Solutions

THEORETICAL PART:

Definition:

- If the equality symbol in a linear equation is replaced with $<, \leq, \geq, >$, the result is a linear inequality.

Properties: Let A, B and C represent algebraic expressions and D represent a nonzero constant. Then

- If $A < B$, then $A + C < B + C$
- If $A < B$ and $D > 0$, then $A \cdot D < B \cdot D$
- If $A < B$ and $D < 0$, then $A \cdot D > B \cdot D$

The inequality symbol $<$ can be replaced with $>, \geq, \leq$.

Definition:

- A **compound inequality** is a statement containing two or more distinct inequalities joined by either of the words "and" (**double inequality**) or "or".
- An **absolute value inequality** is an inequality in which some variable expression appears inside absolute value symbols.

$$|x| < a \Leftrightarrow \quad -a < x < a$$

$$|x| > a \Leftrightarrow \quad x < -a \quad \text{or} \quad x > a$$

Translating inequality phrases:

- "x is no greater than y": $x \leq y$
- "x is at least as large as y": $x \geq y$
- "x does not exceed y": $x \leq y$
- "y exceeds x": $x < y$

PRACTICAL PART:

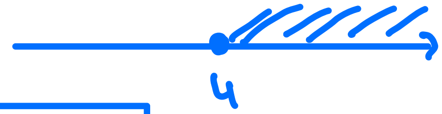
1. Solve the following inequalities, using interval notation to describe the solution set:

(a)

$$5 - 2(x - 3) \leq -(1 - x)$$

$5 - 2x + 6 \leq -1 + x$

$3x \geq 12 \Rightarrow x \geq 4$



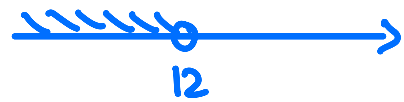
$\boxed{[4, \infty)}$

(b)

$$\frac{3(a-2)}{2} < \frac{5a}{4}$$

$3(a-2) \cdot 2 < 5a$

$6a - 12 < 5a \Rightarrow a < 12$

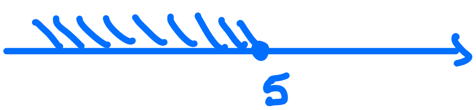


$\boxed{(-\infty, 12)}$

2. Graph the following intervals:

(a) $[-3, 6]$

(b) $(-\infty, 5]$



(c) $[2, 9)$

3. Solve the following double inequalities:

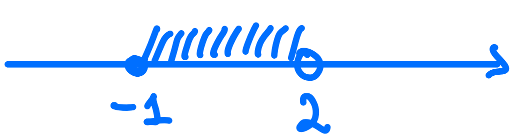
(a)

$$-1 \overset{-3}{<} 3 - 2x \leq 5 \quad -3$$

$-4 < -2x \leq 2 \quad |:(-2)$

$-1 \leq x < 2$

$\boxed{[-1, 2)}$



(b)

$$2(2x - 1) \leq 4x + 2 \leq 4(x + 1)$$

$$\cancel{4x} - 2 \leq \cancel{4x} + 2 \leq \cancel{4x} + 4$$

$$-2 \leq 2 \leq 4$$

$$\boxed{(-\infty, \infty)}$$

4. Solve the following absolute value inequalities:

(a)

$$|4 - 2x| > 6$$

$4 - 2x > 6$

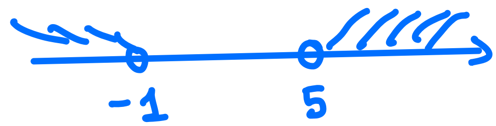
$-2x > 2$

$x < -1$

$4 - 2x < -6$

$-2x < -10$

$x > 5$



$$\boxed{(-\infty, -1) \cup (5, \infty)}$$

(b)

$$|5 + 2s| \leq -3$$

$\emptyset$

The absolute value of a real number cannot be negative

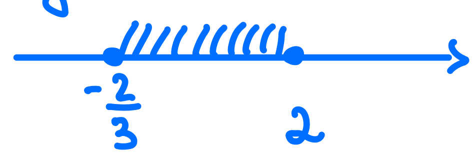
(c)

$$2|3y - 2| + 3 \leq 11$$

$2|3y-2| \leq 8 \Rightarrow |3y-2| \leq 4$

$3y - 2 \leq 4$ or $3y - 2 \geq -4$

$y \leq 2$ $\quad y \geq -\frac{2}{3}$



5. Express the following problem as an inequality, and then solve the inequality.

Problem: The average daily temperature in Santa Fe, NM, over the course of three days exceeded 75. Given that the high on the first day was 72 and the high on the third day was 77, what can we say about the high temperature on the second day?

I day – 72 (°F)
III day – 77 (°F)
II day – x (°F)

average daily temperature:

exceeded

$$\frac{72 + x + 77}{3} > 75$$

$$\Downarrow$$

$$149 + x > 225$$

$$\boxed{x > 76}$$

On the second day the temperature exceeded 76(°F) degrees