Day one

Section 4.1 Max and Min values of the function $f(x)$

Def. 1 Let c be the number in the domain D of the function $f(x)$. Then

- $f(c)$ is an (global) abs. max if

$$f(c) \geq f(x) \text{ for all } x \text{ in } D$$

- $f(c)$ is an (global) abs. min if

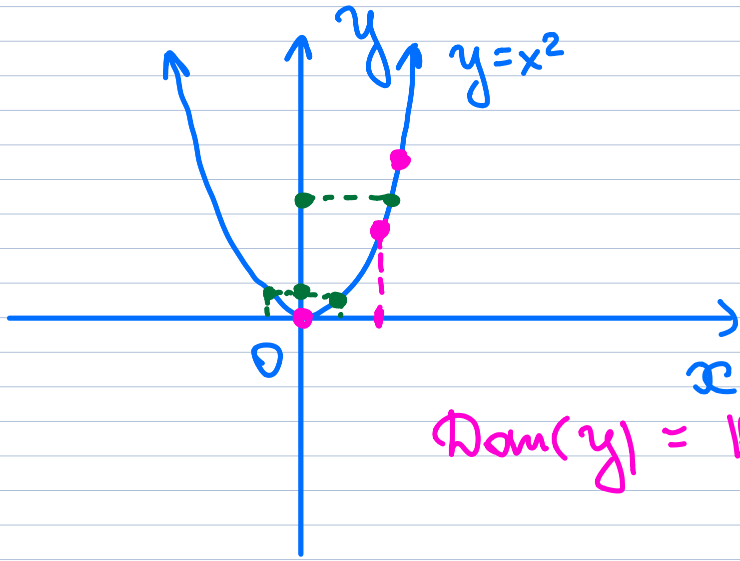
$$f(c) \leq f(x) \text{ for all } x \text{ in } D$$

Def 2

- The number $f(c)$ is a loc. max if $f(c) \geq f(x)$ only for x near c

- The number $f(c)$ is a <u>loc. min</u>

  if $f(c) \leq f(x)$ only for x near c



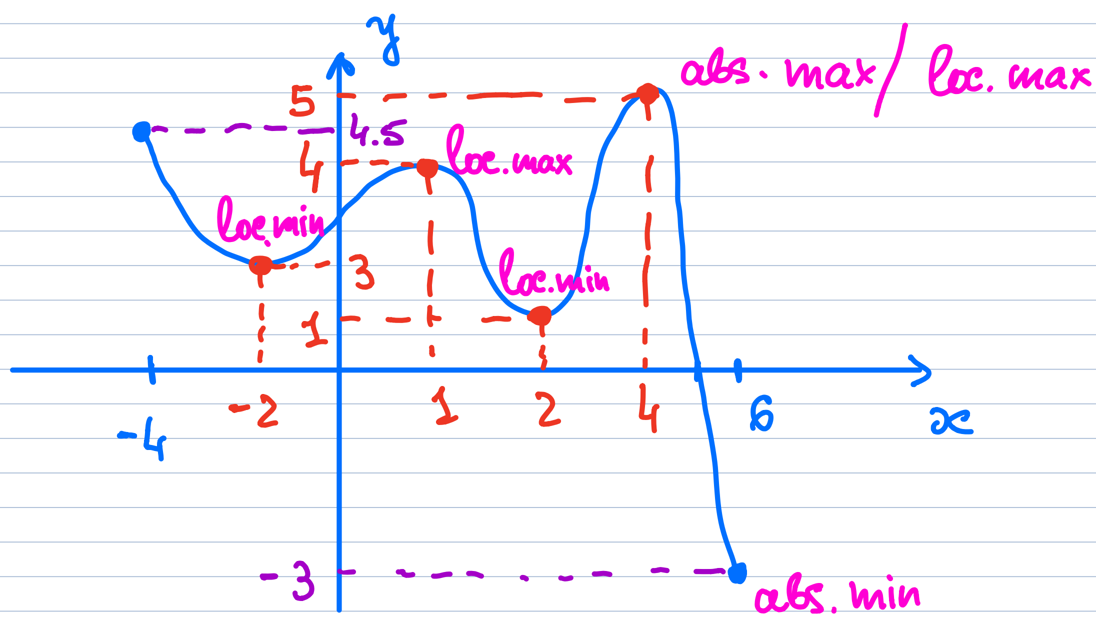
abs. max DNE

abs. min = 0

loc. max DNE

loc. min 0

Dom(y) = ℝ

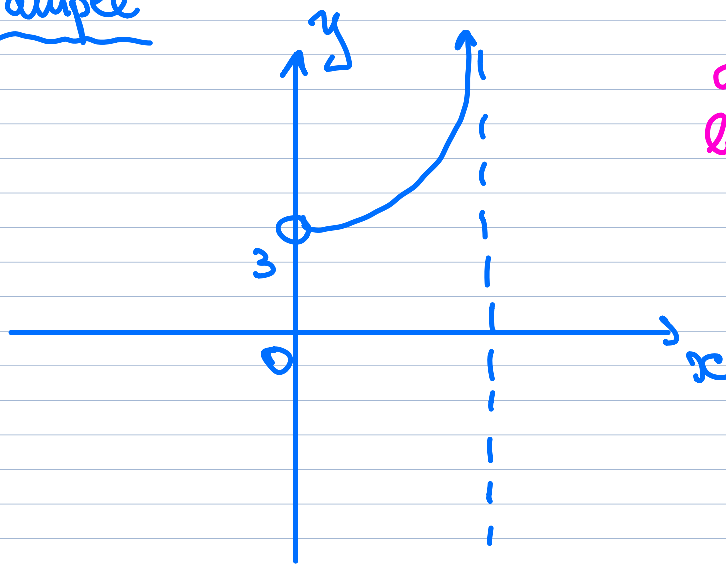


$f(x)$ has an abs. max at $x = 4$

and it is 5 abs. max $f(4)=5$

$f(x)$ has an abs. min at $x=6$
and it is -3
abs. min $f(6)=-3$

Example

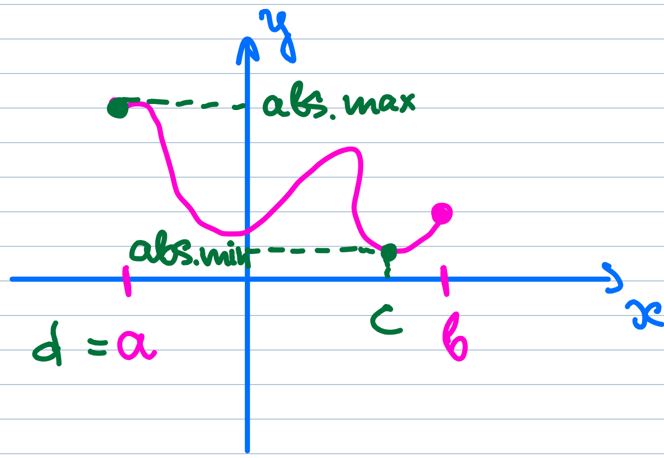


abs max/min DNE
loc max/min DNE

Theorem (The Extreme Value Theorem)
(abs max + min values)

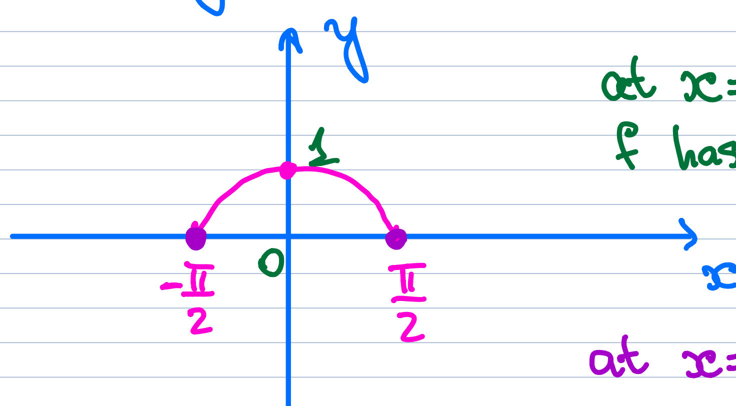
If the $f(x)$ is continuous on the closed interval $[a,b]$,
then $f(x)$ attains its abs. maximum and minimum values

at points $x = c$ and $x = d$ which are in $[a, b]$.



<u>Example</u>

$y = \cos(x)$, continuous $[-\frac{\pi}{2}, \frac{\pi}{2}]$



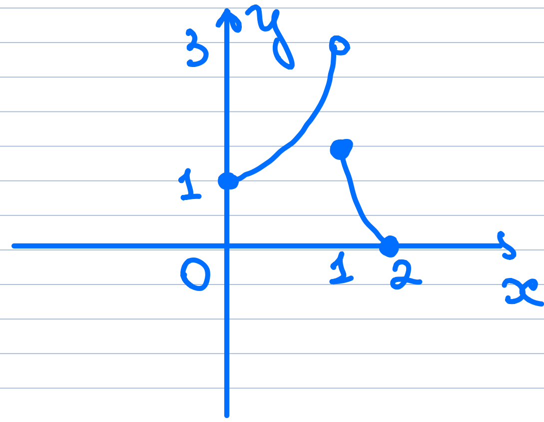
at $x = 0$
f has abs. max
$y = 1$

at $x = -\frac{\pi}{2}, \frac{\pi}{2}$
f has abs. min
$y = 0$

Section 4.1. Day two

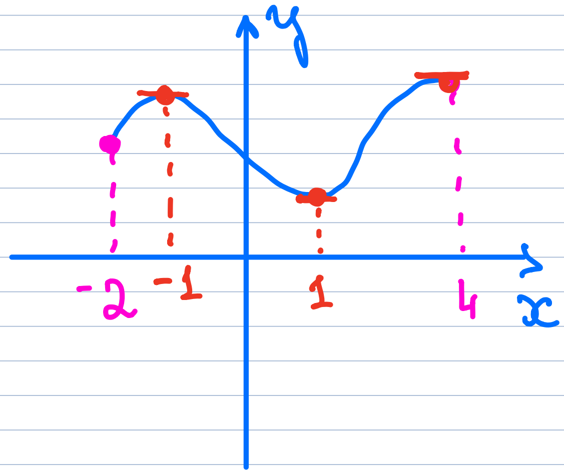
Note: The local max/min values of the function $f(x)$ don't occur at the endpoints of the given interval

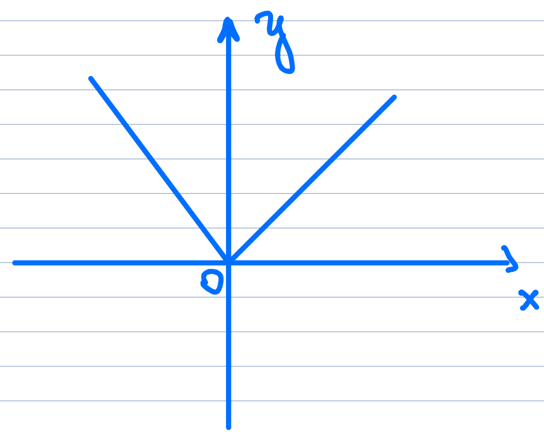
Example

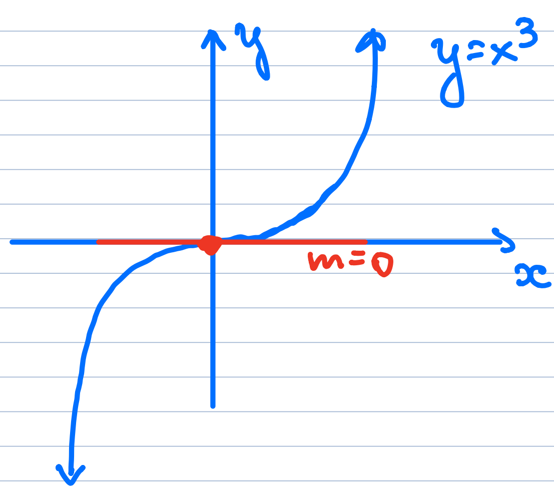


Def 3. A critical number (point)

Example







Fermat's Theorem

The Closed Interval Method:

Section 4.1: Maximum & Minimum Values

1. Sketch a graph $f(x)$ whose domain is the interval $[-1, 4]$ with the following properties:

   (a) f is continuous, has a local minimum at $x = 0$, an absolute minimum at $x = 4$ and an absolute maximum at $x = 2$.

   (b) f has an absolute minimum but no absolute maximum

   (c) f has a critical point at $x = 1$ but no maximum or minimum (of any type) at $x = 1$.

2. Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$. Determine where those absolute maximum and minimum values occur.

3. Find the absolute maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval $[1/5, 4]$. Determine where those absolute maximum and minimum values occur.

4. Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-8, 8]$. Determine where those absolute maximum and minimum values occur.