Section 5.5. day one (ending)

5. Compute $\int \frac{\arctan(x)}{1+x^2}\,dx = \int u \cdot du = \frac{u^2}{2} + C = \boxed{\frac{(\arctan(x))^2}{2} + C}$

- $u = \arctan(x)$
- $du = \frac{1}{1+x^2} \cdot dx$

6. Compute $\int \frac{x^3}{\sqrt{1-x^4}}\,dx = \int \frac{\cancel{x^3}}{\sqrt{u}} \frac{du}{-4\cancel{x^3}} = -\frac{1}{4}\int \frac{du}{\sqrt{u}} = -\frac{1}{4}\int u^{-1/2}\,du =$

$u = 1 - x^4$

$\frac{du}{dx} = -4x^3$

$du = (-4x^3) \cdot dx$

$dx = \frac{du}{-4x^3}$

$= -\frac{1}{4} \frac{u^{-1/2+1}}{-\frac{1}{2}+1} + C =$

$= \boxed{-\frac{1}{4} \frac{(1-x^4)^{1/2}}{1/2} + C}$

7. Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx$. $= \int \frac{\cancel{x}}{\sqrt{1-u^2}} \frac{du}{2\cancel{x}} =$

$u = x^2$

$du = 2x \; dx$

$dx = \frac{du}{2x}$

$= \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin(u) + C$

$= \frac{1}{2}\arcsin(x^2) + C$

8. Compute $\int_0^{\pi/6} \frac{\sin(t)}{(cos(t))^2}\,dt$ two ways: (1) by computing the antiderivative using substitution and then using FTC2 to evaluate using the original bounds; (2) by substituting and changing the bounds to match the substitution.

$\int_0^{\frac{\pi}{6}} \frac{\sin(t)}{(\cos(t))^2}\,dt = \int_{-1}^{-\frac{\sqrt{3}}{2}} \frac{\cancel{\sin(t)}}{(-u)^2} \frac{du}{\cancel{\sin(t)}} = \int_{-1}^{-\frac{\sqrt{3}}{2}} u^{-2}\,du =$

$u = -\cos(t)$

$du = \sin(t)\,dt$

$dt = \frac{du}{\sin(t)}$

$0 \le t \le \frac{\pi}{6}$

$-1 \le u \le -\frac{\sqrt{3}}{2}$

$$= \frac{u^{-1}}{-1} \Big|_{-1}^{-\frac{\sqrt{3}}{2}} = \frac{\left(-\frac{\sqrt{3}}{2}\right)^{-1}}{-1} - \frac{(-1)^{-1}}{-1} =$$

$$= -\left(-\frac{\sqrt{3}}{2}\right)^{-1} + (-1)^{-1} = \boxed{\frac{2}{\sqrt{3}} - 1}$$

1. Compute $\displaystyle\int \frac{\sec^2(x)}{\tan(x)}\,dx$ $\displaystyle = \int \frac{\cancel{u}}{\tan(x)} \; \frac{du}{2\cdot \cancel{u}\cdot \tan(x)} = \frac{1}{2}\int \frac{du}{u-1} =$

$u = \sec^2(x)$

$du = 2\cdot \sec(x)\cdot \sec(x)\cdot \tan(x)\cdot dx$

$du = 2\cdot \sec^2(x)\cdot \tan(x)\cdot dx$

$dx = \dfrac{du}{2\cdot \sec^2(x)\cdot \tan(x)}$

$t = u-1$

$dt = 1\cdot du$

$du = dt$

$\sec^2(x) = \dfrac{1}{\cos^2 x} = \tan^2(x)+1 \Rightarrow \tan^2(x) = \sec^2(x)-1$

$\tan^2(x) = u-1$

2. Compute $\displaystyle\int \sec^2(x)\tan(x)\,dx$

$$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C =$$

$$= \frac{1}{2}\ln|u-1| + C = \boxed{\frac{1}{2}\ln|\sec^2(x)-1| + C}$$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}d\theta$

Verification:

$$\left(\frac{1}{2}\ln|\sec^2(x)-1|\right)' =$$

$$= \frac{1}{\cancel{2}} \frac{1}{\sec^2(x)-1} \cdot \cancel{2}\sec(x)\cdot\sec(x)\cdot\tan(x) =$$

$$= \frac{1}{\sec^2(x)-1} \cdot \sec^2(x)\cdot\tan(x) =$$

$$= \frac{1}{\tan^{\cancel{2}}(x)} \sec^2(x)\cdot\cancel{\tan(x)} =$$

$$= \boxed{\frac{\sec^2(x)}{\tan(x)}}$$

Alternative approach

$$\int \frac{\sec^2(x)}{\tan(x)}\,dx = \int \frac{\cancel{\sec^2(x)}}{u}\,\frac{du}{\cancel{\sec^2(x)}} = \int \frac{1}{u}\,du =$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$dx = \dfrac{du}{\sec^2(x)}$

$$= \ln|u| + C =$$

$$= \boxed{\ln|\tan(x)| + C}$$

Verification:

$$\left(\ln|\tan(x)|\right)' = \frac{1}{\tan(x)} \cdot \sec^2(x)$$

4. Compute $\int \frac{1}{x\ln(x)}\, dx$

5. Compute $\int \frac{\sin(4/x)}{x^2}\, dx$

6. Compute $\int \frac{e^x}{e^x - 3}\, dx$

7. Compute $\displaystyle\int \frac{1}{9+x^2}\,dx$

8. Compute $\displaystyle\int \sqrt{x}(x^4+x)\,dx$

9. Compute $\displaystyle\int \cos(x)\sin(\sin(x))\,dx$

10. Compute $\frac{d}{dx}\left[x\ln(x) - x\right]$. Then compute $\int s^2\ln(s^3)\ ds$

11. Compute $\displaystyle\int x\sqrt{x-1}\ dx$ (Hint: Let $u = x - 1$. What is x in terms of u?)

12. Compute $\displaystyle\int_1^3 \frac{(\ln(x))^3}{x}\ dx$