How to find the Inverse Function of a One-to-One Function f(x):

- write $y = f(x)$
- solve for x
- to express f^{-1} as a function of x, interchange x and y.

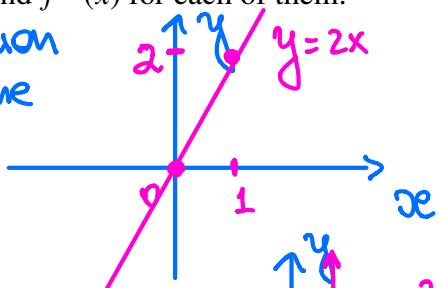
1. Check if the following functions are one-to-one and find $f^{-1}(x)$ for each of them:

   (a) $f(x) = 2x$

   Linear function is one-to-one

   $y = 2x$

   $x = \frac{y}{2} \Rightarrow y = \frac{x}{2} = f^{-1}(x)$

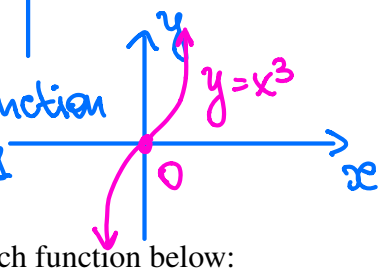
   

   (b) $f(x) = x^3$

   $y = x^3$

   $x = \sqrt[3]{y} \Rightarrow y = \sqrt[3]{x} = f^{-1}(x)$

   Cubic function is 1-to-1

2. Without explicitly finding a formula for $f^{-1}(x)$, find $f^{-1}(1)$ for each function below:

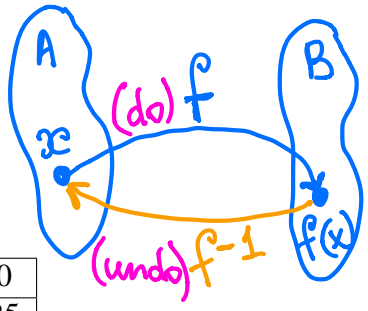
   (a) $f(x) = x - 20$

   $f^{-1}(1) = ?$ ← x-value

   ↑ y-value

   $1 = x - 20$

   $\boxed{x = 21} = f^{-1}(1)$

   (b)

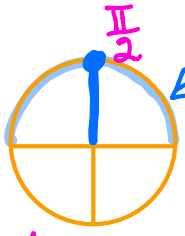
x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2.0
$f(x)$	20	10	5	3	2.5	2	1.5	1	0.25

   $y = 1$ and corresponded x value is $1.75 = f^{-1}(1)$

3. Evaluate $\cos^{-1}(0)$. $= \arccos(0)$

   $\cos\alpha = 0$

   $\alpha = \arccos(0) \Rightarrow \alpha = \frac{\pi}{2} \Rightarrow \cos^{-1}(0) = \frac{\pi}{2}$

   ← domain of $\cos^{-1}(x)$

4. Find the exact value of each expression.

   (a) $\log_2 16 = \log_2 2^4$

   $= 4 \log_2 2 = \boxed{4}$

   (b) $e^{\ln 5} = e^{\log_e 5} = \boxed{5}$

5. Solve each equation below for x.

(a) $10 = 2e^{x+1}$

$5 = e^{x+1}$

$\ln(5) = \ln(e^{x+1})$

$\ln 5 = (x+1)\ln e$

$\ln 5 = x+1$

$\boxed{x = \ln 5 - 1}$

(b) $\ln(x) + \ln(x-1) = 2$

$\ln(x(x-1)) = 2$

$e^{\ln(x^2-x)} = e^2$

$x(x-1) = e^2$

$x^2 - x - e^2 = 0$

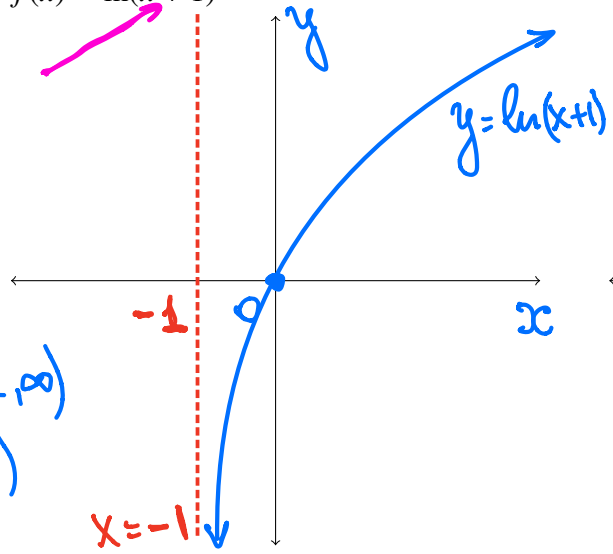
$D = 1 + 4e^2$

$x_1 = \frac{1+\sqrt{1+4e^2}}{2}$; $x_2 = \frac{1-\sqrt{1+4e^2}}{2}$

6. Sketch each function. Include domain, range, intercepts and asymptotes.

(a) $f(x) = \ln(x+1)$

Shift $f(x)=\ln x$ per (1) unit to the left



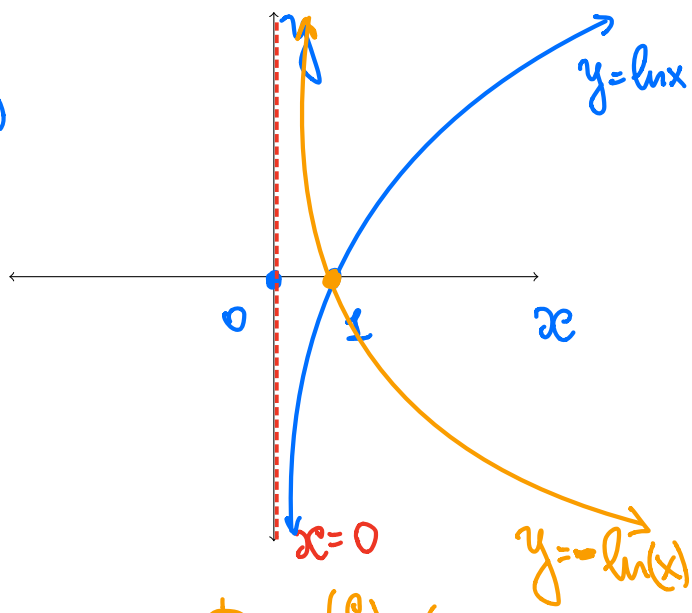
$\text{Dom}(f) = (-1, \infty)$

$E(f) = (-\infty, \infty)$

VA: $x = -1$

$(0,0)$ is a y-intercept point

(b) $f(x) = -\ln x$ ← reflect $y=f(x)$ about x-axis



$\text{Dom}(f) = (0, \infty)$

$E(f) = (-\infty, \infty)$

VA: $x = 0$

$(1,0)$ is an x-intercept point