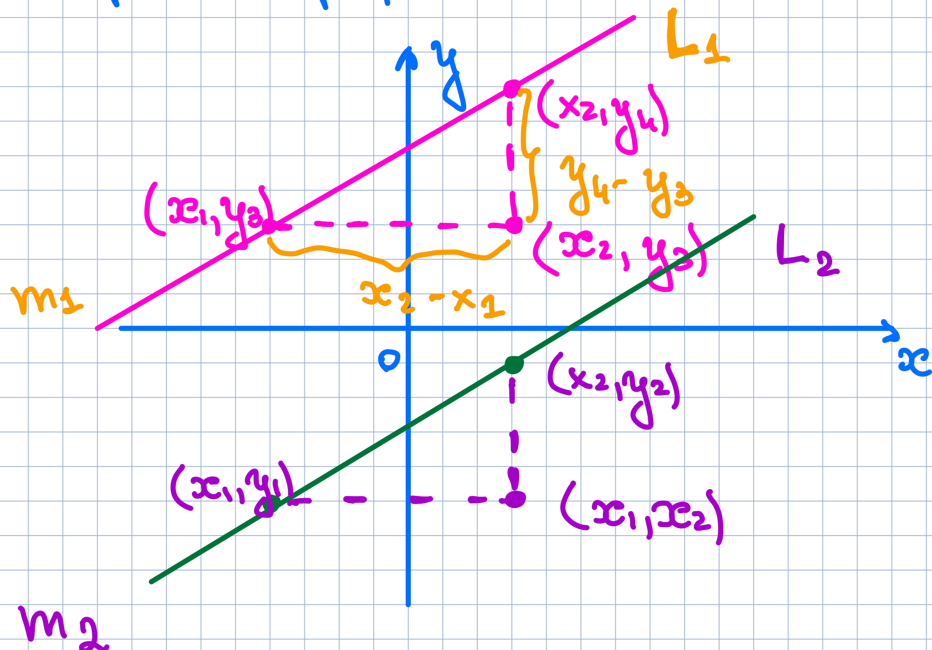
Section 2.5. Parallel and Perpendicular lines

1. Slopes of parallel lines.
2. Slopes of perpendicular lines.

$\boxed{1.}$



Theorem (Slopes of parallel lines)

Two nonvertical lines with slopes m_1 and m_2 are <u>parallel</u> if and only if $\boxed{m_1 = m_2}$.

Also, two vertical lines (with undefined slopes) are always parallel to each other.

Example

Find equations for two lines $\parallel$ to each other of the given lines:

- $y = -\frac{2}{3}x + 4$

Answer: $y = -\frac{2}{3}x - 1$

$y = -\frac{2}{3}x + 10$

Example

Find the equation, in slope-intercept form, for the line that is $\parallel$ to the line $3x + 5y = 23$ and passes through the point $(-2, 1)$.

Answer: $L: \quad y = mx + b$

$m = ?$

$b - ?$

$3x + 5y = 23$

$5y = 23 - 3x \Rightarrow y = -\frac{3}{5}x + \frac{23}{5}$

Hence, $\quad m = -\frac{3}{5}$.

We have that $(-2, 1) \in L$. Therefore,

$$1 = m \cdot (-2) + b$$

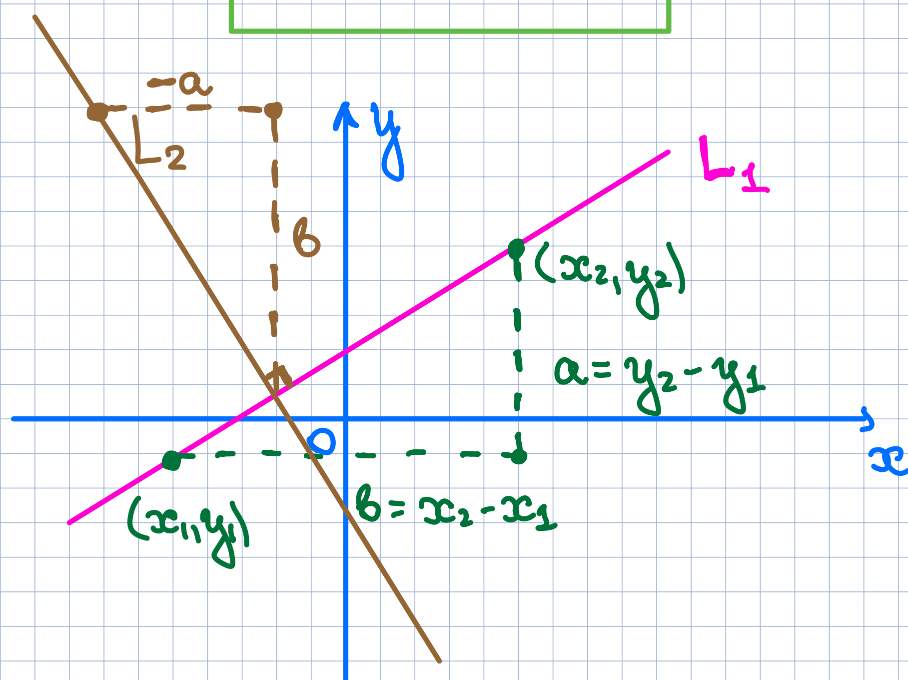
$$1 = -\frac{3}{5} \cdot (-2) + b$$

$$1 = \frac{6}{5} + b \Rightarrow b = -\frac{1}{5}$$

Hence,

$$\boxed{y = -\frac{3}{5}x - \frac{1}{5}}$$

$\boxed{2.}$



L_2 has a slope $m_2 = -\frac{b}{a}$

L_1 has a slope $m_1 = \frac{a}{b}$

Theorem (Slopes of $\perp$ lines)

Suppose m_1 and m_2 represent slopes of two lines, neither of which is vertical. The two lines are $\perp$ if and only if

$$m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1 m_2 = -1$$

or $$m_2 = -\frac{1}{m_1}.$$

If one of two perpendicular lines is vertical, the other is horizontal, and their slopes are undefined and zero.

Example (p. 180)

- $y = -\frac{4}{9}x + 2$, $m_1 = -\frac{4}{9}$

$$m_2 = -\frac{1}{m_1} = \frac{-1}{-\frac{4}{9}} = \frac{9}{4}$$

$$\tilde{y} = \frac{9}{4}x$$

Example

Find the equation, in standard form, of the line that passes through the point $(-3, 13)$ and is $\perp$ to the line

$y=-7.$

Answer: $x = c$ is $\perp$ to $y=-7$

$-3 = c \Rightarrow c = -3$

$$\boxed{x = -3}$$

Example

l_1: $3x-7y=12 \Rightarrow y = \frac{3}{7}x - 12 \Rightarrow m_1 = \frac{3}{7}$

l_2: $14x+6y=-5 \Rightarrow y = -\frac{14}{6}x + 5 \Rightarrow m_2 = -\frac{14}{6}$

$$m_1 = \frac{3}{7} \quad , \quad m_2 = -\frac{7}{3}$$

$$\Downarrow$$

$$m_1 = -\frac{1}{m_2}$$

$$\Downarrow$$

$$l_1 \perp l_2$$