Section 4.2. Properties of functions

1. Symmetry of functions and equations.
2. Intervals of monotonicity.
3. Local extrema.
4. Average rate of change.

$\boxed{1.}$

Def. (y-Axis Symmetry)

The graph of a function f has y-axis symmetry, or is symmetric with respect to the y-axis, if

$$f(-x) = f(x)$$

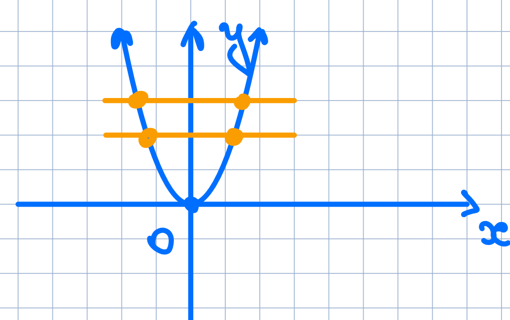
for all x in the domain of f.

Such functions are called <u>even</u> functions.

Example

$y = x^2$

$y(-x) = (-x)^2 = x^2$

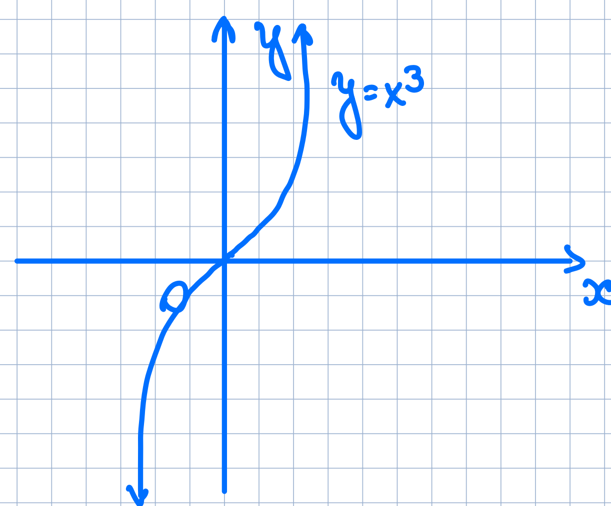


Def. The graph of a function f has origin symmetry, or is symmetric with respect to the origin, if $f(-x) = -f(x)$ for all x in the domain of f. Such functions are called odd functions.

Example

$y = x^3$

$y(-x) = (-x)^3 = -x^3$



Def.

We say that an equation in x and y is symmetric with respect to

1. the y-axis if replacing x with $-x$ results in an equivalent equation;
2. the x-axis if replacing y with $-y$ results in an equivalent equation;
3. the origin if replacing x with $-x$ and y with $-y$ results in an equivalent

equation.

$\boxed{2.}$ **Def.** (Increasing, decreasing and constant)

We say that the function f is

1. increasing on an interval if for any x_1 and x_2 in the interval with $x_1 < x_2$, it is the case that $f(x_1) < f(x_2)$.

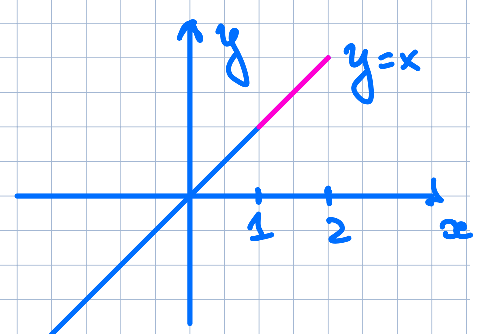
2. decreasing on an interval if for any x_1 and x_2 in the interval with $x_1 < x_2$, it is the case that $f(x_1) > f(x_2)$.

3. constant on an interval if for any x_1 and x_2 in the interval, it is the case that $f(x_1) = f(x_2)$.

Example

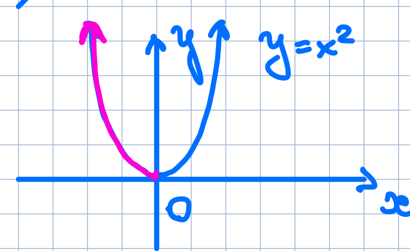
- $f(x) = x$, $x \in (1, 2)$

  f is increasing

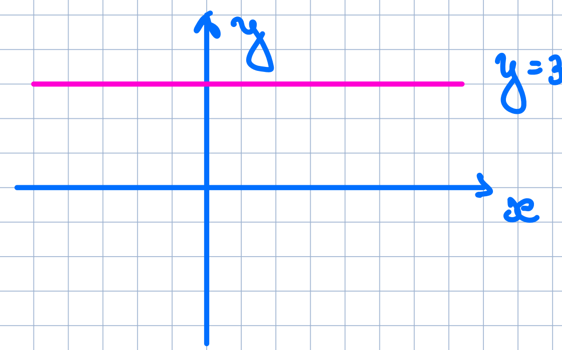


- $f(x) = x^2$, $x \in (-\infty, 0)$

  f is decreasing



- $f(x) = 3 \ , \quad x \in \mathbb{R}$



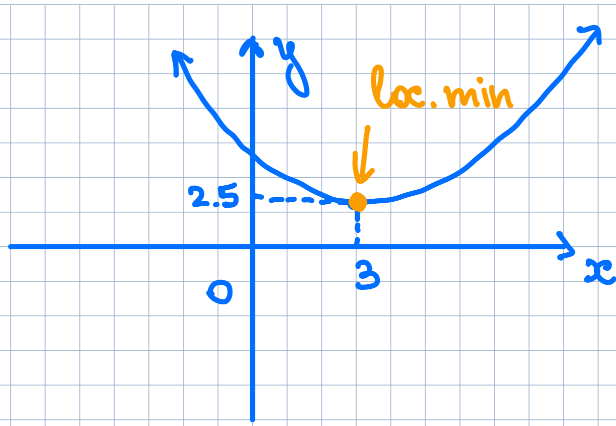
$\boxed{3.}$ <u>Def.</u> (Local Extrema)

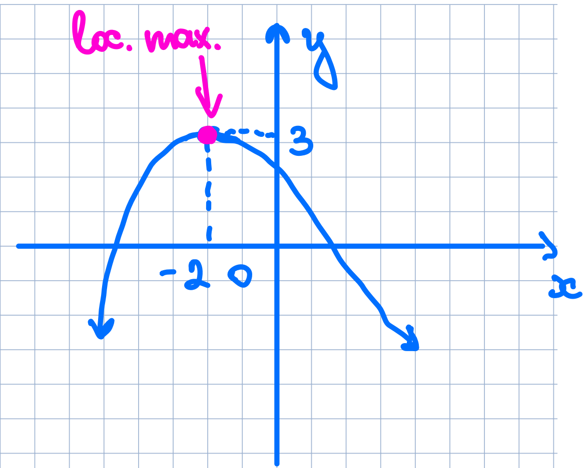
- A function f has a <u>loc. maximum at c</u> if there is an open interval (a,b) containing c for which $f(x) \leq f(c)$ for all x in (a,b).
  We say then that <u>$f(c)$ is a loc. max.</u> value of f.

- f has a <u>loc. minimum at c</u> if there is an open interval (a,b) containing c for which $f(x) \geq f(c)$ for all x in (a,b).
  We say then that <u>$f(c)$ is a loc. min.</u> value of f.

The <u>loc. min. and loc. max.</u> of f are referred to as <u>loc. extrema.</u>

<u>Example</u>





$\boxed{4.}$

Def. (Average rate of change)

Given a function f defined on an interval $[a,b]$, $a \neq b$, the <u>average rate of change</u> of f over $[a,b]$ is

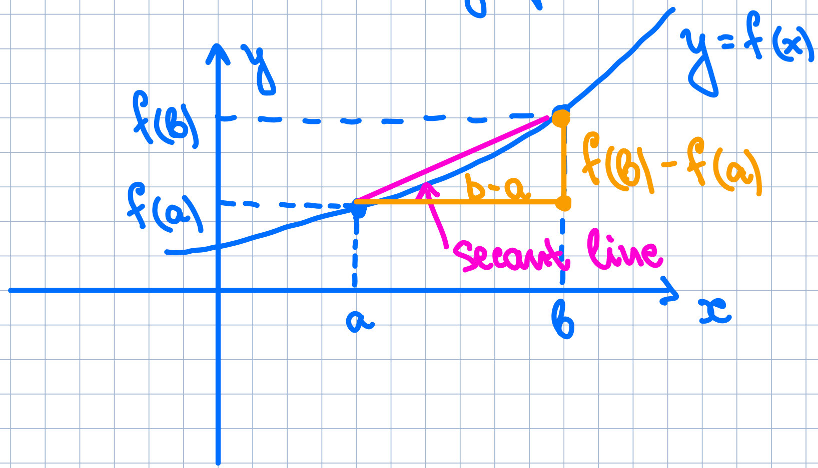
$$\frac{f(b)-f(a)}{b-a}.$$

If $y=f(x)$, then any of the following expressions may be used to represent the average rate of change of f over $[a,b]$:

$$r = \frac{\text{change } f}{\text{change in } x} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}.$$

The average rate of change of f over $[a,b]$ represents the slope of a <u>secant line</u>

drawn between the points $(a, f(a))$ and $(b, f(b))$ on the graph of f.



Example

$f(x) = 3x^2 - 5x + 2$

r - ? on $[\overset{a}{1}, \overset{b}{3}]$

$$r = \frac{\Delta f}{\Delta x} = \frac{f(3) - f(1)}{3-1} = \frac{3(3)^2 - 5\cdot 3 + 2 - (3 - 5 + 2)}{2} =$$

$$= \frac{27 - 15 + 2 - 3 + 5 - 2}{2} = \frac{14}{2} = 7$$

Def. The computation of the average rate of change of a function over an interval of the form $[c, c+h]$ occurs frequently in calculus, and the ratio

$$\frac{f(c+h) - f(c)}{h}$$

is the difference quotient of f at c with increment h.