

**THEORETICAL PART:****Theorem (The division algorithm):**

Let  $p(x)$  and  $d(x)$  be polynomials such that  $d(x) \neq 0$  and with the degree of  $d$  less than or equal to the degree of  $p$ . Then there are unique polynomials  $q(x)$  and  $r(x)$ , called the **quotient** and the **remainder**, respectively, such that

$$p(x) = q(x) \cdot d(x) + r(x).$$

Either the degree of the remainder  $r$  is less than the degree of the **divisor**  $d$ , or the remainder is 0, in which case we say  $d$  **divides evenly** into the polynomial  $p$ . If the remainder is 0, the two polynomials  $q$  and  $d$  are factors of  $p$ .

**Theorem (Zeros and linear factors):**

The number  $c$  is a **zero** of a polynomial  $p$  if and only if the linear polynomial  $x - c$  is a factor of  $p$ . In this case,  $p(x) = q(x)(x - c)$  for some quotient polynomial  $q$ . This also means that  $c$  is the solution of a polynomial equation  $p(x) = 0$ , and if  $p$  is a polynomial with real coefficients and if  $c$  is a real number, then  $c$  is an  $x$ -intercept of  $p$ .

**Theorem (The remainder theorem):**

If the polynomial  $p(x)$  is divided by  $x - c$ , the remainder is  $p(c)$ .

$$p(x) = q(x)(x - c) + p(c)$$

**Procedure (Polynomial long division):**

1. Arrange the terms of each polynomial in descending order.
2. Divide the first term in the dividend by the first term in the divisor. This gives the first term of the quotient.
3. Multiply the entire divisor by the result (the first term of the quotient) and write the beneath the dividend so that like terms line up.
4. Subtract the product from the dividend.
5. Bring down the rest of the original dividend, forming a new dividend.
6. Repeat the process with the new dividend. Continue until the degree of the remainder is less than the degree of the divisor.

**Procedure (Synthetic division):**

1. If the divisor is  $x - c$ , write down  $c$  followed by the coefficients of the dividend.
2. Write the leading coefficient of the dividend on the bottom row.



3. Divide  $p(x) = x^4 + 1$  by  $d(x) = x^2 + i$ .

4. For the polynomial  $p$  below, divide  $p$  by  $x - c$  using synthetic division. Use the result to determine if the given  $c$  is a zero. If not, determine  $p(c)$ .

$$p(x) = -2x^4 + 11x^3 - 5x^2 - 3x + 15; \quad c = 5$$

5. Construct a polynomial that has the given properties:

- (a) Third degree, zeros of -3, 2, and 5, and goes to  $-\infty$  as  $x \rightarrow \infty$ .
- (b) Fourth degree, zeros of -5, -2, 1, and 3, and y-intercept at (0, 15).