Solutions

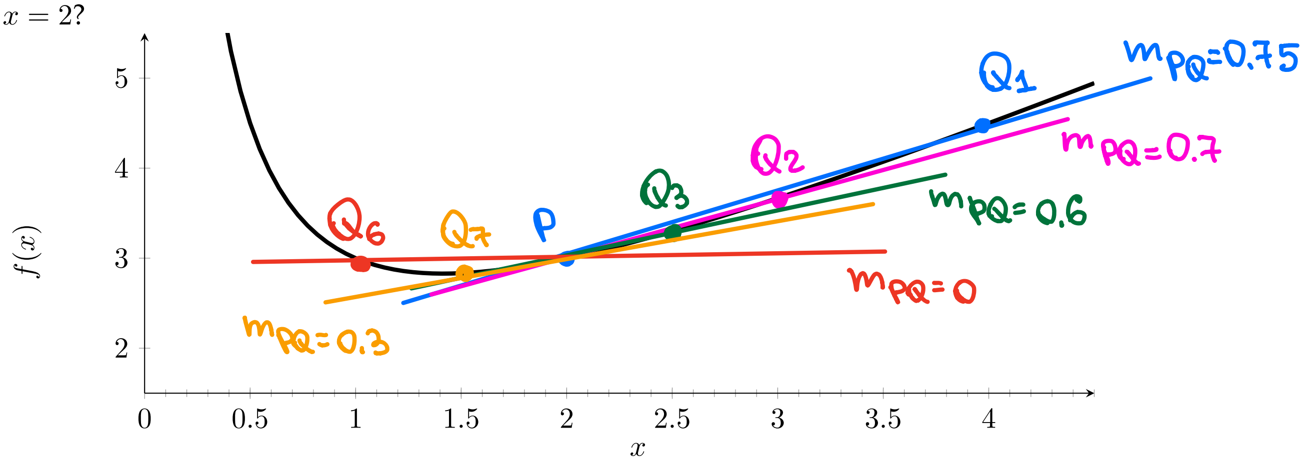
1. The point $P(2,3)$ lies on the graph of $f(x) = x + \frac{2}{x}$.

(a) If possible, find the slope of the secant line between the point P and each of the points with x values listed below. For each estimate the slope to 4 decimal places. NOTE: You do not need the graph of the function to answer this numerical question.

$P(2,3)$

	point Q		slope of secant
	x-value	y-value	line PQ
Q_1	$x = 4$	4.5	$1.5/2 = 0.75$
Q_2	$x = 3$	3.6667	$0.6667/1 = 0.6667$
Q_3	$x = 2.5$	3.3	$0.3/0.5 = 0.6$
Q_4	$x = 2.2$	3.1091	$0.1091/0.2 = 0.5455$
Q_5	$x = 2.1$	3.0524	$0.0524/0.1 = 0.524$
Q_6	$x = 1$	3	0
Q_7	$x = 1.5$	2.8333	$-0.1667/-0.5 = 0.3334$
Q_8	$x = 1.7$	2.8765	$-0.1235/-0.3 = 0.4117$
Q_9	$x = 1.9$	2.9526	$-0.0474/-0.1 = 0.474$

(b) Now, sketch the secant lines on the graph of $f(x)$ shown below. Label the secant lines with their respective slopes. What can you conclude about the slope of the tangent line to $f(x)$ at $x = 2$?



(c) Write a best guess for the equation of the line tangent to $f(x)$ at point P. Is your equation plausible?

$m = \dfrac{y - y_1}{x - x_1}$ $\quad P(2,3) \Rightarrow x_1 = 2,\ y_1 = 3$

We observe that when the point Q is sufficiently close to P, then $m_{PQ} \approx 0.5 = \frac{1}{2}$.

Hence, $m \approx \frac{1}{2}$. From this it follows that

$$\frac{1}{2} = \frac{y-3}{x-2} \Rightarrow \boxed{y = \frac{1}{2}(x-2) + 3}$$

2. The table shows the position of a cyclist after accelerating from rest.

t (minutes)	0	30	60	90	120	150	180	210	240
d (miles)	0	9.2	18.7	23.1	38.1	46.6	59.7	72.6	80

(a) Estimate the cyclist's average velocity in miles per hour during:

i. the first hour $[0, 60]$

$$v = \frac{d(t_2)-d(t_1)}{t_2-t_1} = \frac{18.7-0}{1} = 18.7 \text{ miles/hour}$$

ii. the second hour $[60, 120]$

$$v = \frac{d(120)-d(60)}{1} = \frac{38.1-18.7}{1} = 19.4 \text{ miles/h}$$

iii. the third hour $[120, 180]$

$$v = \frac{59.7-38.1}{1} = 21.6 \text{ miles/h}$$

iv. the fourth hour $[180, 240]$

$$v = \frac{80-59.7}{1} = 20.3 \text{ miles/h}$$

(b) Estimate the cyclist's average velocity (in miles per hour) in the time period $[60, 90]$.

$$\frac{\Delta d}{\Delta t} = \frac{23.1-18.7}{1/2} = 8.8 \text{ miles/h}$$

(c) Estimate the cyclist's average velocity (in miles per hour) in the time period $[90, 120]$.

$$\frac{\Delta d}{dt} = \frac{38.1-23.1}{1/2} = 30 \text{ miles/h}$$

(d) Estimate how fast the cyclist was going 1.5 hours into the ride.

We can look at average velocities on the 1/2 h either side of 90 min = 1.5h and average them.

$$(8.8+30)/2 = 19.4 \text{ miles/h}$$

(e) During what period do you estimate the cyclist was riding the fastest on average?

Probably on $[19, 120]$ with $v = 30$ miles/h

(f) What does any this have to do with secant lines and tangent lines?

- average velocity = slope of a secant line (a, b, c)
- instantaneous velocity = slope of a tangent line (d)