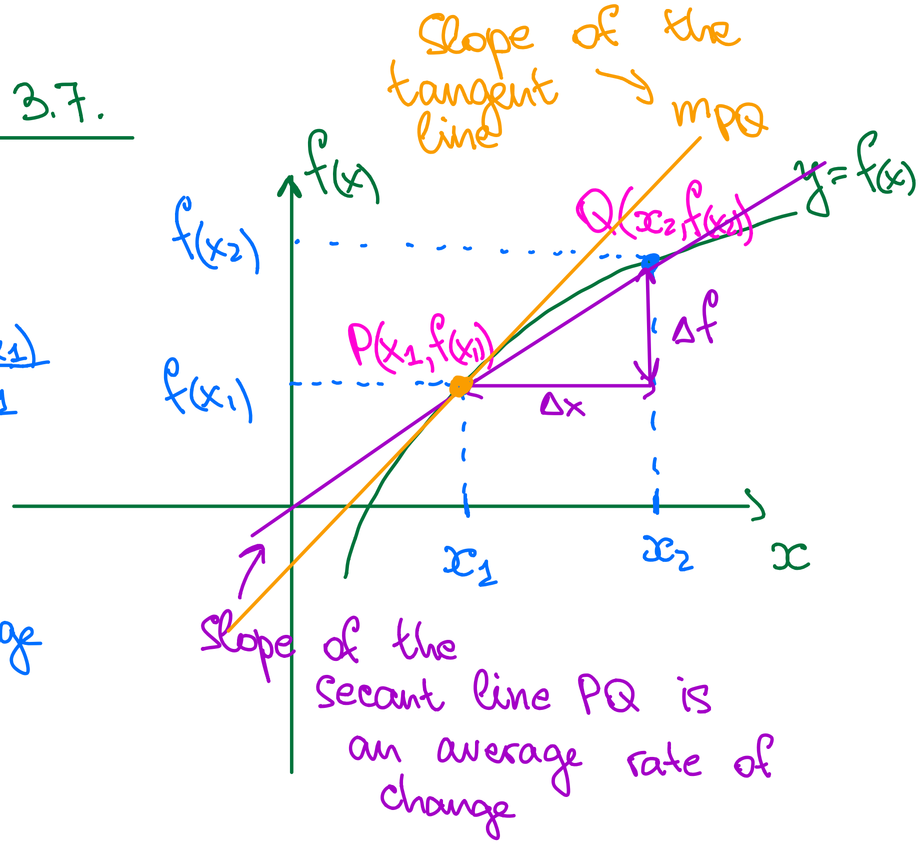
Section 3.7.

$[x_1, x_2]$

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

average rate of change



$$\boxed{\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

Instantaneous rate of change:

$$\boxed{f'(x) = \lim_{x_2 \to x_1} \frac{\Delta f}{\Delta x} = \lim_{x_2 - x_1 \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. A stone is thrown in a pond and a circular ripple travels outward at a speed of 60 cm/s. Determine the rate of change of area inside the ripple at time $t = 1$ second and at time $t = 2$ seconds. (Hint: draw a picture!)

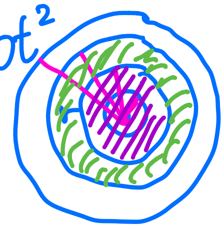
$[1, 2]$

$r = r(t)$ $\quad A = \pi r^2$

$v = 60\,cm/s$ $\quad A = \pi(60t)^2 = \pi \cdot 3600 t^2$

$v = \frac{r}{t} = 60\,cm/s \Rightarrow r = 60 \cdot t\,(cm)$

$r(t) = 60t$



$A'(1) = ?$ $\quad A'(t) = 3600 \cdot 2 \cdot \pi \cdot t = 7200\pi \cdot t$

$A'(1) = 7200\pi\ (cm^2/s)$

3. A population of caribou is growing, and its population is

$$P(t) = 4000\frac{3e^{t/5}}{1 + 2e^{t/5}}.$$

(a) What is the population at time $t = 0$? (initial population)

$$P(0) = 4000\,\frac{3 \cdot e^0}{1 + 2 \cdot e^0} = 4000\,\frac{3 \cdot 1}{1 + 2 \cdot 1} = 4000\ (\text{caribou})$$

(b) Determine the rate of change of the population at any time t.

$$P'(t) = 4000\,\frac{(3e^{t/5})'(1+2e^{t/5}) - (1+2e^{t/5})'(3e^{t/5})}{(1+2e^{t/5})^2} = 4000\,\frac{\frac{3}{5}e^{t/5}(1+2e^{t/5}) - \frac{2}{5}e^{t/5} \cdot 3e^{t/5}}{(1+2e^{t/5})^2}$$

(c) Determine the rate of change of the population at time $t = 0$ years. (initial)

$$P'(0) = 4000\,\frac{\frac{3}{5} \cdot 3 - \frac{2}{5} \cdot 3}{9}\ (\text{caribou/years})$$

(d) Determine the long term population.

$$\lim_{t \to \infty} P(t) = \lim_{t \to \infty} 4000\,\frac{3e^{t/5}}{1 + 2e^{t/5}} = 4000 \lim_{t \to \infty} \frac{3e^{t/5}}{1 + 2e^{t/5}} =$$

$$= 4000 \lim_{t \to \infty} \frac{\cancel{e^{t/5}} \cdot 3}{\cdots} = 4000 \cdot \frac{3}{2}$$

$e^{-15}\left(\cancel{e^{t/5}}\right) + (2)$ (caribou)

1. A particle moves according to the law of motion $s(t) = 2 - 15t + 4t^2 - \frac{1}{3}t^3$, for $t \geq 0$, where t is measured in seconds and s is measured in feet.

   (a) Find the velocity at time t.

   $v(t) = s'(t) = -15 + 8t - t^2$

   (b) What is the velocity after 1 second?

   $v(1) = -15 + 8 - 1 = -8 \ (ft/s)$

   (c) When is the particle at rest?

   $v(t) = 0 \Rightarrow -15 + 8t - t^2 = 0 \mid \cdot(-1)$

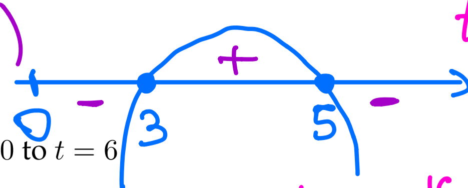
   $t^2 - 8t + 15 = 0 \Rightarrow (t-5)(t-3) = 0$

   $t_1 = 5\,(s),\ t_2 = 3\,(s)$

   (d) When is the particle moving in the positive direction?

   $v(t) > 0$ on $(3,5)$

   $t^2 - 8t + 15 > 0$

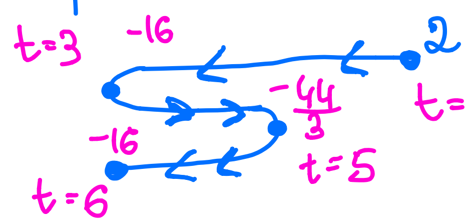


   (e) Draw a diagram of the particle from $t = 0$ to $t = 6$.

   $s(t) = 2 - 15t + 4t^2 - \frac{1}{3}t^3$

   $s(0) = 2$, $s(3) = 2 - 30 + 4 \cdot 9 - \frac{1}{3} \cdot 27 = -16$

   $s(5) = -\frac{44}{3}$, $s(6) = -16$



   (f) Find the *displacement* of the particle during the first 6 seconds.

   $s(6) - s(0) = -16 - 2 = -18$

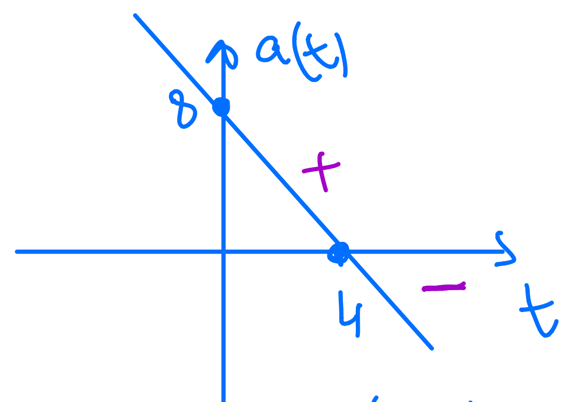
   (g) Find the *total distance traveled* by the particle during the first 6 seconds.

   Total distance $= |-18| + 2\left|-\frac{44}{3} + 16\right| = 20\frac{2}{3}$

   (h) Find the acceleration of the particle.

   $a(t) = v'(t) = -2t + 8$

   (i) Graph the acceleration function.



   (j) When is the particle speeding up?

   $v(t) > 0$ on $(3,5)$
   $a(t) > 0$ on $(0,4)$ $\Rightarrow \boxed{(3,4)}$ Speeding up

   or $v(t) < 0$ on $(0,3) \cup (5,\infty)$
   $a(t) < 0$ on $(4,\infty)$ $\Rightarrow \boxed{(5,\infty)}$ Speeding up

2. The height (in meters) of a projectile shot vertically upward from a point 10 meters above ground lever with an initial velocity of 20 meters per second is $h = 10 + 20t - 4.9t^2$.

   (a) When does the projectile reach its maximum height?

   (b) What is its maximum height?

   (c) When does the projectile hit the ground?

   (d) What what velocity does it hit the ground?

3. A tank holds 1000 gallons of a fluid, which drains from the bottom of the tank in 30 minutes. The function below give the volume of fluid remaining in the tank after t minutes:

$$V(t) = 1000\left(1 - \frac{1}{30}t\right)^2 \text{ for } 0 \leq y \leq 30$$

   Find the rate at which the fluid is draining from the tank after 10 minutes. When is the fluid flowing the fastest? Slowest?