APPM4605 Homework6

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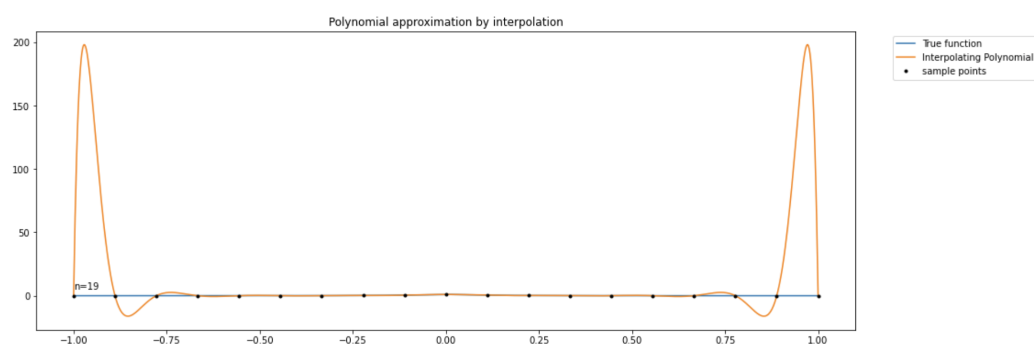
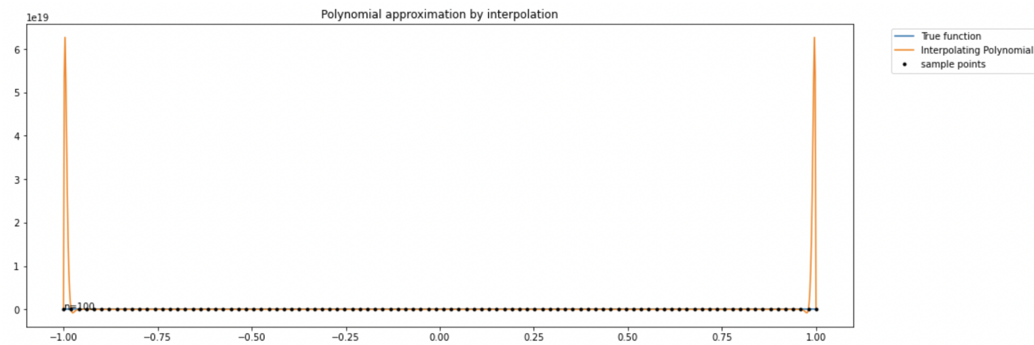
1. (a) $p(x) = c_n + c_{n-1}x + c_{n-3}x^2 + \cdots + c_1x^n$
$y_1 = c_n + c_{n-1}x_1 + c_{n-3}x_1^2 + \cdots + c_1x_1^n$
$y_2 = c_n + c_{n-1}x_2 + c_{n-3}x_2^2 + \cdots + c_1x_2^n$
$\dots$
$y_n = c_n + c_{n-1}x_n + c_{n-3}x_n^2 + \cdots + c_1x_n^n$
This can be rewritten in matrix form $Vc = y$

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \dots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{bmatrix} [c_1, c_2, ..c_n]^T = [y_1, y_2, ..y_n]^T$$

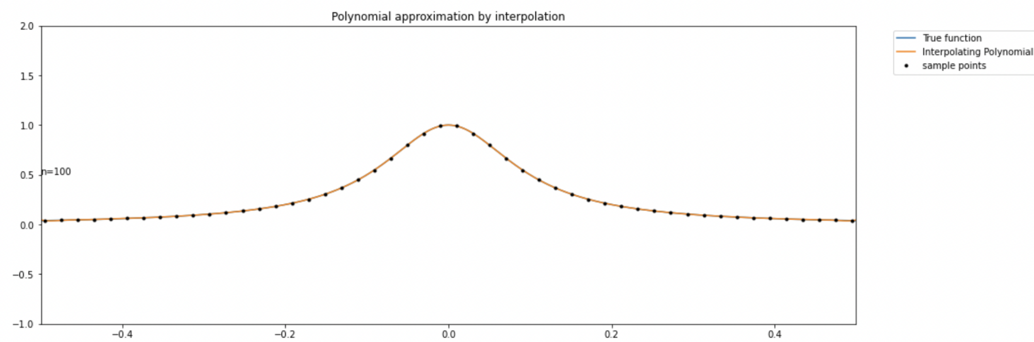
Here, $V = \begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \dots \\ 1 & x_n & x_n^2 & \dots & x_n^n \end{bmatrix}$



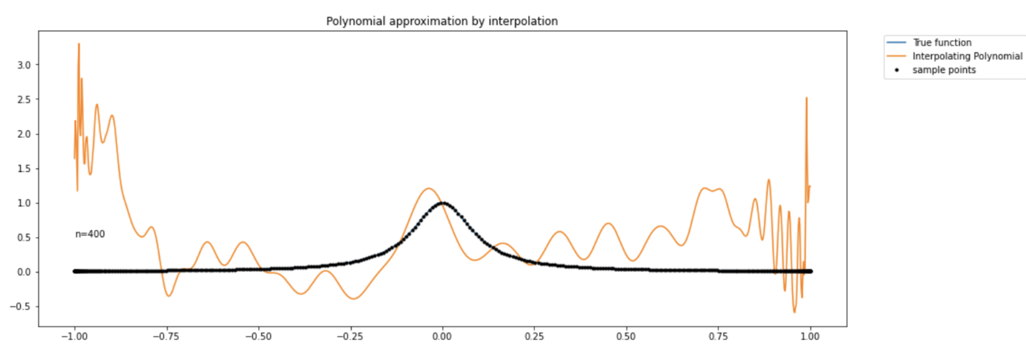
(b) It is obvious that the end points preform badly due to Runge's phenomena.

2. 

   Even after using Lagrange interpolation, the behavior is still bad near the endpoints.

   However, the approximation is well behaved for small xs and large Ns as seen above.

3. When n=400, the interpolation fails.