

Name: \_\_\_\_\_

**Solutions**

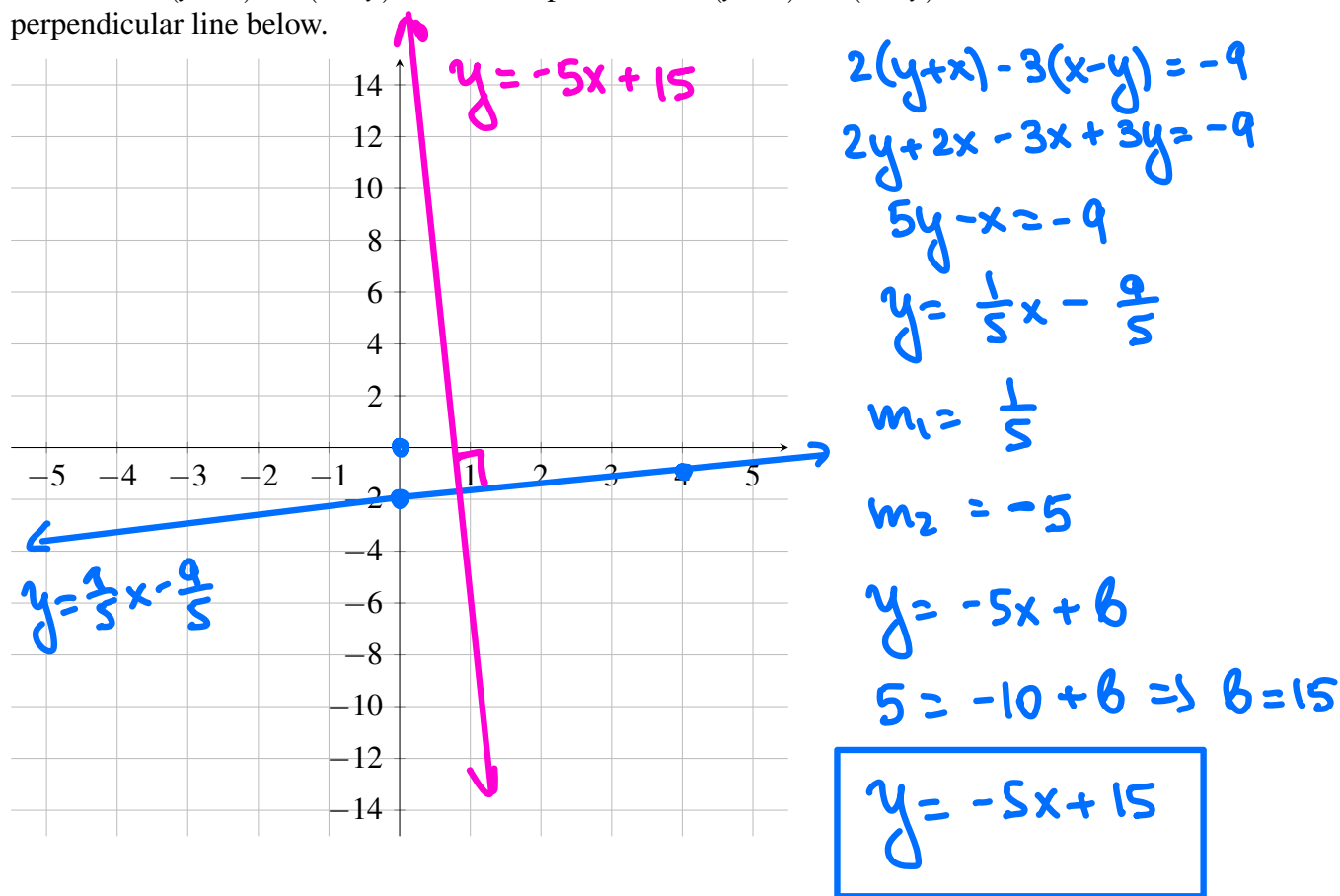
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Assessment 2 Instructions:

- The AS-2 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-2.
- The AS-2 is closed book and closed notes.
- **Calculators are allowed only for the problem # 10** on the AS-2.
- Show all your work for full credit and box your final answer.

## 1. [4 points]

Find the equation of the straight line that passes through the point  $(2, 5)$  and is **perpendicular** to the line  $2(y+x) - 3(x-y) = -9$ . Graph the line  $2(y+x) - 3(x-y) = -9$  and the obtained perpendicular line below.

2. [4 points] Given the two points  $(5, -4)$  and  $(4, 3)$ :

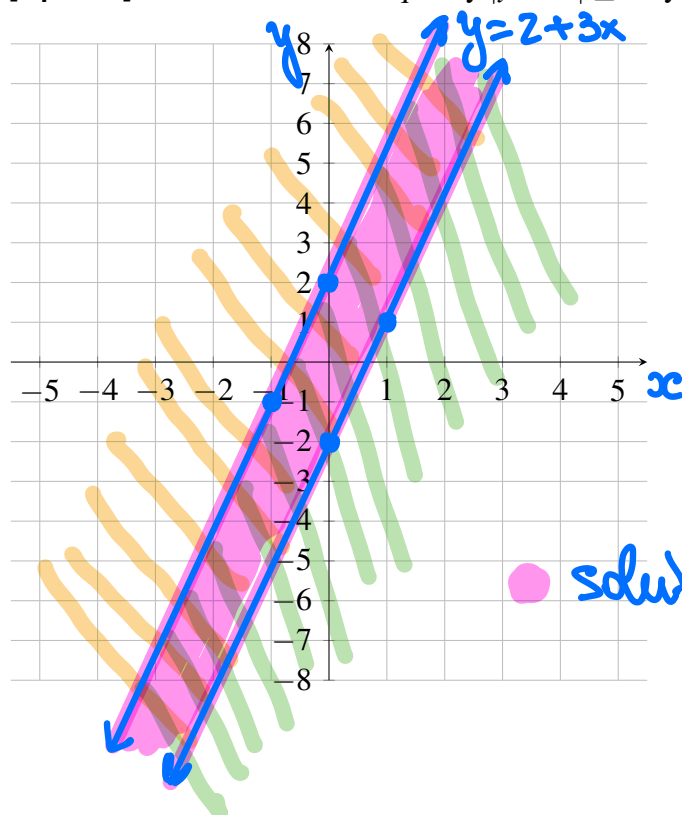
- a. find the length of the line segment (distance) between the points

$$d = \sqrt{(4-5)^2 + (3+4)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

- b. find the midpoint of the segment

$$M\left(\frac{5+4}{2}, \frac{-4+3}{2}\right) = M\left(\frac{9}{2}, -\frac{1}{2}\right)$$

3. [4 points] Solve the linear inequality  $|y - 3x| \leq 2$  by graphing its solution set



$$y - 3x \leq 2 \quad \text{and} \quad y - 3x \geq -2$$

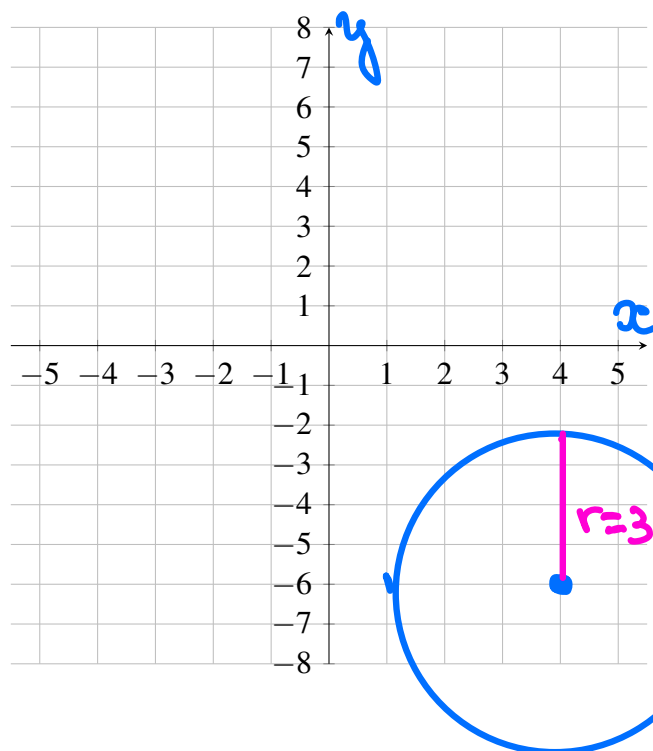
$$y \leq 2 + 3x \quad \text{and} \quad y \geq -2 + 3x$$

$$y = 2 + 3x \quad \quad y = -2 + 3x$$

4. [4 points] Find the standard form for the equation of the circle

$$x^2 + y^2 - 8x + 12y + 43 = 0$$

**Precisely state** the coordinates of the center point and the radius. Graph the obtained circle.



$$(x^2 - 8x) + (y^2 + 12y) + 43 = 0$$

$$(x - 4)^2 - 16 + (y + 6)^2 - 36 + 43 = 0$$

$$(x - 4)^2 + (y + 6)^2 = 9 = 3^2$$

Center  $(4, -6)$   
 Radius  $r = 3$

5. [4 points] For each of the following relations, determine the **domain** and **range**:

a.  $R = \{(3, 3), (-4, 3), (3, 8), (3, -2)\}$

$\text{Dom}(R) = \{3, -4\}$        $\text{Ran}(R) = \{3, 8, -2\}$

b.  $y = 7\pi^2$  — horizontal line

$\text{Dom}(y) = \mathbb{R}$        $\text{Ran}(y) = 7\pi^2$

6. [4 points] Determine the implied domain of the following function

$$f(x) = \frac{5}{\sqrt{3-x^2}}$$

$$3-x^2 \neq 0$$

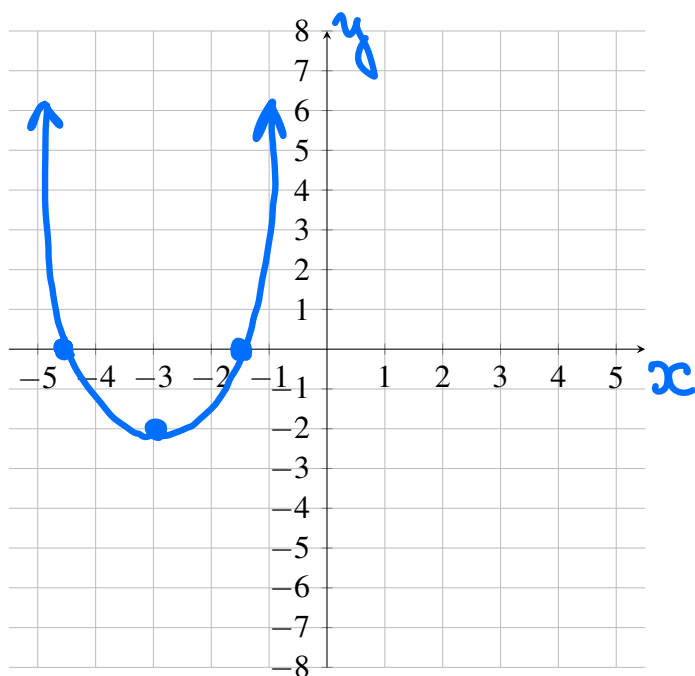
$$x^2 \neq 3$$

$$x \neq \pm\sqrt{3}$$

$$\text{Dom}(f) = \mathbb{R} \setminus \{\pm\sqrt{3}\}$$

7. [4 points] Graph the following quadratic function and **state the coordinates of its vertex**.

$$y = (x+3)^2 - 2$$



vertex:  $(-3, -2)$

$y=0$ :  $(x+3)^2 - 2 = 0$

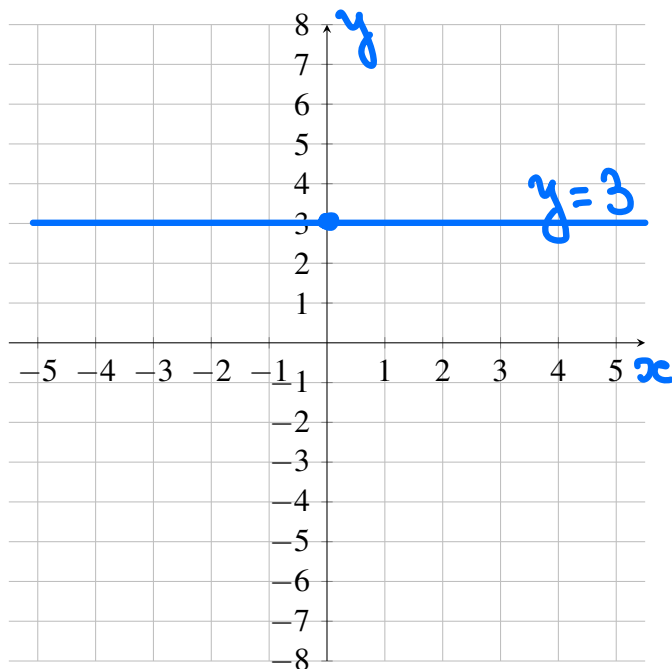
$$(x+3)^2 = 2$$

$$x = \pm\sqrt{2} - 3$$

$$x \approx -1.6$$

$$x \approx -4.4$$

8. [4 points] Graph the following linear function  $f(x) = 3\left(1 - \frac{1}{3}x\right) + x$ . State precisely its slope and y-intercept point.



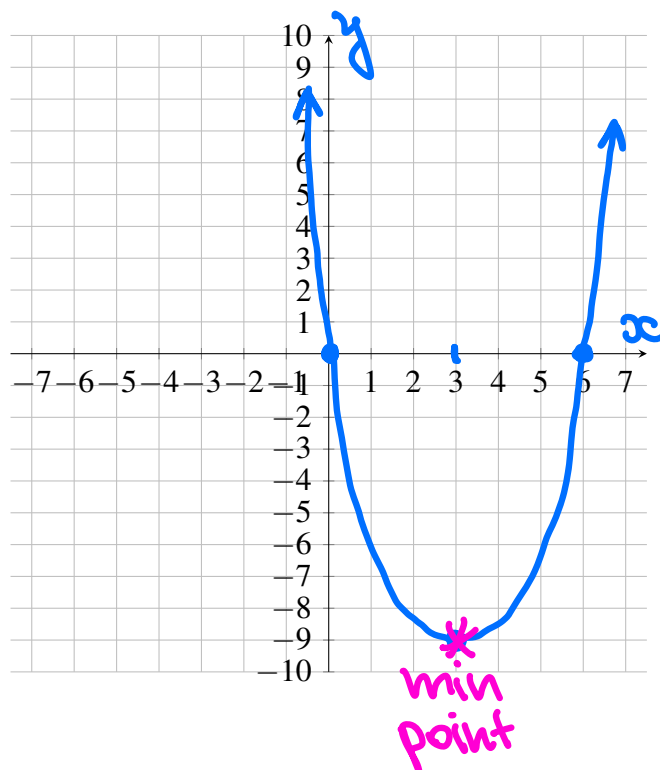
$$f(x) = 3 - x + x = 3$$

$$y = 3$$

$$m = 0$$

y-intercept : None

9. [4 points] Among all pairs of numbers with a difference of 6, find the pair whose product is minimum. Write your answer in the form:  $x = \dots, y = \dots, x \cdot y = \text{min product}$ . (Hint: you need to create a quadratic function, graph it with indicating its vertex coordinates)



$$x - y = 6 \Rightarrow y = x - 6$$

$$x \cdot y \rightarrow \text{min}$$

$$x(x-6) = x^2 - 6x \rightarrow \text{min}$$

$$f(x) = x(x-6)$$

$$0 = x(x-6) \Rightarrow x = 0 \text{ or } x = 6$$

$$x = 3 : f(3) = 3 \cdot (-3) = -9$$

$$x = 3$$

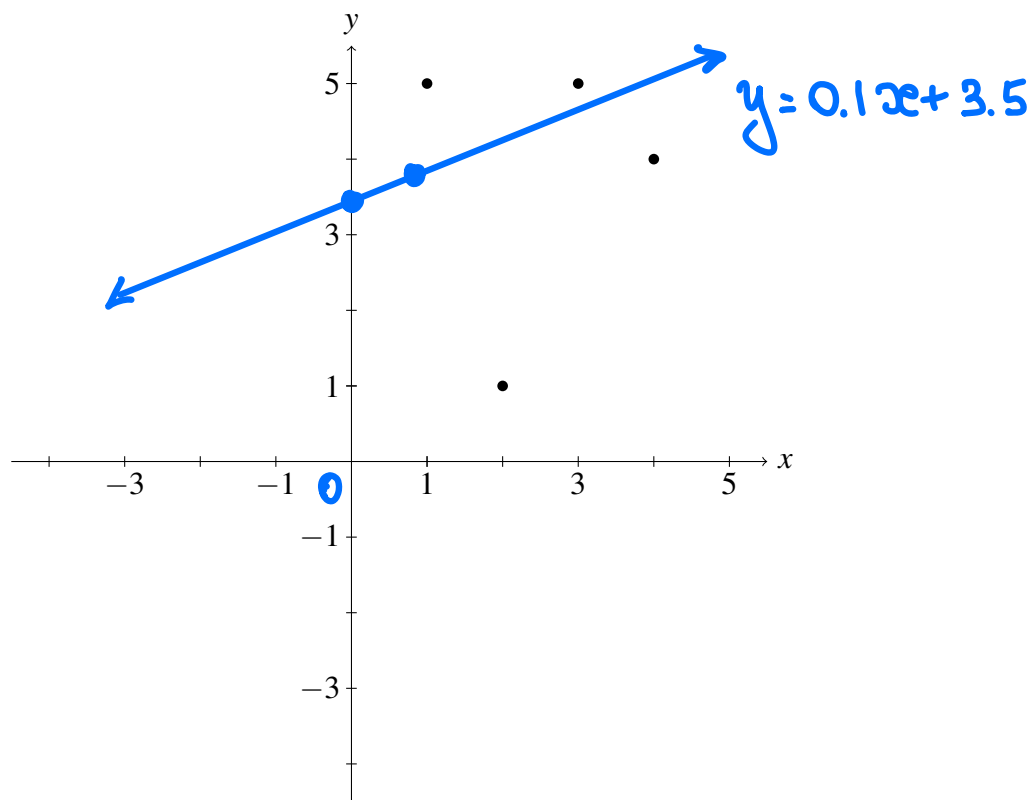
$$\text{Thus } y = x - 6$$

$$y = -3$$

$$x \cdot y = -9$$

10. [4 points] The coordinates of the four graphed points are

$$\{(1,5), (2,1), (3,5), (4,4)\}.$$



We also know the following information:

- $\bar{x} = 2.5, \bar{y} = 3.75$
- $\sum(\Delta x)^2 = 5, \sum \Delta x \Delta y = 0.5, \sum(\Delta y)^2 = 10.75$

a. Using the following formulas

$$m = \frac{\sum \Delta x \Delta y}{\sum (\Delta x)^2} \quad \text{and} \quad b = \bar{y} - m\bar{x}$$

find the equation of the line  $y = mx + b$  of the best fit. Sketch the obtained straight line on the coordinate plane given above.

$$m = \frac{0.5}{5} = 0.1 \quad b = 3.75 - 0.1 \cdot 2.5$$

$$b = 3.5$$

Thus

$$y = 0.1x + 3.5$$

b. Using the following formula

$$r = \frac{\sum \Delta x \Delta y}{\sqrt{\sum (\Delta x)^2} \sqrt{\sum (\Delta y)^2}}$$

calculate the Pearson correlation coefficient  $r$ . Make a conclusion about the linear dependence of the variable  $y$  on the variable  $x$ .

$$r = \frac{0.5}{\sqrt{5} \cdot \sqrt{10.75}} \approx \frac{0.5}{7.33} \approx \boxed{0.07}$$

We can conclude that there is almost no linear dependence of  $x$  on  $y$ .  
 $y$  on  $x$