SECTION 5-3: THE FUNDAMENTAL THEOREM OF CALCULUS, PART 1

1. Suppose f is the function whose graph is shown and that $g(x) = \int_0^x f(t)dt$.

   (a) Find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, $g(5)$, and $g(6)$. Then, sketch a rough graph of g.

   (a) $g(0) =$ ______________

   (b) $g(1) =$ ______________

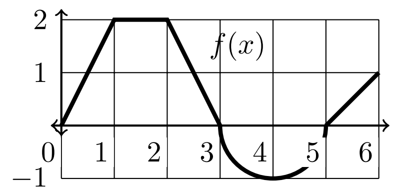
   (c) $g(2) =$ ______________

   (d) $g(3) =$ ______________

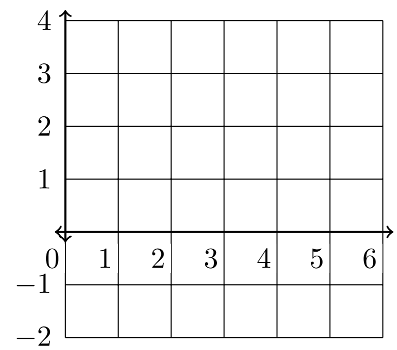
   (e) $g(4) =$ ______________

   (f) $g(5) =$ ______________

   (g) $g(6) =$ ______________



Sketch of $g(x)$



   (i) Where is $g(x)$ increasing? ______________

   (ii) Describe f when $g(x)$ is increasing. ______________

   (iii) Where is $g(x)$ decreasing? ______________

   (iv) Describe f when $g(x)$ is decreasing. ______________

   (v) Where does $g(x)$ have a local maximum? ______________

   (vi) Describe f when $g(x)$ has a local max. ______________

   (vii) Where does $g(x)$ have a local minimum? ______________

   (viii) Describe f when $g(x)$ has a local min. ______________

   (b) Make a guess: what is the relationship between $g(x)$ and $f(x)$?

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, the function g defined by

$$g(x) = \int_a^x f(t)\,dt \quad a \le x \le b$$

is continuous on $[a, b]$ and differentiable on (a, b) and $g'(x) = f(x)$.

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2. Find the derivative of $g(x) = \int_2^x t^2\,dt$.

3. The Fresnel function $S(x) = \displaystyle\int_0^x \sin(\pi t^2/2)\,dt$ first appeared in Fresnel's theory of the diffraction of light waves. Recently it was be applied to the design of highways. Find the derivative of the Fresnel function.

4. Consider $g(x) = \displaystyle\int_1^{x^4} \sec t\,dt$.

   Let $u = x^4$ and $h(x) = \int_1^x \sec t\,dt$.

   (a) Write $g(x)$ as a composition.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

5. Consider $g(x) = \displaystyle\int_{2x+1}^{2} \sqrt{t}\,dt$.

   (a) Write $g(x)$ as a composition.

   (b) Use FTC1 and the chain rule to differentiate $g(x)$.

6. Consider the function $g(x) = \displaystyle\int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt$. Observe that

$$\int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt = \int_{\tan x}^{0} \frac{1}{\sqrt{2+t^4}}\,dt + \int_{0}^{x^2} \frac{1}{\sqrt{2+t^4}}\,dt.$$

   Use properties of definite integrals, FTC1, and the chain rule to determine $g'(x)$.

SECTION 5-3: THE FUNDAMENTAL THEOREM OF CALCULUS, PART 2

The Fundamental Theorem of Calculus (Part 2) If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is **any antiderivative** of f, that is, is a function such that $F' = f$. To evaluate, we write

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^1 x^2\,dx$

   (b) $\displaystyle\int_1^4 (1 + 3y - y^2)\,dy$

2. Review from §4.9: To compute integrals effectively you **must** have your basic antidifferentiation formulas down. You should know that antiderivatives to the following functions. We are using the $\int$ symbol to mean "find the antiderivative" of the function right after the symbol.

Antiderivatives of common functions:

- $\int x^n\,dx =$ ________________
- $\int \sin x\,dx =$ ________________
- $\int \cos x\,dx =$ ________________
- $\int \sec^2 x\,dx =$ ________________
- $\int \sec x \tan x\,dx =$ ________________
- $\int \csc^2 x\,dx =$ ________________
- $\int \csc x \cot x\,dx =$ ________________
- $\int e^x\,dx =$ ________________
- $\int a^x\,dx =$ ________________
- $\int \frac{1}{1+x^2}\,dx =$ ________________
- $\int \frac{1}{\sqrt{1-x^2}}\,dx =$ ________________
- $\int \frac{1}{x}\,dx =$ ________________

3. Evaluate the following integrals.

   (a) $\displaystyle\int_2^5 \frac{3}{x}\,dx$

   (b) $\displaystyle\int_0^{\pi/2} \cos x\,dx$

4. Evaluate the following integrals.

(a) $\int_1^8 \sqrt[3]{x}\, dx$ (b) $\int_{\pi/6}^{\pi/2} \csc x \cot x\, dx$ (c) $\int_0^1 \frac{9}{1+x^2}\, dx$

5. We do not have any product or quotient rules for antidifferentiation. To evaluate an integral that is expressed as a product or quotient you must try to manipulate the integrand (the stuff inside the $\int$ sign) to look like something you know how to antidifferentiate. The following integrals are examples of this. Evaluate the following integrals.

(a) $\int_1^3 \frac{x^3+3x^6}{x^4}\, dx$ (b) $\int_0^1 x(3+\sqrt{x})\, dx$

6. Evaluate the following integrals.

(a) $\int_0^2 (5^x + x^5)\, dx$ (b) $\int_{1/2}^{\sqrt{2}/2} \frac{1}{\sqrt{1-x^2}}\, dx$

7. What is wrong with the following calculation? (Hint: draw a picture!)

$$\int_{-1}^3 \frac{1}{x^2}\, dx = \left.\frac{x^{-1}}{-1}\right|_{-1}^{3} = -\frac{1}{3} - 1 = -\frac{4}{3}$$