THEORETICAL PART:

Solutions

Definition:

Let a be a fixed positive real number not equal to 1. The **logarithmic function with base** a is defined to be the inverse of the exponential function with base a, and is denoted $\log_a x$. In symbols, if $f(x) = a^x$, then $f^{-1}(x) = \log_a x$.

In equation form, the definition of logarithm means that the equations

$$x = a^y \quad \text{and} \quad y = \log_a x$$

are equivalent. Note that a is the base in both equations: either the base of the exponential function or the base of the logarithmic function.

Properties:

1. $\log_a 1 = 0$, because $a^0 = 1$
2. $\log_a a = 1$, because $a^1 = a$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$

Definition:

- The function $\log_{10} x$ is called the **common logarithm** and is usually written $\log x$.
- The function $\log_e x$ is called the **natural logarithm** and is usually written $\ln x$.

Properties of Natural Logarithms:

$$\ln x = y \Leftrightarrow e^y = x$$

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

PRACTICAL PART:

1. Use the definition of logarithmic functions to rewrite the following exponential equations as logarithmic equations:

   (a) $8 = 2^3$
   (b) $5^4 = 625$
   (c) $7^x = z$

(a) $8 = 2^3$

$y = a^x$

$\log_a y = \log_a a^x = x$

$\log_a y = x$

$y = 8 \quad x = 3 \quad a = 2$

$\log_2 8 = 3$

(b) $5^4 = 625 \iff \log_5 625 = 4$

(c) $7^x = z$

$\log_7 7^x = \log_7 z$

$\log_7 z = x$

2. Rewrite the following logarithmic equations as exponential equations:

   (a) $\log_3 9 = 2$

   (b) $3 = \log_8 512$

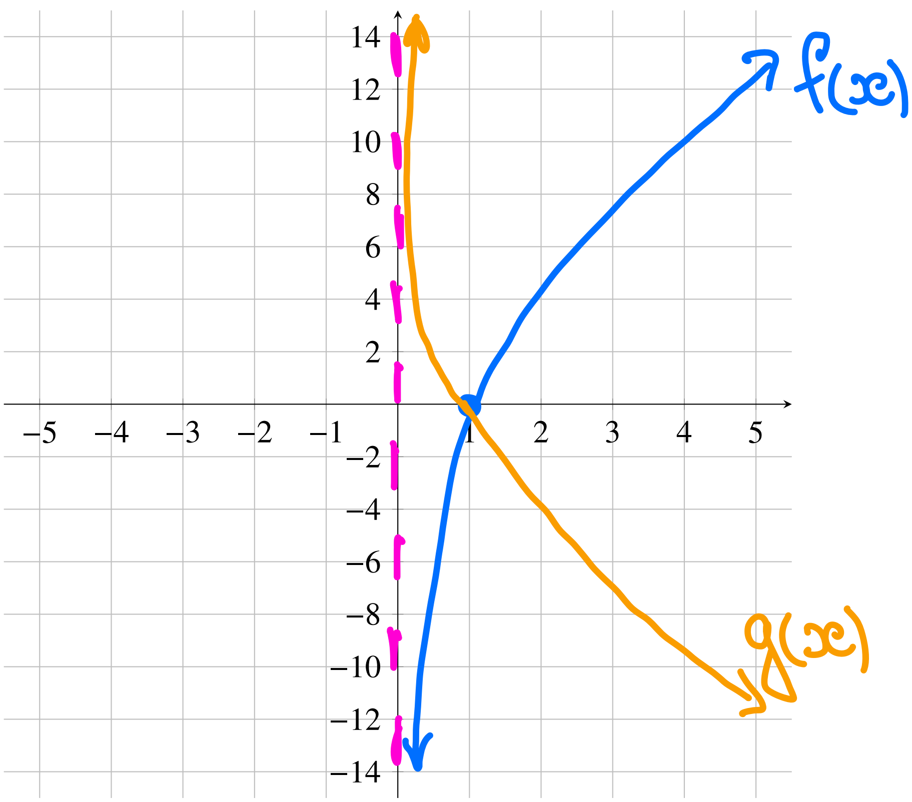
(a) $\log_3 9 = 2 \iff 3^2 = 9$

(b) $\log_8 512 = 3 \iff 8^3 = 512$

3. Sketch the graphs of the following logarithmic functions:

   (a) $f(x) = \log_3 x$ $a = 3$

   (b) $g(x) = \log_{\frac{1}{2}} x$

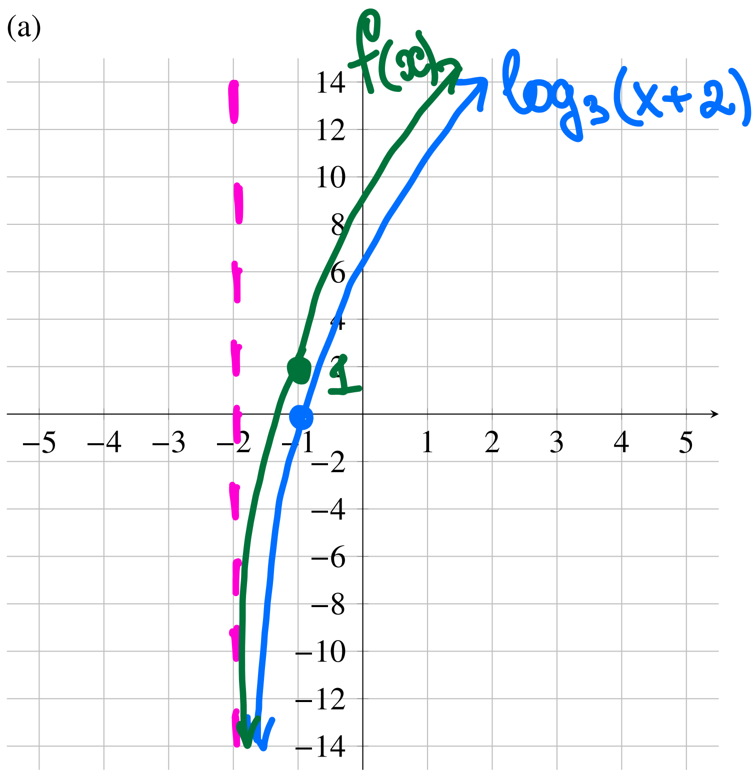


4. Sketch the graph of the following functions. State their domain and range.

   (a) $f(x) = \log_3 (x + 2) + 1$

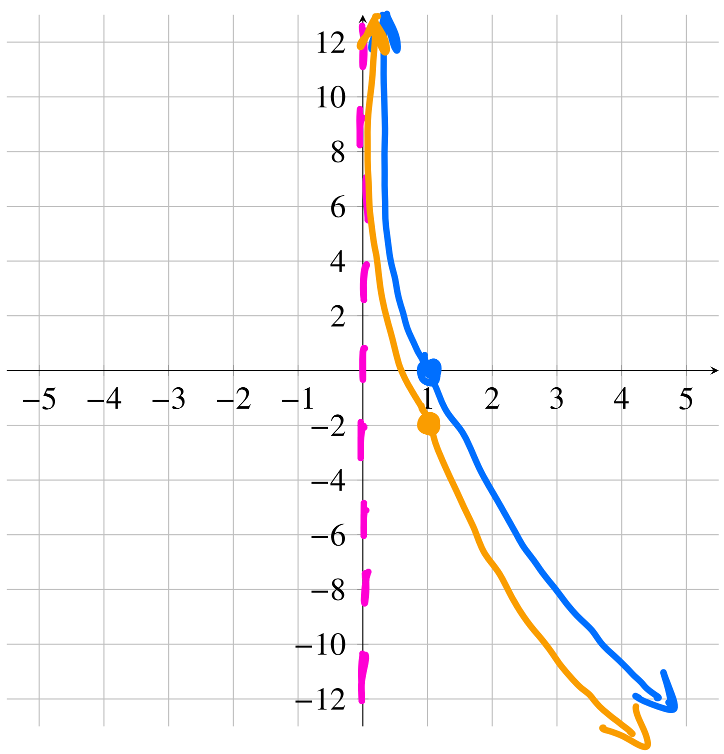
   (b) $g(x) = \log_{\frac{1}{2}} x - 2$

(a)



- $\log_3(x+2)$
- $\log_3(x+2)+1$

(b)



- $\log_{\frac{1}{2}} x$
- $\log_{\frac{1}{2}} x - 2$

5. Evaluate the following logarithmic equations:

   (a) $\log_5 25 =$ 2

   (b) $\log_{\frac{1}{2}} 2 =$ -1

   (c) $\log_{16} 4 =$ $\frac{1}{2}$

   (d) $\log_{10} \frac{1}{100} =$ -2

6. Use elementary properties of exponents and logarithms to solve the following equations.

   (a) $\log_6 (2x) = -1$

   (b) $3^{\log_{3x} 2} = 2$

   (c) $\log_2 8^x = 5$

(a) $\log_6(2x) = -1$

$6^{-1} = 2x \Rightarrow \frac{1}{6} = 2x \Rightarrow \boxed{x = \frac{1}{12}}$

(b) $3^{\log_{3x} 2} = 2$

$\log_{3x} 2 = \log_3 2$

$3x = 3$ $\boxed{x = 1}$

(c) $\log_2 8^x = 5$

$$2^5 = 8^x$$

$$2^5 = 2^{3x}$$

$$5 = 3x$$

$$\boxed{x = \frac{5}{3}}$$

7. Evaluate the following logarithmic expressions.

   (a) $\ln(\sqrt[3]{e}) =$

   (b) $\log 1000 =$

   (c) $\ln(4.78) =$

(a) $\ln(\sqrt[3]{e}) = \ln e^{\frac{1}{3}} = \frac{1}{3} \ln e = \frac{1}{3}$

$\boxed{\ln e = 1}$

(b) $\log 1000 = \log_{10} 10^3 = 3$

$\boxed{\log_{10} 10 = 1}$

(c) $\ln(4.78) \approx 1.564$