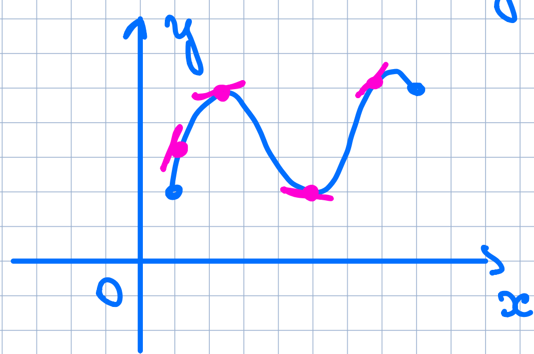
Section 4.3. How derivatives affect the shape of a graph

- What does f' say about f ?



Increasing / decreasing test

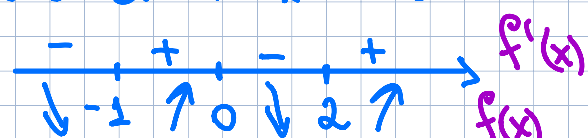
(a) if $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval

(b) if $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval

Example $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$$f'(x) = 12x^3 - 12x^2 - 24x = 0$$

$$x = 0 \quad \text{or} \quad x^2 - x - 2 = 0$$



- Local extreme values

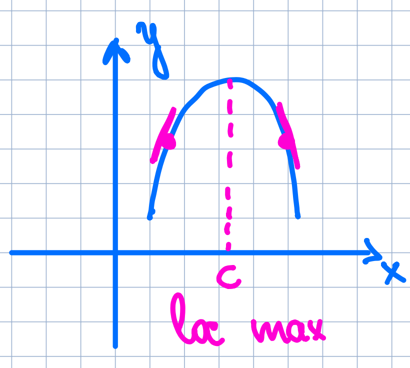
The first derivative test

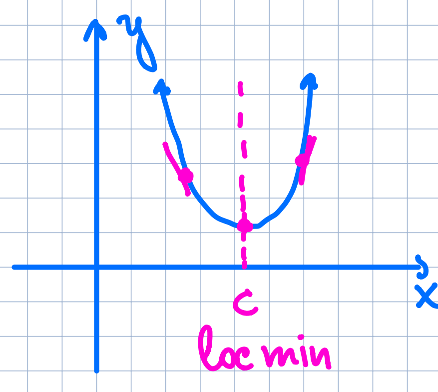
Suppose that c is a critical number of a continuous function f.

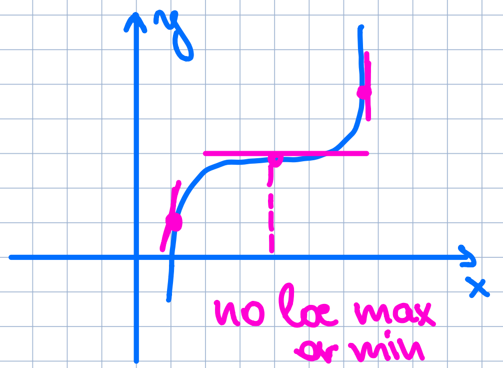
(a) if f' changes from + to - at c, then $f(c)$ = loc max

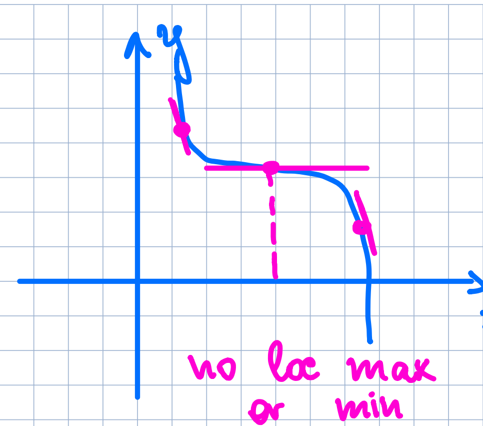
(b) if f' changes from - to + at c, then $f(c)$ = loc min

(c) if f' is + to the left and right of c, or - to the left and right of c, then f has no loc max or min at c.

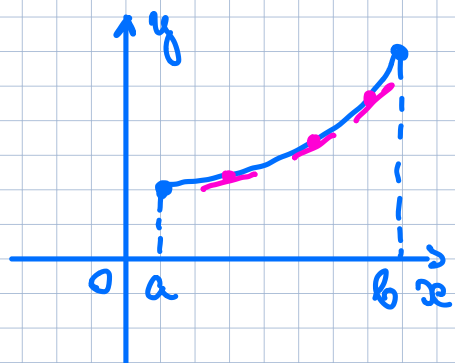


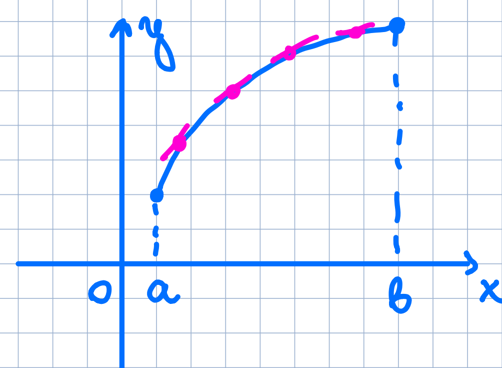






- What does f'' say about f ?





Def. If the graph of f lies above all of its tangents on an interval (a,b), then it is called <u>concave upward</u> on (a,b). If the graph of f lies below all of its tangents on (a,b), it is called <u>concave downward</u> on (a,b).

Concavity Test

(a) if $f''(x) > 0$ for all x in (a,b), then the graph f is concave upward on (a,b).

(b) if $f''(x) < 0$ for all x in (a,b), then the graph f is concave downward on (a,b).

Def. A point P on the curve $y = f(x)$ is called an <u>inflection point</u> if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

Example Sketch a possible graph of a function f:

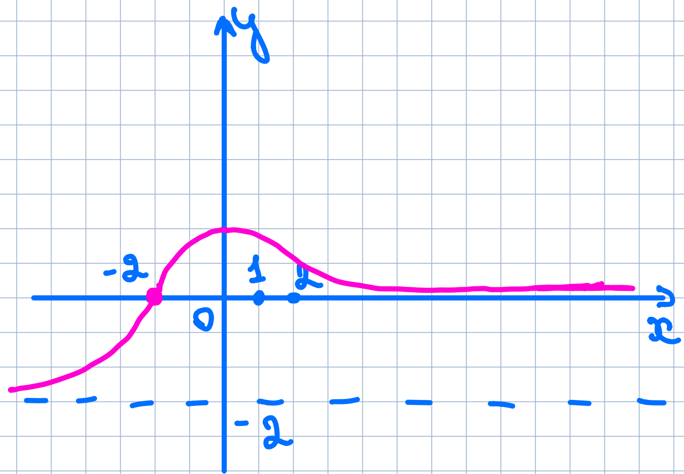
$f'(x) > 0$ on $(-\infty, 1)$

$f'(x) < 0$ on $(1, \infty)$

$f''(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$

$f''(x) < 0$ on $(-2, 2)$

$\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to \infty} f(x) = 0$



The Second Derivative Test

Suppose f'' is continuous near c.

(a) if $f'(c)=0$ and $f''(c)>0$, then f has a local min at c.

(b) if $f'(c)=0$ and $f''(c)<0$, then f has a local max at c.

Example

$$y = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$4x^2(x-3) = 0$$

$$x = 0 \quad \text{or} \quad x = 3$$

$f''(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0$ or $x = 2$

$x=0$ - inflection point (no loc max or min)

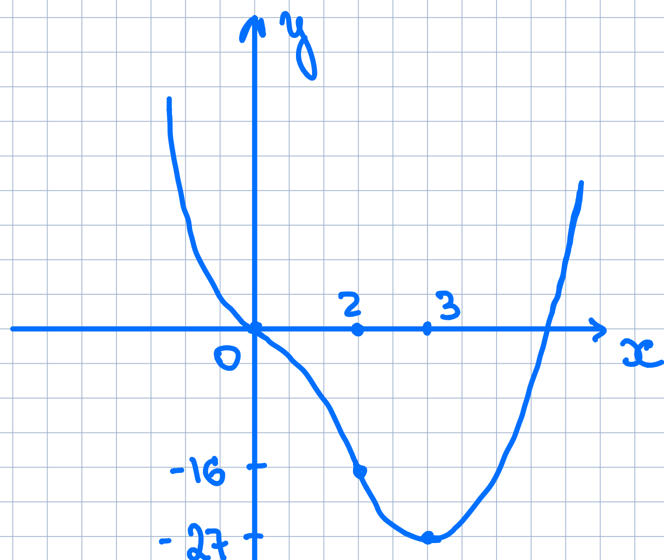
$x=2$ - inflection point

$f''(3) = 12 \cdot 9 - 24 \cdot 3 = 108 - 72 > 0$

$x=3$ - loc. min

$f(2) = -16$

$f(3) = -27$



Note: The Second Derivative Test is inconclusive when $f''(c) = 0$, that is, at such a point there might be a max, min or neither. This test also fails when $f''(c)$ DNE.

In such cases the 1st Derivative test must be used.

In fact, even when both tests