Section 3.1 : Derivatives of elementary functions

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

limit exists

Example $f(x) = c$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0$$

$$\boxed{\begin{array}{c} (c)' = 0 \\ \frac{d}{dx}(c) = 0 \end{array}}$$

d - differential

Example 2 $f(x) = x^n$

$$(x^n)' = \frac{d}{dx}(x^n) = ?$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} =$$

$$= \lim_{h \to 0} \frac{h^n + n \cdot x \cdot h^{n-1} + \ldots + \cancel{x^n} - \cancel{x^n}}{h} =$$

$$(a+b)^n = a^n \cdot b^0 +$$

$$(a+b)^n = \sum_{k=0}^{n} C_n^k \, a^k b^{n-k} =$$

$$= C_n^0 \, a^0 \cdot b^n + C_n^1 \, a^1 \cdot b^{n-1} + \ldots +$$

$$+ C_n^n \, a^n \cdot b^0 = \underline{a^0 \cdot b^n + n \cdot a \cdot b^{n-1} + \ldots + a^n \cdot b^0}$$

$$C_n^0 = \frac{n!}{0! \cdot (n-0)!} = \frac{n!}{n!} = 1$$

$$0! = 1$$

$$C_n^1 = \frac{n!}{1!\,(n-1)!} = \frac{n!}{(n-1)!} = \frac{\cancel{1 \cdot 2 \cdot 3 \ldots (n-1)}\, n}{\cancel{1 \cdot 2 \cdot 3 \ldots (n-1)}}$$

$$1! = 1$$

$$= \boxed{n \cdot x^{n-1}}$$

$$\boxed{(x^n)' = \frac{d}{dx}(x^n) = n \cdot x^{n-1}}$$

Example. $f(x) = x^2$ $\qquad$ $f(x) = 100$

$$\frac{d}{dx}(x^2) = 2x \qquad (100)' = 0$$

$$f(x) = 2x^3$$

$$\frac{d}{dx}(2x^3) = 6x^2$$

Example $\quad g(x) = c \cdot f(x)$

$$\frac{d}{dx}(c \cdot f(x))$$

$$(c \cdot f(x))' = \lim_{h \to 0} \frac{c \cdot f(x+h) - c \cdot f(x)}{h} =$$

$$= \lim_{h \to 0} \frac{c(f(x+h) - f(x))}{h} = c \underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{f'(x)} =$$

$$= c \cdot f'(x)$$

$$\boxed{(c \cdot f(x))' = c \cdot f'(x)}$$

Worksheet: Section 3.1 (Derivatives of Polynomials and Exponential Functions)

1. Fill in the derivative rules: All you are being asked to do is write down the rest of the rules that you have learned about from the textbook and the Intro Video.

   Then practice using the rules to find the derivative for the given function.

   $(x)' = (x^1)' = 1 \cdot x^{1-1} = 1 \cdot x^0 = \boxed{1}$

   (a) $\frac{d}{dx}[c] =$ 0 $\quad h(x) = 5 \quad h'(x) =$ 0

   (b) $\frac{d}{dx}[x^n] =$ nx^{n-1} $\quad h(x) = x^{50} \quad h'(x) =$ $50x^{49}$

   (c) $\frac{d}{dx}[c\,f(x)] =$ $c \cdot \frac{d}{dx}(f(x))$ $\quad h(x) = 3x^2 \quad h'(x) =$ $(3x^2)' = 3 \cdot (x^2)' = 3 \cdot 2x = 6x$

   (d) $\frac{d}{dx}[f(x) + g(x)] =$ $\frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$ $\quad h(x) = 5x^6 + x^7 \quad h'(x) =$ $30 \cdot x^5 + 7x^6$

   (e) $\frac{d}{dx}[f(x) - g(x)] =$ $\frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x))$ $\quad h(x) = 6x^3 - x \quad h'(x) =$ $(6x^3 - x)' = (6x^3)' - (x)' = 18x^2 - 1$

   (f) $\frac{d}{dx}[e^x] =$ e^x $\quad h(x) = \frac{1}{2}e^x \quad h'(x) =$ $\left(\frac{1}{2}e^x\right)' = \frac{1}{2}e^x$

2. Compute the derivatives of the following functions using the above derivative rules.

   Do not simplify your answers. (If you already know what these are, DO NOT USE THE PRODUCT RULE, THE QUOTIENT RULE OR THE CHAIN RULE. If you don't know what they are, presumably you won't be using them either!)

   (a) $f(x) = (x-2)(2x+3)$ (You will need to do algebraic pre-processing first.)

   $f(x) = 2x^2 + 3x - 4x - 6 = 2x^2 - x - 6$

   $\frac{d}{dx}(f(x)) = \frac{d}{dx}(2x^2 - x - 6) = (2x^2)' - (x)' - (6)' = 4x - 1 - 0$

   $= \boxed{4x - 1}$

   (b) $g(x) = \frac{x^2}{2} - \frac{2}{x^2} + \frac{1}{\sqrt{2}}$

   $\frac{2}{x^2} = 2 \cdot x^{-2}$

   $(g(x))' = \left(\frac{x^2}{2} - \frac{2}{x^2} + \frac{1}{\sqrt{2}}\right)' = \left(\frac{x^2}{2}\right)' - \left(\frac{2}{x^2}\right)' + \left(\frac{1}{\sqrt{2}}\right)' =$

   $= \frac{1}{2} \cdot 2x - (-4) \cdot x^{-3} + 0 = x + \frac{4}{x^3}$

   (c) $f(t) = \sqrt{t} - e^t + t^{0.3}$

   $\sqrt{t} = t^{1/2}$

   $\frac{d}{dt}(f(t)) = (f(t))' = (\sqrt{t} - e^t + t^{0.3})' = (\sqrt{t})' - (e^t)' + (t^{0.3})' =$

   $= \frac{1}{2} \cdot t^{-1/2} - e^t + 0.3 \cdot t^{-0.7}$

   (d) $f(x) = \frac{x^2 + x - 1}{\sqrt{x}}$ $= (x^2 + x^1 - 1) \cdot x^{-1/2} = x^{3/2} + x^{1/2} - x^{-1/2}$

   $\frac{d}{dx}(f(x)) = (x^{3/2} + x^{1/2} - x^{-1/2})' = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$

   $\frac{3}{2}x^{3/2-1} \quad \frac{1}{2}x^{1/2-1} \quad -\frac{1}{2}x^{-\frac{1}{2}-1}$

$\pi \approx 3.14$

(e) $V(r) = \frac{4}{3}\pi r^3$

$$\frac{d}{dr}(V(r)) = \frac{d}{dr}\left(\underbrace{\frac{4}{3}}_{\text{constant}}\pi r^3\right) = \frac{4}{3}\pi(r^3)' = \frac{4}{\cancel{3}}\pi\cdot\cancel{3}\cdot r^2 =$$

$$= 4\pi r^2$$

(f) $f(x) = e^{x-3} = e^x \cdot \underbrace{e^{-3}}_{\text{constant}}$ $\quad (e^x)' = e^x$

$e \approx 2.7$

$$\frac{d}{dx}(e^{x-3}) = (e^x \cdot e^{-3})' = e^{-3}(e^x)' = e^{-3}\cdot e^x = e^{x-3}$$

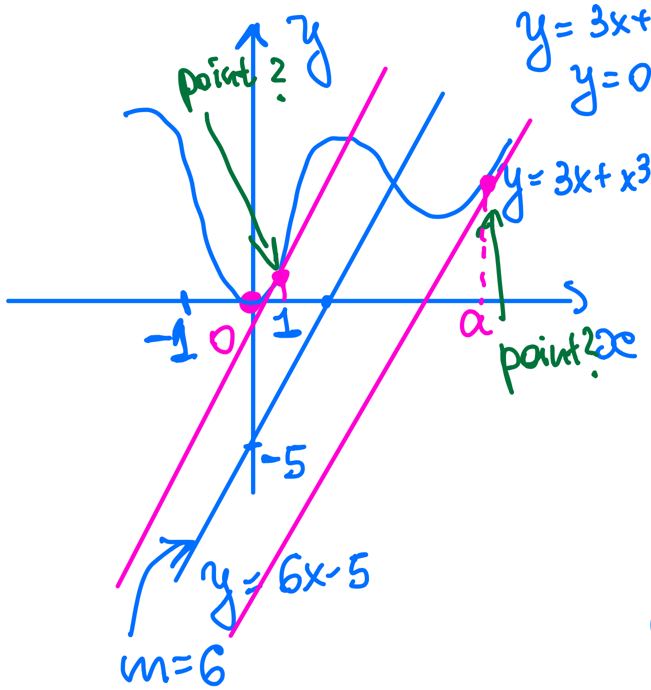
$$f(x) = e^x + e^{-3} \qquad (e^x + e^{-3})' = (e^x)' + (e^{-3})' = e^x + 0$$

(g) $H(r) = a^2r^2 + br + c$

$$\frac{d}{dr}(H(r)) = \frac{d}{dr}(a^2r^2 + br + c) = (a^2r^2)' + (br)' + (c)' =$$

$$= a^2\cdot 2\cdot r + b\cdot 1 + 0 = a^2\cdot 2r + b$$

3. At what point(s) on the curve $y = 3x + x^3$ is the tangent to the curve parallel to the line $y = 6x - 5$?



$y = 3x + x^3 = x(3 + x^2)$

$y = 0 \Rightarrow x(3 + x^2) = 0$

$x = 0$

$\parallel m$

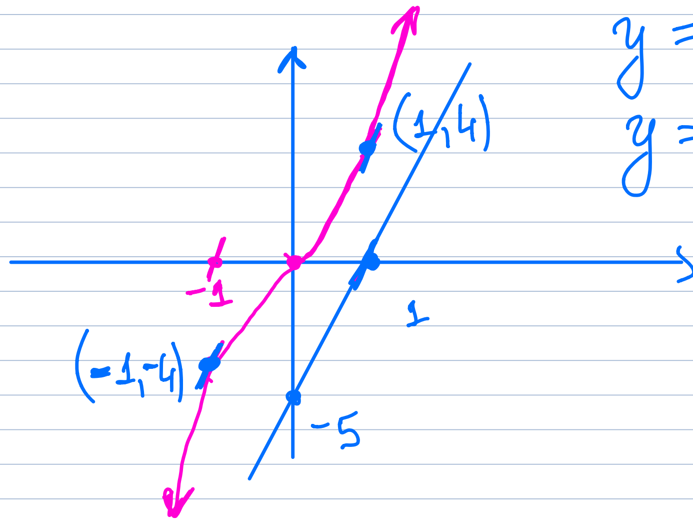
$y = 6x - 5$

$m = 6$

$m = f'(x) = (3x + x^3)'$

$6 = (3x + x^3)' = 3 + 3x^2$

$6 = 3 + 3x^2$



$$\underset{-3}{3} = \underset{-3}{3x^2}$$

$$x^2 = 1$$

$$x = \pm 1$$

$$y = 3x + x^3$$

$$y = 3 \cdot 1 + 1^3 = 4$$

$$y = 3 \cdot (-1) - 1 = -4$$

$(1, 4)$

$(-1, -4)$