 Solutions

1. Fill out the table of Derivatives of Trigonometric Functions below

   (a) $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

   (b) $\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$

   (c) $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

   (d) $\frac{d}{dx}(\text{arccot } x) = -\frac{1}{1+x^2}$

   (e) $\frac{d}{dx}(\text{arcsec } x) = \frac{1}{x\sqrt{x^2-1}}$

   (f) $\frac{d}{dx}(\text{arccsc } x) = -\frac{1}{x\sqrt{x^2-1}}$

2. Find dy/dx by implicit differentiation.

   (a) $x^4 + x^2y^2 + y^3 = 5$

   $$\frac{d}{dx}(x^4 + x^2y^2 + y^3) = \frac{d}{dx}(5)$$

   $$4x^3 + 2x \cdot y^2 + x^2 \cdot 2y \cdot y' + 3y^2 \cdot y' = 0$$

   $$y'(2x^2y + 3y^2) = -4x^3 - 2xy^2 \Rightarrow y' = \frac{-4x^3 - 2xy^2}{2x^2y + 3y^2}$$

   (b) $\tan(x-y) = \frac{y}{1+x^2}$

   $$\frac{d}{dx}(\tan(x-y)) = \frac{d}{dx}\left(\frac{y}{1+x^2}\right)$$

   $$\sec^2(x-y) \cdot (1-y') = -\frac{1}{(1+x^2)^2} \cdot 2x \cdot y + y' \cdot \frac{1}{1+x^2}$$

   $$y'\left(-\frac{1}{1+x^2} - \sec^2(x-y)\right) = -\frac{2xy}{(1+x^2)^2} - \sec^2(x-y)$$

   $$y' = \frac{-\frac{2xy}{(1+x^2)^2} - \sec^2(x-y)}{-\frac{1}{1+x^2} - \sec^2(x-y)}$$

   (c) $x\sin(y) + y\sin(x) = 1$

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$$\frac{d}{dx}\left(x\sin(y) + y\sin(x)\right) = \frac{d}{dx}(1)$$

$$\sin y + x\cos(y)\cdot y' + y'\sin(x) + y\cos(x) = 0$$

$$y'\left(x\cos(y) + \sin(x)\right) = -y\cos(x) - \sin y$$

$$y' = \frac{-y\cos(x) - \sin(y)}{x\cos(y) + \sin(x)}$$

3. Use implicit differentiation to find an equation of the tangent line to the curve at the given point

$$x^2 + 2xy + 4y^2 = 12, \quad (2, 1) \quad \text{(ellipse)}$$

$$\frac{d}{dx}(x^2 + 2xy + 4y^2) = \frac{d}{dx}(12)$$

$$2x + 2y + 2xy' + 8y \cdot y' = 0$$

$$y'(2x + 8y) = -2(x+y)$$

$$y'(x + 4y) = -(x+y)$$

$$y = y'(2)(x-2) + 1$$

$$y' = -\frac{(x+y)}{x+4y}$$

$$y'(2,1) = \frac{-(2+1)}{2+4} = \frac{-3}{6} = \frac{-1}{2}$$

TL: $y = -\frac{1}{2}(x-2) + 1$

4. Find the derivative of the given function. Simplify where possible.

(a) $y = (\arctan x)^2$

$$y' = 2(\arctan x) \cdot \frac{1}{1+x^2}$$

(b) $y = x \arcsin x + \sqrt{1 - x^2}$

$$y' = 1 \cdot \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)$$

5. Find y'' by implicit differentiation for the given curve $x^2 + 4y^2 = 4$.

$$\frac{d}{dx}(x^2 + 4y^2) = \frac{d}{dx}(4)$$

$$2x + 8y \cdot y' = 0$$

$$y' = -\frac{2x}{8y} = -\frac{x}{4y}$$

$$y'' = (y')' = \left(-\frac{x}{4y}\right)' =$$

$$= \frac{(1 \cdot 4y - x \cdot 4y')}{16y^2}$$