Section 4.9. Antiderivatives.

Def. A function $F(x)$ is an antiderivative of $f(x)$ on the interval I if

$$\boxed{F'(x) = f(x)} \text{ for all } x \text{ from } I$$

Example $f(x) = x$

$$F(x) = \frac{x^2}{2} \leftarrow \text{Antiderivative}$$

$$F'(x) = \left(\frac{x^2}{2}\right)' = \frac{2x}{2} = \underline{\underline{x}}$$

$$F(x) = \frac{x^2}{2} + 5$$

$$F'(x) = \left(\frac{x^2}{2} + 5\right)' = x + 0 = \underline{\underline{x}}$$

Theorem If F is an antiderivate of $f(x)$ on the interval I, then

the most general antiderivative of $f(x)$ on I is

$$\boxed{F(x) + C}$$

If C is chosen to be a particular number $\left(\begin{array}{l} C = 2, \\ C = 10, \\ C = -20 \end{array}\right)$

then

$F(x) + 2$
$F(x) - 20$ will be a

particular antiderivative.

Example $f(x) = \cos(x)$

$F(x) = \sin(x) + C$ ← the most general antideriv.

SECTION 4.9: ANTIDERIVATIVES

1. Find a particular antiderivative of $f(x) = 9 + x - x^2$.

$F(x) = 9x + \frac{x^2}{2} - \frac{x^3}{3} + 5$

$F'(x) = 9 + x - x^2$

2. Find all antiderivatives of $f(x) = 9 + x - x^2$.

$F(x) = 9x + \frac{x^2}{2} - \frac{x^3}{3} + C$

particular

3. Find an antiderivative of $f(x) = \frac{1}{x^2}$.

$F(x) = -x^{-1} = -\frac{1}{x}$

$F'(x) = \left(-x^{-1}\right)' = +x^{-2} = \frac{1}{x^2}$

4. To find *all* antiderivatives of a function $f(x)$, do you always just add a $+C$? Explain how to construct a "generic" piecewise function where you're not using just $+C$ to describe *all* antiderivatives.

5. For each of the following functions, find a particular antiderivative.

Function	Antiderivative
x	$\frac{x^2}{2}$
x^2	$\frac{x^3}{3}$
x^3	$\frac{x^4}{4}$
x^k $(k \neq -1)$	$\frac{x^{k+1}}{k+1}$
x^{-1} for $x > 0$	$\ln(x)$
x^{-1} for $x < 0$	$\ln(-x)$
x^{-1} for all x	$\ln\lvert x\rvert$

Function	Antiderivative
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
e^x	e^x
$1/(1+x^2)$	$\arctan(x)$
$(\sec(x))^2$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
1	x
$1/\sqrt{1-x^2}$	$\arcsin(x)$

6. Compute an antiderivative of $f(x) = 15x^{20} + 44x^{10} + 8$

$$F(x) = 15 \cdot \frac{x^{21}}{21} + 44\frac{x^{11}}{11} + 8x$$

$$F'(x) = 15x^{20} + 44 \cdot x^{10} + 8$$

7. Compute an antiderivative of $f(t) = \dfrac{5\sec t \tan t}{3} - 4\sin t - \dfrac{1}{t} + e^2$

$$F(t) = \frac{5}{3}\sec(t) + 4\cos(t) - \ln|t| + t \cdot e^2$$

$$F'(t) = \frac{5}{3}\sec(t)\cdot\tan(t) - 4\sin(t) - \frac{1}{t} + e^2$$

8. Compute an antiderivative of $f(x) = \cos(3x)$.

9. Compute the antiderivative of $f(t) = t^2$ that equals 5 when $t = 2$.