Day one

Section 4.1 Max and Min values of the function $f(x)$

Def. 1 Let c be the number in the domain D of the function $f(x)$. Then

- $f(c)$ is an (global) abs. max if

  $f(c) \geq f(x)$ for all x in D

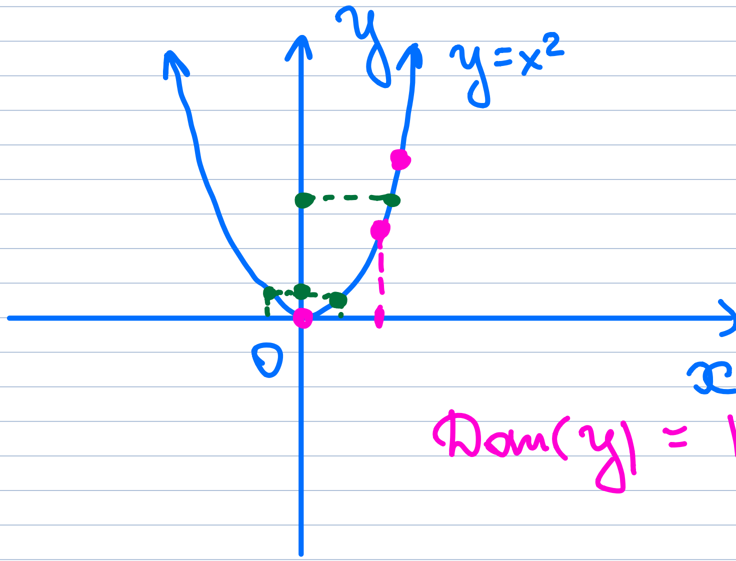
- $f(c)$ is an (global) abs. min if

  $f(c) \leq f(x)$ for all x in D

Def 2

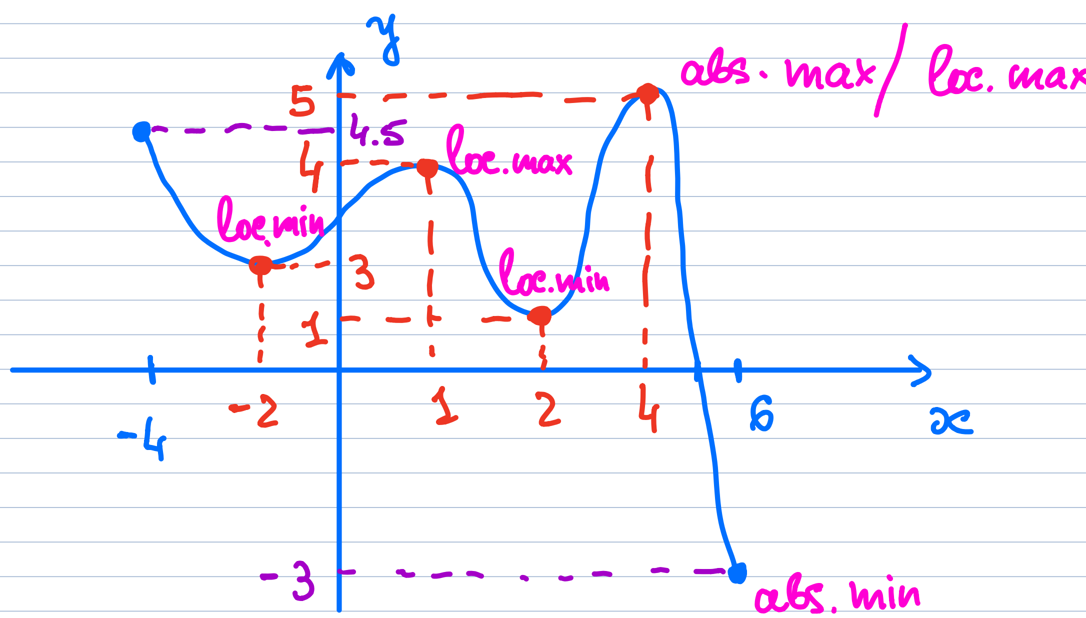
- The number $f(c)$ is a loc. max if $f(c) \geq f(x)$ only for x near c

- The number $f(c)$ is a loc. min if $f(c) \le f(x)$ only for x near c



abs. max DNE
abs. min = 0
loc. max DNE
loc. min 0

Dom(y) = ℝ

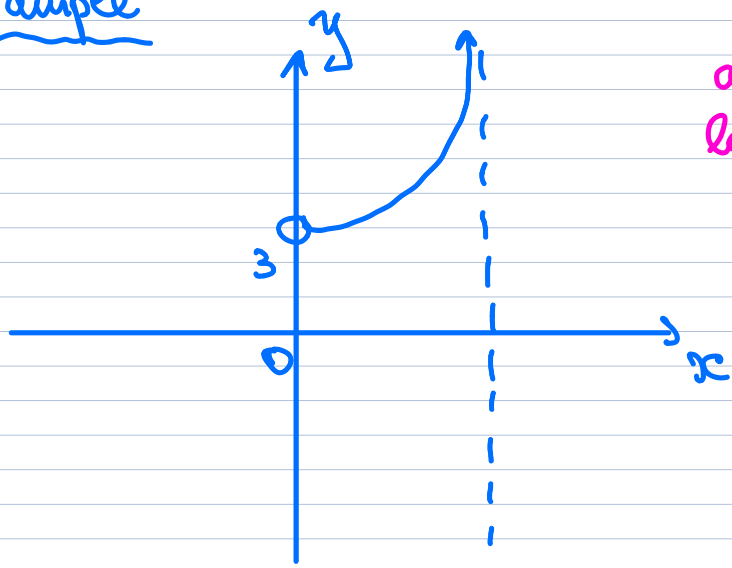


$f(x)$ has an abs. max at $x=4$

and it is 5 abs. max $f(4)=5$

$f(x)$ has an abs. min at $x=6$
and it is -3
abs. min $f(6)=-3$

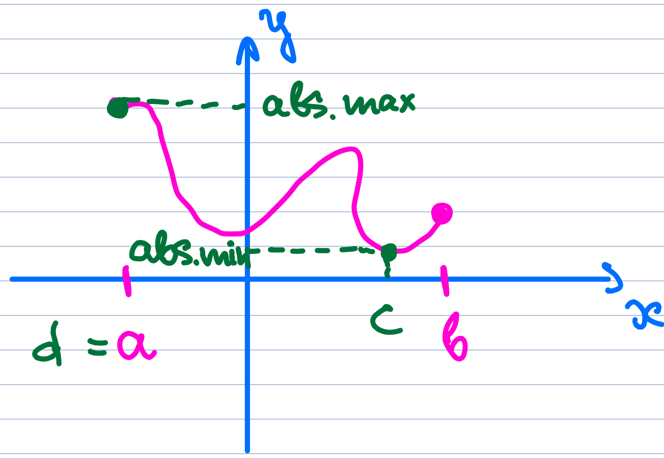
Example



abs max/min DNE
loc max/min DNE

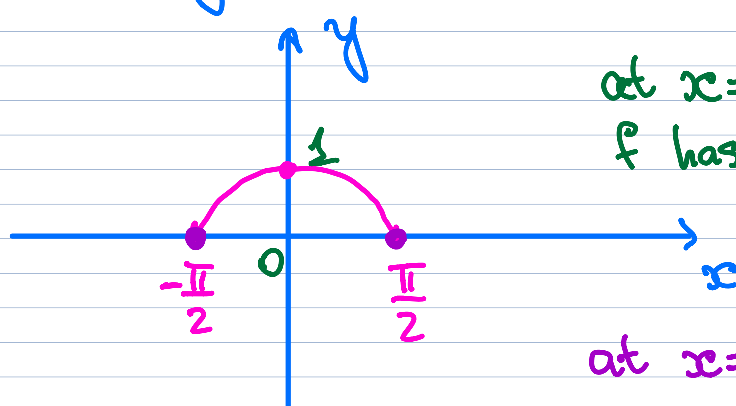
Theorem (The Extreme Value Theorem)
(abs max + min values)

If the $f(x)$ is continuous on the closed interval $[a,b]$, then $f(x)$ attains its abs. maximum and minimum values at points $x = c$ and $x = d$ which are in $[a, b]$.



Example

$y = \cos(x)$, continuous $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



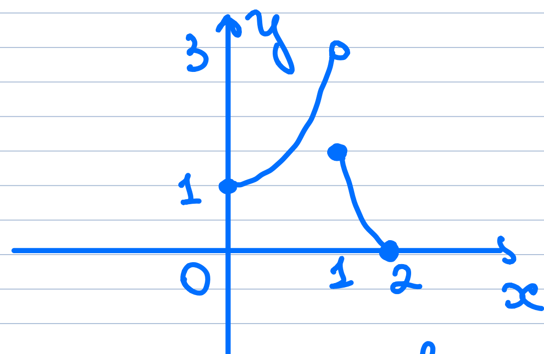
at $x = 0$
f has abs. max
$y = 1$

at $x = -\frac{\pi}{2}, \frac{\pi}{2}$
f has abs. min
$y = 0$

Section 4.1. Day two

Note: The local max/min values of the function $f(x)$ don't occur at the endpoints of the given interval

Example

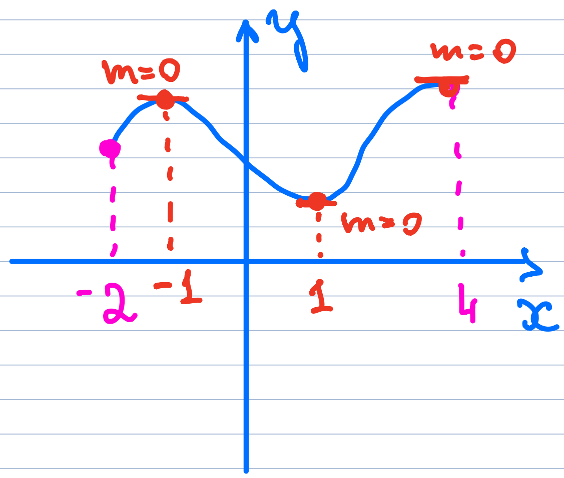


abs. max	DNE	DNE
abs. min	0	$f(2)=0$
loc. max	DNE	
loc. min	DNE	

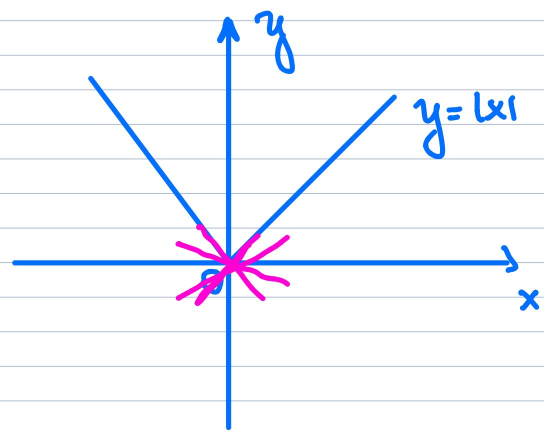
Def 3. A critical number (point) of the function $f(x)$ is a number c in the domain of $f(x)$ such that

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ DNE}$$

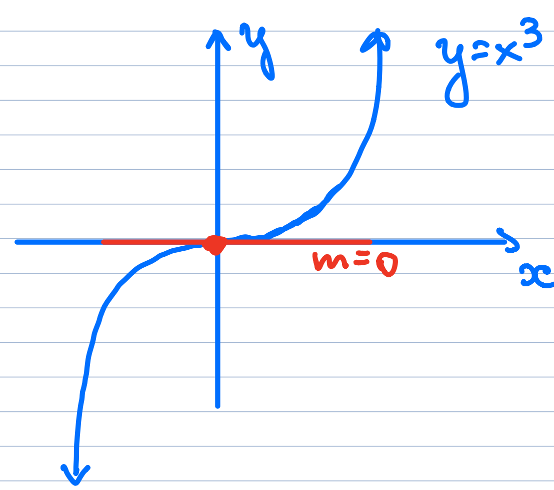
Example



$x = -1, 1, 4$ critical points



$x = 0$ critical point because $f'(0)$ DNE



$x = 0$ is a critical point since $f'(0) = 0$

$f'(x) = 3x^2 = 0$

$3x^2 = 0$

$x^2 = 0 \Rightarrow \underline{x = 0}$

The Closed Interval Method:

Let f be a continuous function on the closed interval $[a, b]$.

To find an abs. max and abs. min values of $f(x)$ we need to:

1. find all critical points of $f(x)$ and evaluate our function $f(x)$ at these points
2. evaluate function $f(x)$ at endpoints of the given closed interval

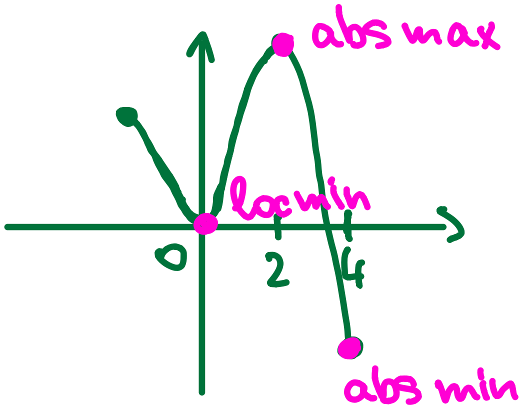
$$f(a), \quad f(b)$$

3. abs max of $f(x) = \max\{1, 2\}$

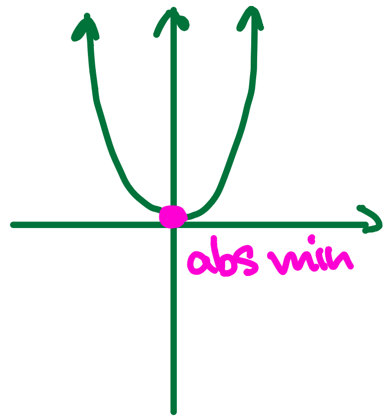
   abs min of $f(x) = \min\{1, 2\}$

1. Sketch a graph $f(x)$ whose domain is the interval $[-1, 4]$ with the following properties:

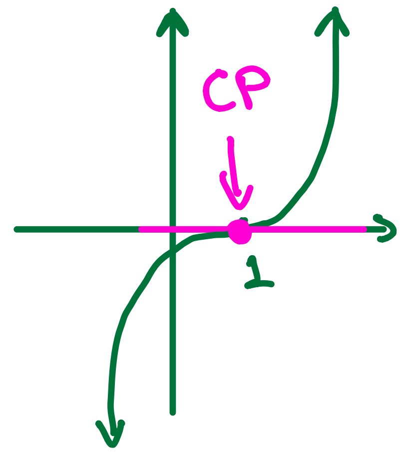
(a) f is continuous, has a local minimum at $x = 0$, an absolute minimum at $x = 4$ and an absolute maximum at $x = 2$.



(b) f has an absolute minimum but no absolute maximum



(c) f has a critical point at $x = 1$ but no maximum or minimum (of any type) at $x = 1$.



2. Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$. Determine where those absolute maximum and minimum values occur.

$f(x) = x - x^{1/3}$, $\quad Dom(f) = [-1, 4]$

1. Find critical points:

$f'(x) = 0$ or $f'(x)$ DNE

$f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3}\frac{1}{x^{2/3}}$

$\boxed{x = 0}$ critical point since $f'(0)$ DNE

$1 - \frac{1}{3}\frac{1}{x^{2/3}} = 0$

$\frac{1}{3}\frac{1}{x^{2/3}} = 1$

$\frac{1}{x^{2/3}} = 3$

$x^{2/3} = \frac{1}{3} \Rightarrow \boxed{x = \frac{1}{\sqrt{3^3}}}$ critical point since $f'\left(\frac{1}{\sqrt{3^3}}\right) = 0$

$f(0) = \boxed{0}$

$f\left(\frac{1}{\sqrt{3^3}}\right) = \frac{1}{\sqrt{3^3}} - \left(\frac{1}{\sqrt{3^3}}\right)^{1/3} = \boxed{\frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}}}$

2. $f(-1) = -1-(-1)^{1/3} = \boxed{0}$

$f(4) = \boxed{4-\sqrt[3]{4}}$

3. $f_{min}^{abs} = \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{1}{3}-1\right)$ at $x = \frac{1}{\sqrt{3^3}}$

$f_{max}^{abs} = 4-\sqrt[3]{4}$ at $x=4$

3. Find the absolute maximum and minimum values of $f(x) = x + \dfrac{1}{x}$ on the interval $[1/5, 4]$. Determine where those absolute maximum and minimum values occur.

$f(x) = x + \frac{1}{x}$, $x \neq 0$ $[\frac{1}{5}, 4]$

1. $f'(x) = 0$ or $f'(x)$ DNE

$f'(x) = 1 - \frac{1}{x^2} = 0$

$1 - \frac{1}{x^2} = 0$

$x^2 = 1$

$x = \pm 1$

$\boxed{x = 1}$ is a CP

$\boxed{x = -1}$ is not a CP since $x = -1$ is not in $[\frac{1}{5}, 4]$

at $\boxed{x = 0}$ $f'(0)$ DNE

is not a CP since $x = 0$ is not in $[\frac{1}{5}, 4]$

4. Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-8, 8]$. Determine where those absolute maximum and minimum values occur.

$f(x) = \sqrt[3]{x^2}$, $[-8, 8]$

1. CP:

$f'(x) = 0$ or $f'(x)$ DNE

$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3} \frac{1}{x^{1/3}}$

$f'(x) = 0$

$\frac{2}{3} x^{-1/3} \cancel{=} 0$ Never

$f'(x)$ DNE at $x = 0$

and $x = 0$ in $[-8, 8]$.

Hence, $x = 0$ is a CP

$f(0) = 0$

2. $f(-8) = \sqrt[3]{64} = 4$

$f(8) = \sqrt[3]{64} = 4$

$f^{abs}_{max} = 4$ at $x = \pm 8$

$f^{abs}_{min} = 0$ at $x = 0$

Problem (continue)

3. $x = 1$ is a CP

$$f(1) = 1 + \frac{1}{1} = \boxed{2}$$

2. $$f\left(\frac{1}{5}\right) = \frac{1}{5} + \frac{1}{\frac{1}{5}} = \boxed{5 + \frac{1}{5}}$$

$$f(4) = \boxed{4 + \frac{1}{4}}$$

3. $f^{abs}_{max} = 5 + \frac{1}{5}$ at $x = \frac{1}{5}$

$f^{abs}_{min} = 2$ at $x = 1$