Section 3.5. Implicit Differentiation

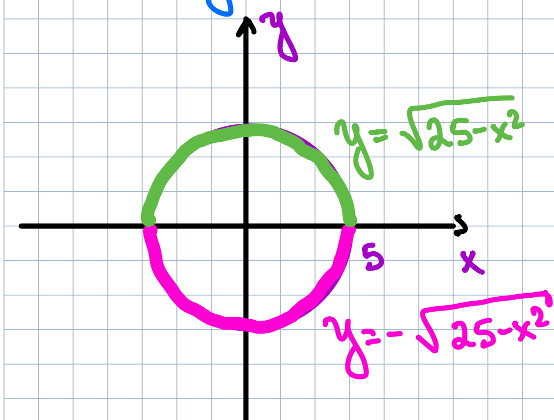
Previously, we just considered the following functions

$$y = \sqrt{x^3+1} \quad \text{or} \quad y = x\sin x$$

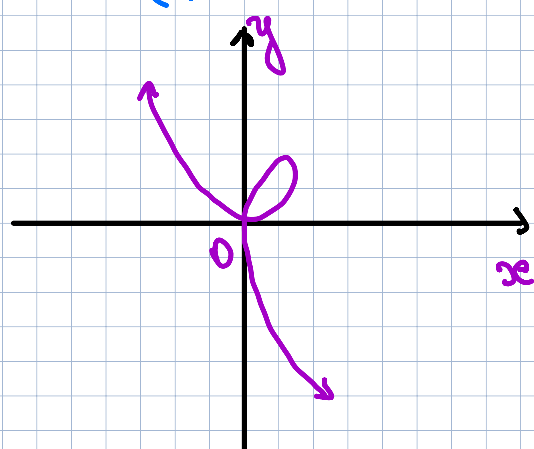
in general $\quad y = f(x)$.

Some functions are defined implicitly:

1) $x^2 + y^2 = 25$



2) $x^3 + y^3 = 6xy$
(folium of Descartes)



$x^3 + y^3 = 6xy$ is defined implicitly:

$$x^3 + [f(x)]^3 = 6x\,[f(x)]$$

is true for all values of x in the domain of $f(x)$.

Implicit Differentiation:

Example $\quad x^2 + y^2 = 25 \qquad y = y(x)$

$\frac{dy}{dx}$?

$$2x + 2y \cdot y' = 0$$

$$x + y \cdot y' = 0$$

$$y' = -\frac{x}{y}$$

Derivatives of Inverse Trigonometric Functions:

If $f(x)$ is 1-1 differ. function, then $f^{-1}(x)$ is differ., except where its tangents are vertical.

- $y = \arcsin(x)$

$\sin y = x, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$$\cos y \cdot \frac{dy}{dx} = 1 \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{\cos y}$$

$\cos y \geq 0$, since $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - x^2}$$

Thus, $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$$\boxed{\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}}$$

- $y = \arctan(x)$

$\tan y = x$ $\qquad \sec^2 y \cdot \frac{dy}{dx} = 1$

$$\Downarrow$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$$

$$\boxed{\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}}$$

<u>Example</u> Differentiate:

$$y = \frac{1}{\arcsin(x)}$$

$$y' = -\frac{1}{(\arcsin(x))^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

Derivatives of Inverse Trigonometric Functions:

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\operatorname{arccsc} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\operatorname{arccot} x) = -\frac{1}{1+x^2}$$