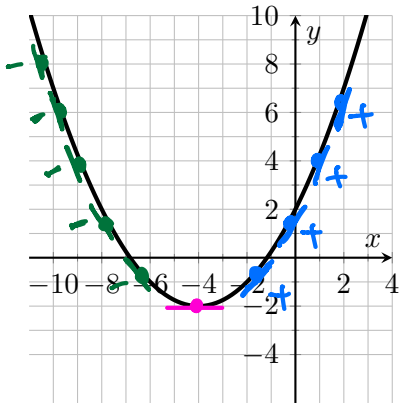
WORKSHEET: SECTION 2.8

Solutions

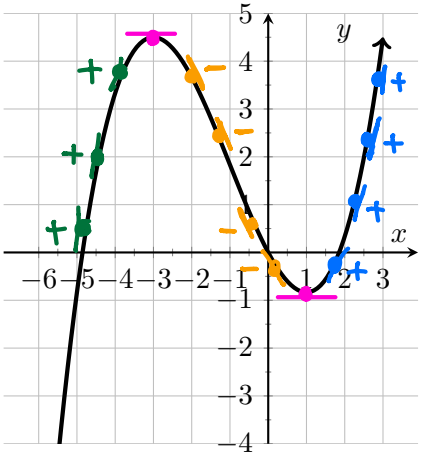
When you are asked to sketch the derivative on the provided axes, I am interested in the qualitative behavior of the derivative: Where does it cross the x-axis? Is it positive or negative? Is it a lot positive or a little positive? Are the slopes growing steeper or getting less steep? (This is why the y-axis is unmarked on the answer graphs.)

Exercise 1. Sketch the derivatives of the following graphs.

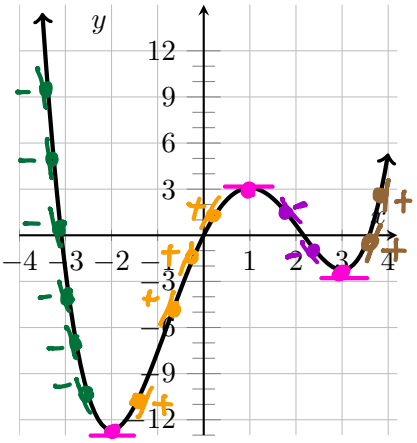
Graph 1

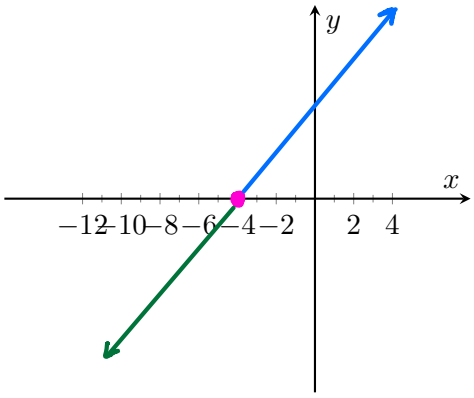


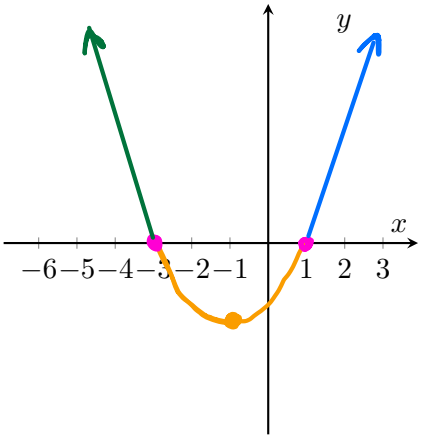
Graph 2

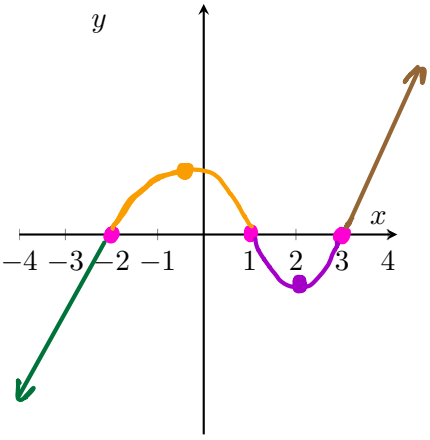


Graph 3









Exercise 2. Each of the graphs in Exercise 1 are polynomials. Fill in the blanks:

(a) Graph 1 looks like a quadratic polynomial (the degree is 2) and the derivative of graph 1 looks like a linear polynomial (the degree is 1).

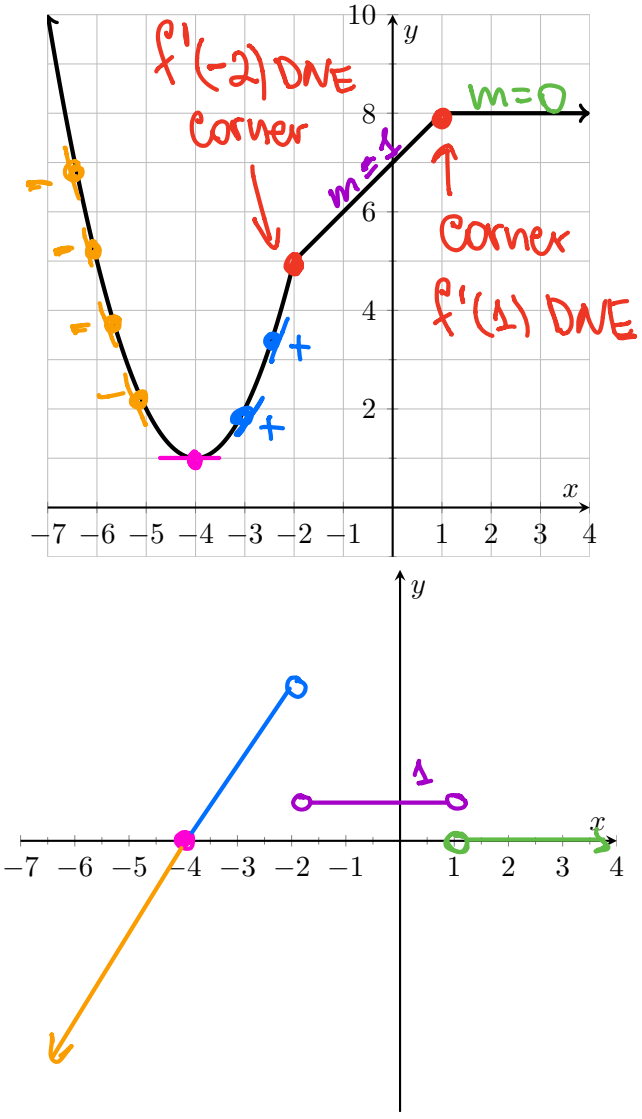
(b) Graph 2 looks like a cubic polynomial (the degree is 3) and the derivative of graph 2 looks like a quadratic polynomial (the degree is 2).

(c) Graph 3 looks like a quartic polynomial (the degree is 4) and the derivative of graph 3 looks like a cubic polynomial (the degree is 3).

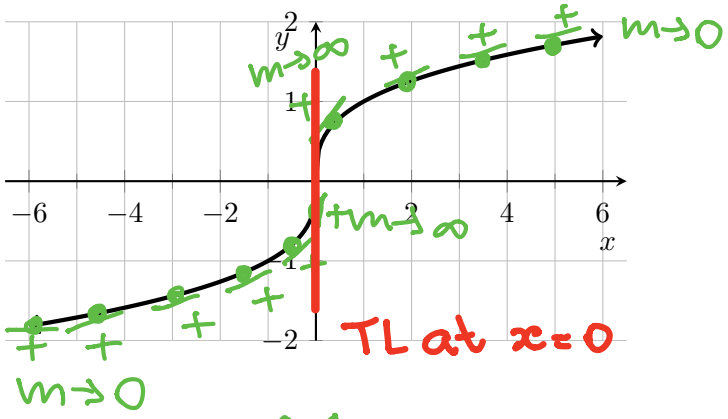
(d) **Make a guess:** If $f(x)$ is a degree n polynomial, then $f'(x)$ is a degree $n-1$ polynomial.

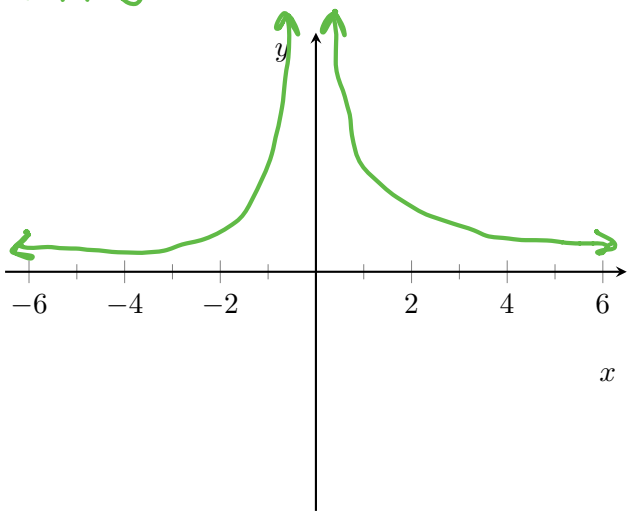
Exercise 3. Sketch the derivatives of graphs 4 and 5.

Graph 4



Graph 5





Exercise 4. What is an important difference between the derivative of graph 3 (from Exercise 1) and the derivative of graph 4? Use terminology from calculus.

The derivative on Graph 4 is not continuous.
$f'(-2)$ and $f'(1)$ DNE.

Exercise 5. Explain why Graph 5 has a tangent line at x = 0, even though the derivative is undefined at $x = 0$.

The TL at $x=0$ is vertical, so the slope is undefined. ($m \to \infty$ at $x=0$).