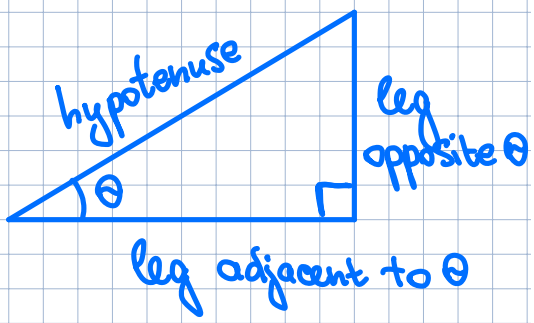
Section 7.2. Trigonometric functions and right angles

1. The trigonometric functions.
2. Evaluating trigonometric functions.
3. Applications of trigonometric functions.

1.

Def. (Sine, cosine, and tangent)



Assume θ is one of the acute angles in the right triangle, and let adj and opp stand for, respectively, the lengths of the legs adjacent to and opposite θ. Let hyp stand for the length of the hypotenuse of the right triangle. Then the **sine**, **cosine**, and **tangent** of θ, are the ratios

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

Def. (Cosecant, Secant, and cotangent)

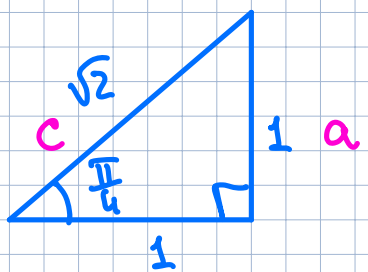
Assume θ is one of the acute angles in a right triangle. Then the cosecant, secant, and cotangent of θ, are the reciprocals of $\sin\theta$, $\cos\theta$, and $\tan\theta$. That is,

$$\csc\theta = \frac{1}{\sin\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}$$

2.

Example (Evaluating Trigonometric Functions)

$\tan(\theta) = ?$
$\sec(\theta) = ?$
$\theta = \frac{\pi}{4}$



$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a/c}{b/c} = \frac{a}{b}$$

$$\tan\theta = \frac{1}{1} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{c}{b} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Def. (Degree, minute, and second notation)

In the context of angle measure,

$$1' = \text{one minute} = \left(\frac{1}{60}\right)(1^\circ)$$

and

$$1'' = \text{one second} = \left(\frac{1}{60}\right)(1') = \left(\frac{1}{3600}\right)(1^\circ).$$

Example

$$14^\circ\, 37'\, 23'' = 14 + \frac{37}{60} + \frac{23}{3600} \approx 14.6231^\circ$$

3.

Example

The manufacturer of a certain brand of 16-foot ladder recommends that, when in use, the angle between the ground and the ladder should equal 75°. What distance should the foot of the ladder be from the base of the wall it is leaning against?

Solution

$$\cos 75^\circ = \frac{x}{16}$$

$$x = 16 \cdot \cos(75^\circ)$$

$$x \approx 4.14 \text{ (feet)}$$