1. Find all critical points of the function $f(x) = \sin(x)^{1/3}$.

Critical points:

$f'(x) = 0$ or $f'(x)$ DNE.

$f'(x) = \cos(x)^{1/3} \cdot \frac{1}{3} x^{-2/3} = 0$

$\frac{\cos(x)^{1/3}}{3x^{2/3}} = 0 \iff \cos(x)^{1/3} = 0 \iff x^{1/3} = \pm\frac{\pi}{2} + \pi n$, n in $\mathbb{Z}$

$x \neq 0$

$\boxed{x = \left(\pm\frac{\pi}{2} + \pi n\right)^3}$ CP

$f'(x)$ DNE at $\boxed{x = 0}$ CP

Therefore, we have the following CP:

$x = \left(\pm\frac{\pi}{2} + \pi n\right)^3$, n is an integer

$x = 0$

2. Find the absolute maximum and minimum values (y-values) of $f(x) = e^{-x^2}$ on the interval $[-2, 3]$, and the locations (x-values) where those values are attained.

$f(x) = e^{-x^2}$, Dom$(f) = \mathbb{R}$, $[-2,3] \subset \mathbb{R}$

1. Critical points:

$f'(x) = 0$ or $f'(x)$ DNE

$f'(x) = e^{-x^2} \cdot (-2x) = 0$

$\underbrace{e^{-x^2}}_{\neq 0} \cdot (-2x) = 0 \Rightarrow x = 0$ in $[-2,3]$

$\boxed{x = 0}$ is a CP

2. $f'(x)$ is defined for all x in $[-2,3]$.

3. $f(0) = 1$

$f(-2) = e^{-4} = \frac{1}{e^4}$

$f(3) = e^{-9} = \frac{1}{e^9}$

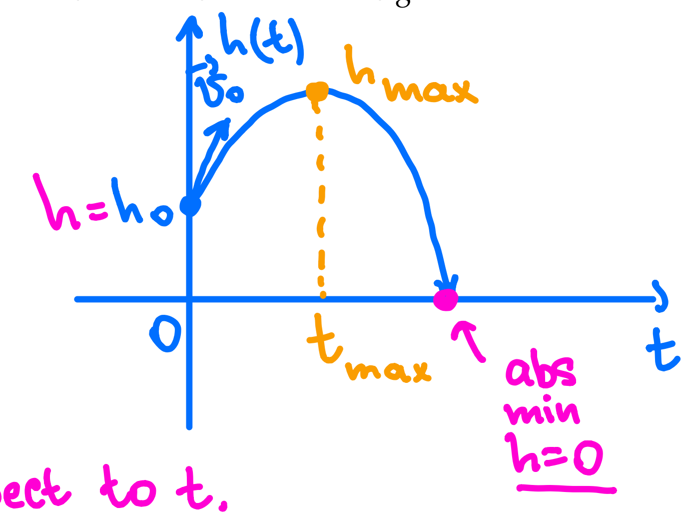
$f_{abs\ max} = 1$ at $x = 0$

$f_{abs\ min} = \frac{1}{e^9}$ at $x = 3$

3. A ball thrown in the air at time $t = 0$ has a height given by

$$h(t) = h_0 + v_0 t - \frac{1}{2} g_0 t^2$$

meters where t is measured in seconds, h_0 is the height at time 0, v_0 is the velocity (in meters per second) at time 0 and g_0 is the constant acceleration due to gravity (in m/s^2). Assuming $v_0 > 0$, find the time that the ball attains its maximum height. Then find the maximum height.



1. CP:

$h'(t) = 0$

Let us remark that $h'(t)$ is defined for all $t \geq 0$ since it is a linear function with respect to t.

$h'(t) = v_0 - g_0 t = 0$

$v_0 = g_0 t \Rightarrow t = \frac{v_0}{g_0}$

$\boxed{t = \frac{v_0}{g_0}}$ is the only one CP

$$h\left(\frac{v_0}{g_0}\right) = h_0 + \frac{v_0^2}{g_0} - \frac{1}{2} g_0 \frac{v_0^2}{g_0^2} = h_0 + \frac{v_0^2}{g_0} - \frac{1}{2}\frac{v_0^2}{g_0} =$$

$$= h_0 + \frac{v_0^2}{2g_0}$$

Therefore, h(t) attains its maximum height at $t = \frac{v_0}{g_0}$ (seconds)

The maximum height attained is

$h(t) = h_0 + \frac{v_0^2}{2g_0}$ (meters).