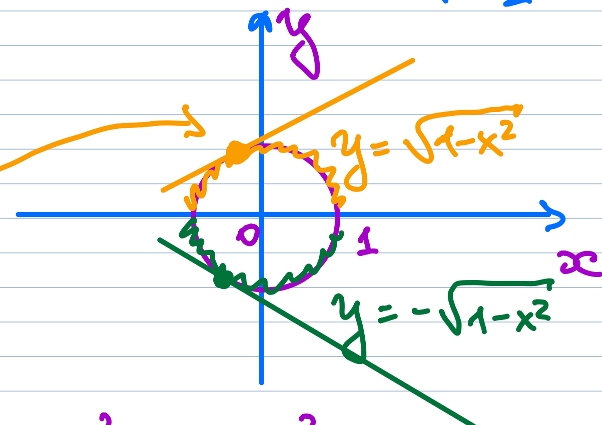
Implicit Differentiation

Explicitely defined func.
have the representation
$y = f(x)$

- $y = x^2$
- $y = \cos(x)$

Curve:
$x^2 + y^2 = 1$

circle $(0,0)$
$r = 1$



$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

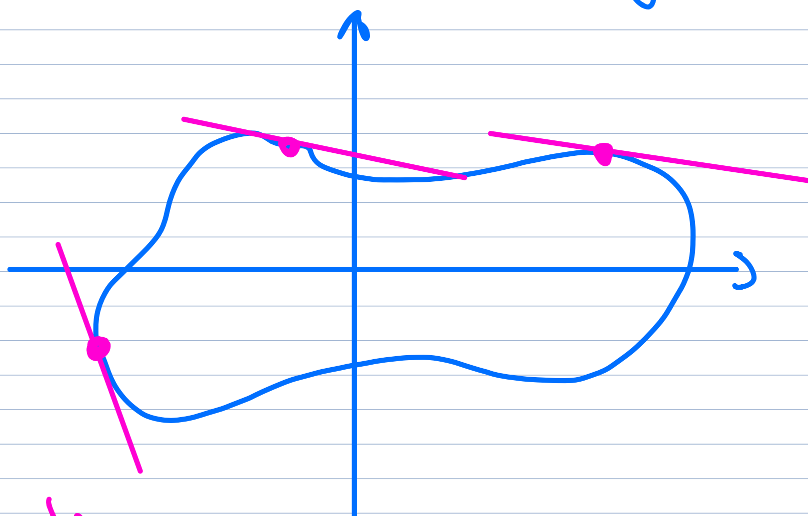
- $y = \sqrt{1-x^2}$

$y' = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)$

- $y = -\sqrt{1-x^2}$

$y' = -\frac{1}{2}(1-x^2)^{-1/2}(-2x)$

Given curve: $x^4 + 3y^3 + \cos(y) = \sin(x)$



Find:

$$\frac{dy}{dx} = y'(x) = ?$$

1. $$\frac{d}{dx}\left(x^4 + 3y^3 + \cos(y)\right) = \frac{d}{dx}\left(\sin(x)\right)$$

independent variable

function of x

$y = y(x)$

2. $$4 \cdot x^3 + 3 \cdot 3 \cdot (y(x))^2 \cdot y'(x) +$$

$$+ \left(-\sin(y(x))\right) \cdot y'(x) = \cos(x)$$

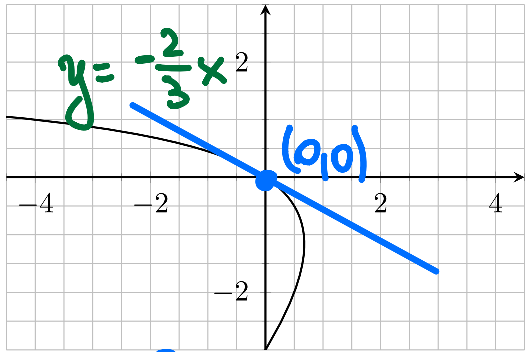
3. Solve for y':

$$y'(9y^2 - \sin(y)) + 4x^3 = \cos(x)$$

$$y'(9y^2 - \sin(y)) = \cos(x) - 4x^3$$

$$\boxed{y' = \frac{\cos(x) - 4x^3}{9y^2 - \sin(y)}}$$

1. Find $\frac{dy}{dx}$ for $2x + 3y = xy - y^2$ and find the equations of tangents to the graph when $x = 0$. Use the portion of the curve shown below as an aid and to determine the plausibility of your answers.



$y'(x) - ?$

1. $\frac{d}{dx}(2x+3y) = \frac{d}{dx}(xy - y^2)$

chain rule

Product rule

2.

$y' = -\frac{2}{3}$

$y = -\frac{2}{3}(x-0)+0$

$2 + 3\cdot y'(x) = 1\cdot y + y'(x)\cdot x - 2y\cdot y'(x)$

$3y' - x\cdot y' + 2y\cdot y' = y - 2$

$y'(3 - x + 2y) = y - 2 \Rightarrow y' = \frac{y-2}{3-x+2y}$

2. Find $\frac{da}{db}$ for $a^3 \sin(3b) = a^2 - b^2$. (Pay attention here: b is the independent variable (like x) and a is the dependent variable (like y).

3. Find $\frac{dy}{dx}$ for $e^{xy} = x + y + 1$

4. You are going to derive the formula for the derivative of inverse tangent the way we found the derivative of inverse sine in the video.

   (a) Find dy/dx for the expression $x = \tan(y)$.

   (b) Use the identity $1 + (\tan(\theta))^2 = (\sec(\theta))^2$ to rewrite you answer in part (a) and *write your dy/dx in terms of x only*.

   (c) Now fill in the blank $\dfrac{d}{dx}[\arctan(x)] =$

   (d) Use your knowledge of the *graph* of $f(x) = \arctan(x)$ to decide if your answer seems plausible...

5. Find the derivative of $f(x) = x \arctan x$.

6. Find the derivative of $f(x) = \arctan(4 - x^2)$.