APPM4605-Homework5

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1. (a) i. Lazy Newton's Method does not converge with initial value $x = 1, y = 1$
   ii. Lazy Newton's Method converges to $[1.00416874, -1.72963729]$ after 21 iterations starting at $x = 1, y = -1$.
   iii. Lazy Newton's Method does not work starting at $x = 0, y = 0$, gives singular matrix.

   (b) i. Broyden's Method converges to $[-1.81626407, 0.8373678]$ with initial value $x = 1, y = 1$ after 16 iterations.
   ii. Broyden's Method gives $[1.00416874, -1.72963729]$ after 10 iterations starting at $x = 1, y = -1$.
   $[1.0041687384746592, -1.7296372870258698]$ after 11 iterations
   iii. Broyden's Method does not work, gives singular matrix.

   (c) i. Newton's Method converges to $[-1.81626407, 0.8373678]$ after 10 iterations with starting values $x = 1, y = 1$.
   ii. Newtons method converges to $[1.00416874, -1.72963729]$ after 10 iterations with starting values $x = 1, y = -1$.
   iii. Newton's Method does not work starting at $x = 0, y = 0$, gives singular matrix.
   iv. Here, Newton has the best performance, Broyden has the 2nd best, and Lazy Newton has the worst.

2. (a) Newton's method converges to $[0., 0.1, 1.]$ after 5 iterations.

   (b) Steepest Descent converges to $[1.85971612e{-}079.99998023e{-}021.00000004e{+}00]$ after 10 iterations.

   (c) This method gives $[2.05199588e-17, 1.00000000e-01, 1.00000000e+00])$ after 2 iterations in Steepest Descent and 4 iterations in Newton's method.

   (d) Newton's method has the best performance. It is obvious that Steepest Descent has worst performance than Newton's Method, so combining them will result in a slower performance than Newton's Method alone. I think this is because Steepest Descent has linear convergence (found through error analysis).