

Day one

Section 4.1

Max and Min

values of the function $f(x)$

Def. 1 Let c be the number in the domain D of the function $f(x)$. Then

- $f(c)$ is an abs. max if

$$f(c) \geq f(x) \text{ for all } x \in D$$

- $f(c)$ is an abs. min if

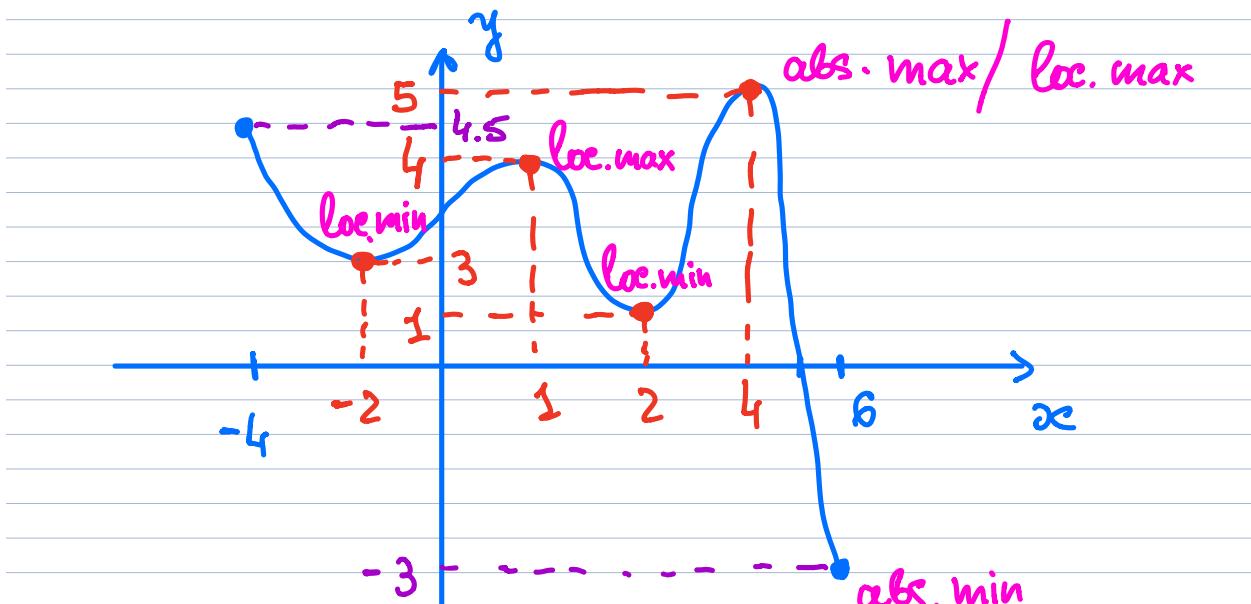
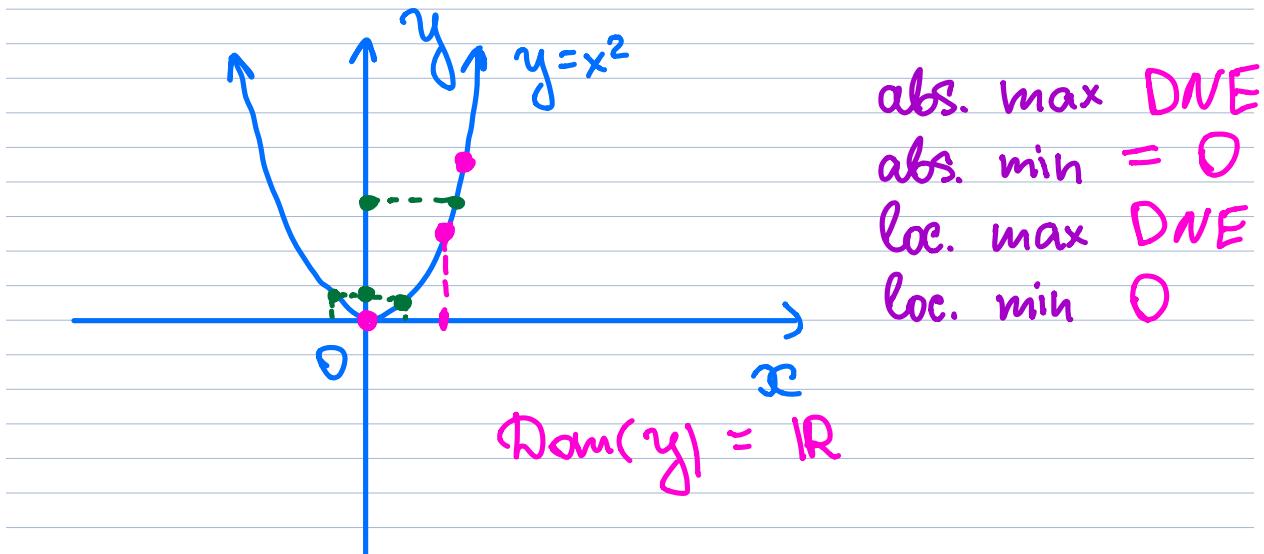
$$f(c) \leq f(x) \text{ for all } x \in D$$

Def 2

- The number $f(c)$ is a loc. max

if $f(c) \geq f(x)$ only for x near c

- The number $f(c)$ is a loc. min
 if $f(c) \leq f(x)$ only for
 x near c



$f(x)$ has an abs. max at $x = 4$

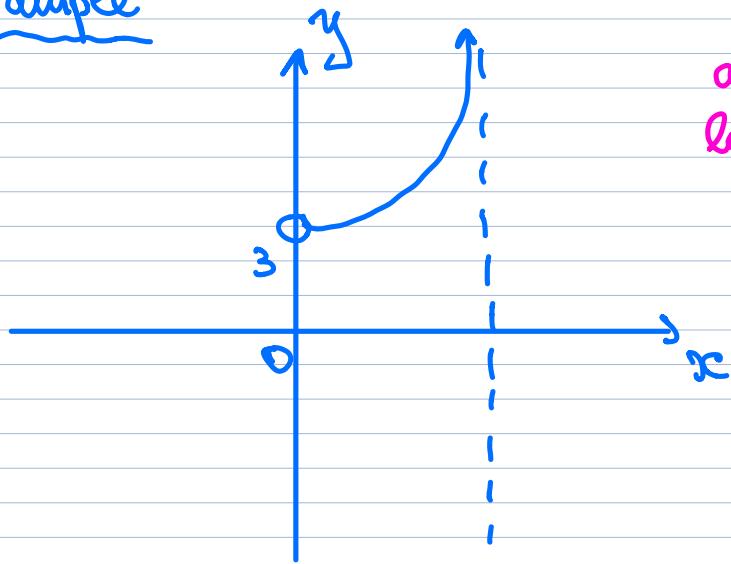
and it is ⑤ $\text{abs. max } f(4)=5$

$f(x)$ has an abs. min at $x=6$

and it is -3

$\text{abs. min } f(6)=-3$

Example



abs max/min DNE
loc max/min DNE

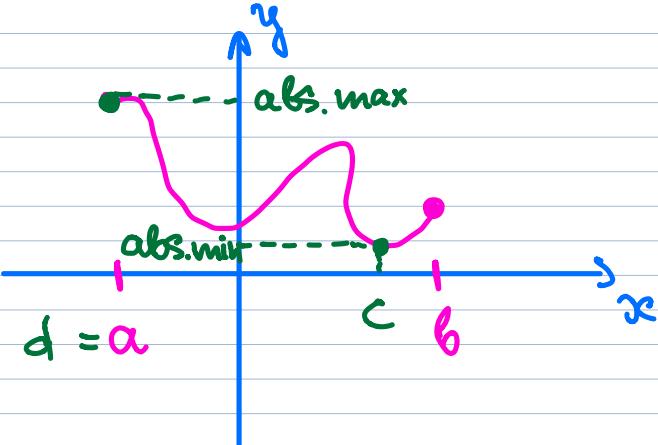
Theorem (The Extreme Value Theorem)
(abs max + min)
values

If the $f(x)$ is continuous on

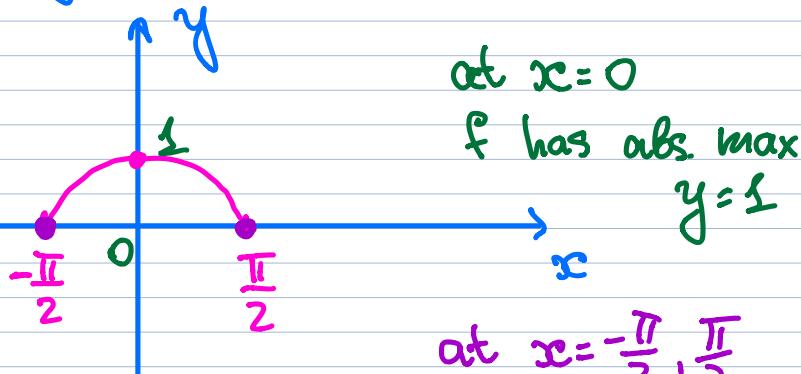
the closed interval $[a, b]$,

then $f(x)$ attains its abs.
maximum and minimum values

at points $x=c$ and $x=d$
 which are in $[a, b]$.



Example $y = \cos(x)$, $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ continuous



at $x=0$
 f has abs. max
 $y=1$

at $x = -\frac{\pi}{2}, \frac{\pi}{2}$
 f has abs. min

$$y=0$$

Section 4.1. Day two

Note:

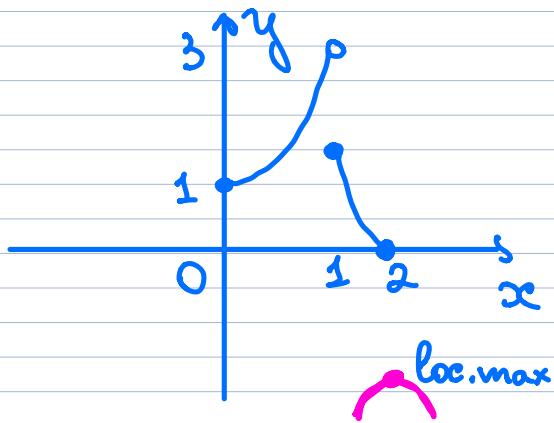
The local max/min values

of the function $f(x)$

don't occur at the endpoints

of the given interval

Example



abs. max	DNE	DNE
abs. min	0	$f(2) = 0$
loc. max	DNE	
loc. min	DNE	

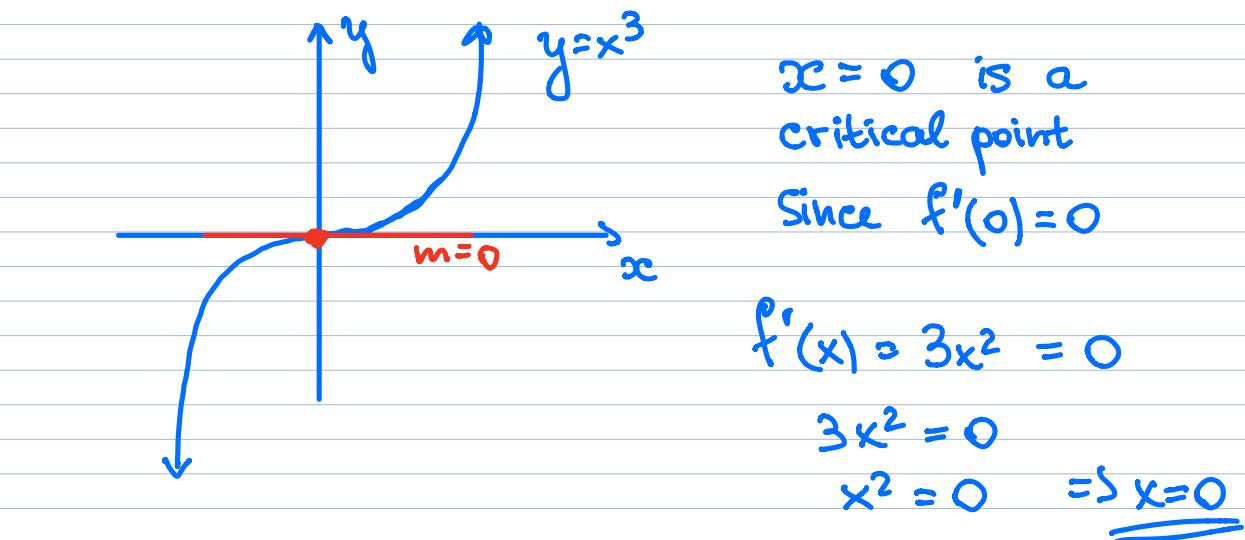
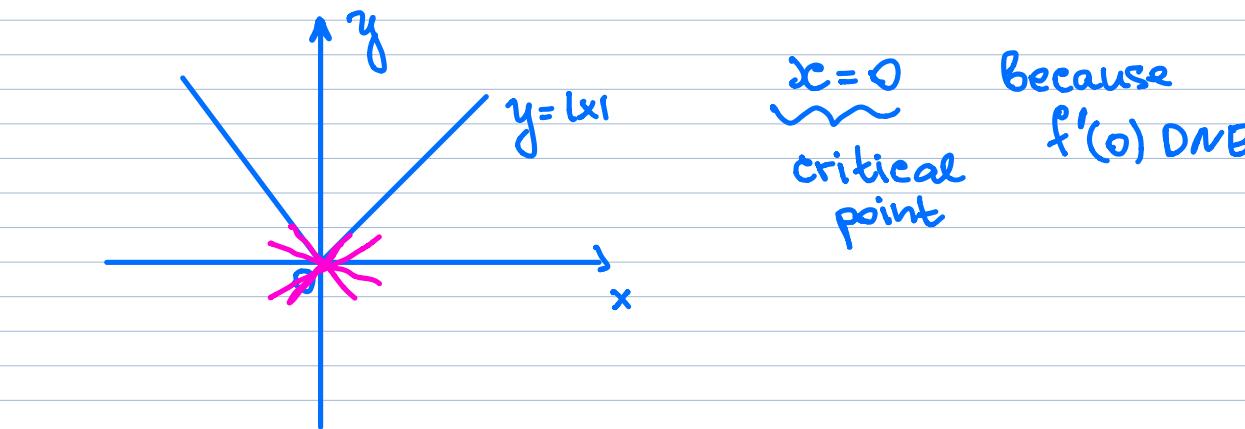
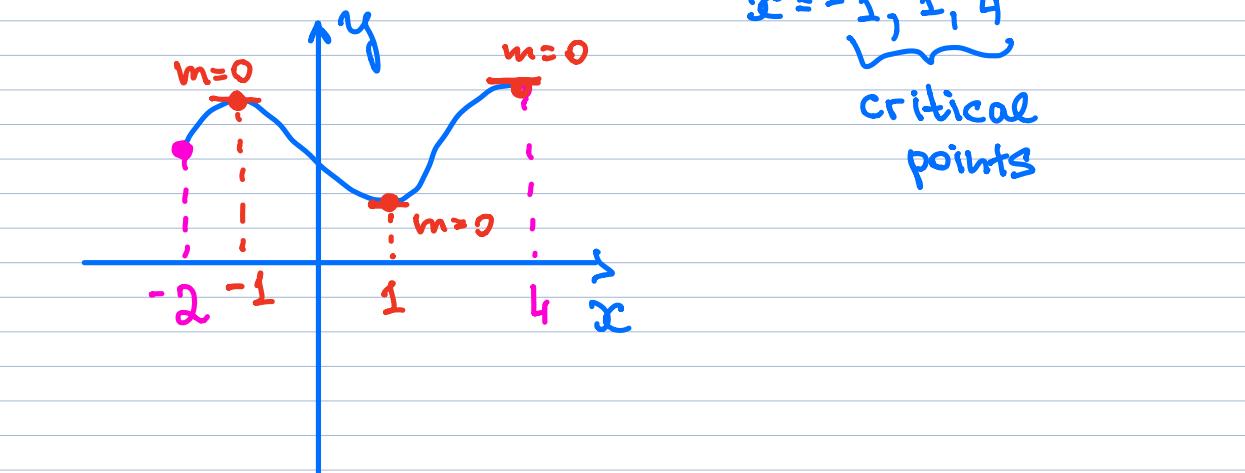
Def 3. A critical number (point)

of the function $f(x)$ is a number

c in the domain of $f(x)$ such that

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ DNE}$$

Example



The Closed Interval Method :

Let f be a continuous function on the closed interval $[a, b]$.

To find an abs. max and abs. min values of $f(x)$ we need to:

1. find all critical points of $f(x)$ and evaluate our function $f(x)$ at these points

2. evaluate function $f(x)$ at endpoints of the given closed interval

$$f(a), f(b)$$

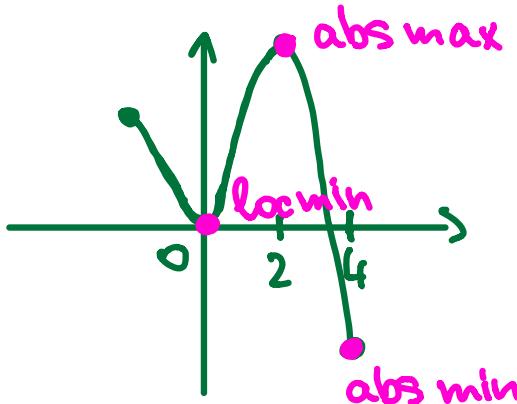
3. abs max of $f(x) = \max\{1, 2\}$

abs min of $f(x) = \min\{1, 2\}$

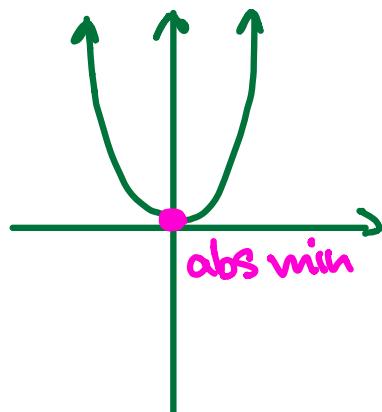
SECTION 4.1: MAXIMUM & MINIMUM VALUES

1. Sketch a graph $f(x)$ whose domain is the interval $[-1, 4]$ with the following properties:

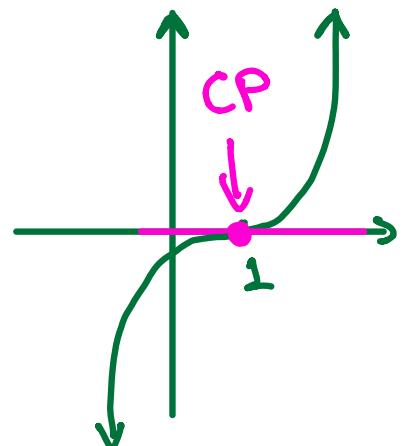
- (a) f is continuous, has a local minimum at $x = 0$, an absolute minimum at $x = 4$ and an absolute maximum at $x = 2$.



- (b) f has an absolute minimum but no absolute maximum



- (c) f has a critical point at $x = 1$ but no maximum or minimum (of any type) at $x = 1$.



2. Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$. Determine where those absolute maximum and minimum values occur.

$$f(x) = x - x^{1/3}, \quad \text{Dom}(f) = [-1, 4]$$

1. Find critical points:

$$f'(x) = 0 \quad \text{or} \quad f'(x) \text{ DNE}$$

$$f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3}\frac{1}{x^{2/3}}$$

$$1 - \frac{1}{3}\frac{1}{x^{2/3}} = 0$$

$$\frac{1}{3}\frac{1}{x^{2/3}} = 1$$

$$\frac{1}{x^{2/3}} = 3$$

$$x^{2/3} = \frac{1}{3}$$

$$x = \frac{1}{\sqrt[3]{3^2}}$$

$$x = 0$$

critical point
since
 $f'(0)$ DNE

critical point
since $f'\left(\frac{1}{\sqrt[3]{3^2}}\right) = 0$

$$f(0) = 0$$

$$f\left(\frac{1}{\sqrt[3]{3^2}}\right) = \frac{1}{\sqrt[3]{3^2}} - \left(\frac{1}{\sqrt[3]{3^2}}\right)^{1/3}$$

$$\boxed{\frac{1}{3\sqrt[3]{3}} - \frac{1}{\sqrt[3]{3}}}$$

$$2. \quad f(-1) = -1 - (-1)^{1/3} = \boxed{0}$$

$$f(4) = \boxed{4 - \sqrt[3]{4}}$$

$$3. \quad f_{\min}^{\text{abs}} = \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{1}{3} - 1\right) \text{ at } x = \frac{1}{\sqrt{3^3}}$$

$$f_{\max}^{\text{abs}} = 4 - \sqrt[3]{4} \text{ at } x = 4$$

3. Find the absolute maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval $[1/5, 4]$. Determine where those absolute maximum and minimum values occur.

$$f(x) = x + \frac{1}{x}, \quad x \neq 0 \quad [\frac{1}{5}, 4]$$

1. $f'(x) = 0$ or $f'(x)$ DNE

$$f'(x) = 1 - \frac{1}{x^2} = 0$$

$$1 - \frac{1}{x^2} = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$x=1$ is a CP

$x=-1$ is not a CP since $x=-1$ is not in $[\frac{1}{5}, 4]$

at $x=0$ $f'(0)$ DNE

↑
is not a CP since
 $x=0$ is not in $[\frac{1}{5}, 4]$

4. Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-8, 8]$. Determine where those absolute maximum and minimum values occur.

$$f(x) = \sqrt[3]{x^2}, \quad [-8, 8]$$

1. CP:

$$f'(x)=0 \text{ or } f'(x) \text{ DNE}$$

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3} \frac{1}{x^{1/3}}$$

$$f'(x)=0$$

$$\frac{2}{3}x^{-1/3}=0 \Rightarrow x=0$$

$x=0$ in $[-8, 8]$

$x=0$ is a CP

$f'(x)$ DNE at $x=0$.

$$f(0)=0$$

2. $f(-8) = \sqrt[3]{64} = 4$

$$f(8) = \sqrt[3]{64} = 4$$

$f_{\max}^{\text{abs}} = 4$ at $x = \pm 4$

$f_{\min}^{\text{abs}} = 0$ at $x = 0$

Problem (continue)

3. $x=1$ is a CP

$$f(1) = 1 + \frac{1}{1} = \boxed{2}$$

$$2. f\left(\frac{1}{5}\right) = \frac{1}{5} + \frac{1}{\frac{1}{5}} = \boxed{5 + \frac{1}{5}}$$

$$f(4) = \boxed{4 + \frac{1}{4}}$$

$$3. f_{\max}^{\text{abs}} = 5 + \frac{1}{5} \quad \text{at } x = \frac{1}{5}$$

$$f_{\min}^{\text{abs}} = 2 \quad \text{at } x = 1$$

