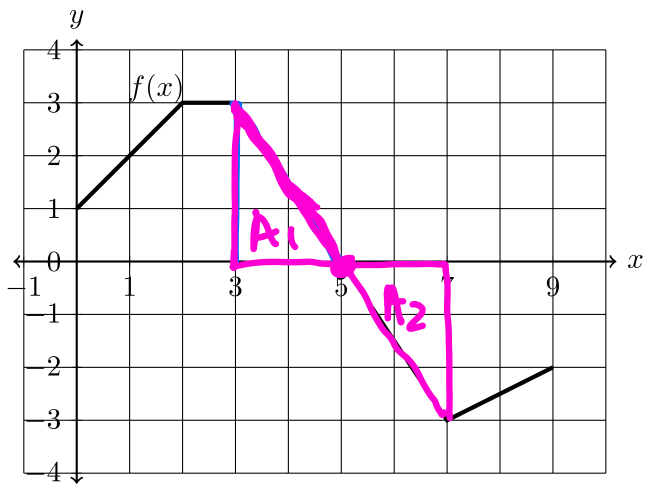
Section 5.2 (day two)

6. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



(a) $\int_2^5 f(x)dx = 3+3 = \boxed{6}$

$A_1 = 3$

$A_2 = \frac{1}{2}\cdot 2\cdot 3 = 3$

(b) $\int_5^9 f(x)dx = -3-4-1 = -8$

$A_1 = 3,\ A_2 = 4,\ A_3 = 1$

(c) $\int_3^7 f(x)dx = 0$

Properties of the Definite Integral:

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b c\,dx = c(b-a)$
- $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$
- $\int_a^b [f(x)\pm g(x)]dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_a^b f(x) + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

7. Using the fact that $\int_0^1 x^2 dx = \frac{1}{3}$ and $\int_1^2 x^2\,dx = \frac{7}{3}$, evaluate the following using the properties of integrals.

(a) $\int_1^0 x^2\,dx =$

$= -\int_0^1 x^2 dx = -\frac{1}{3}$

(b) $\int_0^1 5x^2\,dx =$

$= 5\cdot\int_0^1 x^2 dx =$

$= 5\cdot\frac{1}{3}$

(c) $\int_0^1 (4+3x^2)\,dx =$

$= \int_0^1 4\,dx + 3\int_0^1 x^2 dx =$

$= 4(1-0) + 3\cdot\frac{1}{3} =$

$= 4+1 = 5$

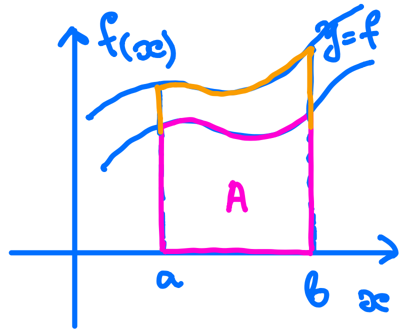
(d) $\int_0^2 x^2\,dx. =$

$= \int_0^1 x^2 dx + \int_1^2 x^2 dx =$

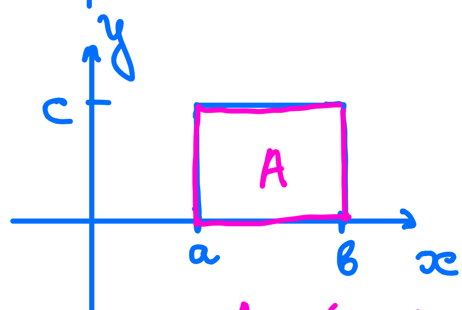
$= \frac{1}{3} + \frac{7}{3} = \frac{8}{3}$

Definite integral properties:

1. $$\int_a^b c \cdot f(x) = c \underbrace{\int_a^b f(x)\,dx}_{= A}$$

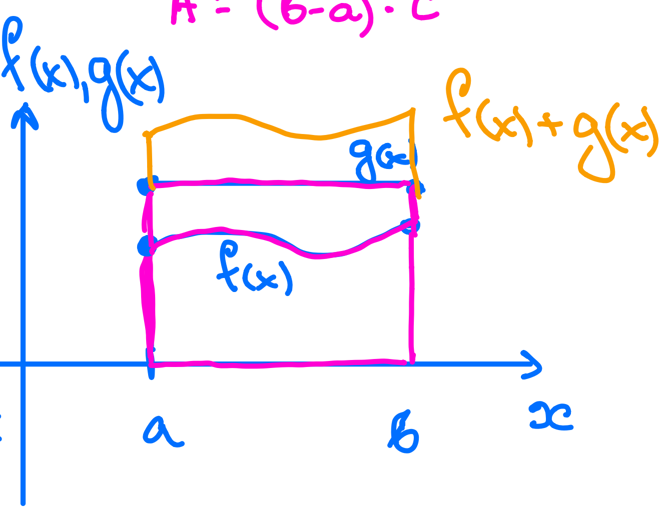


$C = 2$

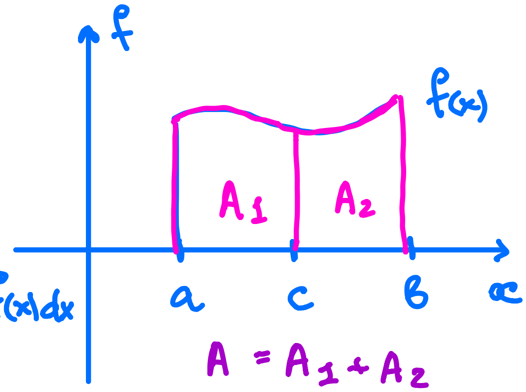


$A = (b-a) \cdot c$

$$\int_a^b \left(f(x) + g(x)\right)dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$



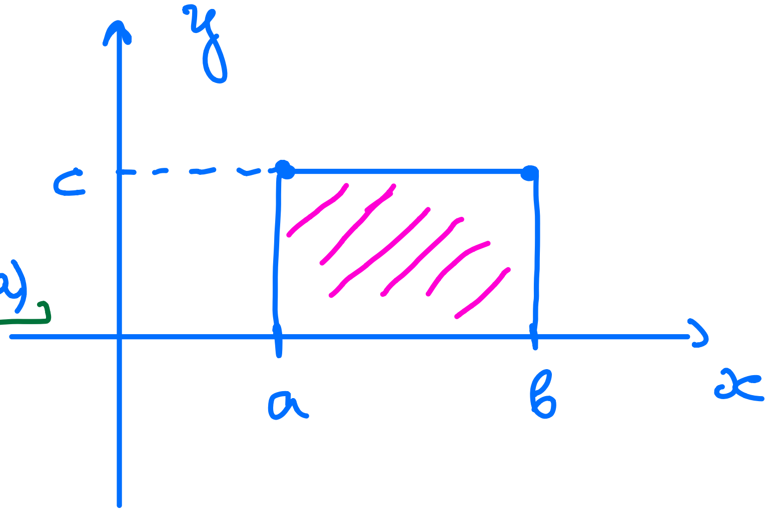
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$A = A_1 + A_2$

$\underline{f(x) = c}$

$$\int_a^b c\,dx = \underbrace{c(b-a)}_{}$$

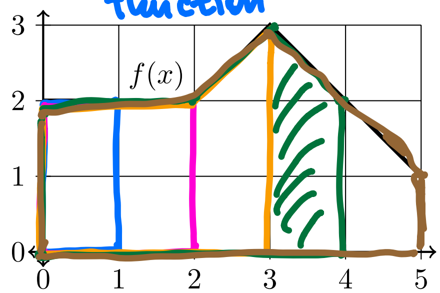


$$\int_b^a c\,dx = c(a-b) = -\underbrace{c(b-a)}_{}$$

"Area So Far" functions

1. Let $f(x)$ be given by the graph below and define $A(x) = \int_0^x f(t)dt$. $\int_a^b f(x)dx$

function



Compute the following using the graph. Hint: $A(1) = \int_0^1 f(x)\,dx$, which calculates the area accumulated under the graph from $x = 0$ to $x = 1$.

$A(1) =$ $\int_0^1 f(t)dt = 1\cdot 2 = 2$

$A(2) =$ $\int_0^2 f(t)\,dt = 4$

$A(3) =$ $\int_0^3 f(t)dt = 6\frac{1}{2}$

$A(4) =$ $6\frac{1}{2} + 2\frac{1}{2}$

$A(5) =$ $10\frac{1}{2}$

$f(1) =$ 2

$f(2) =$ 2

$f(3) =$ 3

$f(4) =$ 2

$f(5) =$ 1

The x-value in the interval $[0, 5]$ at which $A(x)$ attains its maximum is $x = 5$

The maximum value of $A(x)$ on $[0, 5]$ is $10\frac{1}{2}$

The x-value in the interval $[0, 5]$ at which $f(x)$ attains its maximum is $x = 3$

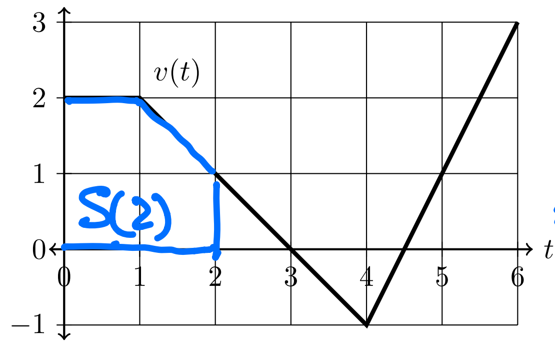
The maximum value of $f(x)$ on $[0, 5]$ is 3

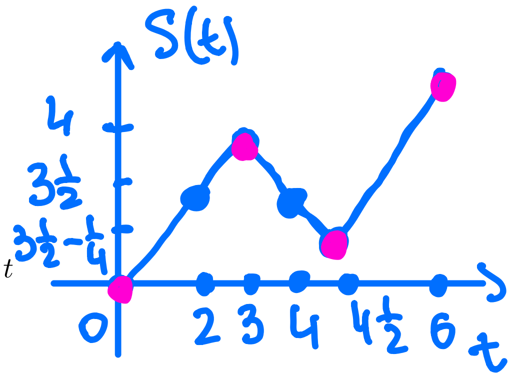
What can you say about the **rate of change** of $A(x)$? always positive

rate of change $= \frac{\Delta A}{\Delta x} = \frac{\Delta A}{1} = \Delta A$ $\overbrace{\left(2, 4, 6\frac{1}{2}, 9, 10\frac{1}{2}\right)}^{\text{always positive}}$

A is increasing

2. A toy car is travelling on a straight track. Its velocity $v(t)$, in meters per second, is given by the graph below. Define $s(t)$ to be the position of the car in meters, and suppose that $s(0) = 0$. Note that $s(t) = \int_0^t v(x)\,dx$. (Here, x is called the "dummy variable of integration".)





Compute the following:

$s(2) =$ $\int_0^2 v(x)\,dx = 3\frac{1}{2}$ $\quad s(4) =$ $\int_0^4 v(x)\,dx = 4 - \frac{1}{2} = 3\frac{1}{2}$ $\quad s(6) =$ $\int_0^6 v(x)\,dx = 4 - \frac{1}{2} \cancel{- \frac{1}{4} + \frac{1}{4}} + 1 + 1 = 5\frac{1}{2}$

$v(2) =$ 1 $\quad v(4) =$ −1 $\quad v(6) =$ 3

The t-value in the interval $[0, 6]$ at which $s(t)$ attains its maximum is $t = 6$

The maximum value of $s(t)$ on $[0, 6]$ is $5\frac{1}{2}$

The t-value in the interval $[0, 6]$ at which $s(t)$ attains its minimum is $t = 4.5$

The minimum value of $s(t)$ on $[0, 6]$ is $3\frac{1}{2} - \frac{1}{4}$

The t-value in the interval $[0, 6]$ at which $v(t)$ attains its maximum is $t = 6$

The maximum value of $v(t)$ on $[0, 6]$ is 3

The t-value in the interval $[0, 6]$ at which $v(t)$ attains its minimum is $t = 4$

The minimum value of $v(t)$ on $[0, 6]$ is −1

Describe the position of the car over the 6 seconds. The car move forward for the first 3 sec, then it moves backward for the next 1.5 sec, the it again moves forward for the next 1.5 sec

Describe the velocity of the car over the 6 seconds. v(t) is stable for the 1st 1 sec, then it is decreasing for the next 3 sec, then it starts increasing for the remaining time.