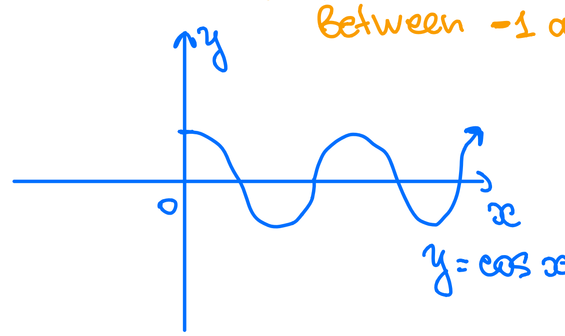
Limits at Infinity

Problem # 1. Compute the limit:

$$\lim_{x\to\infty}\left(e^{-x}+7\cos(9x)\right)=\lim_{x\to\infty}\frac{1}{e^{x}}+\lim_{x\to\infty}7\cos(9x)=$$

$$=\lim_{x\to\infty}\underbrace{\frac{1}{e^{x}}}_{\to 0}+7\lim_{x\to\infty}\underbrace{\cos(9x)}_{\text{oscillates when } x\to\infty \text{ and takes values between } -1 \text{ and } 1.}=\boxed{DNE}$$

DNE

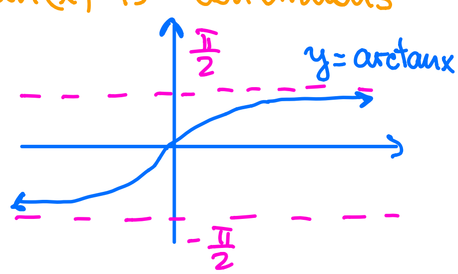


Problem #2. Compute the limit:

$$\lim_{x\to\infty}\arctan(e^{x})=\arctan\left(\lim_{x\to\infty}e^{x}\right)=$$

we can switch the order of lim and arctan x since y = arctan(x) is continuous

$$=\arctan\left(\lim_{x\to\infty}e^{x}\right)=\boxed{\frac{\pi}{2}}.$$



Problem #3. Find Horizontal and Vertical asymptotes of the given function:

$$y = \frac{e^x - 4}{5e^x}$$

$\boxed{VA}$ • $x=a$ is a VA if $\lim\limits_{x\to a^+} f(x) = \pm\infty$ or

$\lim\limits_{x\to a^-} f(x) = \pm\infty$

$y = \frac{e^x-4}{5e^x}$ has a domain $\boxed{x \text{ in } (-\infty, \infty)}$.

We see that $5e^x \neq 0$ for any x in $\mathbb{R}$.

Answer: There is no VA.

$\boxed{HA}$ • $y = L$ is a HA if $\lim\limits_{x\to\pm\infty} f(x) = L$.

$$\lim_{x\to\infty} \frac{e^x-4}{5e^x} = \lim_{x\to\infty} \frac{\frac{1}{e^x}(e^x-4)}{5\cdot\frac{1}{e^x}e^x} = \lim_{x\to\infty} \frac{1 - \overbrace{\frac{4}{e^x}}^{\to 0}}{5} = \boxed{\frac{1}{5}}$$

$$\lim_{x\to-\infty} \frac{e^x-4}{5e^x} = \lim_{x\to\infty} \frac{e^{-x}-4}{5e^{-x}} = \lim_{x\to\infty} \frac{\frac{1}{e^x}-4}{5\cdot\frac{1}{e^x}} =$$

$$= \lim_{x\to\infty} \frac{1-4e^x}{5} = \lim_{x\to\infty} \left(\frac{1}{5} - \frac{4}{5}\underbrace{e^x}_{\infty}\right) = -\infty$$ is not a finite number

Answer: the HA is $y = \frac{1}{5}$.