Derivatives of trig. functions:

1. $(\sin(x))' = \cos(x)$

2. $(\cos(x))' = -\sin(x)$

3. $(\tan(x))' = \sec^2(x)$

4. $(\cot(x))' = -\csc^2(x)$

5. $(\sec(x))' = \tan(x) \cdot \sec(x)$

6. $(\csc(x))' = -\cot(x) \cdot \csc(x)$

Product rule:

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{g^2(x)}$$

Example,

$\underline{\underline{F(x)}}$ is a composition of two functions $\underline{\underline{f}}$ and $\underline{\underline{g}}$

$$F = f(g(x)) = f \circ g$$

outside inside

$$F(x) = \underline{\underline{\sqrt{x-1}}}$$

outside → $f(x) = \sqrt{x}$

inside → $g(x) = x - 1$

Checking:

$$f(g(x)) = \sqrt{g(x)} =$$

$$= \underline{\underline{\sqrt{x-1}}}$$

$$F(x) = \cos x^2$$

outside → $f(x) = \cos(x)$

inside → $g(x) = x^2$

 Solutions

1. Complete the Chain Rule (using both types of notation)

- If $F(x) = f(g(x))$,

  then $F'(x) =$ $f'(g(x)) \cdot g'(x)$

- If $y = f(u)$ and $u = g(x)$,

  then $\frac{dy}{dx} =$ $\frac{df}{du} \cdot \frac{du}{dx}$

2. For each function below, write it as a nontrivial composition of functions in the form $f(g(x))$. Then use the chain rule to compute the derivative.

(a) $H(x) = \sqrt[3]{4 - 2x}$

outside $= f(x) =$ $\sqrt[3]{x} = x^{1/3}$

inside $= g(x) =$ $4-2x$

$H'(x) = \frac{1}{3}(4-2x)^{-2/3} \cdot (4-2x)' = \frac{1}{3}(4-2x)^{-2/3} \cdot (-2)$

(b) $H(x) = \tan(2 - x^4)$

outside $= f(x) =$ $\tan(x)$

inside $= g(x) =$ $2-x^4$

$H'(x) = \sec^2(2-x^4) \cdot (-4x^3)$

(c) $H(x) = e^{2-2x^3}$

outside $= f(x) =$ e^{x}

inside $= g(x) =$ $2-2x^3$

$H'(x) = e^{2-2x^3} \cdot (-6x^2)$

(d) $H(x) = \frac{4}{x+\sin(x)}$

outside $= f(x) =$ $\frac{4}{x}$ $\quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

inside $= g(x) =$ $x+\sin(x)$

$H'(x) = \frac{-4}{(x+\sin(x))^2} \cdot (1+\cos(x))$

3. For each problem below, find the derivative.

(a) $z(x) = (2x^3 - 5x)^7$

$f = x^7$

$g = 2x^3 - 5x$

$z'(x) = 7 \cdot (2x^3 - 5x)^6 \cdot (6x^2 - 5)$

(b) $x(\theta) = (\cos(\theta))^3$

$f(\theta) = \theta^3$

$g(\theta) = \cos(\theta)$

$x'(\theta) = 3\cos^2(\theta) \cdot (-\sin(\theta))$

(c) $y = x^2 - 3\sin(x^3)$

$f = \sin(x)$

$g = x^3$

$y' = 2x - 3 \cdot \cos(x^3) \cdot 3x^2$

(d) $y = 10e^{\sqrt{t}}$

$f = 10e^t$

$g = \sqrt{t}$

$y' = 10 \cdot e^{\sqrt{t}} \cdot \frac{1}{2 \cdot \sqrt{t}}$

$f = e^t$

$g = \sqrt{t}$

$y' = 10 \cdot (e^{\sqrt{t}})' = 10 \cdot e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$

(e) $f(x) = \dfrac{\sqrt{2}}{\sqrt{x^2 - 4}}$

$f = \frac{\sqrt{2}}{\sqrt{x}}$

$g = x^2 - 4$

$f'(x) = \sqrt{2} \cdot \left(-\frac{1}{2}\right)(x^2-4)^{-3/2} \cdot (x^2-4)' = -\frac{1}{\sqrt{2}}(x^2-4)^{-3/2} \cdot 2x$

(f) $g(x) = \dfrac{\sec(x^2 + 2)}{12}$

$f = \sec(x)$

$g = x^2 + 2$

$g'(x) = \frac{1}{12}\tan(x^2+2)\sec(x^2+2) \cdot 2x$

(g) $k(s) = \dfrac{A^2}{B + Cs}$ (A, B, C are constants!)

$f = \frac{A^2}{s}$

$g = B + Cs$

$k'(s) = -\frac{A^2}{(B+Cs)^2} \cdot (B+Cs)' = -\frac{A^2}{(B+Cs)^2} \cdot C$