Name: ______________________________ ________ / 12

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Be sure to include constants of integration when appropriate.
- Circle your final answer.

Compute the following integrals.

$$\frac{x^{-3+1}}{-3+1} = \frac{1}{-2x^2}$$

1. $\int_1^2 \frac{x^4+1}{x^3}\,dx = \int_1^2 x\,dx + \int_1^2 x^{-3}\,dx = \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)\Big|_1^2 =$

$$= \left(\frac{2^2}{2} - \frac{1}{2\cdot 2^2}\right) - \underbrace{\left(\frac{1}{2} - \frac{1}{2}\right)}_{=0} = 2 - \frac{1}{8}$$

2. $\int \frac{2-3\ln t}{t}\,dt = \int \frac{u}{\cancel{t}}\cdot\left(-\frac{\cancel{t}}{3}\right)du = -\frac{1}{3}\int u\,du = -\frac{1}{3}\,\frac{u^2}{2} + C =$

Substitution:

$u = 2 - 3\ln t$

$du = -3\cdot\frac{1}{t}\,dt$

$dt = -\frac{t}{3}\,du$

$$= -\frac{1}{3}\,\frac{(2-3\ln t)^2}{2} + C$$

$$\int\left(\frac{2}{t} - \frac{3\ln t}{t}\right)dt = \int \frac{2}{t}\,dt - 3\int\frac{\ln t}{t}\,dt$$

$$= 2\ln|t| - 3\cdot \ldots$$

3. $\int_\pi^{2\pi} (\cos\theta - 4)\,d\theta =$

$$= \left(\sin(\theta) - 4\theta\right)\Big|_\pi^{2\pi} = \left(\underbrace{\sin(2\pi)}_{=0} - 4\cdot 2\pi\right) - \left(\underbrace{\sin(\pi)}_{=0} - 4\pi\right) =$$

$$= -8\pi + 4\pi = -4\pi$$

$\boxed{4.}$ $\int z\sqrt{z+2}\,dz$ $= \int (u-2)\sqrt{u}\,du = \int \left(u\sqrt{u} - 2\sqrt{u}\right)du =$

$z+2=u$

$du = dz$

$z = u-2$

$$= \int \left(u \cdot u^{1/2} - 2\cdot u^{1/2}\right)du =$$

$$= \int \left(u^{3/2} - 2u^{1/2}\right)du =$$

$$= \frac{u^{5/2}}{5/2} - 2\,\frac{u^{3/2}}{3/2} + C =$$

$$= \frac{(z+2)^{5/2}}{5/2} - 2\,\frac{(z+2)^{3/2}}{3/2} + C$$

5. $\int \tan^2 x \sec^2 x\, dx$ $=$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$dx = \frac{du}{\sec^2(x)}$

$$= \int u^2 \cdot \cancel{\sec^2}(x) \cdot \frac{du}{\cancel{\sec^2(x)}} = \int u^2\,du =$$

$$= \frac{u^3}{3} + c = \frac{(\tan(x))^3}{3} + c$$

$\boxed{1}$

$\boxed{6.}$ $\int \left(\frac{4}{1+x^2} + \frac{1+x^2}{4}\right)dx$ $= \int \frac{4}{1+x^2}\,dx + \int \frac{1+x^2}{4}\,dx = 4\arctan(x) +$

$$+ \frac{1}{4}\int (1+x^2)\,dx = 4\arctan(x) +$$

$$+ \frac{1}{4}\left(x + \frac{x^3}{3}\right) + C$$

$\boxed{1}$

7. $\int t\cos(5-3t^2)\,dt = -\frac{1}{6}\sin(5-3t^2)+C$

$\boxed{1}$

$u = 5-3t^2$

$du = -6t\,dt$

$dt = \frac{du}{-6t}$

$$\int t\cos(5-3t^2)\,dt = \int \cancel{t}\cdot\cos(u)\cdot\frac{du}{-6\cancel{t}} = -\frac{1}{6}\int\cos(u)\,du = -\frac{1}{6}\sin(u)+C$$

$$= \underline{-\frac{1}{6}\sin(5-3t^2)+C}$$

8. $\int(\sin\theta)e^{\cos\theta}\,d\theta = \int \cancel{\sin\theta}\cdot e^u\cdot\frac{du}{-\cancel{\sin\theta}} =$

$\cos\theta = u$

$du = -\sin\theta\,d\theta$

$d\theta = \frac{du}{-\sin\theta}$

$$= -\int e^u\,du = -e^u + C =$$

$$= -e^{\cos\theta} + C$$

$\boxed{1}$

9. $\int_{-1}^{1}(x+3)(x-4)\,dx = \int_{-1}^{1}(x^2+3x-4x-12)\,dx =$

$$= \int_{-1}^{1}(x^2-x-12)\,dx = \left(\frac{x^3}{3}-\frac{x^2}{2}-12x\right)\Big|_{-1}^{1}$$

$\boxed{1}$

$$= \left(\frac{1}{3}-\frac{1}{2}-12\right)-\left(-\frac{1}{3}-\frac{1}{2}+12\right)$$

$\boxed{10.}$ $\displaystyle\int \frac{t^2}{t^3-9}\,dt = \int \frac{\cancel{t^2}}{u}\cdot\frac{1}{3\cancel{t^2}}\,du = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\cdot \ln|u| + C =$

$t^3-9=u$

$du = 3t^2\,dt$

$dt = \frac{1}{3t^2}\,du$

$= \frac{1}{3}\cdot \ln|t^3-9| + C$

$\boxed{1}$

11. $\displaystyle\int \sqrt[3]{x^4} - \sqrt[3]{5}\,dx = \int \left(x^{4/3} - 5^{1/3}\right) dx = \frac{x^{7/3}}{7/3} - 5^{1/3}x + C$

$\boxed{1}$

12. $\displaystyle\int \left(3e^w - \frac{1}{w^5}\right) dw = \int 3e^w\,dw - \int \frac{1}{w^5}\,dw =$

$= \int 3e^w\,dw - \int w^{-5}\,dw =$

$\boxed{1}$

$= 3e^w - \frac{w^{-4}}{-4} + C$