

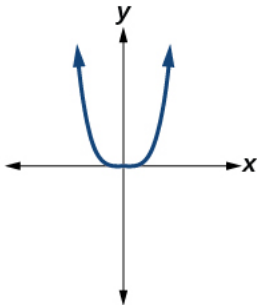
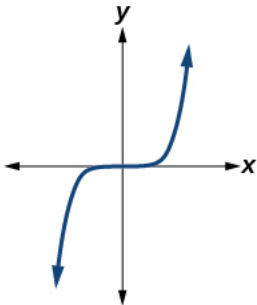
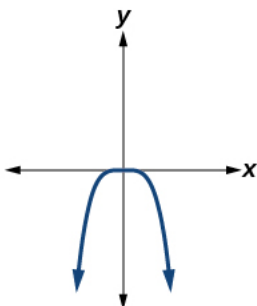
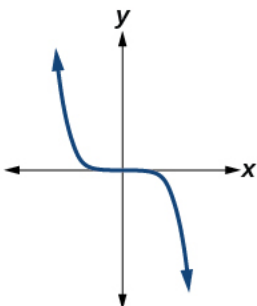
THEORETICAL PART:

Definition (Zeros of a polynomial):

The number c (which may be a complex number) is a **zero** of a polynomial function $p(x)$ if $p(c) = 0$. This is also expressed by saying that c is a **root** of the polynomial or a **solution** of the equation $p(x) = 0$.

Definition (Polynomial equations):

A **polynomial equation in one variable**, say x , is an equation that can be written in the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$, where n is a nonnegative integer and a_0, a_1, \dots, a_n are constants. Assuming $a_n \neq 0$, we say such an equation is of **degree n** and call a_n as a **leading coefficient**.

	Even power	Odd power
Positive constant $k > 0$	 <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$ and $x \rightarrow \infty, f(x) \rightarrow \infty$</p>	 <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$ and $x \rightarrow \infty, f(x) \rightarrow \infty$</p>
Negative constant $k < 0$	 <p>$x \rightarrow -\infty, f(x) \rightarrow -\infty$ and $x \rightarrow \infty, f(x) \rightarrow -\infty$</p>	 <p>$x \rightarrow -\infty, f(x) \rightarrow \infty$ and $x \rightarrow \infty, f(x) \rightarrow -\infty$</p>

Definition (Polynomial Inequalities):

A **polynomial inequality** is any inequality that can be written in the form

$$p(x) > 0, \quad p(x) < 0, \quad p(x) \geq 0, \quad p(x) \leq 0,$$

where $p(x)$ is a polynomial function.

Procedure (Solving polynomial inequalities using the sign-test method):

To solve the polynomial inequality $p(x) < 0$, $p(x) > 0$, $p(x) \leq 0$, or $p(x) \geq 0$, perform the following steps:

1. Find the real zeros of $p(x)$. Equivalently, find the real solutions of the equation $p(x) = 0$.
2. Place the zeros on a number line, splitting it into intervals.
3. Within each interval, select a **test point** and evaluate p at that number. If the result is positive, then $p(x) > 0$ for all x in the interval. If the result is negative, then $p(x) < 0$ for all x in the interval.
4. Write the solution set, consisting of all the intervals that satisfy the given inequality. If the inequality is not strict (\geq or \leq), then the zeros are included in the solution set as well.

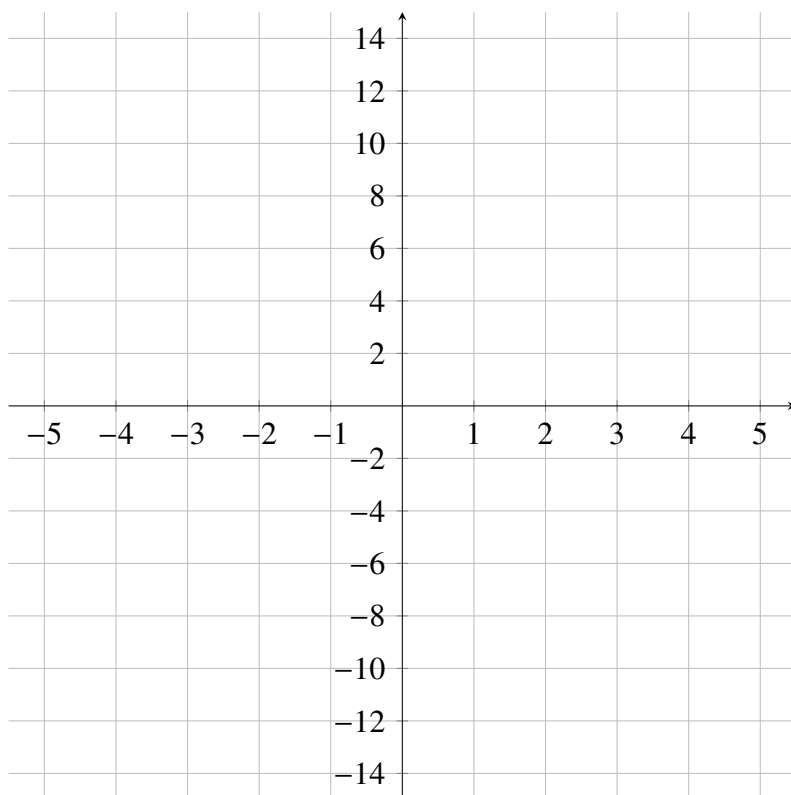
PRACTICAL PART:

1. Verify that the given values of x solve the corresponding polynomial equations:
 - (a) $6x^2 - x^3 = 12 + 5x$, $x = 4$
 - (b) $x^2 = 2x - 5$, $x = 1 + 2i$

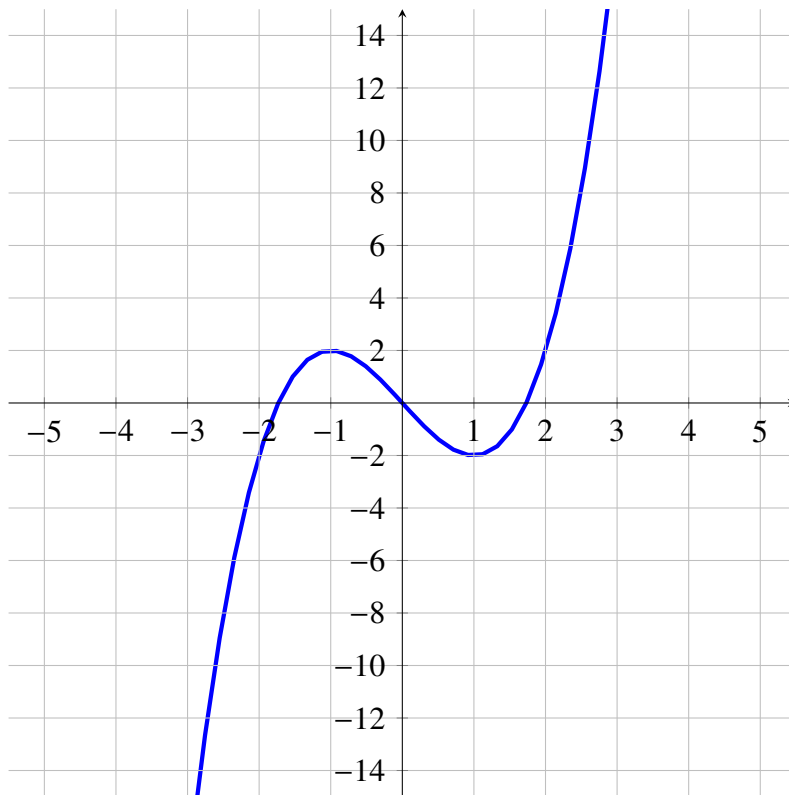
2. Sketch the graphs of the following polynomial functions, paying attention to the x -intercept(s), y -intercept, and the behaviour as $x \rightarrow \pm\infty$:

(a) $f(x) = -x(2x + 1)(x - 2)$

(b) $g(x) = x^2 + 2x - 3$



3. Find the polynomial of lowest possible degree that corresponds to the graph below:



4. Solve the following polynomial inequalities:

(a) $(x + 3)(x + 1)(x - 2) < 0$

(b) $(x + 3)(x + 1)(x - 2) \geq 0$

5. Solve the polynomial inequality:

$$(x + 2)(x - 1)^2(x - 3) \leq 0$$