APPM4650 Homework12

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November 29, 2021

1. (a) Since H is Hermitian, let $Hu = \lambda u$.
$\lambda < u, u >=< \lambda u, u >=< Hu, u >=< u, H^* u >=< u, Hu >=< u, \lambda u >= \bar{\lambda} < u, u >$. Here, $\dot{\lambda} = \lambda$ so λ must be a real number.
Since H is unitary, let λ be the eigenvalue of H with the corresponding eigenvector $v \neq 0$, then
$< v, v >=< HH^* v, v >=< Hv, Hv >=< \lambda v, \lambda v >= \lambda\bar{\lambda} < v, v >$
Therefore, $(1 - \lambda\bar{\lambda}) < v, v >= 0$, since $< v, v >\neq 0$ $\lambda\bar{\lambda} = 1, |\lambda|^2 = 1, \lambda = \pm 1$

(b) From c), H has only one eigen value of -1 and $(n-1)$ eigen values for 1. The trace is the sum of the eigen values, $1(n-1) - 1 = (n-2)$. The two eigen values are $1, -1$.

(c) $Hw = (I - 2ww^*)w = w - 2ww^*w = w - 2w = -w$
If $v^*w = 0$, then $Hv = (I - 2ww^*)v = v - 2ww^*v = v - 0 = v$

(d) Since $Hw = -w$, H has one -1 eigen value. H has $n-1$ 1 eigen values since H has $n-1$ linearly independent eigen vectors orthogonal to w $(Hv = v)$.

(e) The determinant of H is the product of it's eigen values, $H = (1)^{n-1}(-1) = -1$

(f) The matrix $H = I - 2ww^*$
$det(H) = det(I - 2ww^*) = det(I) - det(2ww^*)$
Using $det(AB) = det(A)det(B) = det(B)det(A) = det(BA)$,
$det(I) - det(2ww^*) = det(I) - 2det(w)det(w^*) = det(I) - 2det(w^*w)$
Since $w^*w = 1$, $det(I) - 2det(w^*w) = det(I) - 2(1) = 1 - 2 = -1$

2. $\begin{bmatrix} 12 & 10 & 4 \\ 10 & 8 & -5 \\ 4 & -5 & 3 \end{bmatrix}$

$\alpha = -\sqrt{10^2 + 4^2} = -2\sqrt{29}$

$r = \sqrt{(\frac{1}{2}(-2\sqrt{29})^2 - \frac{1}{2}(10)(-2\sqrt{29})} = \sqrt{58 + 10\sqrt{29}}$

$w_1 = 0, w_2 = \frac{10+2\sqrt{29}}{2\sqrt{58+10\sqrt{29}}} = \frac{\sqrt{29}\sqrt{58+10\sqrt{29}}}{58}, w_3 = \frac{4}{2\sqrt{58+10\sqrt{29}}} = \frac{2}{\sqrt{58+10\sqrt{29}}}$

$w = (0, \frac{\sqrt{29}\sqrt{58+10\sqrt{29}}}{58}, \frac{2}{\sqrt{58+10\sqrt{29}}})^t$

$$P^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2\left(0, \frac{\sqrt{29}\sqrt{58+10\sqrt{29}}}{58}, \frac{2}{\sqrt{58+10\sqrt{29}}}\right)^t \cdot \left(0, \frac{\sqrt{29}\sqrt{58+10\sqrt{29}}}{58}, \frac{2}{\sqrt{58+10\sqrt{29}}}\right) =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{5}{\sqrt{29}} & -\frac{2}{\sqrt{29}} \\ 0 & -\frac{2}{\sqrt{29}} & \frac{5}{\sqrt{29}} \end{bmatrix}$$

$$A^{(2)} = P^{(1)} A^{(1)} P^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{5}{\sqrt{29}} & -\frac{2}{\sqrt{29}} \\ 0 & -\frac{2}{\sqrt{29}} & \frac{5}{\sqrt{29}} \end{bmatrix} \begin{bmatrix} 12 & 10 & 4 \\ 10 & 8 & -5 \\ 4 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{5}{\sqrt{29}} & -\frac{2}{\sqrt{29}} \\ 0 & -\frac{2}{\sqrt{29}} & \frac{5}{\sqrt{29}} \end{bmatrix} =$$

$$\begin{bmatrix} 12 & -2\sqrt{29} & 0 \\ -2\sqrt{29} & \frac{112}{29} & \frac{155}{29} \\ 0 & \frac{155}{29} & \frac{207}{29} \end{bmatrix}$$