Solutions

THEORETICAL PART:

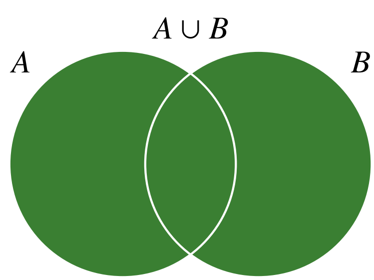
Definitions:

- The natural numbers set: $\mathbb{N} = \{1, 2, 3, 4, 5, \cdots\}$
- The whole numbers set: $\{0, 1, 2, 3, 4, \cdots\}$
- The integers numbers set: $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$
- The rational numbers set: $\mathbb{Q} = \left\{\frac{p}{q} \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0\right\}$
- The irrational numbers set: $\mathbb{I} = \mathbb{R} \backslash \mathbb{Q}$
- The real numbers set: $\mathbb{R}$
- Empty set or the null set notation: $\emptyset$, $\{\ \}$
- The notation $\{x \mid x \text{ has property } P\}$ is used to describe a set of real numbers, all of which have the property P

Basic Set Operations and Venn Diagrams:

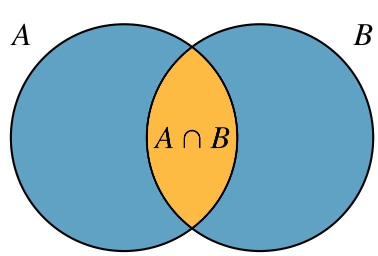
- The **union** of two sets A and B:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



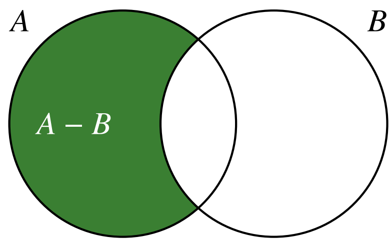
- The **intersection** of two sets A and B:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



- The **difference** of two sets A and B:

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$



Definitions:

- The absolute value of a real number a "$|a|$" is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

- **Properties of Absolute Value:**

  $|a| \geq 0$

  $|-a| = a$

  $a \leq |a|$

  $|ab| = |a||b|$

  $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

  $|a+b| \leq |a| + |b|$ (triangle inequality)

- The given two real numbers a and b, the **distance** between them is defined to be $|a-b|$.

- **Field Properties:**

  Closure:

  additive: $a+b$ is a real number

  multiplicative: ab is a real number

  Commutative:

  additive: $a+b=b+a$

  multiplicative: $ab=ba$

  Associative:

  additive: $a+(b+c)=(a+b)+c$

  multiplicative: $a(bc)=(ab)c$

  Identity:

  additive: $a+0=0+a=a$

  multiplicative: $a \cdot 1 = 1 \cdot a = a$

  Inverse:

  additive: $a+(-a)=0$

  multiplicative: $a \cdot \frac{1}{a} = 1, a \neq 0$

  Distributive:

  $a(b+c)=ab+ac$

- **Cancellation Properties:** Let A, B and C be algebraic expressions. We have

$$A = B \Leftrightarrow A + C = B + C \quad \text{(Additive cancellation)}$$

$$A = C \Leftrightarrow A \cdot C = B \cdot C, \quad \text{where} \quad C \neq 0 \quad \text{(Multiplicative cancellation)}$$

- **Zero-Factor Property:** Let A, B be algebraic expressions. Then we have

$$AB = 0 \Rightarrow A = 0 \quad \text{or} \quad B = 0.$$

PRACTICAL PART:

1. Which elements of the following set $\{5\sqrt{7}, 4\pi, -1, \frac{22}{7}, |-8|, 3.\bar{3}\}$ are
   - natural numbers $(\mathbb{N})$: $|-8|$
   - whole numbers : $|-8|$
   - integers $(\mathbb{Z})$: -1, $|-8|$
   - rational numbers $(\mathbb{Q})$: $\frac{22}{7}$, $3.\bar{3}$, -1, $|-8|$
   - irrational numbers $(\mathbb{I})$: $5\sqrt{7}$, 4π
   - real numbers? $(\mathbb{R})$: $5\sqrt{7}$, 4π, -1, $\frac{22}{7}$, $|-8|$, $3.\bar{3}$

2. Which set the following intervals do represent?
   (a) $(2, 8)$ $= \{x \mid 2 < x < 8\}$
   (b) $[-3, 10)$ $= \{x \mid -3 \leq x < 10\}$
   (c) $(-\infty, \infty)$ $= \{x \mid x \text{ is a real number}\}$

3. Write the following sets as an interval using interval notation:
   (a) $A = \{x \mid -3 \leq x < 19\}$ $= [-3, 19)$
   (b) $B = \{\text{The nonnegative real numbers}\}$ $= [0, \infty)$

4. Using absolute value properties simplify the following expressions:
   (a) $|(-3)(5)| =$ $|-3| \cdot |5| = 3 \cdot 5 = \boxed{15}$

(b) $\left|\frac{-3}{7}\right| =$ $\frac{|-3|}{|7|} = \boxed{\frac{3}{7}}$

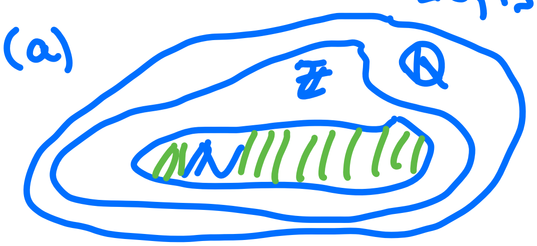
5. Simplify the following set expressions:

   (a) $\mathbb{N} \cap \mathbb{Z} \cap \mathbb{Q}$ $= \mathbb{N}$

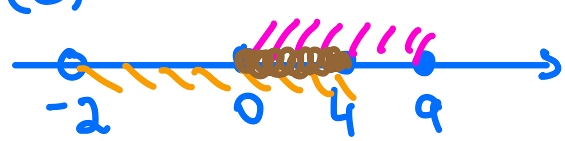
   (b) $(5, 10) \cup \mathbb{Z}$ $= (5,10) \cup \mathbb{Z}$

   (c) $(-2, 4] \cap [0, 9]$ $= [0,4]$

(a)



(c)



6. Evaluate the following algebraic expressions for the given values of the variables:

   (a) for $x = 8$

$$\sqrt{2x} + \frac{3x}{4},$$

   (b) for $x = 2$, $y = -1$, $z = 3$

$$\frac{x^2y^3}{8z} - \frac{|2xy|}{8z}$$

(a) $x=8$: $\sqrt{2\cdot 8} + \frac{3\cdot \cancel{8}^{2}}{\cancel{4}} = \sqrt{16} + 6 = 4+6 = \boxed{10}$

(b) $x=2, y=-1, z=3$: $\frac{2^2\cdot(-1)^3}{8\cdot 3} - \frac{|2\cdot 2\cdot(-1)|}{8\cdot 3} = \frac{-4}{24} - \frac{4}{24} = \frac{-8}{24} = \boxed{-\frac{1}{3}}$

7. Identify the property that justifies each of the following statements.

   (a)

$$4(y - 3) = 4y - 12,$$ Distributive

   (b)

$$25x^3 = 10y \Leftrightarrow 5x^3 = 2y,$$ Multiplicative cancellation

   (c)

$$x^2z = 0 \Rightarrow x^2 = 0 \quad \text{or} \quad z = 0.$$ Zero-factor property

   (d)

$$y + 12 = 18$$ Additive cancellation

(a): Distributive: $a(b+c) = ab+ac$

(b): Multiplicative cancellation:

$$AB = A \cdot C \iff B = C, \quad A \neq 0$$

(c): Zero-factor property:

$$A \cdot B = 0 \Rightarrow A = 0 \text{ or } B = 0$$

(d): Additive cancellation:

$$A+B = A+C \iff B = C$$

" $\iff$ " means "if and only if"

" $\Rightarrow$ " means "implies"