# WORKSHEET: SECTION 2-3 DAY TWO

Evaluate each limit. Show your work or explain your reasoning.

**Solutions**

1. $\lim\limits_{h\to 0} \dfrac{(-9+h)^2-81}{h}$ $= \dfrac{(-9)^2-81}{0} = \dfrac{0}{0}$

We need to do some algebra.

$$\lim_{h\to 0} \frac{\cancel{81}-18h+h^2-\cancel{81}}{h} = \lim_{h\to 0} \frac{\cancel{h}(-18+h)}{\cancel{h}} = \lim_{h\to 0}(-18+h) = -18$$

2. $\lim\limits_{t\to 8}(1+\sqrt[3]{t})(2-t^2)$

Direct Substitution Property If $f$ is a polynomial or a rational function and $a$ is in the domain of $f$, then

$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

$$\lim_{t\to 8}(1+\sqrt[3]{t})(2-t^2) = \lim_{t\to 8}\left(2-t^2+2\sqrt[3]{t}-t^{7/3}\right) =$$

$$= 2-64+2\cdot 2-2^7 = -58-2^7 = -186$$

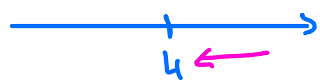
3. $\lim\limits_{\theta\to 4} \dfrac{\theta^2-4\theta}{\theta^2-\theta-12}$ $= \dfrac{16-16}{16-16} = \dfrac{0}{0}$

We cannot apply a direct substitution since the denominator $\to 0$ as $\theta \to 4$. We need to do some algebra.

$$\lim_{\theta\to 4} \frac{\theta\cancel{(\theta-4)}}{\cancel{(\theta-4)}(\theta+3)} = \lim_{\theta\to 4}\frac{\theta}{\theta+3} = \frac{4}{7}$$

4. $\lim\limits_{x\to 4} \dfrac{x^2}{x^2-x-12}$ $= \dfrac{16}{0}$

We need to consider ② cases:



1. $\lim\limits_{x\to 4^+} \dfrac{x^2}{x^2-x-12 \searrow 0^+} = +\infty$

2. $\lim\limits_{x\to 4^-} \dfrac{x^2}{x^2-x-12 \searrow 0^-} = -\infty$

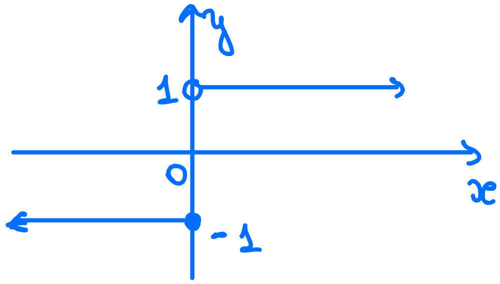
Hence, $\lim\limits_{x\to 4} \dfrac{x^2}{x^2-x-12}$ DNE

5. $\lim\limits_{x\to -3} \dfrac{\frac{1}{3}+\frac{1}{x}}{x+3}$ type "$\frac{0}{0}$". Need algebra.

$$\lim_{x\to -3} \frac{\frac{1}{3}+\frac{1}{x}}{x+3} = \lim_{x\to -3} \frac{\cancel{x+3}}{3x\cancel{(x+3)}} = -\frac{1}{9}$$

6. Write $\frac{|x|}{x}$ as a piecewise-defined function. $\lim_{x\to 0^-} \frac{|x|}{x} = -1$ $\lim_{x\to 0+} \frac{|x|}{x} = 1$

$$\frac{|x|}{x} = \begin{cases} 1, & x > 0 \\ -1, & x \leq 0 \end{cases}$$



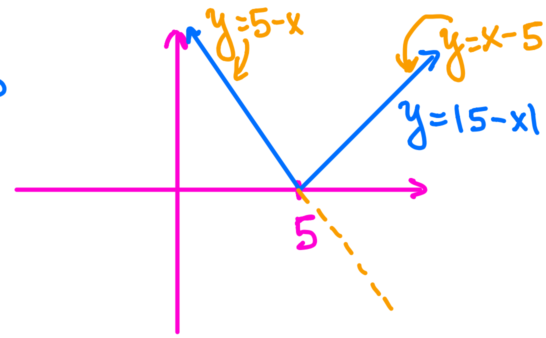

7. $\lim_{x\to 0} \frac{|x|}{x}$

Since $\lim_{x\to 0^+} \frac{|x|}{x} = -1$ and $\lim_{x\to 0^-} \frac{|x|}{x} = 1$, then

$\lim_{x\to 0} \frac{|x|}{x}$ DNE

8. $\lim_{x\to 5^-} \frac{3x-15}{|5-x|} = \lim_{x\to 5^-} \frac{3x-15}{5-x} = \lim_{x\to 5^-} \frac{3(x-5)}{5-x} = -3$

When $x \to 5^-$, then $|5-x| = 5-x$




9. $\boxed{\text{The Squeeze Theorem}}$ If $f(x) \leq g(x) \leq h(x)$ when $x$ is near $a$ (except possibly $a$) and

$$\boxed{\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L}$$

then

$$\boxed{\lim_{x\to a} g(x) = L}$$

Problem: show that

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

. Let $g(x) = x^2 \sin\left(\frac{1}{x}\right)$. Since we know that $|\sin(x)| \leq 1$, then

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

0

We have that $\lim_{x\to 0} (-x^2) = 0 = \lim_{x\to 0} x^2 = 0$.

Therefore, $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.