soln.

(b) This way is less expensive than Gaussian Elimination without taking the zeros into account. The Gaussian Elimination has a cost of $O(n^3)$ but the cost of this method is linear.

3. $6x + 2y + 2z = -2$
$2x + 2/3y + 1/3y = 1$
$x + 2y - z = 0$

(a) $(x, y, z) = (2.6, -3.8, -5)$
$6(2.6) + 2(-3.8) + 2(-5) = -2$
$2(2.6) + 2/3(-3.8) + 1/3(-5) = 1$
$(2.6) + 2(-3.8) - (-5) = 0$

(b) $\begin{pmatrix} 6 & 2 & 2 & -2 \\ 2 & .6667 & .3333 & 1 \\ 1 & 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & .3333 & .3333 & -.3333 \\ 2 & .6667 & .3333 & 1 \\ 1 & 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & .3333 & .3333 & -.3333 \\ 0 & .0001 & -.3333 & 1.667 \\ 0 & 1.667 & -1.333 & .3333 \end{pmatrix} =$
$\begin{pmatrix} 1 & .3333 & .3333 & -.3333 \\ 0 & .0001 & -.3333 & 1.667 \\ 0 & 0 & 5555 & -27790 \end{pmatrix}$
Solving with back substitution
$5555z = -27790$, $z = -5.003$

$.0001y - .3333z = 1.667$, $.0001y + 1.667 = 1.667$, $y = 0$
$x + .3333y + .3333z = -.3333$, $x - 1.667 = -.3333$, $x = x = 1.334$

(c) $\begin{pmatrix} 6 & 2 & 2 & -2 \\ 2 & .6667 & .3333 & 1 \\ 1 & 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 2 & -2 \\ 0 & 0.0001 & -.3333 & 1.667 \\ 0 & 10 & -8 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 2 & -2 \\ 0 & 10 & -8 & 2 \\ 0 & 0.0001 & -.3333 & 1.667 \end{pmatrix} =$
$\begin{pmatrix} 6 & 2 & 2 & -2 \\ 0 & 10 & -8 & 2 \\ 0 & 0 & -33320 & 166700 \end{pmatrix}$
Solving with back substitution
$-33320z = 166700$, $z = -5.003$
$10y - 8z = 2$, $10y + 40.02 = 2$, $y = -3.802$
$6x + 2y + 2z = -2$, $6x - 7.604 - 10.01 = -2$, $x = 2.602$

(d) The parital pivoting was more accurate to the correct answer.