1. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$ $= \int \frac{\cancel{u}}{\tan(x)} \frac{du}{2\cdot\cancel{u}\cdot\tan(x)} = \frac{1}{2}\int \frac{du}{u-1}$ =

$u = \sec^2(x)$

$du = 2\cdot\sec(x)\cdot\sec(x)\cdot\tan(x)\cdot dx$

$du = 2\cdot\sec^2(x)\cdot\tan(x)\cdot dx$

$dx = \frac{du}{2\cdot\sec^2(x)\cdot\tan(x)}$

$t = u - 1$

$dt = 1\cdot du$

$du = dt$

$\sec^2(x) = \frac{1}{\cos^2 x} = \tan^2(x) + 1 \Rightarrow \tan^2(x) = \sec^2(x) - 1$

$\tan^2(x) = u - 1$

2. Compute $\int \sec^2(x)\tan(x)\,dx$

$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C =$

$= \frac{1}{2}\ln|u-1| + C = \boxed{\frac{1}{2}\ln|\sec^2(x) - 1| + C}$

3. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}d\theta$

$\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta = -\int \frac{\cancel{\sin(\theta)}}{u}\cdot\frac{du}{\cancel{\sin(\theta)}} = -\int \frac{1}{u}\,du = -\ln|u| + C$ =

$u = 1 + \cos(\theta)$

$du = -\sin(\theta)\,d\theta$

$d\theta = -\frac{du}{\sin\theta}$

$= -\ln|1+\cos(\theta)| + C$

Verification:

$$\left(\frac{1}{2}\ln\left|\sec^2(x)-1\right|\right)' =$$

$$= \frac{1}{\cancel{2}}\,\frac{1}{\sec^2(x)-1}\cdot\cancel{2}\sec(x)\cdot\sec(x)\cdot\tan(x) =$$

$$= \frac{1}{\sec^2(x)-1}\cdot\sec^2(x)\cdot\tan(x) =$$

$$= \frac{1}{\tan^{\cancel{2}}(x)}\,\sec^2(x)\cdot\cancel{\tan(x)} =$$

$$= \boxed{\frac{\sec^2(x)}{\tan(x)}}$$

Alternative approach

$$\int \frac{\sec^2(x)}{\tan(x)}\,dx = \int \frac{\cancel{\sec^2(x)}}{u}\,\frac{du}{\cancel{\sec^2(x)}} = \int \frac{1}{u}\,du =$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$dx = \dfrac{du}{\sec^2(x)}$

$$= \ln|u| + C =$$

$$= \boxed{\ln|\tan(x)| + C}$$

Verification:

$$\left(\ln|\tan(x)|\right)' = \frac{1}{\tan(x)} \cdot \sec^2(x)$$

2\. (a) $\int \sec^2(x)\tan(x)\,dx = \int \cancel{\sec^2(x)} \cdot u \cdot \frac{du}{\cancel{\sec^2(x)}} =$

$u = \tan(x)$

$du = \sec^2(x) \cdot dx$

$dx = \frac{du}{\sec^2(x)}$

$= \int u^1\,du = \frac{u^2}{2} + C = \frac{(\tan(x))^2}{2} + C$

(b) $\int \sec^2(x)\tan(x)\,dx = \int u \cdot \frac{\cancel{\sec(x) \cdot \tan(x)}}{\cancel{\sec(x) \cdot \tan(x)}} \cdot du =$

$u = \sec(x)$

$du = \sec(x) \cdot \tan(x) \cdot dx$

$dx = \frac{du}{\sec(x) \cdot \tan(x)}$

$= \int u\,du = \frac{u^2}{2} + C = \frac{(\sec(x))^2}{2} + C$

4. Compute $\int \frac{1}{x\ln(x)}\,dx$ $= \int \frac{1}{u} \cdot \frac{1}{\cancel{x}} \cdot \cancel{x} \cdot du = \int \frac{1}{u}\,du = \ln|u| + C$

$u = \ln(x)$

$du = u'(x) \cdot dx$

$du = \frac{1}{x}\,dx$

$dx = x \cdot du$

$= \ln|\ln(x)| + C$

5. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$ $= \int \frac{\sin(u)}{\cancel{x^2}} \cdot \frac{-\cancel{x^2}}{4}\,du = -\frac{1}{4}\int \sin(u)\,du$ $\ominus$

$u = \frac{4}{x}$

$du = 4 \cdot \left(-\frac{1}{x^2}\right)dx$

$dx = -\frac{x^2}{4}\,du$

$\ominus$ $-\frac{1}{4}\left(-\cos(u)\right) + C =$

$= \frac{1}{4}\cos\left(\frac{4}{x}\right) + C$

6. Compute $\int \frac{e^x}{e^x - 3}\,dx$ $= \int \frac{\cancel{e^x}}{u} \cdot \frac{du}{\cancel{e^x}} = \int \frac{1}{u}\,du = \ln|u| + C =$

$u = e^x - 3$

$du = e^x\,dx$

$dx = \frac{du}{e^x}$

$= \ln|e^x - 3| + C$

7. Compute $\int \frac{1}{9+x^2}\,dx$ $= \int \frac{1}{9(1+\frac{x^2}{9})}\,dx = \frac{1}{9}\int \frac{1}{1+\frac{x^2}{3^2}}\,dx = \frac{1}{9}\int \frac{1}{1+(\frac{x}{3})^2}\,dx =$

$u = \frac{x}{3}$

$du = \frac{1}{3}\,dx$

$dx = 3 \cdot du$

$= \frac{1}{9}\int \frac{1}{1+u^2} \cdot 3 \cdot du =$

$= \frac{3}{9}\int \frac{1}{1+u^2}\,du =$

$= \frac{1}{3}\arctan(u) + C =$

$= \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$

8. Compute $\int \sqrt{x}(x^4+x)\,dx$ $=$

$= \int (\sqrt{x} \cdot x^4 + \sqrt{x} \cdot x)\,dx =$

$= \int (x^{9/2} + x^{3/2})\,dx = \frac{2 \cdot x^{11/2}}{11} + \frac{2 \cdot x^{5/2}}{5} + C$

9. Compute $\int \cos(x)\sin(\sin(x))\,dx$ $= \int \cancel{\cos(x)} \cdot \sin(u) \cdot \frac{du}{\cancel{\cos(x)}} = \int \sin(u)\,du =$

$u = \sin(x)$

$du = \cos(x) \cdot dx$

$dx = \frac{du}{\cos(x)}$

$= -\cos(u) + C =$

$= -\cos(\sin(x)) + C$

10. Compute $\frac{d}{dx}[x\ln(x) - x]$. Then compute $\int s^2 \ln(s^3)\, ds$

11. Compute $\int x\sqrt{x-1}\, dx$ (Hint: Let $u = x - 1$. What is x in terms of u?)

$\boxed{u = x - 1} \Rightarrow \boxed{x = u + 1}$

$du = 1 \cdot dx$

$$\int x\sqrt{x-1}\, dx = \int x\sqrt{u}\cdot du = \int (u+1)\sqrt{u}\, du = \int \left(u^{3/2} + u^{1/2}\right) du =$$

$$= \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C =$$

$$= \frac{(x-1)^{5/2}}{5/2} + \frac{(x-1)^{3/2}}{3/2} + C$$

12. Compute $\int_1^3 \frac{(\ln(x))^3}{x}\, dx = \int_{\ln 1}^{\ln 3} u^3 \cdot du =$

$u = \ln(x)$

$du = \frac{1}{x}\, dx$

$dx = x \cdot du$

$1 \leq x \leq 3$

$\ln 1 \leq u \leq \ln 3$

$0 \leq u \leq \ln 3$

FTC part 2

$$= \frac{u^4}{4}\Bigg|_{\ln 1}^{\ln 3} = \frac{(\ln 3)^4}{4} - \frac{(\ln 1)^4}{4}.$$