

Assessment Instructions:**Solutions**

- The Assessment 5 is 10 problems and is worth 40 points. Each numbered problem will earn you a score of 1-4 based on your set up of the function, your use of course methods to solve and prove your solution and your statement of the solution.
- You will have 1 hour to complete AS-5.
- The AS-5 is closed book and closed notes.
- **Calculators are not allowed** on the Assessment.

6.4

1. Use the properties of logarithms to expand the following expressions as much as possible.

(a) $\log_9(9xy^{-3})$

(b) $\log_2\left(\frac{y^2 + z}{16x^4}\right)$

2. Use the properties of logarithms to condense the following expressions as much as possible.

(a) $2 \log a^2b - \log(1/b) + \log(1/a)$

(b) $3 \log_4(x^2) + \log_4(x^6)$

3. Evaluate the following logarithmic expressions.

(a) $\log_2 0.01$

(b) $\log_6 3^4$

6.5

4. Solve the following exponential and logarithmic equations.

(a) $10^{2x+5} = e$

(b) $81^x = 3^{2x+16}$

(c) $\log_4(x - 3) + \log_4 2 = 3$

(d) $\log_2 x = 4$

7.1

5. Convert the radian measure to degrees.

(a) $\frac{7\pi}{20}$

(b) $\frac{-9\pi}{4}$

6. Convert the degree measure to radians.

(a) 154°

(b) 480°

(c) -144°

7. Given a circle of radius 22.5 cm, find the length of the arc subtended by a central angle of 5π .

8. Find the area of the sector of a circle of radius 7 cm with a central angle of 70° .

7.2

9. Problems 1-14 (textbook, pp.527-528).

10. Convert the expression from degrees, minutes, seconds notation to decimal notation.

$$6^\circ 8' 50''$$

11. Find $\cos \theta$ if $\sec \theta = -7/4$.

7.3

12. Determine the point (x, y) on the unit circle associated with each real number s .

(a) $s = \frac{\pi}{6}$

(b) $s = -120^\circ$

13. Determine all real numbers s associated with each point (x, y) on the unit circle.

(a) $(x, y) = (1/2, -\sqrt{3}/2)$

14. Determine the values of the six trigonometric functions of the given angle θ .

(a) $\theta = \frac{5\pi}{2}$

15. Determine the reference angle associated with the given angle.

(a) $\theta = \frac{5\pi}{4}$

16. Given that $\cot \theta = \frac{3}{4}$ and $\sin \theta$ is negative, determine $\sec \theta$.

7.4

17. Determine the amplitude, period, frequency, and phase shift of the following functions.

(a) $f(x) = -3 \cos(x + 7)$

(b) $0.5 \sin(8x + 1)$

18. Sketch the graph of each of the following functions.

(a) $f(x) = 2 \sin(x - \pi/4)$

(b) $g(x) = 2 - \cos(2\pi x)$

7.5

19. Sketch the graph of the following functions

(a) $f(x) = \frac{1}{3} \csc(2x)$

(b) $g(x) = 1 - \cot(x - \pi/2)$

7.6

20. Evaluate each of the following expressions without the use of a calculator.

(a) $\cos^{-1}(\sqrt{2}/2)$

(b) $\cot^{-1}(-\sqrt{3})$

21. Find the value of each of the following expressions without using a calculator.

(a) $\sin(\arctan(\sqrt{3}))$

(b) $\sec(\arcsin(-1/2))$

8.1

22. Use trigonometric identities to simplify the expressions.

(a) $\sin(-x)\tan x$

(b) $\cot^2 x - \cot^2 x \cos^2 x$

(c) $\frac{1}{\cos x \csc(-x)}$

8.2

23. Use the sum and difference identities to determine the exact value of each of the following expressions.

(a) $\sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

(b) $\tan 75^\circ$

24. Use the sum and difference identities to rewrite each of the following expressions as a trigonometric function of one angle, and then evaluate the result.

(a) $\sin 15^\circ \cos 30^\circ + \cos 15^\circ \sin 30^\circ$

6.4

1. (a) $\log_9(9xe y^{-3}) = \log_9 9 + \log_9 x + \log_9 y^{-3} =$
 $= 1 + \log_9 x - 3 \log_9 y$

(b) $\log_2\left(\frac{y^2+z}{16x^4}\right) = \log_2(y^2+z) - \log_2(16x^4) =$
 $= \log_2(y^2+z) - \log_2 16 - \log_2 x^4 =$
 $= \log_2(y^2+z) - \log_2 2^4 - 4 \log_2 x =$
 $= \log_2(y^2+z) - 4 - 4 \log_2 x$

2. (a) $2 \log a^2 b - \log(f) + \log\left(\frac{1}{a}\right) =$
 $= \log\left(\frac{a^4 b^2 \cdot \frac{1}{a}}{f}\right) = \log(a^3 b^3)$

(b) $3 \log_4(x^2) + \log_4(x^6) = \log_4 x^6 + \log_4 x^6 =$
 $= \log_4 x^{12}$

3. (a) $\log_2 0.01 = \log_2 \frac{1}{100} = \log_2 10^{-2} =$

$$= -2 \log_2 10 = -2 \log_2 (2 \cdot 5) = -2 - 2 \cdot \log_2 5$$

$$(b) \quad \log_6 3^4 = 4 \log_6 3 = 4 \frac{\log_3 3}{\log_3 6} = \\ = \frac{4}{\log_3 6}$$

6.5

$$4. (a) \quad 10^{2x+5} = e$$

$$\ln 10^{2x+5} = \ln e$$

$$(2x+5) \ln 10 = 1$$

$$2x+5 = \frac{1}{\ln 10}$$

$$2x = \frac{1}{\ln 10} - 5$$

$$x = \frac{1}{2 \ln 10} - \frac{5}{2}$$

$$(b) \quad 81^x = 3^{2x+16}$$

$$3^{4x} = 3^{2x+16}$$

$$4x = 2x + 16$$

$$2x = 16$$

$$\boxed{x=8}$$

(c) $\log_4(x-3) + \log_4 2 = 3$

$$\log_4(2x-6) = 3$$

$$4^{\log_4(2x-6)} = 4^3$$

$$2x-6 = 64$$

$$2x = 70$$

$$\boxed{x=35}$$

(d) $\log_2 x = 4$

$$2^4 = x$$

$$\boxed{x=16}$$

$$\boxed{7.1}$$

$$5. \text{ (a)} \quad \frac{7\pi}{20} \cdot \left(\frac{180}{\pi}\right)^\circ = \frac{7 \cdot 180}{20}^\circ = 63^\circ$$

$$\text{(b)} \quad -\frac{9\pi}{4} \cdot \left(\frac{180}{\pi}\right)^\circ = -\frac{9 \cdot 180}{4}^\circ = -405^\circ$$

$$6. \text{ (a)} \quad 154^\circ = 154 \cdot \frac{\pi}{180} \text{ rad} = \frac{77\pi}{90}$$

$$\text{(b)} \quad 480^\circ = 480 \cdot \frac{\pi}{180} = \frac{240}{90} \pi = \\ = \frac{80}{30} \pi = \frac{40}{15} \pi = \frac{8}{3} \pi$$

$$\text{(c)} \quad -144^\circ = -144 \cdot \frac{\pi}{180} = -\frac{72}{90} \pi = \\ = -\frac{36}{45} \pi = -\frac{12}{15} \pi$$

$$7. \quad r = 22.5 \text{ cm}$$

$$\Theta = 5\pi$$

$$S = r \cdot \Theta = 112.5 \pi \text{ (cm)}$$

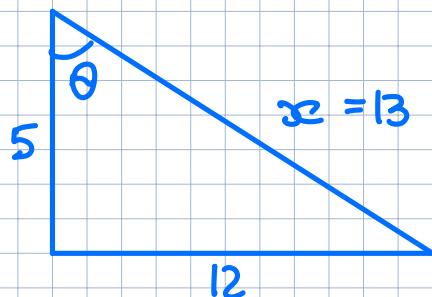
$$8. \quad r = 7 \text{ cm}$$

$$\Theta = 70^\circ = 70 \cdot \frac{\pi}{180} = \frac{35}{90} \pi = \frac{7}{18} \pi \text{ rad}$$

$$A = \frac{r^2 \theta}{2} = \frac{49 \cdot \frac{7\pi}{18}}{2} = \frac{343\pi}{36} (\text{cm}^2)$$

7.2

9. pr. #2



$$x^2 = 25 + 144 = 169$$

$$x = 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = \frac{5}{13}$$

$$\tan \theta = \frac{12}{5}$$

$$\cot \theta = \frac{5}{12}$$

$$\sec \theta = \frac{13}{5}$$

$$\csc \theta = \frac{13}{12}$$

10.

$$6^\circ 8' 50'' = 6^\circ + 8 \cdot \frac{1}{60} + 50 \cdot \frac{1}{3600} =$$

$$= 6 + 0.13 + 0.014 = 6.144$$

11.

$$\cos \theta - ?$$

$$\sec \theta = -\frac{5}{13}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

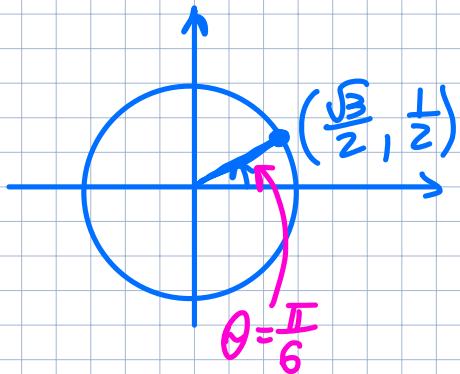
$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = -\frac{1}{\frac{7}{4}} = -\frac{4}{7}$$

7.3

12. (a) $S = \frac{\pi}{6}$

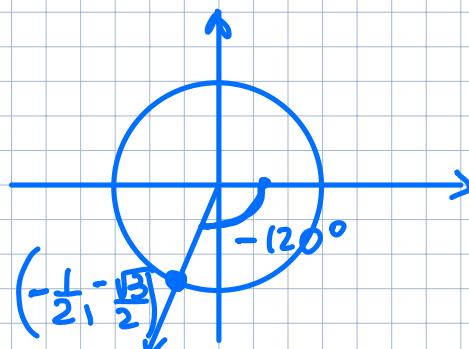
$$(x, y) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$



(b) $S = -120^\circ$

$$S = -90^\circ - 30^\circ$$

$$(x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

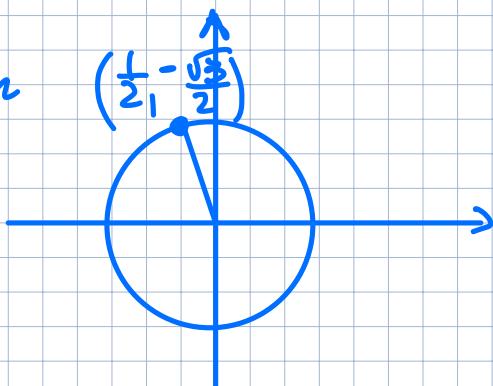


13. (a) $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$S = \frac{\pi}{2} + \frac{\pi}{6} + 2\pi n$$

$$S = \frac{4\pi}{6} + 2\pi n$$

$$S = \frac{2\pi}{3} + 2\pi n$$



14. (a) $\theta = \frac{5\pi}{2}$

$$\cos \frac{5\pi}{2} = \cos \left(2\pi + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\sin \frac{5\pi}{2} = \sin \frac{\pi}{2} = 1$$

$$\tan \frac{5\pi}{2} = \text{DNE}$$

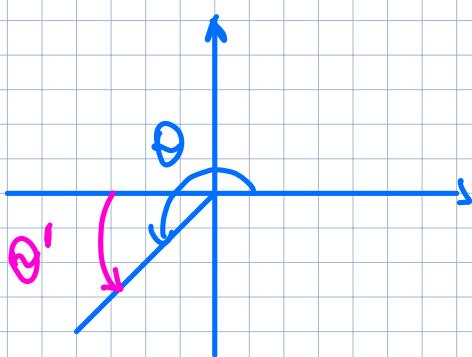
$$\cot \frac{5\pi}{2} = 0$$

$$\sec \frac{5\pi}{2} - \text{DNE}$$

$$\csc \frac{5\pi}{2} = 1$$

15. (a) $\theta = \frac{5\pi}{4}$

$$\theta' = \frac{\pi}{4}$$

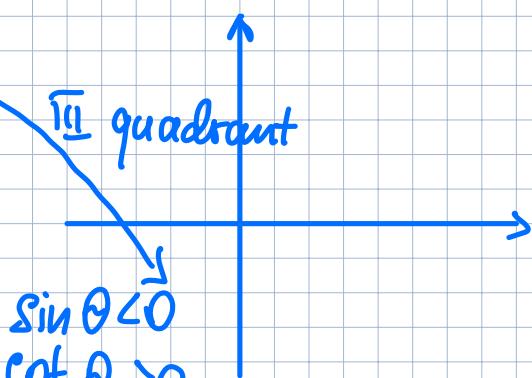


16.

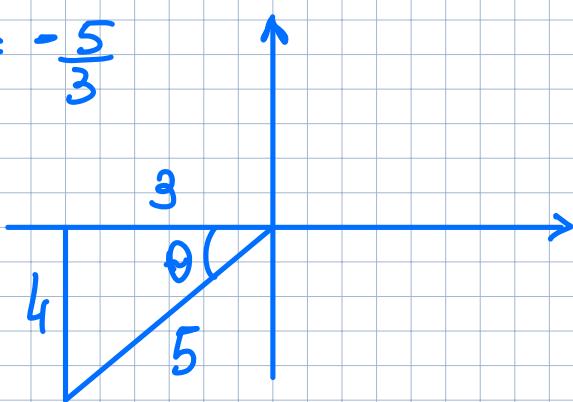
$$\cot \theta = \frac{3}{4}$$

$$\sin \theta < 0$$

$$\sec \theta - ?$$



$$\sec \theta = -\frac{1}{\frac{3}{5}} = -\frac{5}{3}$$



7. 4.

$$17. (a) f(x) = -3 \cos(x + \pi)$$

$$|a| = 3$$

$$D = \frac{1}{2\pi}$$

$$T = \frac{1}{\nu} = 2\pi$$

phase shift left: $\frac{\pi}{8}$

(B) $g(x) = 0.5 \sin(8x + 1)$

$$|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$$

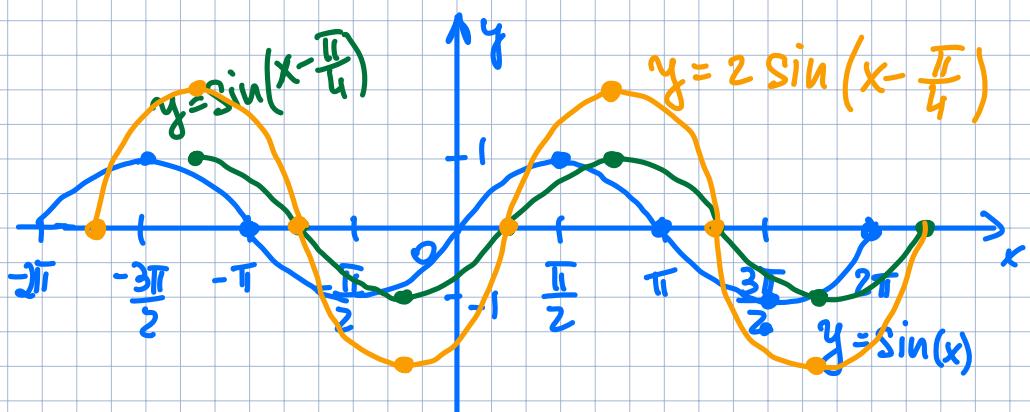
$$\nu = \frac{8}{2\pi} = \frac{4}{\pi}$$

$$T = \frac{1}{\nu} = \frac{\pi}{4}$$

phase shift left: $\frac{1}{8}$

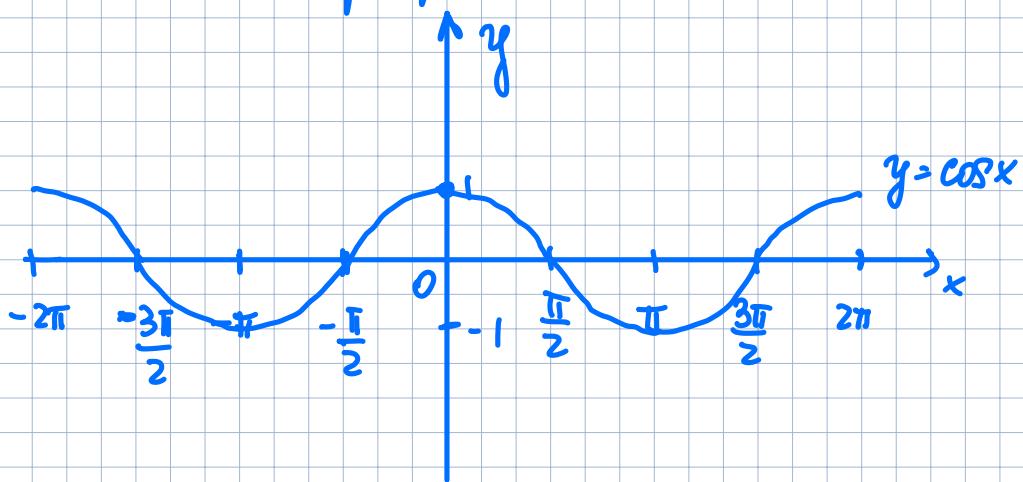
18. (a) $f(x) = 2 \sin\left(x - \frac{\pi}{4}\right)$

- shift right horizontally per $\frac{\pi}{4}$
- stretch vertically per 2



$$(B) \quad g(x) = 2 - \cos(2\pi x)$$

- compress horizontally per 2π
- reflect w.r.t. x-axis
- shift up per 2

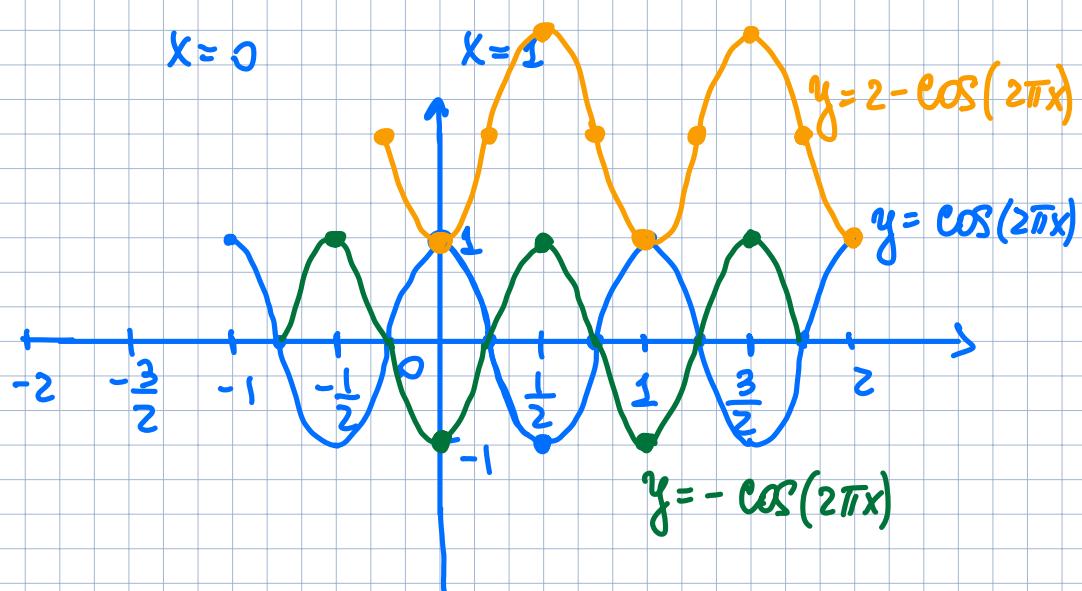


$$2\pi x = 0$$

$$x = 0$$

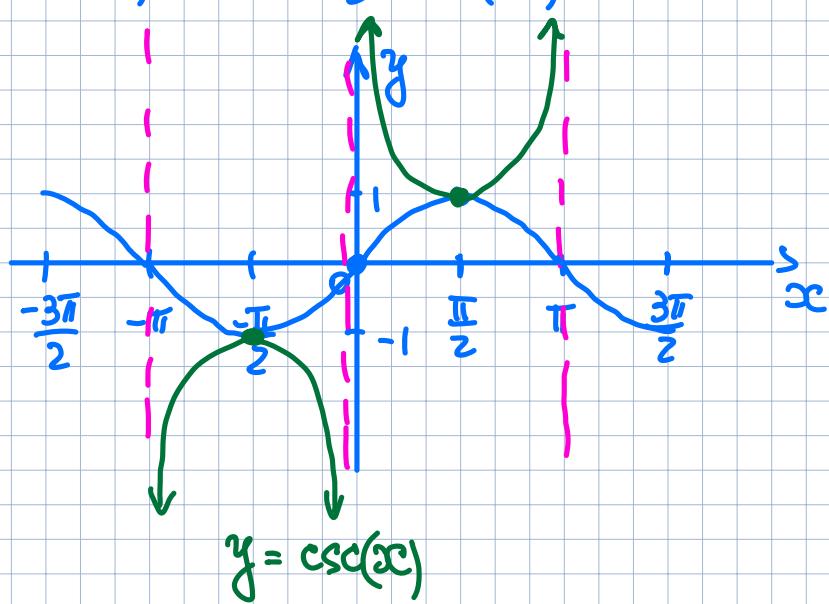
$$2\pi x = 2\pi$$

$$x = 1$$



7.5

19. (a) $f(x) = \frac{1}{3} \csc(2x)$



$$2x = 0$$

$$x = 0$$

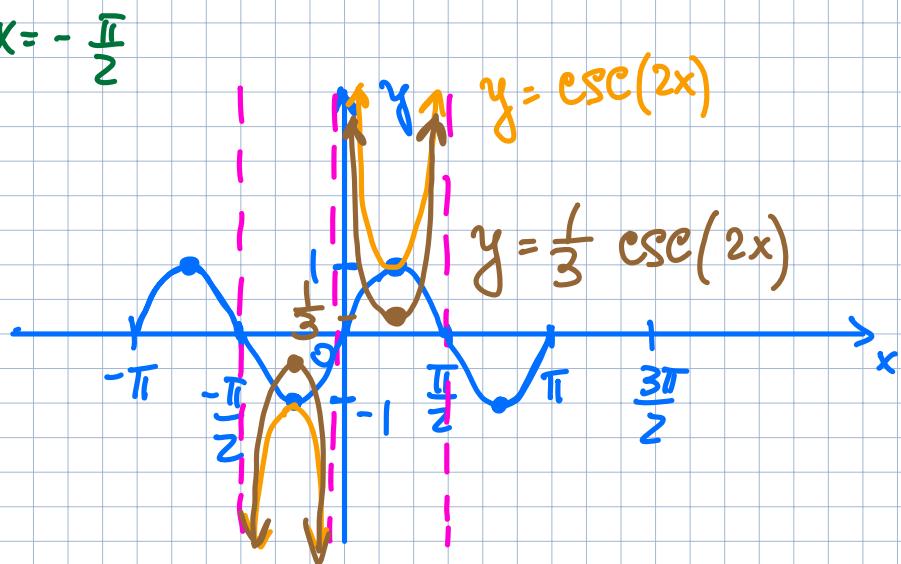
$$2x = -\pi$$

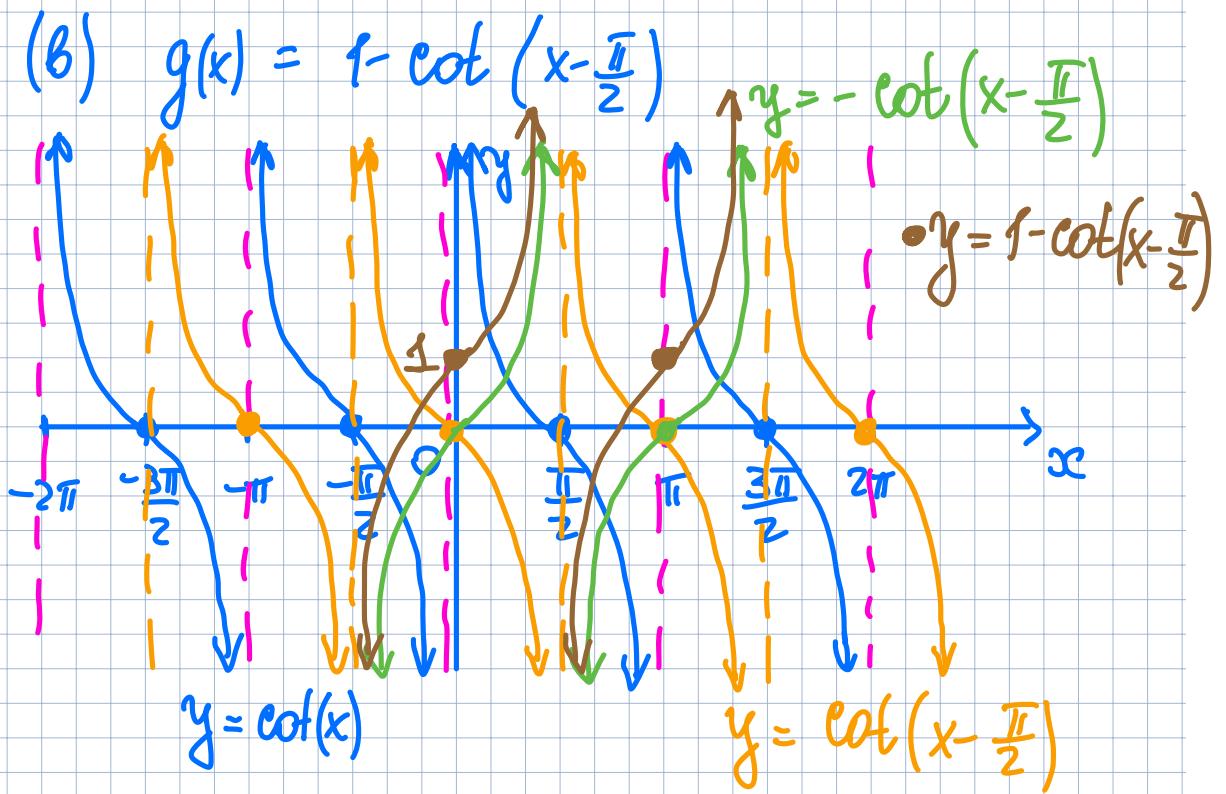
$$x = -\frac{\pi}{2}$$

$$2x = \pi$$

$$x = \frac{\pi}{2}$$

$$y = \csc(2x)$$





7.6

20.

$$(a) \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$(B) \cot^{-1}\left(-\sqrt{3}\right) = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$$

21.

$$(a) \sin(\arctan(\sqrt{3})) = \frac{\sqrt{3}}{2}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$(B) \sec(\arcsin(-\frac{1}{2})) = \frac{2}{\sqrt{3}}$$

$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\sec\left(-\frac{\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

8.1

$$22. (a) \sin(-x)\tan x = -\sin x \cdot \frac{\sin x}{\cos x} = \\ = -\frac{\sin^2 x}{\cos x} = \frac{\cos^2 x - 1}{\cos x} = \cos x - \sec x$$

$$(B) \cot^2 x - \cot^2 x \cdot \cos^2 x =$$

$$= \cot^2 x (1 - \cos^2 x) = \cot^2 x \cdot \sin^2 x =$$

$$= \cos^2 x$$

$$(c) \frac{1}{\cos x \csc(-x)} = \frac{-1 \cdot \sin x}{\cos x \cdot 1} = -\tan x$$

8.2

$$(a) \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \sin \frac{2\pi}{3} \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \sin \frac{\pi}{4} = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \cdot \sin 45^\circ$$

$$\cdot \sin 45^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$(b) \tan 75^\circ = \tan(30^\circ + 45^\circ) =$$

$$= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \cdot 1} =$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

$$24. (a) \sin 15^\circ \cos 30^\circ + \cos 15^\circ \sin 30^\circ =$$

$$= \sin(15^\circ + 30^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$$