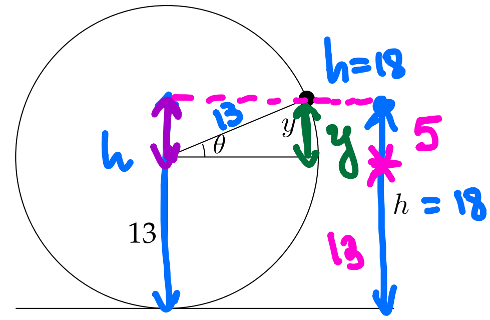
Derivatives Review:

1. $(\cos x)' = -\sin x$
2. $(e^{2x})' = e^{2x} \cdot 2$
3. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
4. $(\sec 3x)' = \tan(3x)\sec(3x) \cdot 3$
5. $\left(-\frac{1}{x}\right)' = \frac{1}{x^2}$
6. $(\arctan x)' = \frac{1}{1+x^2}$
7. $(\ln(5x))' = \frac{1}{5x} \cdot 5$
8. $(\cot(2x))' = -\csc^2(2x) \cdot 2$
9. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

1. A Ferris wheel with a radius of 13 meters is rotating at a rate of one revolution every three minutes. How fast is a rider rising when her seat is 18 meters above the ground? (Assume the wheel is tangent to the ground at the bottom.) *Hint: Label useful things in the diagram sketch.*



(a) In terms of the labels given in the picture and calculus-type language:

- What do we KNOW? (Hint: how many radians in one revolution?)

$r = 13$ meters $\qquad \frac{d\theta}{dt} = \frac{2\pi}{3} \frac{\text{rad}}{\text{min}}$

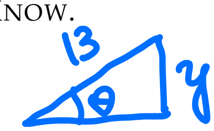
- What do we WANT?

$\frac{dh}{dt}$ when $h = 18$ meters

(b) Determine an equation that relates the variables in your WANT and KNOW.

$h = 13 + y = 13 + 13 \cdot \sin\theta$

$h = 13 + 13\sin\theta$



$\sin\theta = \frac{y}{13}$

$y = 13 \cdot \sin\theta$

(c) Solve the related rates problem.

(Hint: use what you know about right-triangle trigonometry! You don't actually need to know the angle from horizontal she's at when she's 18 feet above the ground.)

$\theta(t)$

$\frac{dh}{dt} = \frac{d}{dt}\left(13 + 13\sin\theta\right) = 0 + 13 \cdot \cos\theta \cdot \frac{d\theta}{dt}$

$\frac{dh}{dt} = 13\cos\theta \cdot \frac{d\theta}{dt} = 13 \cdot \cos\theta \cdot \frac{2\pi}{3}$

$\cos\theta = \frac{x}{13}$

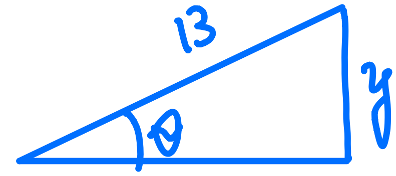
$x^2 + y^2 = 13^2$

$y = 5$

$x^2 + 25 = 169$

$x^2 = 144$

$x = 12$



$\cos\theta = \frac{x}{13} = \frac{12}{13}$

$\frac{dh}{dt} = \cancel{13} \cdot \frac{\cancel{12}^{\,4}}{\cancel{13}} \cdot \frac{2\pi}{\cancel{3}} = 8\pi$ m/min

2. Consider the function $f(x) = x^3$.

(a) At the point $x = 2$, what is $f(x)$? $f(2) =$ 8

(b) Let $L(x)$ be the function that is the tangent line to $f(x)$ at $x = 2$. This tangent line is sometimes called the *linearization* of $f(x)$ at $x = 2$. Finish the equation (you will need to show some work).

$L(x) = f'(a)(x-a) + f(a)$ $\quad L(x) = 12(x-2)+8$

$a = 2$

$f(a) = 8$

$f'(x) = 3x^2 \quad f'(2) = 12$

$L(x) =$ $12(x-2)+8$.

(c) Observe that the value $x = 2.1 = 2 + \frac{1}{10}$ is very close to $x = 2$. Evaluate $L(x)$ at $x = 2.1$. Do not use a calculator until your very last step (that is you can get a decimal approximation of a fraction, but you should compute the fraction by hand).

$(2.1)^3 \approx L(2.1) = 12(2.1-2)+8 = 12 \cdot 0.1 + 8 =$

$= \frac{12}{10} + 8 = \frac{6}{5} + 8 = \frac{46}{5} \approx 9.2$

$L(2.1)$ as a fraction $\frac{46}{5}$ $\quad L(2.1)$ as a decimal approximation. 9.2

(d) Use a calculator or a computer to evaluate $f(2.1)$. $f(2.1) =$ 9.261

(e) What is the error if you use $L(2.1)$ to approximate $f(2.1)$? (That is, what is the difference between the two quantities?) What is the percent error, calculated as (approx value - actual value)/(actual value)?

error $= |9.2 - 9.261| = 0.061$

(f) Draw a rough sketch of $f(x)$ and $L(x)$, and use the picture and your computations to explain, in a sentence or two, why using $L(2.1)$ to approximate the cube of 2.1 is a reasonable thing to do.