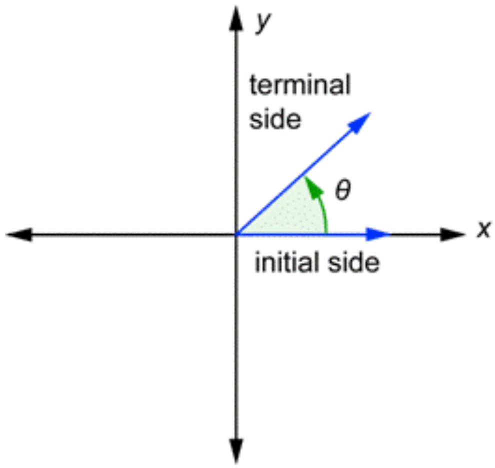
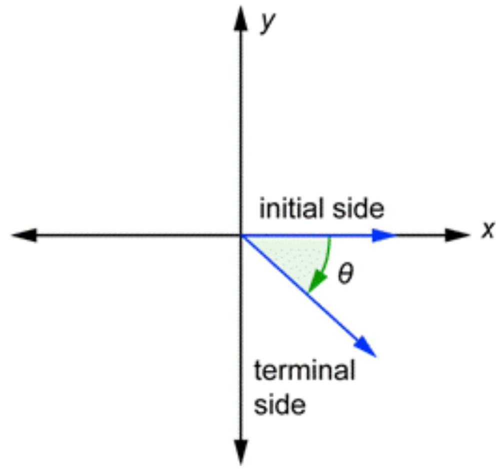
THEORETICAL PART: Solutions

Definition (Radian Measure):

Let θ be an angle at the center of a circle of radius 1 (the unit circle). The measure of θ in **radians** is the length of that portion of the circle subtended by *theta*, which is the portion of the circumference.





Formula (Angle Measurement Conversion)

Since $180^\circ = \pi\ rad$, we know that $1^\circ = \frac{\pi}{180}\ rad$ and $\left(\frac{180}{\pi}\right)^\circ = 1\ rad$. Multiplying both sides of these equations by an arbitrary quantity x, we have

1. $x^\circ = x\left(\frac{\pi}{180}\right)\ rad$.
2. $x\ rad = x\left(\frac{180}{\pi}\right)^\circ$.

Formula (Arc Length):

Given a circle of radius r, the length s of the arc subtended by a central angle θ (in radians) is given by the following formula:

$$s = \left(\frac{\theta}{2\pi}\right)(2\pi r) = r\theta$$

Definition (Angular speed and linear speed):

If an object moves along the arc of a circle defined by a central angle θ in time t, the object is said to have an **angular speed** ω given by

$$\omega = \frac{\theta}{t}.$$

If the circle has a radius of r, the distance traveled in time t is the arc length s, and the **linear speed** v is given by

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Formula (Sector Area):

The area A of a sector with a central angle of θ in a circle of radius r is

$$A = \left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{r^2\theta}{2}.$$

PRACTICAL PART:

1. Convert the following angle measures as directed.

   a. Express $\frac{\pi}{3}$ *rad* in degrees.

   b. Express 270° in radians.

   c. Express -2 *rad* in degrees.

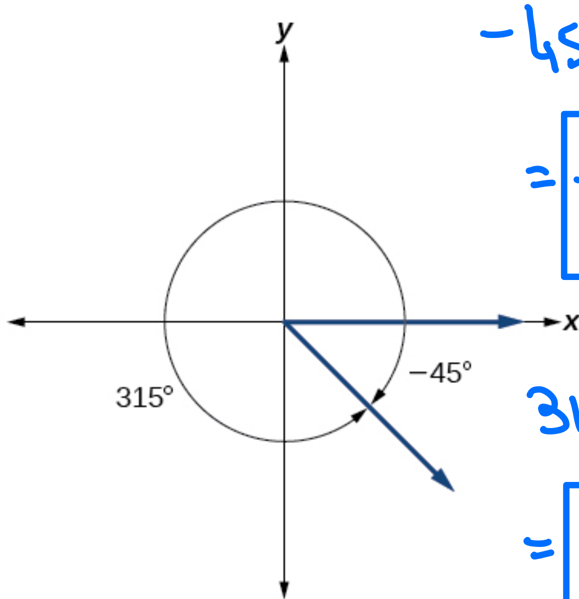
(a) $\frac{\pi}{3}$ rad $= \frac{\pi}{3} \cdot \left(\frac{180}{\pi}\right)^\circ = 60^\circ$

(b) $270^\circ = 270 \cdot \left(\frac{\pi}{180}\right)$ rad $= \frac{3\pi}{2}$ rad

(c) -2 rad $= -2 \cdot \left(\frac{180}{\pi}\right)^\circ \approx -114.592^\circ$

2. Use the information in each diagram to determine the **radian** measure of the indicated angle.

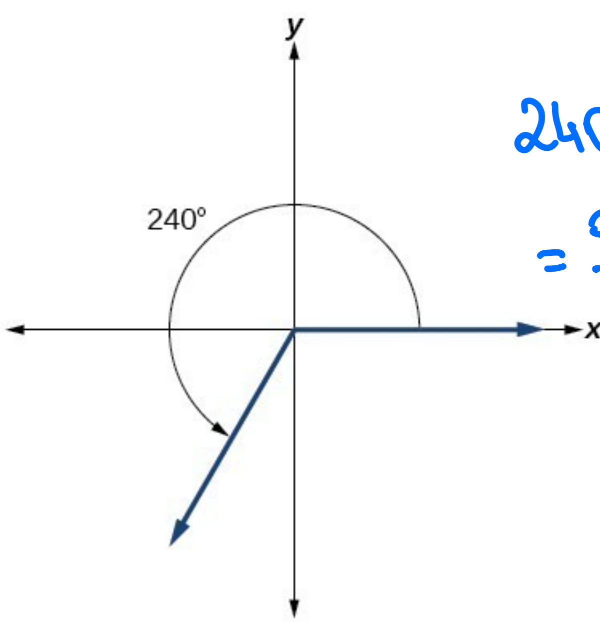
(a)



$$-45^\circ = -45\cdot\frac{\pi}{180} = \boxed{-\frac{\pi}{4}\text{ rad}}$$

$$315^\circ = 315\cdot\frac{\pi}{180} = \boxed{\frac{7\pi}{4}\text{ rad}}$$

(b)



$$240^\circ = 240\cdot\frac{\pi}{180} = \frac{80\pi}{60} = \boxed{\frac{4\pi}{3}\text{ rad}}$$

3. Find the length of the arc subtended by the given central angle θ on a circle of radius r.

   a. $r = 4\ in., \theta = 1.$

   b. $r = 16\ ft, \theta = \frac{\pi}{4}.$

(a) $S = r \cdot \theta = 4 \cdot 1 = 4 (in)$

(B) $S \cdot r \cdot \theta = 16 \cdot \frac{\pi}{4} = 4\pi (ft)$

4. Suppose an ant crawls along the rim of a circular glass with radius 2 inches, and traverses the arc in 20 seconds. What are the angular and linear speeds of the ant, and how far does it travel?

$r = 2 (in)$

$t = 20 (sec)$

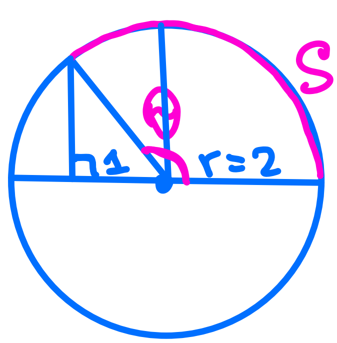
$\omega - ?$ $\quad s - ?$

$v - ?$

$\theta - ?$

$$\boxed{\omega = \frac{\theta}{t}}$$

$$\boxed{v = \frac{s}{t} = r\omega}$$



$\theta = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$

Hence, $w = \frac{2\pi}{3 \cdot 20} = \frac{\pi}{30} \left(\frac{rad}{sec}\right)$

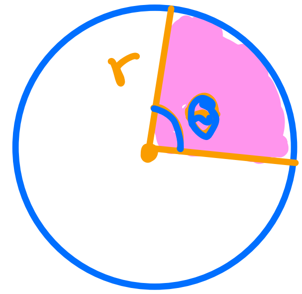
$$V = r \cdot w = 2 \cdot \frac{\pi}{30} = \frac{\pi}{15} \left(in/sec\right)$$

$$S = r \cdot \theta = 2 \cdot \frac{2\pi}{3} = \frac{4\pi}{3} \left(in\right)$$

5. Determine the areas of the sectors defined by the given radii and angles.

   a. Circle of radius 3 *cm*, central angle of 52°.

   b. Circle of radius $\frac{1}{2}$ *ft*, central angle of $\frac{4\pi}{3}$.



θ is a central angle

(a) $A = \frac{r^2 \theta}{2}$

$\theta = 52^0 = 52 \cdot \frac{\pi}{180}$ rad $=$

$= \frac{13\pi}{45}$ rad

$A = \frac{9 \cdot \frac{13\pi}{45}}{2} = \frac{13\pi}{10}$ (cm^2)

(b) $r = \frac{1}{2}(ft)$

$\theta = \frac{4\pi}{3}$

$A = \frac{\left(\frac{1}{2}\right)^2 \frac{4\pi}{3}}{2} = \frac{\pi}{6}$ (ft^2)