# Section 7.6. Inverse Trigonometric Functions

1. The inverse trigonometric functions.
2. Evaluating inverse trigonometric functions.
3. Applications of inverse trigonometric functions.

## 1.

**Def.** (Arcsine)

Given $x \in [-1,1]$, <u>arcsine</u> is defined by either of the following:

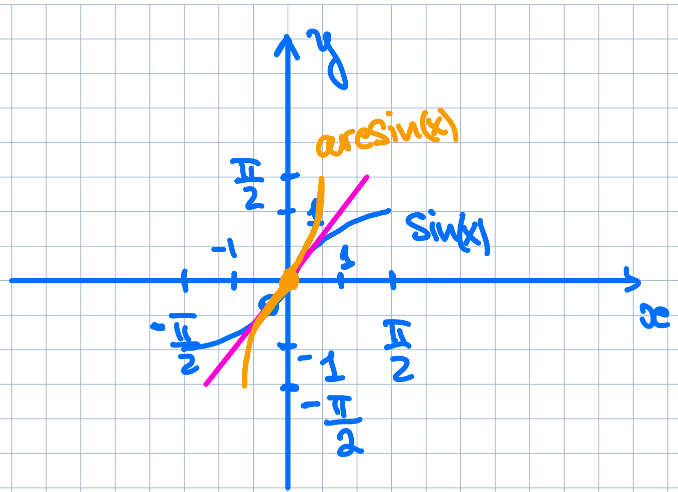
$$\arcsin x = y \iff x = \sin y$$

or

$$\sin^{-1} x = y \iff x = \sin y$$

$\mathrm{Dom}(\arcsin x) = [-1,1]$

$\mathrm{Ran}(\arcsin x) = [-\frac{\pi}{2}, \frac{\pi}{2}]$.

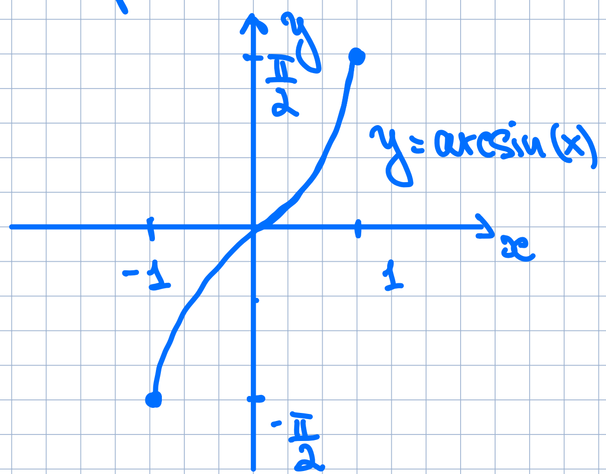



$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$

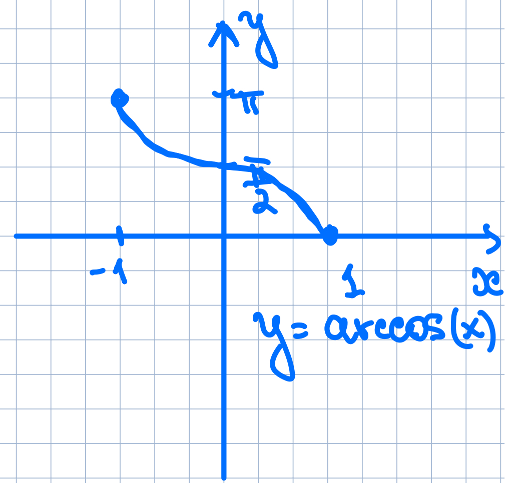
**Def.** (Inverse trigonometric functions)

| Function | Domain | Range | Notation |
|---|---|---|---|
| Inverse sine | $[-1,1]$ | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ | $\arcsin x = y \Leftrightarrow x = \sin y$ |
| Inverse cosine | $[-1,1]$ | $[0,\pi]$ | $\arccos x = y \Leftrightarrow x = \cos(y)$ |
| Inv. tangent | $(-\infty,\infty)$ | $(-\frac{\pi}{2}, \frac{\pi}{2})$ | $\arctan x = y \Leftrightarrow x = \tan y$ |
| Inv. cosecant | $(-\infty,1] \cup [1,\infty)$ | $[-\frac{\pi}{2},0) \cup (0,\frac{\pi}{2}]$ | $\operatorname{arccsc} x = y \Leftrightarrow x = \csc y$ |
| Inv. sec. | $(-\infty,-1] \cup [1,\infty)$ | $[0,\frac{\pi}{2}) \cup (\frac{\pi}{2},\pi]$ | $\operatorname{arcsec} x = y \Leftrightarrow x = \sec y$ |
| Inv. cotangent | $(-\infty,\infty)$ | $(-\frac{\pi}{2},0) \cup (0,\frac{\pi}{2}]$ | $\operatorname{arccot} x = y \Leftrightarrow x = \cot y$ |

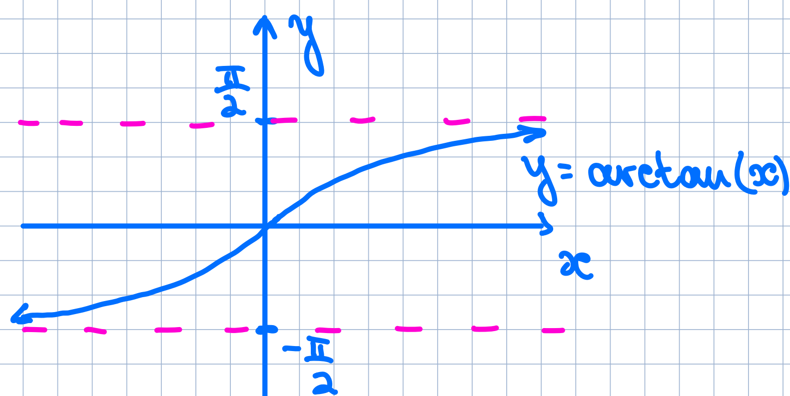
## Graphs:



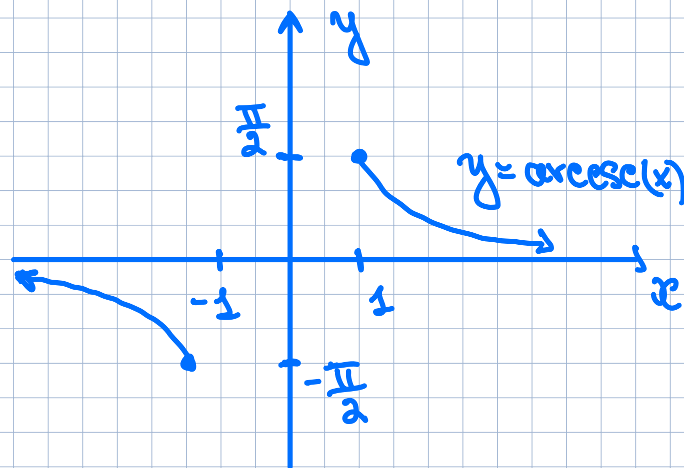

Domain: $[-1, 1]$
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$




Domain: $[-1, 1]$
Range: $[0, \pi]$

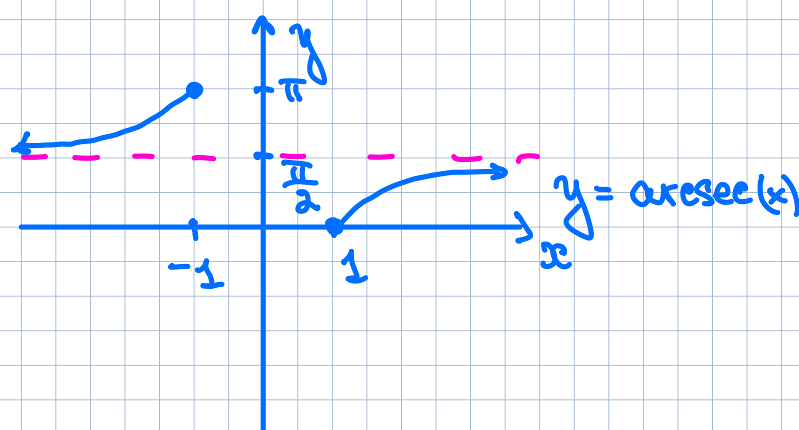



Domain: $(-\infty, \infty)$
Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$



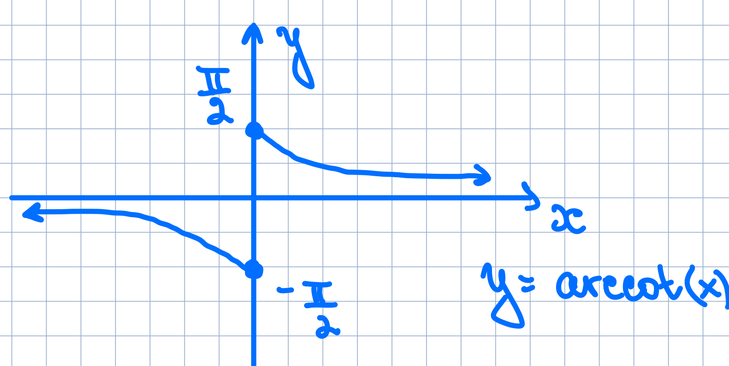

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$




Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$




Domain: $(-\infty, \infty)$

Range: $(-\frac{\pi}{2}, 0) \cup$
$\cup (0, \frac{\pi}{2}]$

$\boxed{2.}$

## Example (Evaluating inverse trigonometric functions)

- $\arctan(-1) = ?$

$\arctan(-1) = x \Leftrightarrow -1 = \tan(x)$

$$\boxed{x = -\frac{\pi}{4}}$$

- $\operatorname{arccsc}(2) = ?$

$\operatorname{arccsc}(2) = x \Leftrightarrow 2 = \csc(x)$

$$2 = \frac{1}{\sin(x)}$$

$$\sin(x) = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}}$$

## Example (Evaluating compositions of trigonometric functions)

- $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right) = ?$

$$\sin\left(\frac{3\pi}{4}\right) = \sin\left(\pi - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

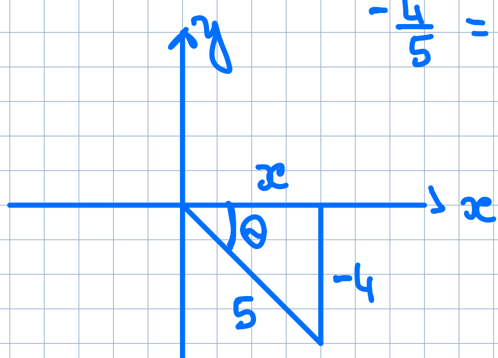
$$\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

<u>Example</u> (Evaluating compositions of trigonometric functions)

- $\tan\left(\sin^{-1}\left(-\frac{4}{5}\right)\right) =$

$$\sin^{-1}\left(-\frac{4}{5}\right) = -\sin^{-1}\left(\frac{4}{5}\right) = \theta$$

$$-\frac{4}{5} = \sin(\theta)$$




By Pythagorean Theorem:

$$x = \sqrt{25 - 16}$$

$$x = 3$$

$$\tan\theta = \frac{-4}{3}$$

$$\Downarrow$$

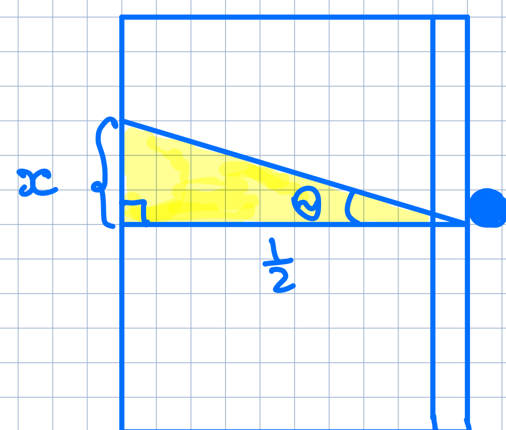
$$\boxed{\tan\left(\sin^{-1}\left(-\frac{4}{5}\right)\right) = -\frac{4}{3}}$$

$\boxed{3.}$

## Example (Using Inverse Trigonometric Functions)

A lighthouse is to be constructed half a mile from a long, straight reef.
In order to ensure the light illuminates certain portions of the reef within specified lengths of time, the engineer needs a formula for $\theta$ in terms of $x$.
Find such a formula.

## Solution




$$\tan\theta = \frac{x}{1/2}$$

$$\tan\theta = 2x$$

$$\Downarrow$$

$$\boxed{\theta = \tan^{-1}(2x)}$$