Derivatives of elementary functions

1. $\frac{d}{dx}(c) = 0$ ✓

2. $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$ ✓

3. $\frac{d}{dx}(e^x) = e^x$ ✓

4. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$$(\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\,\frac{1}{x^{1/2}} = \frac{1}{2\sqrt{x}}$$

5. $\frac{d}{dx}(f \pm g) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

6. $\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$

$\frac{df}{dx}$ means that we find the derivative of $f(x)$ with respect to x variable.

Problems

(1) $f(x) = \pi\sqrt{x} + e^x + 6x^3$

$$\frac{d}{dx}\left(f(x)\right) = \left(\pi\sqrt{x} + e^x + 6x^3\right)' =$$

$$= \left(\pi\sqrt{x}\right)' + \left(e^x\right)' + \left(6x^3\right)' =$$

$$= \frac{\pi}{2\sqrt{x}} + e^x + 18\cdot x^2$$

(2) $$f(x) = \frac{e\cdot x^4 - 4x^{-2} + e^x\cdot x}{x} =$$

$$= e\cdot x^3 - 4\cdot x^{-3} + e^x$$

$$\frac{d}{dx}\left(f(x)\right) = 3\cdot e\cdot x^2 + 12x^{-4} + e^x$$

(3) $$f(x) = (3 + x^2)(4x^{-1} - 5x^{5/2}) =$$

$$= 12x^{-1} - 15x^{5/2} + 4x - 5x^{9/2}$$

$\frac{9}{2} - \frac{2}{2}$

$$\frac{d}{dx}\left(f(x)\right) = -\frac{12}{x^2} - \frac{75\sqrt{x^3}}{2} + 4 - \frac{45\sqrt{x^7}}{2}$$

1. Complete The Product Rule: If f and g are differentiable, then

$$\frac{d}{dx}\left[f(x)g(x)\right] = \frac{d}{dx}(f(x))\cdot g(x) + \frac{d}{dx}(g(x))\cdot f(x) = f'\cdot g + g'\cdot f$$

2. Complete The Quotient Rule: If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}(f(x))\cdot g(x) - \frac{d}{dx}(g(x))\cdot f(x)}{(g(x))^2} = \frac{f'\cdot g - g'\cdot f}{g^2}$$

3. Find the derivatives for each function below. Do not use the Product Rule or the Quotient Rule if you don't have to!

Product rule $\quad \frac{d}{dx}(g\cdot h)$

(a) $f(x) = \underbrace{5x^3}_{g}\underbrace{e^x}_{h}$

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(g\cdot h) = \frac{d}{dx}(5x^3\cdot e^x) = (5x^3)'\cdot e^x + (e^x)'\cdot 5x^3 =$$

$$= 15x^2\cdot e^x + e^x\cdot 5x^3 = \boxed{5x^2e^x(3+x)}$$

(b) $f(x) = \dfrac{2x^2-5}{4-x}$ Quotient rule

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(\frac{\overset{h}{2x^2-5}}{\underset{g}{4-x}}\right) = \frac{h'\cdot g - g'\cdot h}{g^2} = \frac{(2x^2-5)'\cdot(4-x) - (4-x)'(2x^2-5)}{(4-x)^2}$$

$$= \frac{4x\cdot(4-x) - (-1)\cdot(2x^2-5)}{(4-x)^2} =$$

$$= \frac{16x - 4x^2 + 2x^2 - 5}{(4-x)^2} = \boxed{\frac{-2x^2+16x-5}{(4-x)^2}}$$

(c) $f(x) = \underbrace{(1-x^2)}_{g}\underbrace{(e^x+x)}_{h}$

$$\frac{d}{dx}(f(x)) = (1-x^2)'\cdot(e^x+x) + (e^x+x)'(1-x^2) =$$

$$= -2x\cdot(e^x+x) + (e^x+1)(1-x^2) = \boxed{-2xe^x - 2x^2 + e^x - e^xx^2 + 1 - x^2}$$

(d) $g(x) = \underbrace{\frac{\sqrt{x}}{8}}_{f}\underbrace{(1-x\sqrt{x})}_{h}$ Product rule

Approach 1:

$$\frac{d}{dx}(g(x)) = \left(\frac{\sqrt{x}}{8}\right)'\cdot(1-x\sqrt{x}) + (1-x\sqrt{x})'\cdot\frac{\sqrt{x}}{8} =$$

$$= \left(\frac{1}{16}x^{-1/2}\right)(1-x\sqrt{x}) + \left(0-\frac{3}{2}x^{1/2}\right)\cdot\frac{\sqrt{x}}{8}$$

Approach 2: (do algebra at first)

$$g(x) = \frac{\sqrt{x}}{8} - \frac{\sqrt{x}}{8}\cdot x\cdot\sqrt{x} = \frac{\sqrt{x}}{8} - \frac{x^2}{8}$$

$$\frac{d}{dx}(g(x)) = \left(\frac{\sqrt{x}}{8}\right)' - \left(\frac{x^2}{8}\right)' = \frac{1}{16\sqrt{x}} - \frac{x}{4}$$

(e) $h(x) = \dfrac{10x - x^{3/2}}{4x^2}$ (Avoid the quotient rule!)

$$h(x) = \frac{10\cancel{x}}{4x^{\cancel{2}}} - \frac{x^{3/2}}{4x^2} = \frac{10}{4x} - \frac{1}{4x^{1/2}} = \frac{10}{4}x^{-1} - \frac{1}{4}x^{-1/2}$$

$$\frac{d}{dx}(h(x)) = \left(\frac{10}{4}x^{-1}\right)' - \left(\frac{1}{4}x^{-1/2}\right)' = \boxed{-\frac{10}{4}x^{-2} + \frac{1}{8}x^{-3/2}}$$

(f) $y = \dfrac{\sqrt[3]{x}}{2x+1}$

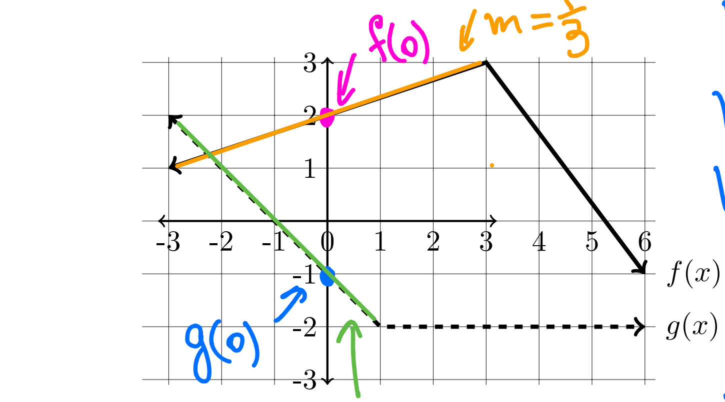
$$\frac{d}{dx}(y(x)) = \frac{(\sqrt[3]{x})'(2x+1) - (2x+1)'\sqrt[3]{x}}{(2x+1)^2} = \boxed{\frac{\frac{1}{3}x^{-2/3}(2x+1) - 2\sqrt[3]{x}}{(2x+1)^2}}$$

(g) $v(t) = \dfrac{2te^t}{t^2+1}$ Quotient Rule + Product Rule

$$\frac{d}{dt}(v(t)) = \frac{(2te^t)'(t^2+1) - (t^2+1)'\,2te^t}{(t^2+1)^2} = \boxed{\frac{(2e^t + 2te^t)(t^2+1) - 2t\cdot 2te^t}{(t^2+1)^2}}$$

Product Rule $(2te^t)' = (2t)'e^t + (e^t)'2t = \boxed{2\cdot e^t + e^t\cdot 2t}$

4. The graphs of $f(x)$ (shown thick) and the graphs of $g(x)$ (shown dashed) are shown below. If $h(x) = f(x)g(x)$, find $h'(0)$.



$h(x) = f(x)\cdot g(x)$

$h'(x) = (f(x)\cdot g(x))' = f'(x)\cdot g(x) + g'(x)\cdot f(x)$

$h'(0) = f'(0)\cdot g(0) + g'(0)\cdot f(0)$

slope at $x=0$

$g(0) = -1$ $\quad$ $f'(0) = m = \frac{1}{3}$

$f(0) = 2$ $\quad$ $g'(0) = m = -1$

$h'(0) = \frac{1}{3}\cdot(-1) + (-1)\cdot 2 = -\frac{1}{3} - 2.$

5. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$. Find the following values.

(a) $(f-g)'(5)$

$(f-g)'(5) = f'(5) - g'(5)$

$= 6 - 2 = \boxed{4}$

(b) $(fg)'(5)$

$(fg)'(5) =$

$= f'(5)g(5) + g'(5)f(5) =$

$= 6\cdot(-3) + 2\cdot 1 = \boxed{-16}$

(c) $(g/f)'(5)$

$\left(\frac{g}{f}\right)'(5) = \frac{g'(5)f(5) - f'(5)g(5)}{f^2(5)} =$

$= \frac{2\cdot 1 - 6\cdot(-3)}{1^2} =$

$= \frac{20}{1} = \boxed{20}$

Remark: In problem #5 we used the following rules:

1. $(f-g)'(a) = f'(a) - g'(a)$

2. $(f+g)'(a) = f'(a) + g'(a)$

3. $(f \cdot g)'(a) = f'(a) \cdot g(a) + g'(a) \cdot f(a)$

4. $\left(\frac{f}{g}\right)'(a) = \frac{f'(a) \cdot g(a) - g'(a) \cdot f(a)}{g^2(a)}$