Worksheet: Section 2.3 day one (Limit Laws)

Solutions

1. Fill in the blanks below.

   Assume a and c are fixed constants. Also, **assume** $\lim_{x\to a} f(x)$ **and** $\lim_{x\to a} g(x)$ **exist.**

   (a) $\lim_{x\to a} c =$ c

   (b) $\lim_{x\to a} x =$ a

   Sum Law

   (c) $\lim_{x\to a} (f(x)+g(x)) =$ $\lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

   i. What do the rules above imply about $\lim_{x\to 12} (x+\pi)$?

   $\lim_{x\to 12} (x+\pi) = x+12$

   (d) $\lim_{x\to a} (f(x)-g(x)) =$ $\lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ Difference Law

   (e) $\lim_{x\to a} cf(x) =$ $c \lim_{x\to a} f(x)$ Constant Multiple Law

   i. What do the rules above imply about $\lim_{x\to 5} 2x+3$?

   $\lim_{x\to 5} (2x+3) = 2\lim_{x\to 5} x + \lim_{x\to 5} 3 = 2\cdot 5+3 = 13$

   (f) $\lim_{x\to a} f(x)g(x) =$ $\lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$ Product Law

   (g) $\lim_{x\to a} x^n =$ a^n provided n is a positive integer

   (h) $\lim_{x\to a} (f(x))^n =$ $\left(\lim_{x\to a} f(x)\right)^n$ provided n is a positive integer

   (i) $\lim_{x\to a} \frac{f(x)}{g(x)} =$ $\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ provided $\lim_{x\to a} g(x) \neq 0$ Quotient Law

   (j) $\lim_{x\to a} \sqrt[n]{x} =$ $\sqrt[n]{a}$ provided n is a positive integer; if n is even, then $a>0$

   (k) $\lim_{x\to a} \sqrt[n]{f(x)} =$ $\sqrt[n]{\lim_{x\to a} f(x)}$ provided n is a pos. integer; if n is even, then $\lim_{x\to a} f(x) > 0$

2. If $\lim_{x\to\sqrt{2}} f(x) = 8$ and $\lim_{x\to\sqrt{2}} g(x) = e^2$, then evaluate

$$\lim_{x\to\sqrt{2}} \left(\frac{g(x)}{(3-f(x))^2} + 2\sqrt{g(x)} \right) = \lim_{x\to\sqrt{2}} \frac{g(x)}{(3-f(x))^2} + \lim_{x\to\sqrt{2}} 2\sqrt{g(x)} =$$

$$= \frac{\lim_{x\to\sqrt{2}} g(x)}{\lim_{x\to\sqrt{2}} (3-f(x))^2} + 2\lim_{x\to\sqrt{2}} \sqrt{g(x)} = \frac{e^2}{(3-\lim_{x\to\sqrt{2}} f(x))^2} + 2\cdot\sqrt{\lim_{x\to\sqrt{2}} g(x)} =$$

$$= \frac{e^2}{(3-8)^2} + 2\cdot\sqrt{e^2} = \frac{e^2}{25} + 2e.$$

3. Use the previous rules to evaluate (a) and explain why you *cannot* use the rules to evaluate (b).

(a) $$\lim_{w\to-\frac{1}{2}} \frac{2w+1}{w^3} = \frac{\lim_{w\to-\frac{1}{2}} (2w+1)}{\lim_{w\to-\frac{1}{2}} w^3} = \frac{2\cdot\left(-\frac{1}{2}\right)+1}{-\frac{1}{8}} = \frac{0}{-\frac{1}{8}} = 0.$$

Since $\lim_{w\to-\frac{1}{2}} w^3 = \left(-\frac{1}{2}\right)^3 = -\frac{1}{8} \neq 0$, then we are allowed to use a Quotient Law.

(b) $$\lim_{t\to1} \frac{t^2+t-2}{t^2-1} \neq \frac{\lim_{t\to1} (t^2+t-2)}{\lim_{t\to1} (t^2-1)}$$

Since $\lim_{t\to1} (t^2-1) = 1-1 = 0$, we are not allowed to use a quotient law.

4. (One more super-useful rule!) If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x\to a} f(x) \boxed{=} \lim_{x\to a} g(x)$ provided the limits exist. Use this rule *and what you know about zeros of polynomials* to evaluate

$\lim_{t\to1} \frac{t^2+t-2}{t^2-1}$ Let $f(t) = \frac{t^2+t-2}{t^2-1}$. Then $\mathrm{Dom}(f) = \mathbb{R}\setminus\{\pm1\}$.

Now let $g(t) = \frac{t^2+t-2}{t^2-1} = \frac{(t+2)\cancel{(t-1)}}{\cancel{(t-1)}(t+1)} = \frac{t+2}{t+1}$.

Then $\mathrm{Dom}(g) = \mathbb{R}\setminus\{-1\}$.

Hence, we see that $g(t) = f(t)$ except $x = 1$.

Then we have

$$\lim_{t\to1} \frac{t^2+t-2}{t^2-1} = \lim_{t\to1} \frac{t+2}{t+1} = \boxed{\frac{3}{2}}$$