Section 4.4. Indeterminate Forms and L'Hospital's Rule

$$F(x) = \frac{\ln x}{x-1}$$

$$\lim_{x \to 1} \frac{\ln x}{x-1} = \boxed{\frac{0}{0}}$$

Indeterminate forms:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, ∞^0, 1^∞

$\frac{0}{0}$: L'H

$\frac{\infty}{\infty}$: L'H

$0 \cdot \infty$: $\frac{1}{\infty} \cdot \infty$ L'H; $0 \cdot \frac{1}{0}$ L'H

$\infty - \infty$: Algebra, $\frac{0}{0}$ or $\frac{\infty}{\infty}$ L'H

0^0, ∞^0, 1^∞: take ln of our function; $0 \cdot \infty$; $\frac{\infty}{\infty}$ or $\frac{0}{0}$ L'H

L'Hospital's Rule

Suppose that f and g are differentiable functions and $g'(x) \neq 0$ on some open interval I that contains a.

Suppose that

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

or

$$\lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

exists or is equal $\pm\infty$.

$\frac{0}{0} \quad \frac{\pm\infty}{\pm\infty}$

SECTION 4.4: LIMITS OF INDETERMINATE TYPE AND L'HOSPITAL'S RULE

Evaluate:

1. $\lim_{x\to 2} \dfrac{x^2-4}{x^2-5x+6}$ (type $\dfrac{4-4}{10-10} = \dfrac{0}{0}$)

L'H

$$\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} \overset{L'H}{=} \lim_{x\to 2} \frac{2x}{2x-5} = \frac{2\cdot 2}{2\cdot 2-5} = \frac{4}{-1} = -4$$

2. $\lim_{x\to 0} \dfrac{\sin x}{x}$ (type $\dfrac{0}{0}$)

$$\lim_{x\to 0} \frac{\sin x}{x} \overset{L'H}{=} \lim_{x\to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

3. $\lim_{x\to 0} \dfrac{\tan(5x)}{\sin(3x)}$ (type _____)

4. $\lim_{u\to\infty} \dfrac{e^{u/10}}{u^2}$ (type $\dfrac{\infty}{\infty}$)

$$\lim_{u\to\infty} \frac{e^{u/10}}{u^2} \overset{L'H}{=} \lim_{u\to\infty} \frac{e^{u/10}\cdot\frac{1}{10}}{2u} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{u\to\infty} \frac{e^{\frac{u}{10}}\cdot\frac{1}{100}}{2} =$$

$$= \infty$$

5. $\lim_{x\to 0} \dfrac{\cos(4x)}{e^{2x}}$ (type _____)

6. $\lim_{x\to 0} \dfrac{xe^x}{2^x - 1}$ (type _____)

7. $\lim_{x\to 1^+} \big(\ln(x^4 - 1) - \ln(x^9 - 1)\big)$ (type _____)

8. $\lim_{x\to \infty} \sqrt{x}e^{-x/2}$ (type _____)

9. $\lim_{x\to 0^+} (1 + \sin(2x))^{1/x}$ (type _____)