Solutions

SECTION 3.4 CHAIN RULE (DAY 2)
SECTION 3.5 INTRO

1. Evaluate the derivatives.

(a) $H(x) = \sqrt[3]{\dfrac{4-2x}{5}}$

$(\sqrt[3]{x})' = \frac{1}{3}x^{-2/3}$

$f(x) = \sqrt[3]{x}$

$g(x) = \dfrac{4-2x}{5} = \dfrac{4}{5} - \dfrac{2x}{5}$

$H'(x) = f'(g(x))\cdot g'(x) = \dfrac{1}{3\sqrt[3]{\left(\frac{4-2x}{5}\right)^2}}\cdot\left(-\dfrac{2}{5}\right)$

(b) $y = e^{\sec\theta}$

$f(\theta) = e^{\theta}$

$g(\theta) = \sec(\theta)$

$y'(\theta) = \underbrace{e^{\sec(\theta)}}_{= f'(g(\theta))}\cdot\underbrace{\sec(\theta)\cdot\tan(\theta)}_{g'(\theta)}$

(c) $F(x) = \dfrac{8}{x^2+\sin(x)}$

$f(x) = \dfrac{8}{x}$

$g(x) = x^2+\sin(x)$

$F'(x) = \underbrace{-\dfrac{8}{(x^2+\sin(x))^2}}_{f'(g(x))}\cdot\underbrace{(2x+\cos(x))}_{g'(x)}$

(d) $F(x) = \dfrac{1}{\sqrt{2}}\tan(\dfrac{\pi}{6}-x)$

$f(x) = \dfrac{\tan(x)}{\sqrt{2}}$

$g(x) = \dfrac{\pi}{6} - x$

$F'(x) = \dfrac{\sec^2(\frac{\pi}{6}-x)}{\sqrt{2}}\cdot(-1) = -\dfrac{\sec^2(\frac{\pi}{6}-x)}{\sqrt{2}}$

(e) $y = \dfrac{x e^{-\pi x^2/10}}{100}$

$g(x)$, $f(x)$

1. Product Rule
2. Chain Rule

$y' = (f(x)\cdot g(x))' = f'\cdot g + g'\cdot f = \left(\dfrac{x}{100}\right)'\cdot e^{-\pi x^2/10} + \left(e^{-\pi x^2/10}\right)'\cdot\dfrac{x}{100} = \boxed{\dfrac{1}{100}\cdot e^{-\pi x^2/10} + e^{-\pi x^2/10}\cdot\left(-\dfrac{\pi x}{5}\right)\cdot\dfrac{x}{100}}$

$\left(e^{-\pi x^2/10}\right)' = e^{-\pi x^2/10}\cdot\left(-\dfrac{\pi x}{5}\right)$

$f(x) = e^x$

$g(x) = -\pi x^2/10$

$g'(x) = -\dfrac{\pi x}{5}$

(f) $y = \dfrac{e^2 - x}{5+\cos(5x)}$

f, g

(g) $F(x) = (2re^{rx}+n)^p$ (Assume r, n, and p are fixed constants.)

1. Quotient Ruel

$y' = \dfrac{(e^2-x)'(5+\cos(5x)) - (5+\cos(5x))'\cdot(e^2-x)}{(5+\cos(5x))^2}$

$(e^2-x)' = \boxed{-1}$

$(5+\cos(5x))' = (\cos(5x))' = \boxed{-\sin(5x)\cdot 5}$

2. Chain Rule

$f = \cos(x)$
$g = 5x$

exponential function

2. (a) Complete the **rule**: $\frac{d}{dx}(b^x) =$ $b^x \cdot \ln b$

   (b) Determine the derivative of $f(x) = 2^x - x^3$

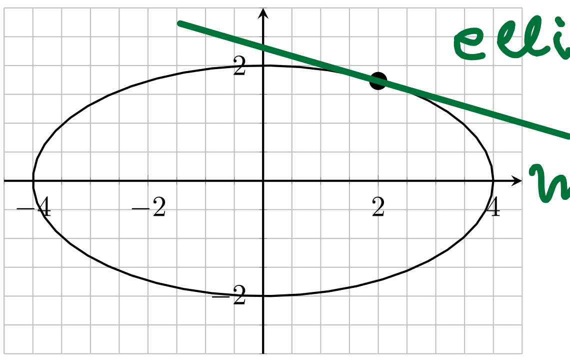
$$(b^x)' = \left(e^{x \ln b}\right)' = e^{x \ln b} \cdot \ln b = \boxed{b^x \cdot \ln b}$$

$f = e^x$
$g = x \cdot \ln b$

$$(2^x - x^3)' = 2^x \cdot \ln 2 - 3x^2$$

3. Consider the curve $x^2 + 4y^2 = 16$.

   (a) Think of y as being some function of x, and differentiate everything in sight with respect to x. Your answer should be an equation that contains x, y, and y'. Because we are thinking of $y = g(x)$, $\frac{d}{dx}(y) = \frac{dy}{dx}$ (or y'). You need to use the chain rule to determine $\frac{d}{dx}(y^2)$.



ellipse

$m = -\frac{1}{2\sqrt{3}}$

   Your first step:

$y = y(x)$

$$\frac{d}{dx}\left(x^2 + 4y^2\right) = \frac{d}{dx}(16) \implies$$

$$2x + 8 \cdot y(x) \cdot y'(x) = 0$$

$$y'(x) = -\frac{2x}{8y(x)}$$

   (b) Solve your previous step for y'.

$$m = y'(x) = -\frac{2x}{8y} = -\frac{x}{4y}$$

   (c) Determine the slope of the tangent line at the point $(2, \sqrt{3})$ by substituting $x = 2$, $y = \sqrt{3}$ into your equation for y'. Draw the tangent line at the point indicated on the graph. Is your computation plausible?

$$m = y' = -\frac{2}{4 \cdot \sqrt{3}} = -\frac{1}{2\sqrt{3}}$$

Write the equation of the tangent line at $(2, \sqrt{3})$: $y = -\frac{1}{2\sqrt{3}}(x - 2) + \sqrt{3}$

$F(x) = (2re^{rx}+n)^p$

$f(x) = x^p$

$g(x) = 2re^{rx}+n$

$F'(x) = f'(g(x)) \cdot g'(x) =$

$= p \cdot (2re^{rx}+n)^{p-1} \cdot 2r^2e^{rx}$

$g'(x) = (2re^{rx}+n)' = (2re^{rx})' = 2r(e^{rx})'$

$\underline{(e^{rx})'} = f'(g(x)) \cdot g'(x) = \underline{e^{rx} \cdot r}$

$f(x) = e^x$

$g(x) = rx$

- $f(x) = \cos(5x)$

  $f'(x) = -\sin(5x) \cdot 5$

- $f(x) = e^{-2x}$

  $f'(x) = e^{-2x} \cdot (-2)$

- $f(x) = \tan(\pi x)$

$$f'(x) = \sec^2(\pi x) \cdot \pi$$