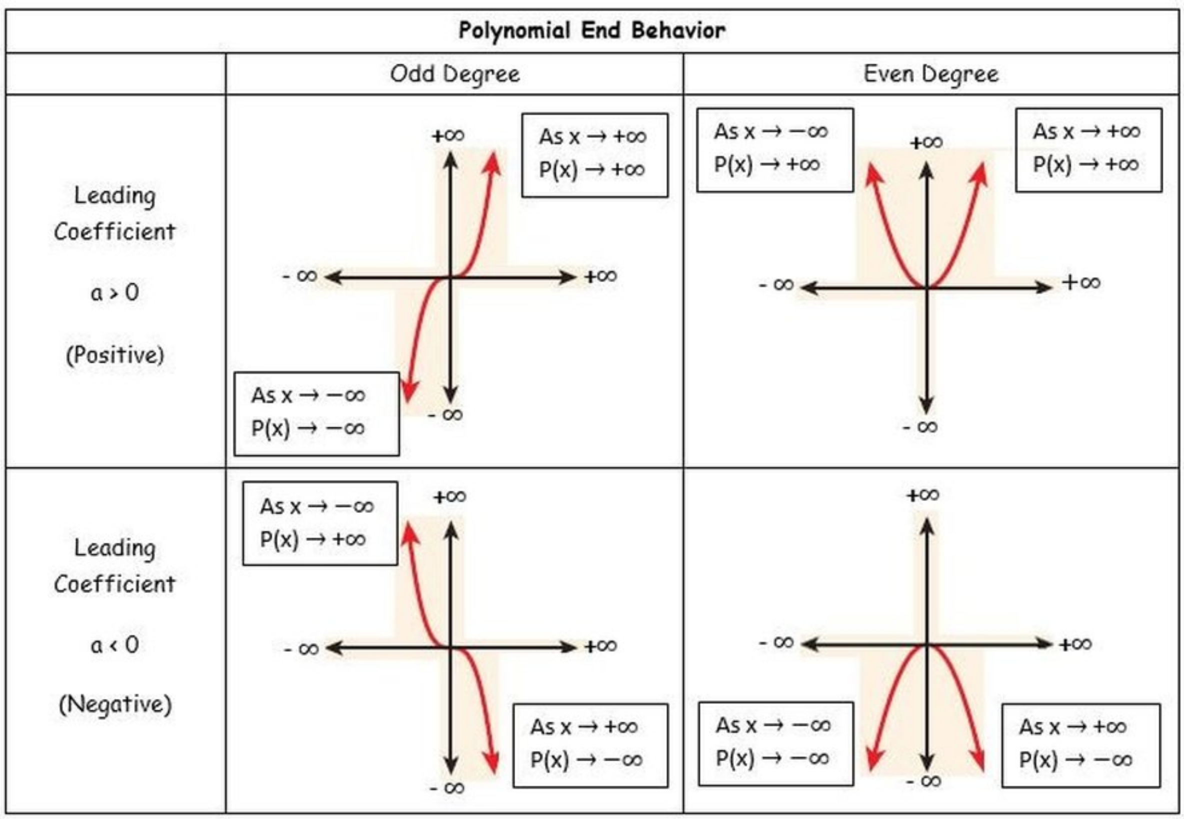
THEORETICAL PART:

Definition (Zeros of a polynomial):

The number c (which may be a complex number) is a **zero** of a polynomial function $p(x)$ if $p(c) = 0$. This is also expressed by saying that c is a **root** of the polynomial or a **solution** of the equation $p(x) = 0$.

Definition (Polynomial equations):

A polynomial equation in one variable, say x, is an equation that can be written in the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$, where n is a nonnegative integer and $a_0, a_1, \cdots, a_n$ are constants. Assuming $a_n \neq 0$, we say such an equation is of **degree** n and call a_n as a **leading coefficient**.



Definition (Polynomial Inequalities):

A **polynomial inequality** is any inequality that can be written in the form

$$p(x) > 0, \quad p(x) < 0, \quad p(x) \geq 0, \quad p(x) \leq 0,$$

where $p(x)$ is a polynomial function.

Procedure (Solving polynomial inequalities using the sign-test method):

To solve the polynomial inequality $p(x) < 0$, $p(x) > 0$, $p(x) \leq 0$, or $p(x) \geq 0$, perform the following steps:

1. Find the real zeros of $p(x)$. Equivalently, find the real solutions of the equation $p(x) = 0$.
2. Place the zeros on a number line, splitting it into intervals.
3. Within each interval, select a **test point** and evaluate p at that number. If the result is positive, then $p(x) > 0$ for all x in the interval. If the result is negative, then $p(x) < 0$ for all x in the interval.
4. Write the solution set, consisting of all the intervals that satisfy the given inequality. If the inequality is not strict ($\geq$ or $\leq$), then the zeros are included in the solution set as well.

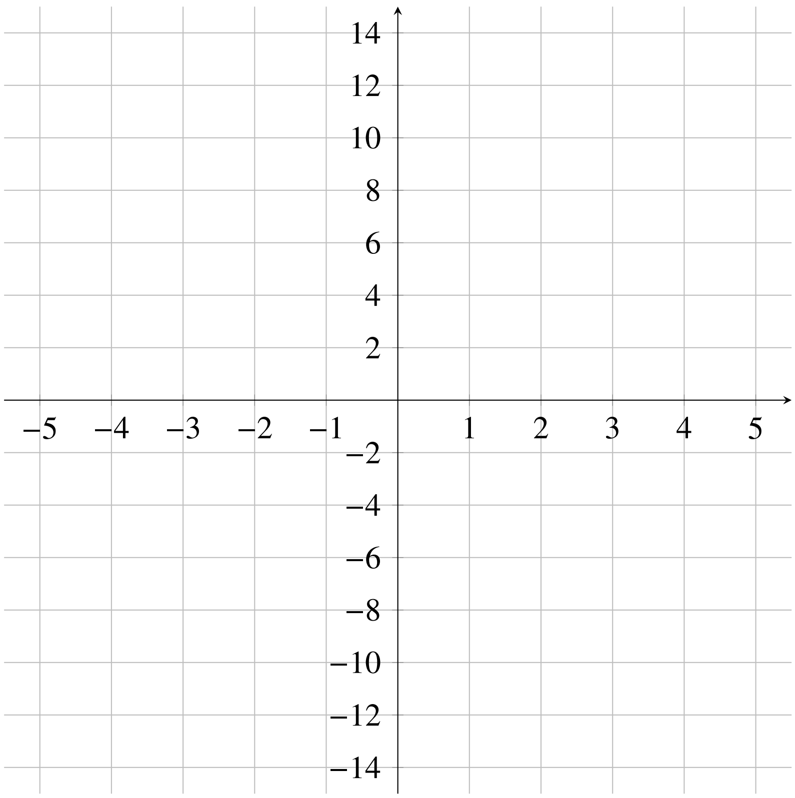
PRACTICAL PART:

1. Verify that the given values of x solve the corresponding polynomial equations:
   (a) $6x^2 - x^3 = 12 + 5x$, $x = 4$
   (b) $x^2 = 2x - 5$, $x = 1 + 2i$

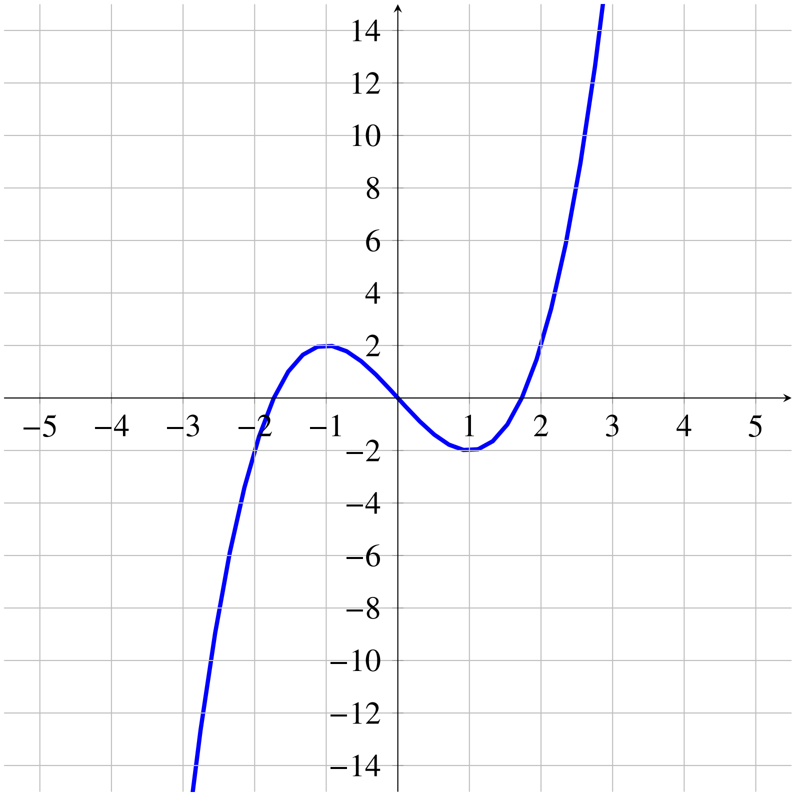
2. Sketch the graphs of the following polynomial functions, paying attention to the x-intercept(s), y-intercept, and the behaviour as $x \to \pm\infty$:

   (a) $f(x) = -x(2x+1)(x-2)$

   (b) $g(x) = x^2 + 2x - 3$



3. Find the polynomial of lowest possible degree that corresponds to the graph below:



4. Solve the following polynomial inequalities:

   (a) $(x + 3)(x + 1)(x - 2) < 0$

   (b) $(x + 3)(x + 1)(x - 2) \geq 0$

5. Solve the polynomial inequality:

$$(x+2)(x-1)^2(x-3) \leq 0$$