APPM4650 Homework 9

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1. (a)

```
def trapezoidal(a,b,n,f):
    h = (b-a)/n
    x1=((f(a)+f(b))/2)
    for i in range(1, n):
        x=a+i*h
        x1+=f(x)
    x1*=h
    return x1

def simpsons(a,b,n,f):
    h=(b-a)/n
    xl0=f(a)+f(b)
    xl1=0
    xl2=0
    for i in range(1,n):
        x=a+i*h
        if (i%2==0):
            xl2+=f(x)
        else:
            xl1+=f(x)
    xl=h*(xl0+2*xl2+4*xl1)/3
    return xl
```

(b) Trapezoidal

error$= -\frac{5+5}{12}h^2 f''(\mu)$

$f''(x) = -\frac{2\left(-3x^2+1\right)}{(1+x^2)^3}$

Through graphing, we find that the max of $| -\frac{2\left(-3x^2+1\right)}{(1+x^2)^3}|$ to be 2.

$|error| = \frac{5+5}{12}h^2 | -\frac{2\left(-3x^2+1\right)}{(1+x^2)^3}| \leq 2(\frac{5}{6}h^2) < 10^{-4}$

$h^2 < 10^{-4}\frac{6}{10}$

$h \approx 0.00774$

$h = \frac{b-a}{n} = \frac{10}{n} \approx 0.00774$

$n \approx 1292$

Simpsons

error$= -\frac{b-a}{180}h^4 f^{(4)}(\mu)$

$f^{(4)}(x) = \frac{24\left(5x^4-10x^2+1\right)}{(x^2+1)^5}$

Through graphing, we find the max of $|\frac{24\left(5x^4-10x^2+1\right)}{(x^2+1)^5}|$ to be 24.

$|error| = |\frac{24\left(5x^4-10x^2+1\right)}{(x^2+1)^5}|\frac{10}{180}h^4 \leq \frac{4}{3}h^4 < 10^{-4}$

$h^3 < 10^{-4}\frac{3}{4}$

$h \approx 0.04217$

$h = \frac{b-a}{n} = \frac{10}{n} \approx 0.04217$

$n \approx 238$

In [18]:
```
def simpsons(a,b,n,f):
    h=(b-a)/n
    xl0=f(a)+f(b)
    xl1=0
    xl2=0
    for i in range(1,n):
        x=a+i*h
        if (i%2==0):
            xl2+=f(x)
        else:
            xl1+=f(x)
    xl=h*(xl0+2*xl2+4*xl1)/3
    return xl
fx=lambda x: 1/(1+x**2)
simpsons(-5,5,238,fx)
```
Out[18]: 2.7468015336718317

In [19]:
```
 def trapezoidal(a,b,n,f):
    h = (b-a)/n
    x1=((f(a)+f(b))/2)
    counter=0
    for i in range(1, n):
        x=a+i*h
        x1+=f(x)
    x1*=h
    return x1
trapezoidal(-5,5,1292,fx)
```
Out[19]: 2.746801386191278

In [15]:
```
class function_counter:
    def __init__(self,f):
        self.f=f
        self.counter=0

    def __call__(self,x):
        self.counter+=1
        return self.f(x)

from scipy import integrate
print('Tol of 10^-6')
fx1=function_counter(lambda x: 1/(1+x**2))
print(integrate.quadrature(fx1, -5, 5, tol=10**(-6)))
print('Func evals:', fx1.counter)
print('Tol of 10^-4')
fx2=function_counter(lambda x: 1/(1+x**2))
print(integrate.quadrature(fx2, -5, 5, tol=10**(-4)))
print('Func evals:', fx2.counter)
```
```
Tol of 10^-6
(2.7468012434694327, 7.225317166792422e-07)
Func evals: 42
Tol of 10^-4
(2.7467673525842766, 8.503832952921897e-05)
Func evals: 30
```

(c) The number of function evaluations using the built in method is much less than previously calculated number of function evaluations for the trapezoidal and Simpson rules. Even for a lower error bound, the built it method uses fewer function evals.

2. $I = I_n + \frac{C_1}{n\sqrt{n}} + \frac{C_2}{n^2} + \frac{C_3}{n^2\sqrt{n}} + \ldots$ (1)
   $I = I_{n/2} + \frac{C_1}{\frac{n}{2}\sqrt{n/2}} + \frac{C_2}{(n/2)^2} + \frac{C_3}{(n/2)^2\sqrt{n/2}} + \ldots$
   Combining with (1) to remove $\frac{1}{n\sqrt{n}}$
   $I = \frac{1}{\frac{1}{2\sqrt{2}}-1}(\frac{1}{2\sqrt{2}}I_{n/2} - I_n + (-1+\sqrt{2})\frac{C_2}{n^2} + \ldots)$ (2)
   $I = I_{n/4} + \frac{8C_1}{n} + \frac{16C_2}{n^2} + \ldots$
   Combining with (1) to remove $\frac{1}{n\sqrt{n}}$
   $I = \frac{1}{-1+\frac{1}{8}}(\frac{I_{n/4}}{8} - I_n + \frac{C_2}{n^2} + \ldots)$ (3)
   Combining (2) and (3)
   $I = (\frac{\frac{1}{2\sqrt{2}}-1}{-1+\sqrt{2}} - \frac{7}{8})(\frac{-1+\sqrt{2}}{8}I_{n/2} + -\sqrt{2}I_n - \frac{1}{8}I_{n/4} + \ldots)$

3. 3 DOF gives
   $\int_0^3 dx = 3 = c_0 + c_1$
   $\int_0^3 xdx = \frac{9}{2} = c_1 x_1$
   $\int_0^3 x^2dx = 9 = c_1 x_1^2$
   $\frac{9}{\frac{9}{2}} = 2 = x_1$
   $\frac{9}{2} = 2c_1$
   $c_1 = \frac{9}{4}$
   $3 = c_0 + \frac{9}{4}$
   $c_0 = \frac{3}{4}$ These solutions do not apply to 3rd degree
   $\int_0^3 x^3dx = \frac{81}{4} \neq 0 + \frac{8*9}{4}$
   $\int_0^3 f(x)dx = \frac{3}{4}f(0) + \frac{9}{4}f(2)$
   For polynomial degree ≤ 2