# Section 1.3. New functions from old functions

- **Translations**

  - Vertical and Horizontal Shifts:

Suppose $c > 0$. To obtain the graph of

1. $y = f(x) + c$, shift $y = f(x)$ per $c$ units upward
2. $y = f(x) - c$, shift $y = f(x)$ per $c$ units downward
3. $y = f(x - c)$, shift $y = f(x)$ per $c$ units to the right
4. $y = f(x - c)$, shift $y = f(x)$ per $c$ units to the left

- **Stretching and Reflecting**

  - Vertical and Horizontal Stretching and Reflecting:

Suppose $c > 1$. To obtain the graph of

1. $y = c\,f(x)$, stretch $y = f(x)$ vertically by $c$
2. $y = \left(\frac{1}{c}\right) f(x)$, shrink $y = f(x)$ vertically by $c$
3. $y = f(cx)$, shrink $y = f(x)$ horizontally by $c$
4. $y = f\left(\frac{x}{c}\right)$, stretch $y = f(x)$ horizontally by $c$

5. $y = -f(x)$, reflect $y = f(x)$ about $x$-axis

6. $y = f(-x)$, reflect $y = f(x)$ about $y$-axis

## Combinations of functions

Let Domain $(f) = A$

Domain $(g) = B$

Then

- $(f+g)(x) = f(x) + g(x)$ and Domain $(f+g) = A \cap B$
- $(f-g)(x) = f(x) - g(x)$ and Domain $(f-g) = A \cap B$
- $(f \cdot g)(x) = f(x) \cdot g(x)$ and Domain $(f \cdot g) = A \cap B$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ and Domain $\left(\frac{f}{g}\right) = \{x \text{ in } A \cap B \mid g(x) \neq 0\}$

**Def.** Given two functions $f$ and $g$, the composite function $f \circ g$ (composition of $f$ and $g$) is defined by

$$(f \circ g)(x) = f(g(x))$$

Domain $(f \circ g) = \{x \text{ in } \mathrm{Dom}(g) \mid g \text{ in } \mathrm{Dom}(f)\}$

Composition $f \circ g$ is defined if and only if $g(x)$ and $f(g)$ are defined.