Solutions

THEORETICAL PART:

Definition:

A **rational equation** is an equation that contains at least one rational expression, while any nonrational expressions are polynomials.

General approach to solve such equations: multiply each term in the equation by the LCD of all the rational expressions.

Applications of Rational Equations:

- *The rate of work is the reciprocal of the time needed to complete the task:* if a given job can be done by a worker in x units of time, the worker works at a rate of $\frac{1}{x}$ jobs per unit of time.
- *Rates of work are additive.*

Definition.

A **radical equation** is an equation that has at least one radical expression containing a variable, while any nonradical expressions are polynomial terms.

Procedure of solving radical equations:

Step 1. Begin by isolating the radical expression on one side of the equation. If there is more than one radical expression, choose one to isolate on one side.

Step 2. Raise both sides of the equation by the power necessary to "undo" the isolated radical. That is, if the radical is an n-th root, raise both sides to the n-th power.

Step 3. If any radical expressions remain, simplify the equation if possible and then repeat steps 1 and 2 until the result is a polynomial equation. When a polynomial equation has been obtained, solve the equation using polynomial methods.

Step 4. Check your solutions in the original equation! Any extraneous solutions must be discarded.

Definition.

Meaning of $a^{\frac{m}{n}}$: If m and n are natural numbers with $n \neq 0$, if m and n have no common factors greater than 1, and if $\sqrt[n]{a}$ is a real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

PRACTICAL PART:

1. Solve the following rational equation:
   - $\frac{x^3+3x^2}{x^2-2x-15} = \frac{4x+5}{x-5}$

$$\frac{x^2\cancel{(x+3)}}{\cancel{(x+3)}(x-5)} = \frac{4x+5}{x-5}$$

$$\frac{x^2}{\cancel{x-5}} = \frac{4x+5}{\cancel{x-5}}$$

$$x^2 = 4x+5$$

$$x^2-4x-5=0$$

$$(x-5)(x+1)=0$$

$$\boxed{x=5 \quad \text{or} \quad x=-1}$$

2. Solve the equation:
   - $|x-5| = \frac{7}{x+1}$

$$x-5 = \frac{7}{x+1} \quad \text{or} \quad x-5 = \frac{-7}{x+1}$$

$$(x+1)(x-5)=7 \quad \text{or} \quad (x-5)(x+1)=-7$$

$$x^2-4x-5=7 \qquad x^2-4x-5=-7$$

$$x^2-4x-12=0 \qquad x^2-4x+2=0$$

$$(x-6)(x+2)=0 \quad \text{or}$$

$$\boxed{\begin{array}{l} x=6 \\ x=-2 \end{array}}$$

$$D=16-8=8$$

$$x_3 = \frac{4+\sqrt{8}}{2}$$

$$x_4 = \frac{4-\sqrt{8}}{2}$$

3. Solve the radical equations:
   - (a) $\sqrt{1-x} - 1 = x$
   - (b) $\sqrt{x+1} + \sqrt{x+2} = 1$

(a)

$$\sqrt{1-x}-1=x$$

$$(\sqrt{1-x})^2=(1+x)^2$$

$$\cancel{1}-x = \cancel{1}+2x+x^2$$

$$x^2+3x=0$$

$$x(x+3)=0$$

$$\boxed{x=0} \quad \text{or} \quad \cancel{\boxed{x=-3}}$$

(b)

$$(\sqrt{x+1})^2=(1-\sqrt{x+2})^2$$

$$\cancel{x}+\cancel{1} = \cancel{1}-2\sqrt{x+2}+\cancel{x}+2$$

$$\cancel{2}\sqrt{x+2} = \cancel{2}$$

$$(\sqrt{x+2})^2=(1)^2$$

$$x+2=1$$

$$\boxed{x=-1}$$

4. Solve the following equation with rational exponents:

(a) • $x^{\frac{2}{3}} - 9 = 0$

(b) • $(32x^2 - 32x + 17)^{\frac{1}{4}} = 3$

(a) $x^{\frac{2}{3}} = 9$

$\left(\sqrt[3]{x^2}\right)^3 = 9^3$

$x^2 = 9^3 = 729$

$\boxed{x = \pm\sqrt{729}}$

(b) $\sqrt[4]{32x^2 - 32x + 17} = 3$

$32x^2 - 32x + 17 = 3^4 = 81$

$32x^2 - 32x - 64 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \quad \text{or} \quad x = -1}$

5. You are given the **escape speed** formula

$$v_e^2 = \left(\sqrt{\frac{2GM}{r}}\right)^2$$

Solve for r.

$v_e^2 = \frac{2GM}{r} \quad \Big| \cdot r$

$r \cdot v_e^2 = 2GM$

$$\boxed{r = \frac{2GM}{v_e^2}}$$