Section 5.5. The Substitution Rule

Mper 1
$$\int (e^{2e} + \sin(x)) dx = \int e^{x} dx + \int \sin(x) dx =$$

$$= e^{2e} - \cos(x) + C$$

Example 2

$$\int_{\mathbb{R}} 2x \cdot \sqrt{1+x^2} dx =$$

Introducing something extra:

$$\frac{du}{dx} = 2x \cdot dx = \sqrt{dx = \frac{du}{2x}}$$

$$= \int u^{1/2} du = \int u du = \int u du = \int u^{3/2} + c = \int u^{1/2} du = \frac{3/2}{3/2} + c = \frac{(1+x^2)^{3/2}}{3/2} + c$$

$$\mathcal{U} = g(x) = 1+x^{2}$$

$$\sqrt{1+x^{2}} = f(g(x))$$

$$\int f(g(x)) \cdot g'(x) dx =$$

$$= \int f(u) du$$

The Substitution Rule If u = g(x)is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x)) \cdot g'(x) dx = \int f(x) dx$$

$$\int 2x \sqrt{1+x^2} dx = \int \sqrt{u} du$$

$$f = \sqrt{1+x^2}$$

$$g(x) = 2x$$

$$du = g'(x) dx$$

$$du = g'(x) dx$$

Example 3 Substitution Rule
$$\int \sqrt{2x+1} \, dx = \int \sqrt{2} \, du =$$

•
$$u = g(x) = 2x + 1$$

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$$du = g'(x) dx = 2 \cdot dx$$

$$du = 2 \cdot dx$$

$$dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int u \, du = \frac{1}{2} \frac{3/2}{3/2} + C = \frac{1}{2} \frac{(2x+1)^{3/2}}{3/2} + C$$

SECTION 5-5: SUBSTITUTION (DAY 1)

$$= \frac{1}{2}(-\cos(\pi t)) + C =$$

$$= \frac{1}{2}(-\cos(t^2 + 1)) + C$$

$$dt = \frac{1}{2t} du = \frac{1}{2} \left(-\cos(t^2 + 1) + C \right)$$
2. Compute $\int e^{4x-9} dx = \int e^{4x-9} dx = \frac{1}{4} \cdot e^{4x-9} + C$

•
$$du = 4 \cdot dx$$

 $dx = \frac{1}{4} du$

3. Compute
$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$
. = $\int \frac{e^{\mathbf{u}}}{\sqrt{x}} \cdot 2\sqrt{x} d\mathbf{u} = 2\int e^{\mathbf{u}} d\mathbf{u} = 2e^{\mathbf{u}} + C = 0$

•
$$du = \frac{1}{2\sqrt{x}} \cdot dx$$

 $dx = 2\sqrt{x} \cdot du$

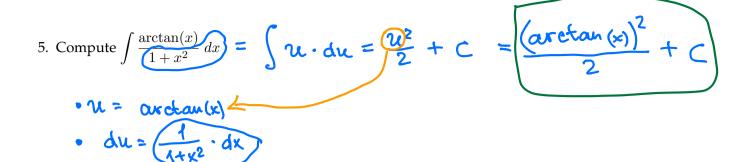
4. Compute
$$\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int_{1}^{2} \frac{e^{u}}{\sqrt{x}} dx = \int_{1}^{2} \frac{e^{u}}{\sqrt$$

•
$$du = \frac{1}{2\sqrt{x}} dx$$

$$dx = 2\sqrt{x} du$$

$$= 2e^{2} - 2e^{1} = 2(e^{2} - e) =$$

$$= 2e(e-1)$$



6. Compute
$$\int \frac{x^3}{\sqrt{1-x^4}} dx$$

7. Compute
$$\int \frac{x}{\sqrt{1-x^4}} dx$$
.

8. Compute $\int_0^{\pi/6} \frac{\sin(t)}{(\cos(t))^2} dt$ two ways: (1) by computing the antiderivative using substitution and then using FTC2 to evaluate using the original bounds; (2) by substituting and changing the bounds to match the substitution.