Section 4.5. Summary of Curve Sketching

Guidelines for sketching a curve:

Let us have a curve $y = f(x)$.

1) Domain
2) Intercepts
3) Symmetry:

$f(-x) = f(x)$: even function, symmetric about the y-axis

$f(-x) = -f(x)$: odd function, symmetric about the origin

$f(x+p) = f(x)$: periodic function, p-period

4) Asymptotes:

- horizontal

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \quad \lim_{x\to-\infty} f(x) = L$$

- vertical

$$\lim_{x\to a^+} f(x) = \infty \quad \text{or} \quad \lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty \quad \text{or} \quad \lim_{x\to a^-} f(x) = -\infty$$

- oblique

$$\lim_{x\to\infty} \left(f(x) - (mx+b) \right) = 0, \quad m \neq 0$$

5) Intervals of increase or decrease
(First Derivative Test)

6) Local max and min values:

(a) find critical points

$f'(c) = 0$ or $f'(c)$ DNE

(b) $f''(c) \neq 0$

$f''(c) > 0$ loc min

$f''(c) < 0$ loc max

7) Concavity and Points of inflection:

$f''(c) > 0$ $\cup$ Concave upward

$f''(c) < 0$ $\cap$ Concave downward

8) Sketch the curve

Example $f(x) = \dfrac{2x^2}{x^2-1}$

1) $\text{Dom}(f) = \mathbb{R} \setminus \{\pm 1\}$

$x^2 - 1 \neq 0, \quad x \neq \pm 1$

2) $x=0,\ f=0 \quad (0,0)$

$y=0,\ 0=\dfrac{2x^2}{x^2-1},\ x=0 \quad (0,0)$

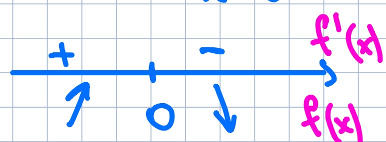
3) $f(-x)=f(x)$ - symmetric about the y-axis

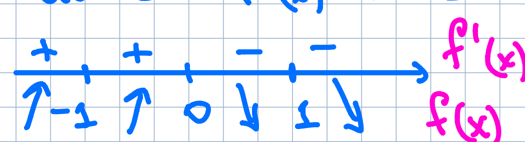
4) $\lim_{x\to\infty} \dfrac{2x^2}{x^2-1} = 2$ - horizontal asymptote

$\lim_{x\to\pm1} f(x)=\infty$ - vertical asymptote

5) $f'(x) = \dfrac{-4x}{(x^2-1)^2} = 0$

$x=0$ at ±1 $f'(x)$ DNE





f is increasing on $(-\infty,-1)\cup(-1,0)$

f is decreasing on $(0,1)\cup(1,\infty)$

6) $f''(x) = \dfrac{12x^2+4}{(x^2-1)^3}$

$f''(x)>0 \iff x^2-1>0 \iff |x|>1$

$f''(x)<0 \iff x^2-1<0 \iff |x|<1$

$f(x)$ is concave upward on $(-\infty,-1)\cup(1,\infty)$.

$f(x)$ is concave downward on $(-1,1)$.

$f(x)$ has no inflection points since $-1, 1$ are not in the domain of $f(x)$.