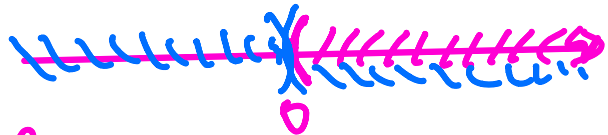
Section 4.5 Curve Sketching & Section 4.7 Applied Optimization (Day 1)

1. Follow the guidelines from the previous worksheet to sketch the graph of

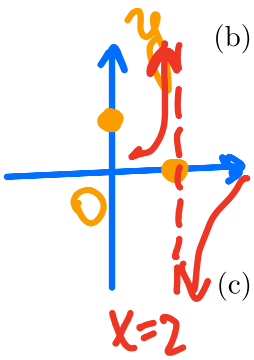
$$f(x) = \frac{2}{x} + \ln(x). \qquad \left(\text{Note: } f'(x) = \frac{x-2}{x^2} \text{ and } f''(x) = \frac{4-x}{x^3}\right)$$

(a) What is the function's domain?

Domain: $x > 0$ or $(0, \infty)$



$f_1(x) = \frac{2}{x}$; $\text{Dom}(f_1) = \mathbb{R} \setminus \{0\}$ $\qquad$ $f_2(x) = \ln(x)$; $\text{Dom}(f_2) = (0, \infty)$



(b) (if defined) Determine the y-intercept. Determine the x-intercepts if it's not too hard.

y-intercept: $x = 0$ $\quad$ None

x-intercept: $y = 0$ $\quad$ $\frac{2}{x} + \ln(x) = 0$ $\quad$ None

(c) (if defined) What behavior occurs for this function as $x \to \pm\infty$?

$$\lim_{x\to\infty}\left(\frac{2}{x} + \ln(x)\right) = \underbrace{\lim_{x\to\infty}\frac{2}{x}}_{\to 0} + \underbrace{\lim_{x\to\infty}\ln(x)}_{\to\infty} = 0 + \infty = \infty$$

$$\cancel{\lim_{x\to-\infty}\left(\frac{2}{x} + \ln(x)\right)}$$

(d) Does the function have any vertical asymptotes? Where?

VA: $\lim_{x\to a} f(x) = \pm\infty$, then $x = a$ is VA

$x = 0$

$$\lim_{x\to 0^+}\left(\frac{2}{x} + \ln(x)\right) = \lim_{x\to 0^+}\frac{2}{x}\left(1 + \frac{\ln x}{\frac{2}{x}}\right) = \boxed{+\infty}$$

(e) Find intervals where f is increasing/decreasing and identify critical points.

CP: $f'(x) = 0$ or $f'(x)$ DNE

$$\lim_{x\to 0^+}\frac{\ln x}{\frac{2}{x}} = \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{2}{x^2}} = \lim_{x\to 0^+}\frac{1}{-\frac{2}{x}} = \lim_{x\to 0^+} -\frac{x}{2} \to 0$$

$$f'(x) = -\frac{2}{x^2} + \frac{1}{x} = 0$$

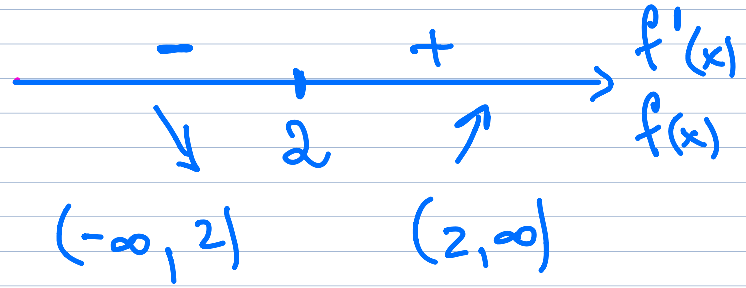
$$\frac{-2 + x}{x^2} = 0$$

$$-2 + x = 0$$

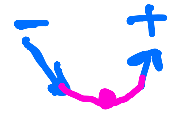
$x = 2$ CP

$x = 0$ is not a CP since it is not in

D(f).



(f) Classify each critical point as a local min/max/neither.



1st Derivative Test:

$x = 2$ f': $-$ to $+$, then at $x = 2$

f attains its loc. min value

(g) Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = \left(-\frac{2}{x^2} + \frac{1}{x}\right)' = \left(+\frac{4}{x^3} - \frac{1}{x^2}\right) = 0$$

$$\frac{4-x}{x^3} = 0 \qquad 4 - x = 0 \Rightarrow x = 4$$

$x = 4$ is an inflection point

f'': $+$ on the left of 4, $-$ on the right of 4

CU ∪ 4 ∩ CD

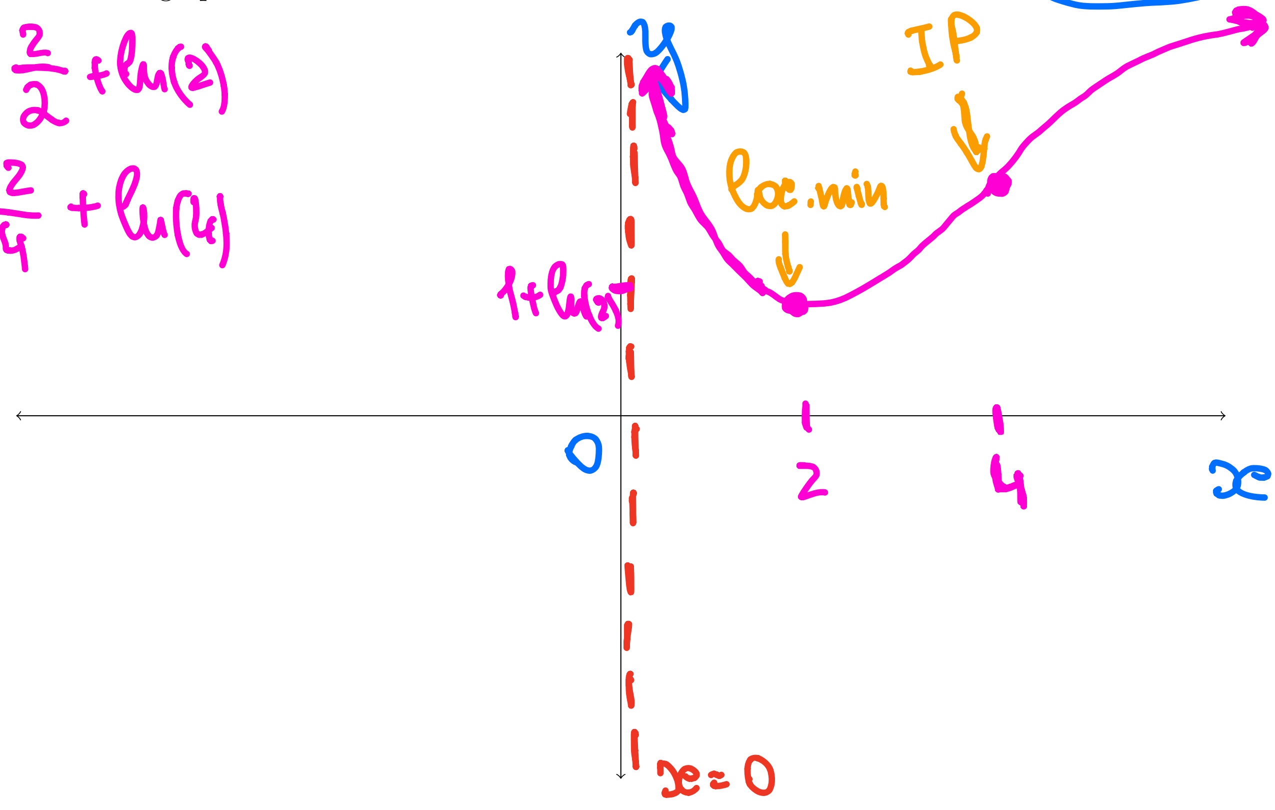
(h) Collect all the information you have determined into a handy list.

1. $\mathrm{Dom}(f) = (0, \infty)$
2. $x = 0$ is VA
3. $f \nearrow$ on $(2, \infty)$ and $f \searrow$ on $(-\infty, 2)$
4. $f(2)$ has loc. min
5. f is CU on $(-\infty, 4)$
   f is CD on $(4, \infty)$
   $x = 4$ is an IP

(i) Sketch the graph of the function

$$f(2) = \frac{2}{2} + \ln(2)$$

$$f(4) = \frac{2}{4} + \ln(4)$$



Section 4.7 Applied Optimization (Day 1)

Algorithm for Approaching Optimization

1. Read the problem two or three times. Draw pictures. Label them. Pick specific numerical examples, to make the problem concrete. Be creative. Try more than just one approach.
2. Identify the quantity to be minimized or maximized (and which one... min or max).
3. Chose notation and explain what it means.
4. Write the thing you want to maximize or minimize as a function of one variable, including a reasonable domain.
5. Use calculus to answer the question and justify that your answer is correct.

(a) Why does justification matter?

(b) Find two positive numbers whose sum is 110 and whose product is a maximum.

(c) A rancher has 800 feet of fencing with which to enclose three adjacent rectangular corrals. See figure below. What dimensions should be used so that the enclosed area will be a maximum?