THEORETICAL PART: Solutions

Properties (Summary of Logarithmic Properties):

1. The equations $x = a^y$ and $y = \log_a x$ are equivalent, and are, respectively, the exponential form and the logarithmic form of the same statement.
2. The inverse of the function $f(x) = a^x$ is $f^{-1}(x) = \log_a x$, and vice versa.
3. A consequence of the last point is that $\log_a(a^x) = x$ and $a^{\log_a x} = x$. In particular, $\log_a 1 = 0$ and $\log_a a = 1$.
4. $\log_a(xy) = \log_a x + \log_a y$.
5. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$.
6. $\log_a(x^r) = r\log_a x$.

PRACTICAL PART:

1. Solve the equation $3^{2-5x} = 11$. Express the answer exactly and as a decimal approximation.

$$3^{2-5x} = 11$$
$$\log_3 3^{2-5x} = \log_3 11$$
$$2-5x = \log_3 11$$
$$5x = 2-\log_3 11$$
$$x = \frac{1}{5}(2-\log_3 11) \approx \boxed{-0.037}$$

2. Solve the equation $5^{3x-1} = 2^{x+3}$. Express the answer exactly and as a decimal approximation.

$$\ln 5^{3x-1} = \ln 2^{x+3}$$
$$(3x-1)\ln 5 = (x+3)\ln 2$$
$$3x\ln 5 - \ln 5 = x\ln 2 + 3\ln 2$$
$$x(3\ln 5 - \ln 2) = 3\ln 2 + \ln 5$$
$$x = \frac{3\ln 2 + \ln 5}{3\ln 5 - \ln 2} \approx \boxed{0.892}$$

3. Solve the equation $log_7(3x-2) = 2$.

$$7^{\log_7(3x-2)} = 7^2$$

$$3x - 2 = 7^2 = 49$$

$$3x = 2 + 49 = 51$$

$$\boxed{x = \frac{51}{3} = 17}$$

4. Solve the equation $\log_5 x = \log_5(2x+3) - \log_5(2x-3)$.

$$\log_5 x - \log_5(2x+3) + \log_5(2x-3) = 0$$

$$\log_5\left(\frac{x}{2x+3}\right) + \log_5(2x-3) = \log_5 1$$

$$\log_5\left(\frac{x(2x-3)}{2x+3}\right) = \log_5 1$$

$$\frac{x(2x-3)}{2x+3} = 1$$

$$x(2x-3) = 2x+3$$

$$2x^2 - 3x - 2x - 3 = 0$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3)=0$$

$$x=-\frac{1}{2} \quad \text{or} \quad x=3$$

Check: $x=-\frac{1}{2}$

$\log_5\left(-\frac{1}{2}\right)$ is not defined

x has to be a positive real number

Answer: $\boxed{x=3}$

5. (Compounding Interest)

Rita is saving up money for a down payment on a new car. She currently has \$5500 but she knows she can get a loan at a lower interest rate if she can put down \$6000. If she invests her \$5500 in a money market account that earns an annual interest rate of 4.8% compounded monthly, how long will it take her to accumulate the \$6000?

$$\boxed{A = P\left(1+\frac{r}{n}\right)^{nt}}$$

$P = 5500$

$r = 0.048$

$n = 12$

$A = 6000$

$t = ?$ (years)

$$6000 = 5500\left(1+\frac{0.048}{12}\right)^{12t}$$

$$\frac{6000}{5500} = \left(1+\frac{0.048}{12}\right)^{12t}$$

$$\ln\left(\frac{6000}{5500}\right) = 12t\ \ln(1.004)$$

$$t = \frac{\ln\left(\frac{6000}{5500}\right)}{12\ln(1.004)}$$

$t \approx 1.82$ (around a year + 10 months)