Section 3.2. The product and Quotient Rules

• The Product Rule:

Let $u = f(x)$ and $v = g(x)$ be differentiable functions.

		A_2
Δv	$u\Delta v$	$\Delta u \Delta v$
v	uv A_1	$v \Delta u$
	u	Δu

$$\Delta u = f(x+\Delta x) - f(x)$$
$$\Delta v = g(x+\Delta x) - g(x)$$

$$A_1 = u \cdot v$$
$$A_2 = (u+\Delta u)(v+\Delta v)$$

Change in the area:

$$\Delta(uv) = (u+\Delta u)(v+\Delta v) - uv = u\Delta v + v\Delta u + \Delta u \Delta v$$

$$\frac{\Delta(uv)}{\Delta x} = u\frac{\Delta v}{\Delta x} + v\frac{\Delta u}{\Delta x} + \Delta u \frac{\Delta v}{\Delta x}$$

Let $\Delta x \to 0$. Then

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} + 0\cdot\frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

The Product Rule:

If f and g are both differentiable, then

$$\boxed{\frac{d}{dx}(f\cdot g) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))}$$

The Quotient Rule:

Let $u = f(x)$ and $v = g(x)$ be differentiable functions. Then

$$\Delta\left(\frac{u}{v}\right) = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v} = \frac{v\Delta u - u\Delta v}{v(v+\Delta v)}$$

So

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \lim_{\Delta x\to 0}\frac{\Delta\left(\frac{u}{v}\right)}{\Delta x} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

The Quotient Rule:

If f and g are differentiable, then

$$\boxed{\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{g^2}}$$

Table of Differentiation Formulas

1. $\frac{d}{dx}(c) = 0$
2. $(cf)' = cf'$
3. $(f \cdot g)' = fg' + f'g$
4. $\frac{d}{dx}(x^n) = nx^{n-1}$
5. $(f+g)' = f' + g'$
6. $(f-g)' = f' - g'$
7. $\frac{d}{dx}(e^x) = e^x$
8. $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$