THEORETICAL PART:

Definition:

A **quadratic equation in one variable**, say the variable x, is an equation that can be transformed into the form

$$ax^2 + bx + c = 0,$$

where $a, b, c \in \mathbb{R}$ and $a \neq 0$.

We also call such equations as **second-degree** equations.

Completing the Square Procedure:

Step 1. Write the equation $ax^2 + bx + c = 0$ in the form $ax^2 + bx = -c$.

Step 2. Divide by $a \neq 1$, so that the coefficient of x^2 is 1: $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

Step 3. Divide the coefficient of x by 2, square the result, and add this to both sides:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2.$$

Step 4. The trinomial on the left side will now be a perfect square trinomial. That is, it can be written as the square of a binomial.

The Quadratic Formula:

The solutions of the general quadratic equation $ax^2 + bx + x = 0$, with $a \neq 0$, are given by the **quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We call $D = b^2 - 4ac$ the **discriminant**. Its value determines the number and type (real or complex) of solutions.

- $b^2 - 4ac > 0$: we have 2 real distinct solutions.
- $b^2 - 4ac = 0$: we have 1 repeated real solution.
- $b^2 - 4ac < 0$: we have 2 complex solutions (complex conjugate).

Definition: An equation is **quadratic-like**, or **quadratic in form**, if it can be written in the form

$$aA^2 + bA + c = 0,$$

where a, b, c are constants, $a \neq 0$, and A is an algebraic expression. Such equations can be solved by using a **substitution** method.

PRACTICAL PART:

1. Solve the quadratic equation by factoring:
   - $s^2 + 9 = 6s$

2. Solve the quadratic equation by taking square roots:
   - $(2x + 3)^2 = 8$

3. Solve the quadratic equation by completing the square:
   - $x^2 - 2x - 6 = 0$

4. Solve the quadratic equation using the quadratic formula:
   - $8x^2 - 4x = 1$

5. For each of the following quadratic equations, calculate the discriminant and determine the number and type of solutions:
   - $-2x^2 + 12x - 18 = 0$
   - $5x^2 + 7x + 2 = 0$
   - $x^2 - 4x + 9 = 0$

6. Solve the quadratic-like equation:

    - $(x^2 + 2x)^2 - 7(x^2 + 2x) - 8 = 0$
    - $y^{\frac{2}{3}} + 4y^{\frac{1}{3}} - 5 = 0$

7. Solve the equation by factoring:

    - $8t^3 - 27 = 0$
    - $x^{\frac{7}{3}} + x^{\frac{4}{3}} - 2x^{\frac{1}{3}} = 0$