Section 1.7. Linear inequalities in one variable

1. Solving linear inequalities
2. Solving double linear inequalities.
3. Solving linear absolute value inequalities.
4. Applications of linear inequalities.

1.

$=$ $\Rightarrow$ $>, <, \geq, \leq$

then the result is a linear inequality

The solution : some interval of real numbers

Example (Multiplying inequalities by negative numbers)

① $-3 < 2$ $\quad | \cdot(-1)$

$3 < -2$ False

There is something wrong!

② $x < 0$ $\quad | \cdot (-1)$

$-x < 0$

} can't both be true!

If $x = -1$, then $-1 < 0$, but $1 \not< 0$.

Properties (cancellation)

1. If $A < B$, then $A + C < B + C$
2. If $A < B$ and $D > 0$, then $A \cdot D < B \cdot D$
3. If $A < B$ and $D < 0$, then $A \cdot D > B \cdot D$.

Example

Solve the following inequality:

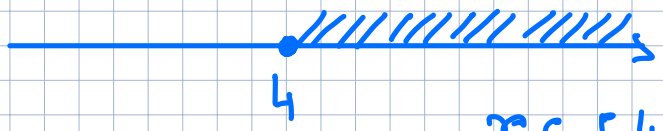
$$5 - 2(x-3) \leq -(1-x)$$

$$5 - 2x + 6 \leq -1 + x$$

$$-2x - x \leq -1 - 6 - 5$$

$$-3x \leq -12$$

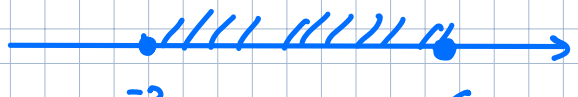
$$x \geq 4$$



$x \in [4; +\infty)$

Example (Graphing intervals of real numbers)

(a) $[-3, 6]$



(b) $(-\infty, 5]$

(c) $[2, 9)$

2.

Def. A **compound inequality** is a statement containing two or more distinct inequalities joined by "and" or "or".

Double inequality – two inequalities are joined by "and".

Example

$$2(2x-1) \leq 4x+2 \leq 4(x+1)$$

$$4x-2 \leq 4x+2 \leq 4x+4$$

$$-2 \leq 2 \leq 4$$

The above inequality holds true for

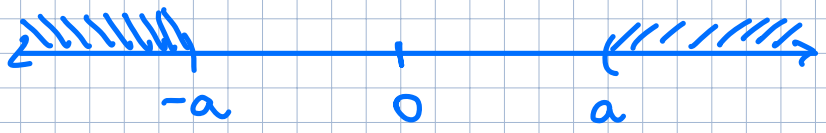
all values of x. Therefore, the solution is $(-\infty, \infty)$.

3. **Def.** An **absolute value inequality** is an inequality in which some variable expression appears inside absolute value symbols.

$$|x| < a \iff -a < x < a$$

$|x| < a$

$$|x| > a \iff a < x < -a$$



$|x| > a$

We also consider cases

$$|x| \leq a, \quad |x| \geq a$$

Example

$$|4 - 2x| > 6 \iff 6 < 4 - 2x < -6$$

$$6 < 4 - 2x < -6$$
$$6 - 4 < -2x < -6 - 4$$
$$2 < -2x < -10$$
$$5 < x < -1$$
$$x > 5 \text{ and } x < -1$$

4. Applications of linear inequalities.

Translating inequality phrases:

"x is no greater than y" $\quad x \leq y$

"x is at least as large as y" $\quad x \geq y$

"x does not exceed y" $\quad x \leq y$