Solutions

THEORETICAL PART:

Theorem (Slopes of Parallel Lines): "$\parallel$" - parallel

Two nonvertical lines with slopes m_1 and m_2 are parallel if and only if $m_1 = m_2$. Also, two vertical lines are always parallel to each other.

Theorem (Slopes of Perpendicular Lines): "$\perp$" - perpendicular

Suppose m_1 and m_2 represent the slopes of two lines, neither of which is vertical. The two lines are perpendicular if and only if

$$m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_2 = -\frac{1}{m_1} \quad \text{or} \quad m_1 m_2 = -1.$$

If one of two perpendicular lines is vertical, the other is horizontal, and their slopes are, respectively, undefined and zero.

PRACTICAL PART:

1. Find equations for two lines parallel to each of the given lines:

   (a) $y = -\frac{2}{3}x + 4$

   (b) $10x - 2y = 14$

$m_1 = m_2$

(a) $l_1: y = -\frac{2}{3}x + 4$

$m_1 = -\frac{2}{3}$

$l_2: y = -\frac{2}{3}x + 10$ (10 can be any number)

$m_2 = -\frac{2}{3}$

(b) $10x - 2y = 14$

$-2y = 14 - 10x$

$l_1: y = 5x - 7$

$m_1 = 5$

$l_2: y = 5x - 1000$

2. Find the equation, in slope-intercept form, for the line that is parallel to the line $3x + 5y = 23$ and passes through the point $(-2, 1)$.

Slope-intercept form: $y = mx + b$

$l_1: y = m_1 x + b$

$l_2: 3x + 5y = 23 \Rightarrow y = -\frac{3}{5}x + \frac{23}{5}$

$m_2 = -\frac{3}{5}$

$l_1 \parallel l_2 \Rightarrow m_1 = m_2 = -\frac{3}{5}$

parallel

Thus, $m_1 = -\frac{3}{5}$.

$l_1: y = -\frac{3}{5}x + b$

$(-2, 1)$ belongs to l_1. Therefore,

$$1 = -\frac{3}{5}(-2) + b$$

$$1 = \frac{6}{5} + b \Rightarrow b = -\frac{1}{5}$$

Hence,

$$l_2: \boxed{y = -\frac{3}{5}x - \frac{1}{5}}$$

3. For each line given, find the equation of a perpendicular line:

   - $y = -\frac{4}{9}x + 2$.
   - The line passing through the points $(-1, 3)$ and $(4, 1)$.

$l_1 \perp l_2$ if and only if $m_1 \cdot m_2 = -1$

- $y = -\frac{4}{9}x + 2$

$m_1 = -\frac{4}{9}$

$m_2 \cdot \left(-\frac{4}{9}\right) = -1 \Rightarrow m_2 = \frac{9}{4}$

$y = \boxed{\frac{9}{4}}x + 5$

- $(-1,3)$, $(4,1)$

$m_1 = \frac{1-3}{4+1} = -\frac{2}{5}$

$m_2 = -\frac{1}{m_1} = \frac{5}{2}$

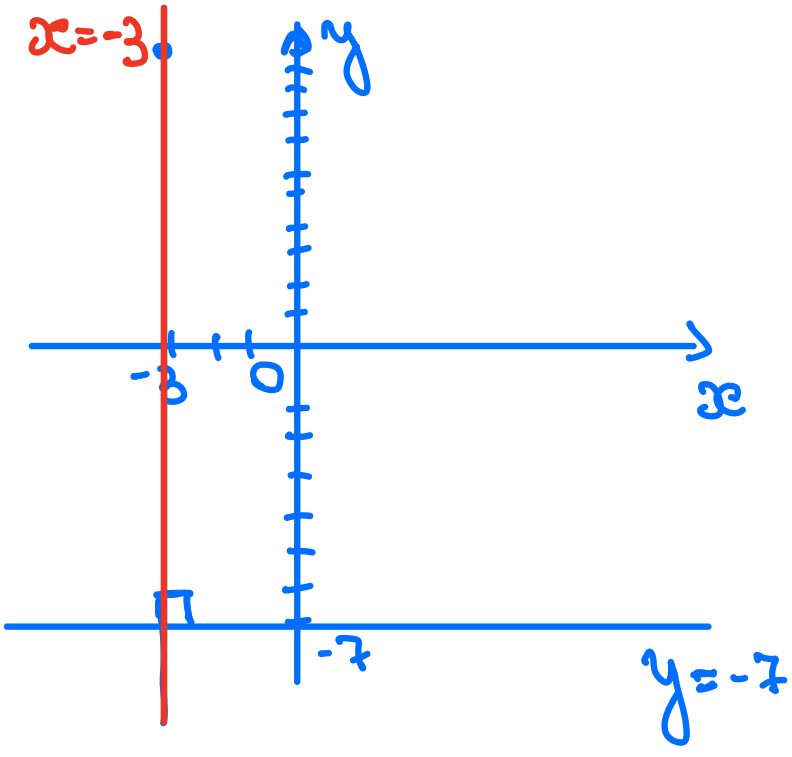
$y = \boxed{\frac{5}{2}}x + 100$

4. Find the equation, in standard form, of the line that passes through the point $(-3, 13)$ and is perpendicular to the line $y = -7$.

Stan. form: $ax + by = c$

$y = -7$

We see that a line $\perp$ to $y = -7$ is a vertical line. Since it also passes through $(-3, 13)$ we see that this line is $\boxed{x = -3}$



5. For each pair of lines, determine if the lines are parallel, perpendicular, or neither:

   - $3x - 7y = 12$ and $14x + 6y = -5$
   - $y = \frac{3}{4}x + 1$ and $y = \frac{4}{3}x - 5$

- $3x - 7y = 12$

$-7y = 12 - 3x$

$y = \boxed{\frac{3}{7}}x - \frac{12}{7}$

$14x + 6y = -5$

$6y = -5 - 14x$

$y = -\frac{14}{6}x + \frac{5}{6}$

$y = \boxed{-\frac{7}{3}}x + \frac{5}{6}$

$m_1 \cdot m_2 = \frac{3}{7} \cdot \left(-\frac{7}{3}\right) = -1$

Therefore, two lines are perpendicular

- $y = \frac{3}{4}x + 1$

$m_1 = \frac{3}{4}$

$y = \frac{4}{3}x - 5$

$m_2 = \frac{4}{3}$

$m_1 \neq m_2 \Rightarrow$ two lines are not parallel

$m_1 \cdot m_2 = 1 \neq -1 \Rightarrow$ two lines are not $\perp$