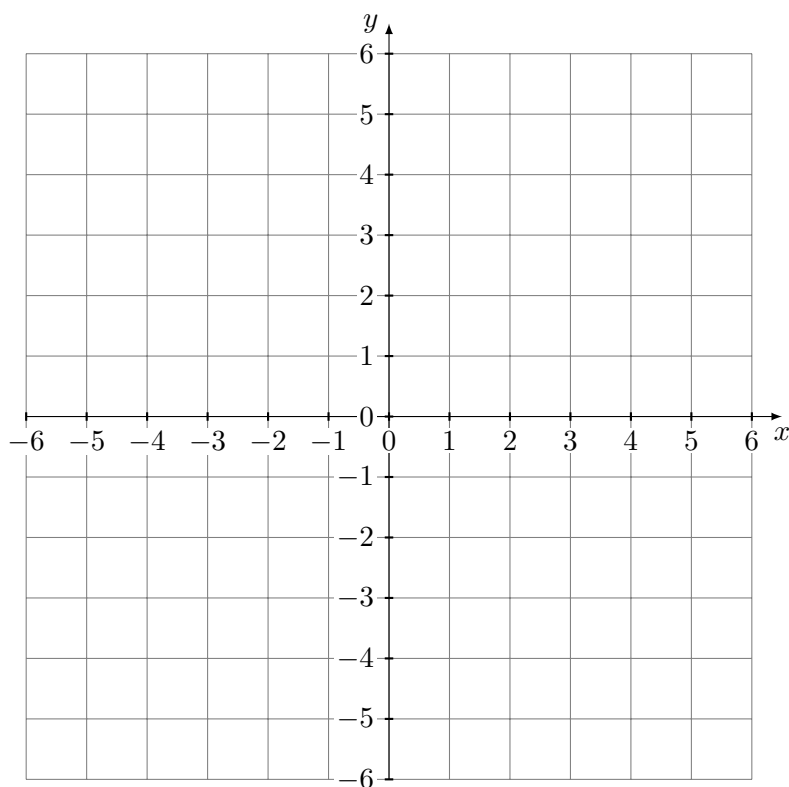


MIDTERM EXAM 1 REVIEW

Exercise 1. Sketch the graph of an example of a function f that satisfies all of the given conditions:

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) = 2, \quad \lim_{x \rightarrow 0^+} f(x) = 0, \quad \lim_{x \rightarrow 4^-} f(x) = 3, \quad \lim_{x \rightarrow +\infty} f(x) = 5, \\ \lim_{x \rightarrow 4^+} f(x) = 0, \quad f(0) = 2, \quad f(4) = 1, \quad \lim_{x \rightarrow -\infty} f(x) = -\infty. \end{aligned}$$



Exercise 2. Evaluate the limit, if it exists.

(a) $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 16} - 4}{t^2}$

(b) $\lim_{x \rightarrow 3^-} \frac{\sqrt{x}}{(x-3)^5}$

(c) $\lim_{x \rightarrow \infty} \frac{1 + e^x}{1 + 2e^x}$

(d) $\lim_{x \rightarrow \infty} \ln \left(\frac{1 + e^x}{1 + 2e^x} \right)$

(e) $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x^4 + 1}}$

Exercise 3. In the first few years after a coal mine's operation, the total deposit of coal (in millions of tons) t years after opening is approximately

$$C(t) = 300 - \frac{t^{3/2}}{2}.$$

(a) Find the average rate of change of the amount of coal in the deposit from the opening of the mine to year 4. Include correct units in your answer.

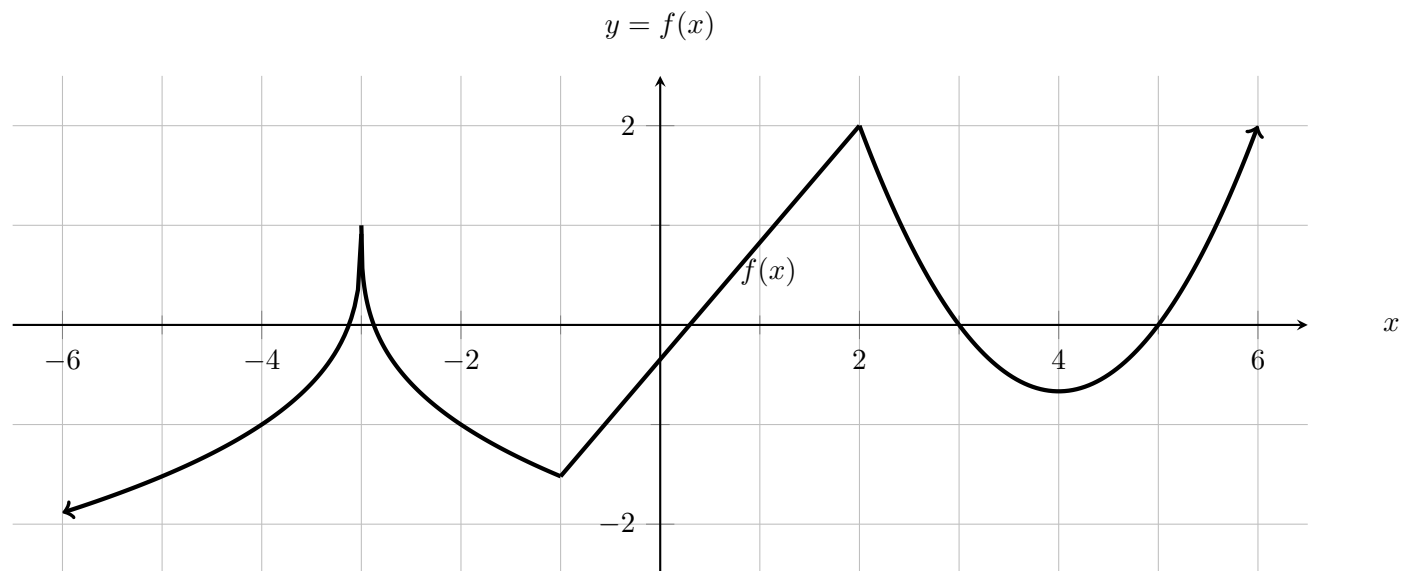
(b) It is a fact that $C'(t) = -\frac{3}{4}\sqrt{t}$. Compute $C'(4)$ and indicate what this quantity tells us about the mine. Write your answer in a sentence. Again, include correct units in your description.

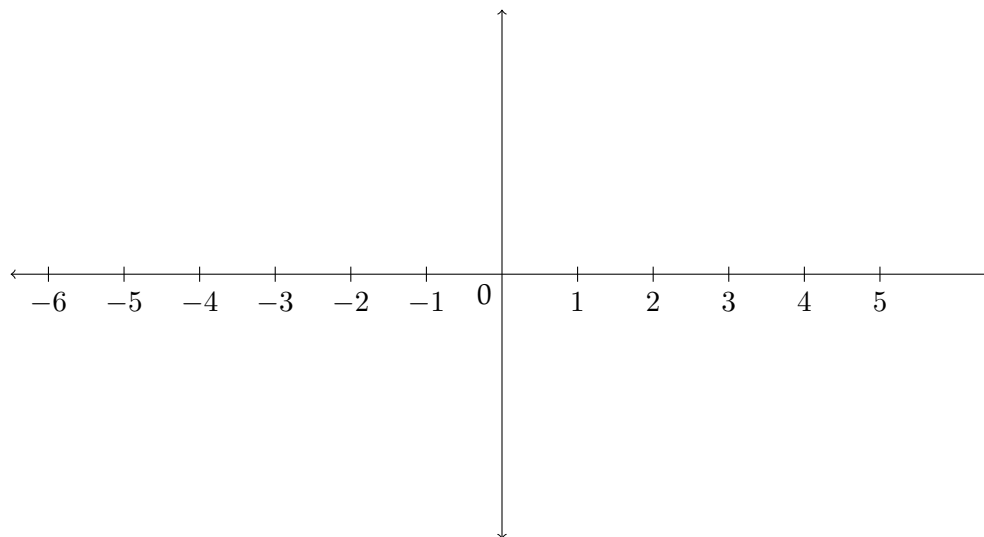
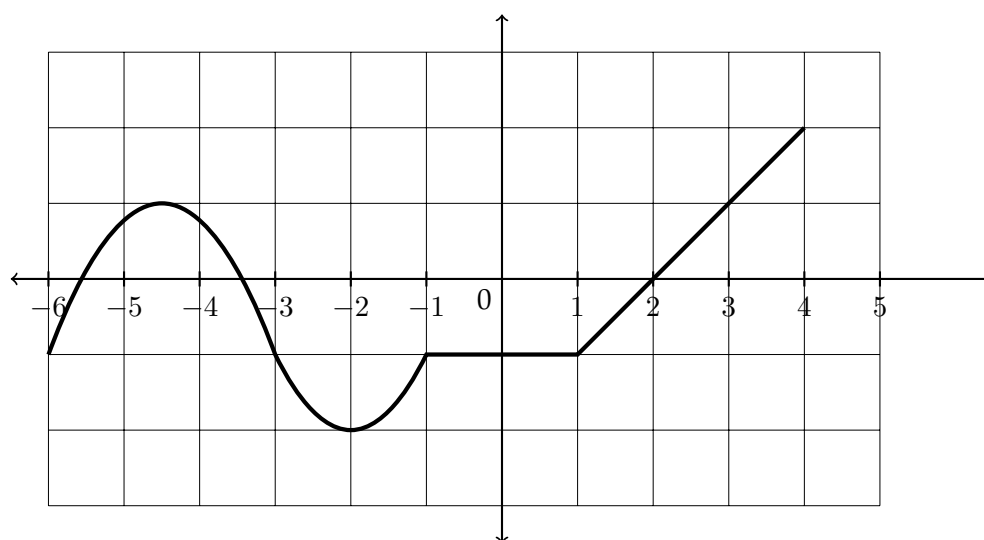
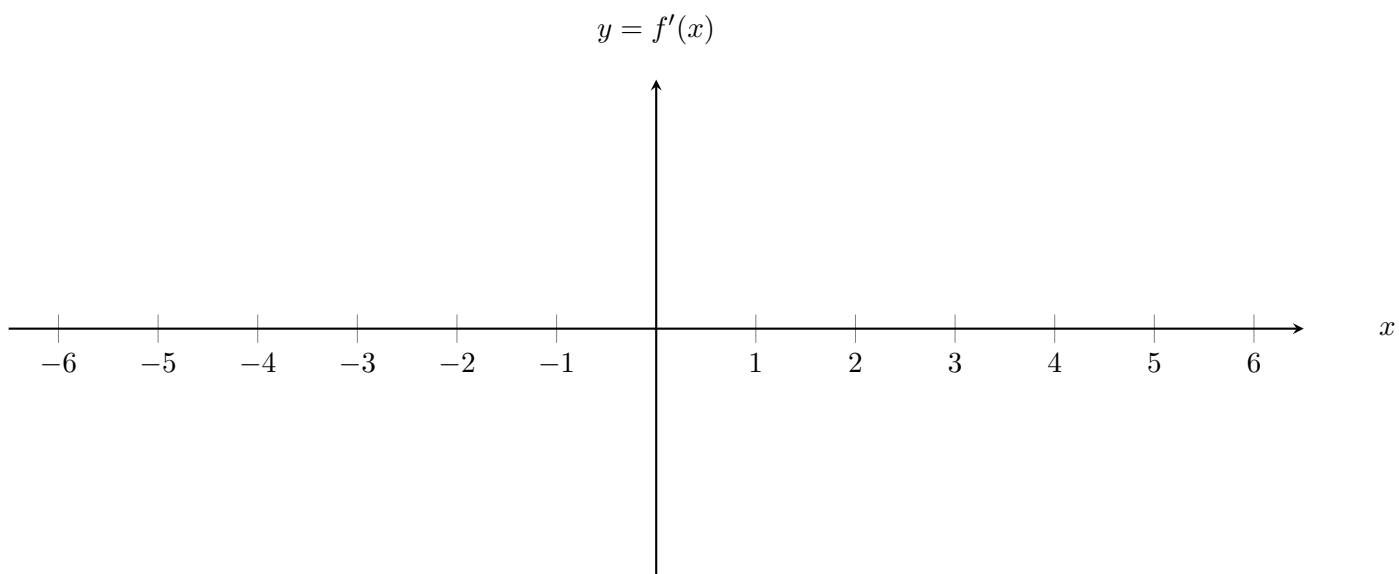
Exercise 4. Consider the function

$$f(x) = \frac{1}{x+2}.$$

Using the **definition of the derivative**, find $f'(5)$.

Exercise 5. The graph of $f(x)$ is shown on the top set of axes. Sketch the derivative of $f(x)$ on the second set of axes.

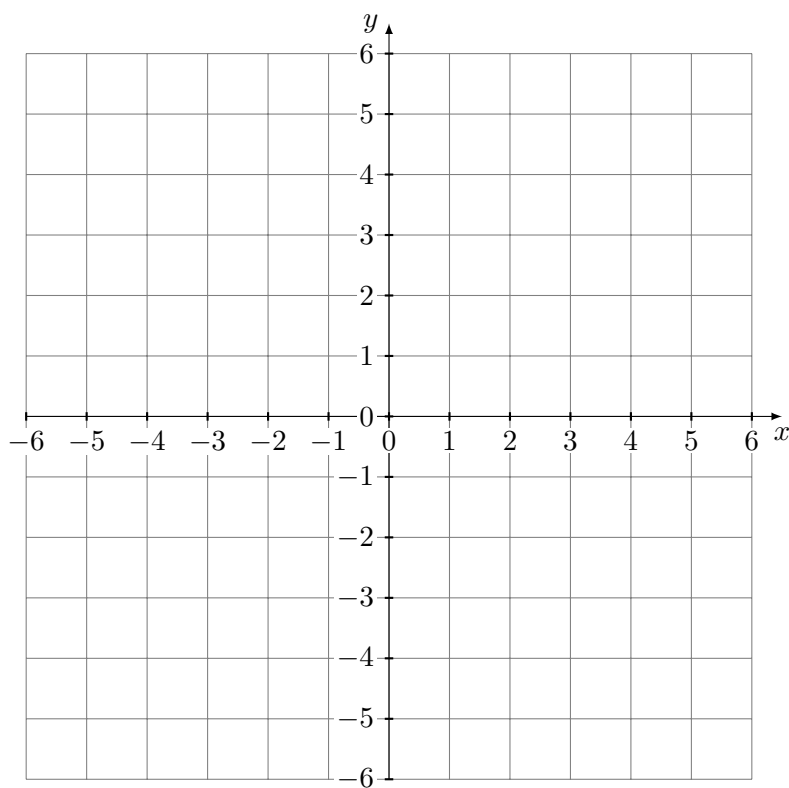




Exercise 6. Find the equation of the tangent line to the curve at the given point:

$$y = 4x - 3x^2, \quad P = (2, -4).$$

Sketch both your function $f(x)$ and the tangent line to it at point P on axes below.



Exercise 7. The limit represents the derivative of some function f at some point a . State such a function f and a point a .

$$\lim_{h \rightarrow 0} \frac{e^{-2+h} - e^{-2}}{h}$$