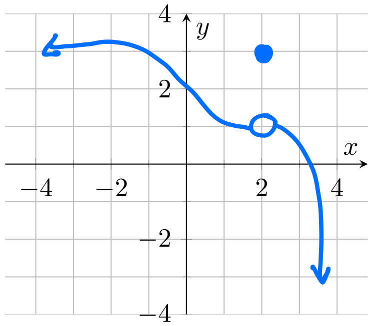
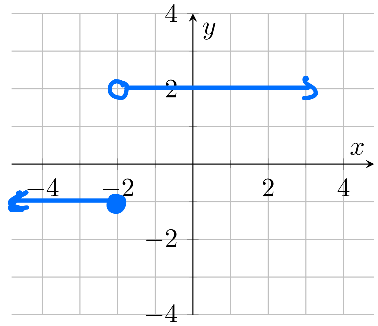
1. Sketch the graphs of three functions with
   (a) a removable discontinuity at $x = 2$,
   (b) a jump discontinuity at $x = -2$,
   (c) an infinity discontinuity at $x = 3$

   and that are continuous for all other real numbers:

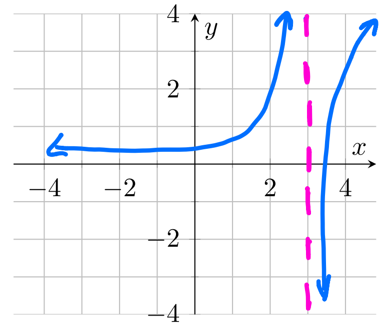
(a) removable discontinuity



(b) jump discontinuity

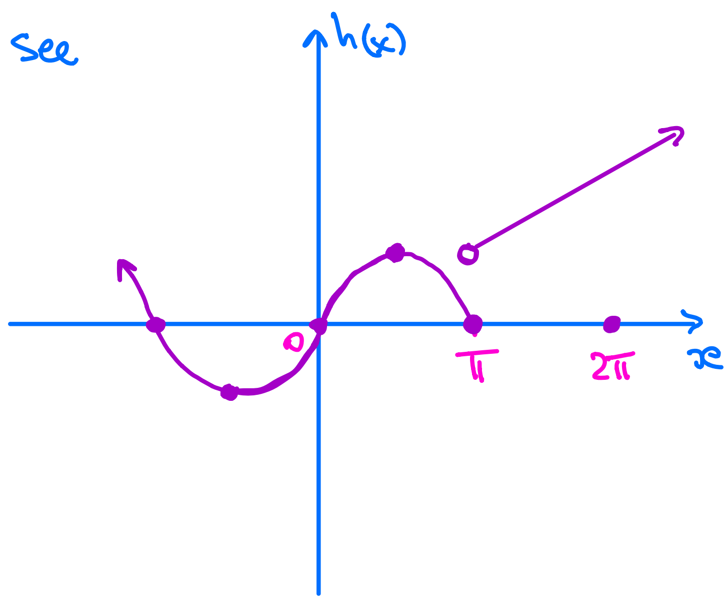


(c) infinity discontinuity



2. Determine where the function $h(x) = \begin{cases} \sin x & x < \pi \\ 0 & x = \pi \\ x + 1 - \pi & x > \pi \end{cases}$ is not continuous and **justify** your answer. Sketch the graph of the function.

From the graph we see that h(x) is not continuous at $x=\pi$ (jump discontinuity)



$$\lim_{x\to\pi^-} h(x) = h(\pi) = 0$$

$$\neq$$

$$\lim_{x\to\pi^+} h(x) = 1$$

3. Use continuity to evaluate the limit $\lim_{x\to 10} \frac{x^2}{\sqrt{x-5}}$.

$f(x) = \frac{x^2}{\sqrt{x-5}}$ is continuous at $x=10$.

$$\lim_{x\to10^+} \frac{x^2}{\sqrt{x-5}} = \frac{100}{\sqrt{5}} = f(10) = \lim_{x\to10^-} \frac{x^2}{\sqrt{x-5}}$$

4. Determine the value of c that will make $f(x) = \begin{cases} c - x^2 & x \leq 1 \\ 5x - 2 & x > 1 \end{cases}$ continuous everywhere.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (5x-2) = 5-2 = 3$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (c-x^2) = c-1$$

$$f(1) = c-1$$

$$c-1 = 3 \Rightarrow c = 4$$

Hence, for $c=4$: $f(x) = \begin{cases} 4-x^2, & x\leq 1 \\ 5x-2, & x>1 \end{cases}$ our function is continuous everywhere.

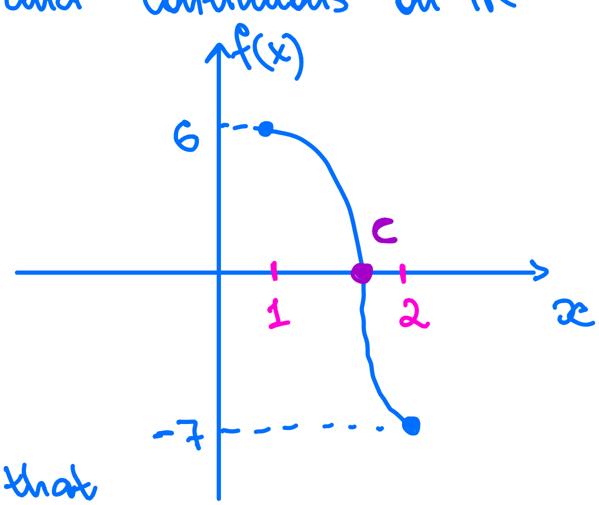
5. Use the Intermediate Value Theorem to show that there is a root of the equation $5+2x-x^4 = 0$ in the interval $(1, 2)$. To do so, explain how you are verifying that the hypotheses of the IVT hold, and then explaining what the IVT lets you conclude.

Let us first define a function $f(x) = 5+2x-x^4$

- $f(x)$ is a polynomial function and continuous on $\mathbb{R}$
- $(a,b) = (1,2)$

$f(a) = f(1) = 5+2-1 = 6 > 0$

$f(b) = f(2) = 5+4-16 = -7 < 0$



Since f is cont. and $f(1)>0$, $f(2)<0$, by the IVT there exists value c in $(1,2)$ such that

$$f(c) = 0$$

The value c is a root we are looking for.