Name: Solutions ______ / 40

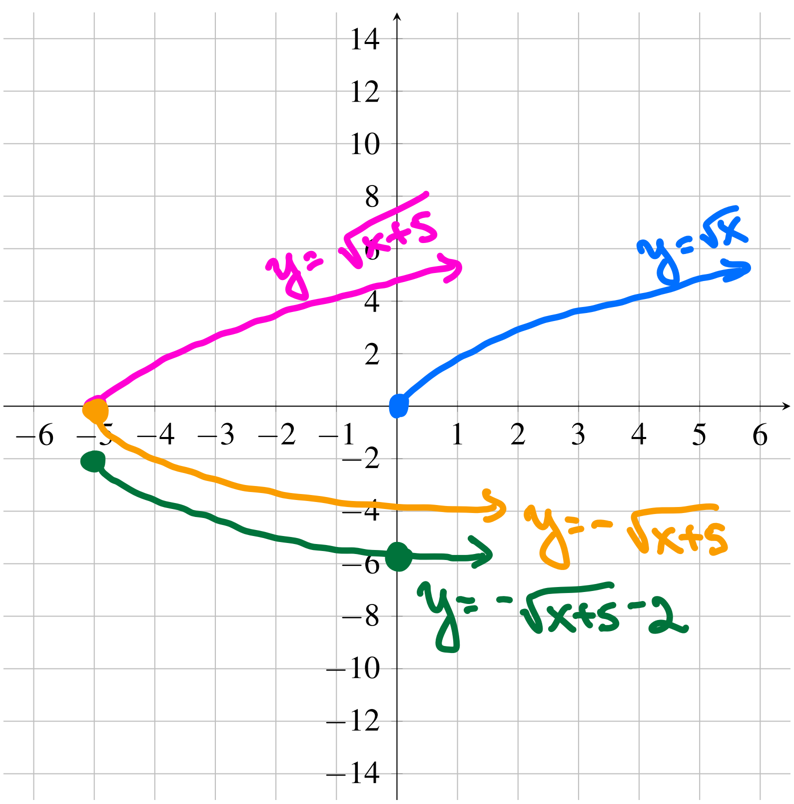
Assessment 3 Instructions:

- The AS-3 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-3.
- The AS-3 is closed book and closed notes.
- **Calculators are not allowed** on the AS-3.
- Show all your work for full credit and box your final answer.

1. [4 points] Graph the function

$$f(x) = -\sqrt{x+5} - 2$$

by making the appropriate transformations of a basic curve. State the basic function, the transformations and find all intercepts that exist.



① horiz. shift to the left by ⑤

② reflection w.r.to x-axis

③ vertical shift down by ②

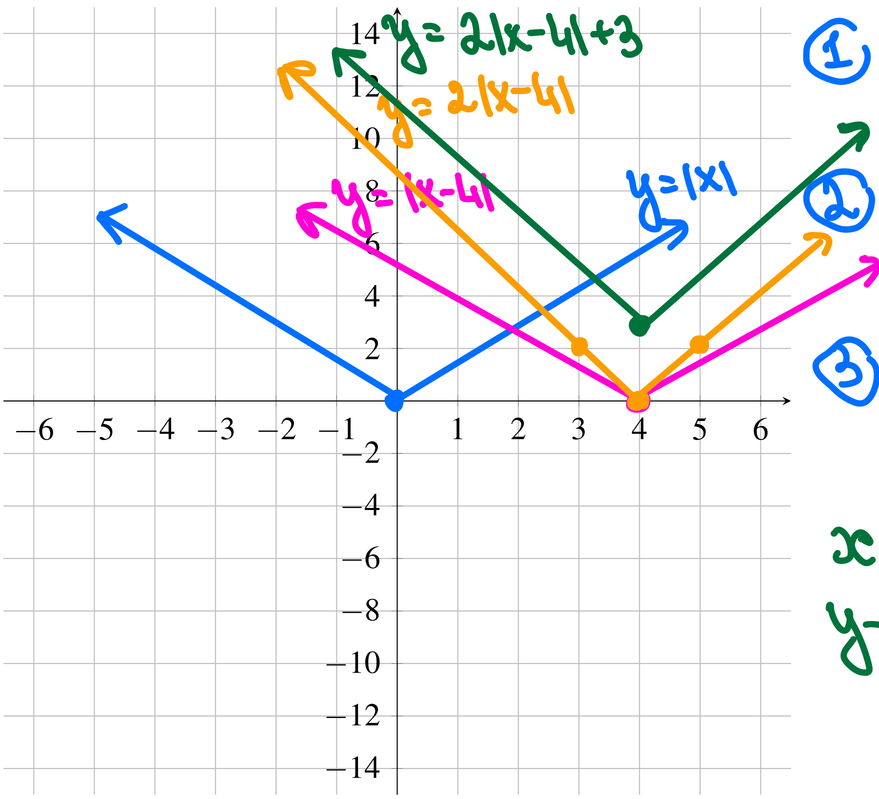
x-intercept: None

y-intercept: $(0, -\sqrt{5} - 2)$

2. [4 points] Graph the function

$$g(x) = 2|x-4| + 3$$

by making the appropriate transformations of a basic curve. State the basic function, the transformations and find all intercepts that exist.



① horizontal shift to the right by 4

② vertical stretching by 2

③ vertical shift up by 3

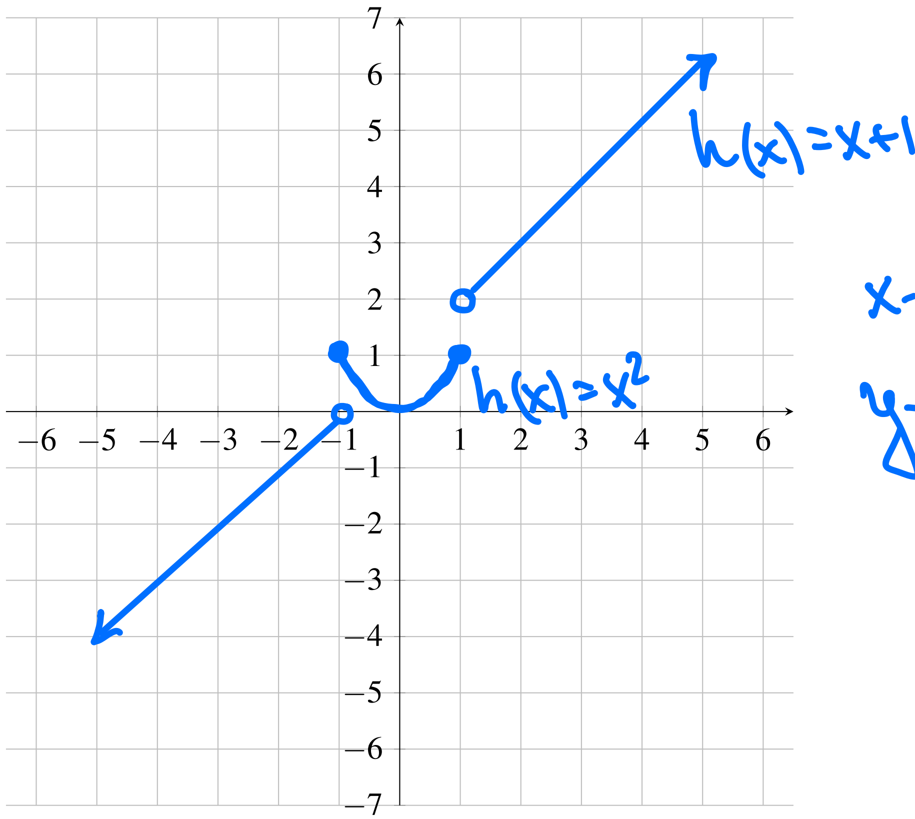
x-intercept: None

y-intercept: (0, 11)

3. [4 points] Graph the function

$$h(x) = \begin{cases} x^2, & -1 \le x \le 1, \\ x+1, & x < -1 \text{ or } x > 1 \end{cases}$$

State all intercept points that exist.



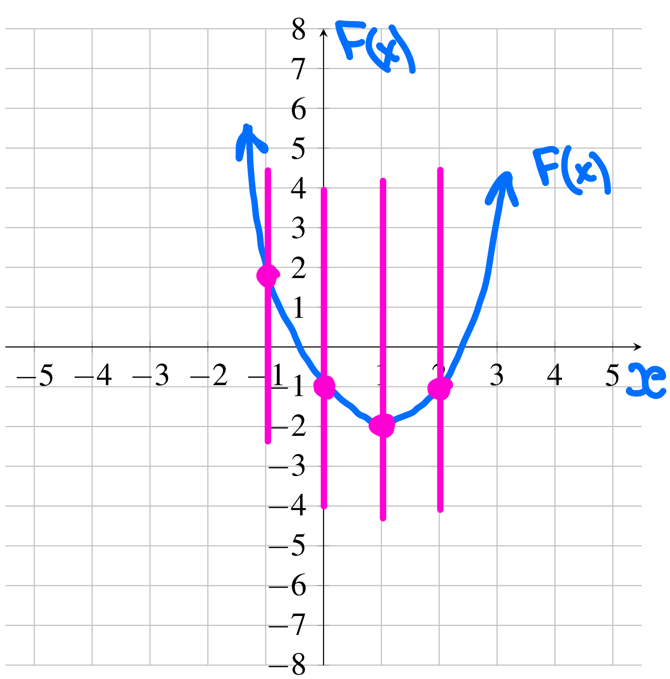
x- intercept: (0,0)

y- intercept: (0,0)

4. [4 points]

a. Determine if the following relation $F(x) = (x-1)^2 - 2$ is a function.

Hint: sketch a graph and use a Vertical Line Test.

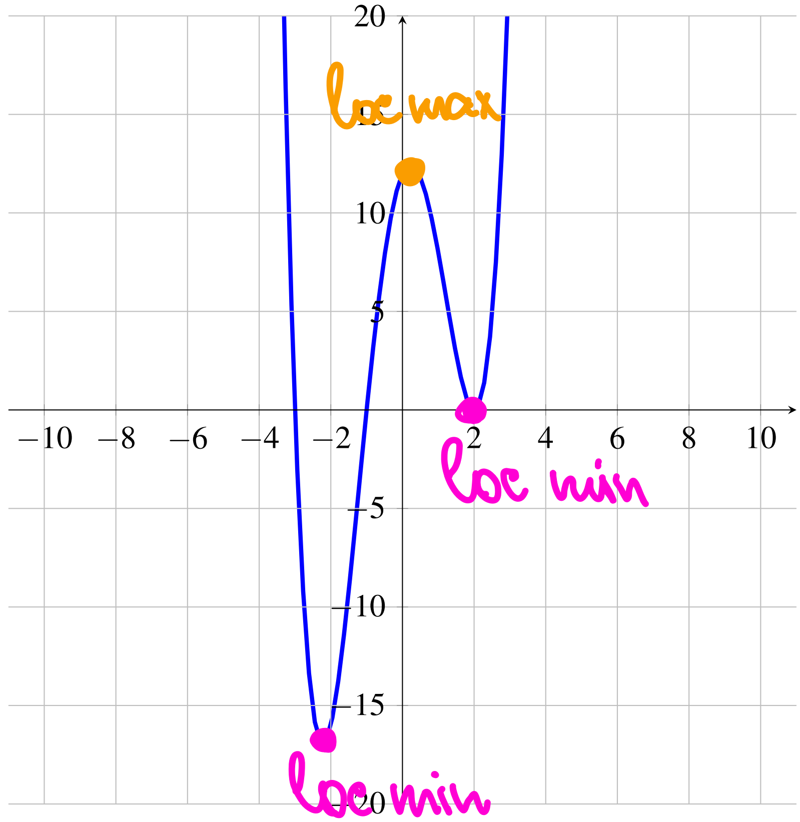


relation F(x) is a function since the vertical line crosses F(x) exactly at one point.

b. If the above relation is a function, then find the open intervals of monotonicity where the function is increasing, decreasing, or constant.

F(x) is increasing on $(1,\infty)$
F(x) is decreasing on $(-\infty,1)$

5. [4 points] Using the graph of the function below determine:



a. the locations and types of the local extrema (local min and max)

at $x=-2.2$ and $x=2$ we have a loc. min

at $x=0.2$ we have loc. max

b. the values of the local extrema

$f(-2.2)=-17$ $f(0.2)\approx 13$

$f(2)=0$

6. [4 points] For the given function determine:

$$f(x) = \frac{3}{x+4}$$

a. domain of f $\quad x \neq -4$

$\mathrm{Dom}(f) = \mathbb{R} \setminus \{-4\}$

b. $f(0) = \frac{3}{4}$

c. $\dfrac{f(x+1)-f(x)}{x} = \dfrac{\frac{3}{x+5} - \frac{3}{x+4}}{x} = \dfrac{3(x+4)-3(x+5)}{(x+5)(x+4)\,x} =$

$f(x+1) = \dfrac{3}{x+5}$

$= \dfrac{\cancel{3x}+12-\cancel{3x}-15}{x(x+5)(x+4)} = \dfrac{-3}{x(x+4)(x+5)}$

7. [4 points] For the given functions

$$g(x) = x^2 - 1, \quad \text{and} \quad h(x) = \sqrt[3]{x}$$

a. find the **formula** $(g+h)(x)$ and **domain** for $\cancel{f}\,h+g$

$\mathrm{Dom}(g) = \mathbb{R}$

$\mathrm{Dom}(h) = \mathbb{R}$

$(g+h)(x) = x^2 - 1 + \sqrt[3]{x}$

$\mathrm{Dom}(h+g) = \mathrm{Dom}(h) \cup \mathrm{Dom}(g) = \mathbb{R}$

b. find the **formula** $(g \cdot h)(x) =$

$(g \cdot h)(x) = (x^2-1)\sqrt[3]{x} = x^{2+\frac{1}{3}} - \sqrt[3]{x} = x^{7/3} - \sqrt[3]{x} = \sqrt[3]{x^7} - \sqrt[3]{x}$

c. find the **formula** $(h \circ g)(x) =$ $h(g(x)) = h(x^2-1) = \sqrt[3]{x^2-1}$

8. [4 points] For the given relation

$$R = \{(4,2),(3,-1),(-2,-1),(2,4)\}$$

a. find the inverse R^{-1} of the given relation

$R^{-1} = \{(2,4),(-1,3),(-1,-2),(4,2)\}$

b. find the domain of the inverse relation R^{-1}

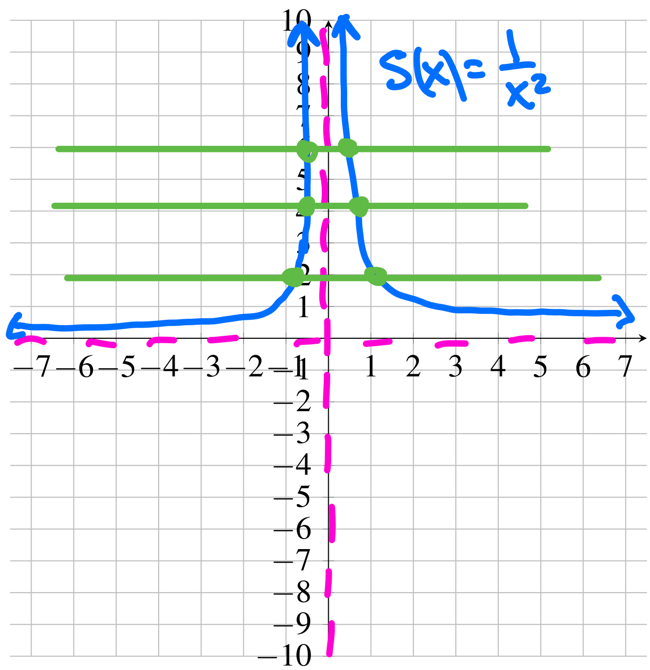
$\text{Dom}(R^{-1}) = \{2,-1,4\}$

c. find the range of the inverse relation R^{-1}

$\text{Ran}(R^{-1}) = \{4,3,-2,2\}$

9. [4 points] Determine if the function $s(x) = \dfrac{1}{x^2}$ has an inverse function $s^{-1}(x)$.

Hint: sketch a graph and use a Horizontal Line Test or use a one-to-one function definition.



The function $S(x)$ does not have an inverse $S^{-1}(x)$ since the horizont. line crosses $S(x)$ at two points:

$S(-1) = \frac{1}{(-1)^2} = 1$

$S(1) = \frac{1}{1} = 1$

$S(x)$ is not one-to-one

10. [4 points] Find a formula for the inverse of the following function

$$f(x) = \sqrt[3]{3x-1}.$$

$y = \sqrt[3]{3x-1}$

$y^3 = 3x - 1$

$y^3 + 1 = 3x$

$\frac{1}{3}(y^3+1) = x$

$\boxed{f^{-1}(x) = \frac{1}{3}(x^3+1)}$

11. [Extra Credit, 4 points points]

Write a formula for the function described below:

Use the function $g(x) = |x|$. Move the function 7 units to the left, reflect across the x-axis, and reflect across the y-axis.

$g(x) = |x|$

$g_1(x) = |x+7|$

$g_2(x) = -|x+7|$

$\boxed{g_3(x) = -|-x+7|}$