Section 1.1. Four ways to present a function

Def. A function is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

D is a domain
E is a range
x is an independent variable
$f(x)$ is a dependent variable

Def. If f is a function with domain D, then its graph is the set of ordered pairs

$$\{(x, f(x)) \mid x \text{ in } D\}$$

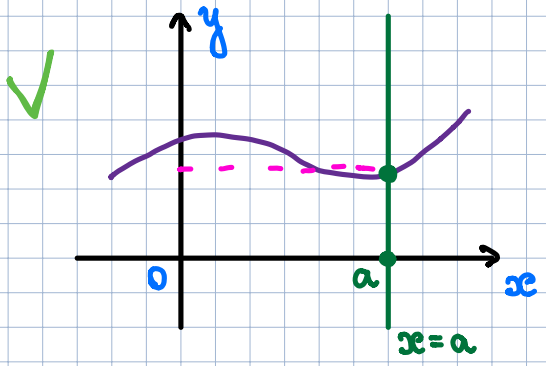
- The graph of a function is a curve in the xy-plane.

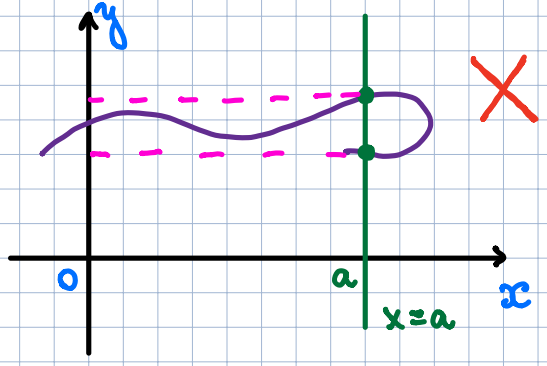
Question: which curves in the xy-plane are graphs of functions?

The vertical line test

A curve in the xy-plane is the graph of a function of x if and only if no vertical

line intersects the curve more than once.

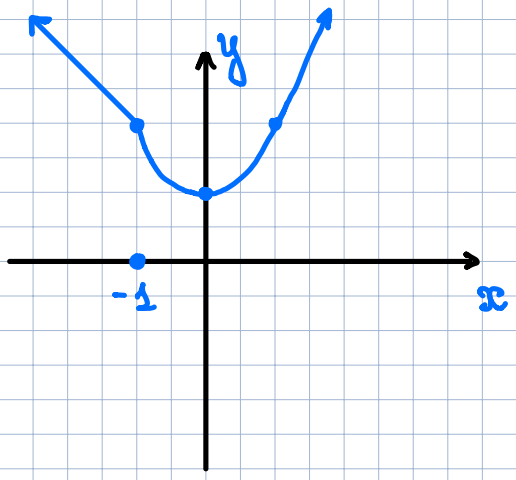




Piecewise Defined Functions

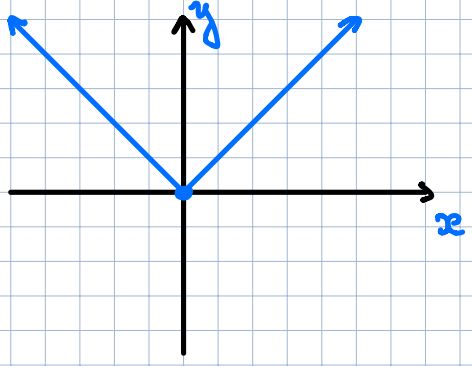
Example 1

$$f(x)=\begin{cases}1-x, & x\le -1\\ x^2+1, & x>-1\end{cases}$$



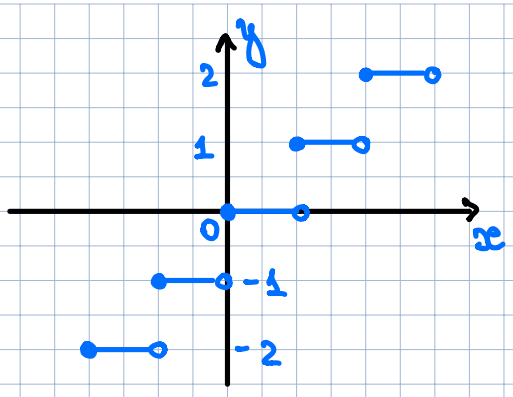
Example 2 (absolute value function)

$$f(x)=|x|=\begin{cases}x, & x\ge 0\\ -x, & x<0\end{cases}$$



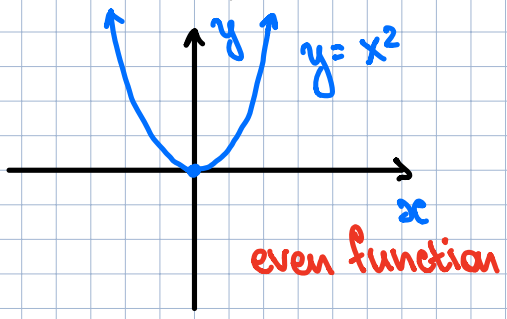
Example 3 (step function)

$$f(x)=[x]$$

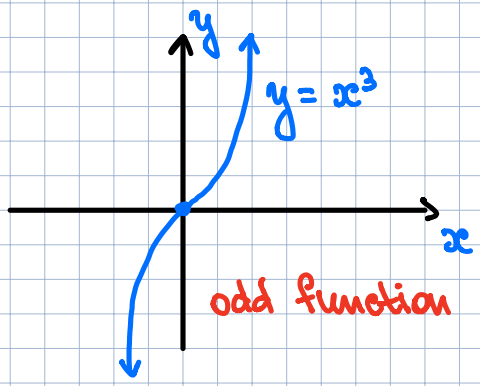


Symmetry

- If a function f satisfies $f(-x)=f(x)$ for every $x \in D$, then f is called an **even** function.
- If a function f satisfies $f(-x)=-f(x)$ for every $x \in D$, then f is called an **odd** function.



even function



odd function

Increasing and decreasing functions

Def. A function f is called increasing on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called decreasing on I if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$