Assessment Instructions:

Solutions

- The Assessment 2 is 10 problems and is worth 40 points. Each numbered problem will earn you a score of 1-4 based on your set up of the function, your use of course methods to solve and prove your solution and your statement of the solution.
- You will have 1 hour to complete AS-2.
- The AS-2 is closed book and closed notes.
- Calculators are not allowed on the Assessment.

1. Find the equation of the line that:
   (a) passes through $(5, 0)$ and is perpendicular to $-5x + 2y = 1$
   (b) passes through $(2, -1)$ and is parallel to $y = -3x + 4$
   (c) is parallel to y-axis and has an x-intercept at $(2, 0)$
   (d) is perpendicular to the x-axis and passes through $(-2, -3)$
   (e) passes through the point $(3, -4)$ and $(2, 9)$
   (f) satisfies $f(1) = -5$ and $f(9) = 4$

2. Given the two points, find the length of the line segment between the points and the midpoint of the segment:
   (a) $(3, 2)$ and $(5, 1)$
   (b) $(2, 2)$ and $(8, 3)$
   (c) $(5, 4)$ and $(4, 3)$

3. Graph the linear inequality:
   (a) $x + y > 0$
   (b) $-(y - x) > -\frac{5}{2} - y$
   (c) $-2y \leq -x + 4$
   (d) $y > -2$ and $2y > -3x - 4$
   (e) $-2y < -3x - 6$ or $-3y \geq -6x - 18$

4. Graph the absolute value linear inequality:

   (a) $|3y - 1| \leq 2$

   (b) $|x + y| \geq 1$

   (c) $|x - 3| > 2$

5. Find the standard form for the equation of the circle:

   (a) $9x^2 + 9y^2 - 18x + 36y + 44 = 0$

   (b) center $(12, -4)$ passing through $(-9, 5)$

   (c) endpoints of the diameter are $(-8, 6)$ and $(1, 11)$

   (d) $5x^2 + 5y^2 + 50x + 40y = -185$

   (d) $x^2 + y^2 - 18x - 8y + 48 = 0$

6. Sketch the graph:

   (a) $(x + 2)^2 + y^2 = 169$

   (b) $(x + 3)^2 + (y - 7)^2 = 64$

   (c) $x^2 + y^2 + 8x = 9$

7. For each of the following relations, determine the domain and range:

   (a) $R = \{(0, 0), (-5, 2), (3, 3), (5, 3)\}$

   (b) $3x - 4y = 17$

   (c) $y = x^2$

   (d) $x = 4x$

8. Rewrite each of the following relations as a function of x and evaluate it at $x = -1$:

   (a) $6x^2 - x + 3y = x + 2y$

   (b) $\dfrac{9y + 2}{6} = \dfrac{3x - 1}{2}$

9. Identify the domain, the codomain, and the range of the following functions:

   (a) $f : \mathbb{N} \to \mathbb{N}$ and $f(x) = x + 5$

   (b) $h : [0, \infty) \to \mathbb{R}$ and $h(x) = \sqrt{x}$

10. Determine the implied domain of the following functions:

    (a) $g(x) = \dfrac{2x}{1 - 3x}$

    (b) $h(x) = \sqrt{3 - x}$

    (c) $f(x) = \dfrac{5}{\sqrt{3 - x^2}}$

11. Graph the following linear functions:

    (a) $y = -2$

    (b) $f(x) = 3 - 2x$

    (c) $g(x) = \dfrac{2x - 8}{4}$

12. For the given points use linear regression to find and graph the line of best fit along with the points and find the Pearson correlation coefficient r

    (a) $\{(1, 5), (2, -1), (3, 5), (4, 0), (5, 4)\}$

13. Graph the following quadratic functions (parabolas) and state the coordinates of its vertex:

    (a) $y = (x - 2)^2 + 3$

    (b) $f(x) = -3x^2 - 1$

    (c) $g(x) = 4x^2 - 6$

    (d) $f(x) = x^2 + 2x + 4$

14. Among all the pairs of numbers with a sum of 10, find the pair whose product is maximum.

15. The total revenue for Thompson's Studio Apartments is given by the function

    $$R(x) = 100x - 0.1x^2,$$

    where x is the number of rooms rented.

    What number of rooms rented produces the maximum revenue?

1. (a) $A(5,0)$

$\ell \perp -5x+2y=1$

$m_1 = -\frac{1}{m_2}$

$2y = 1+5x$

$y = \frac{5}{2}x + \frac{1}{5}$

$m_2 = \frac{5}{2}$

$m_1 = -\frac{1}{\frac{5}{2}} = -\frac{2}{5}$

$y = -\frac{2}{5}x + b$

$A(5,0) \rightarrow \quad 0 = -\frac{2}{5}\cdot 5 + b \Rightarrow b = 2$

Hence, $\boxed{y = -\frac{2}{5}x + 2}$

(b) $A(2,-1)$ $\quad \ell \parallel y = -3x+4$

$m_1 = m_2$

$m_2 = -3 = m_1$

$y = -3x + b$

$-1 = -3\cdot 2 + b$

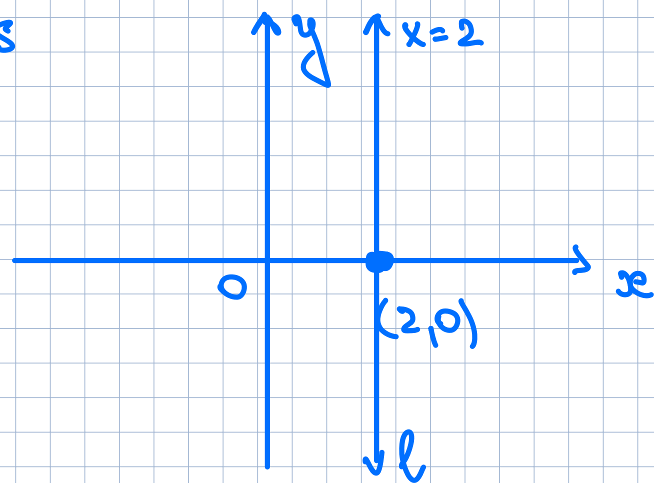
$-1 = -6 + b$

$b = 5$

Hence, $\boxed{y = -3x + 5}$

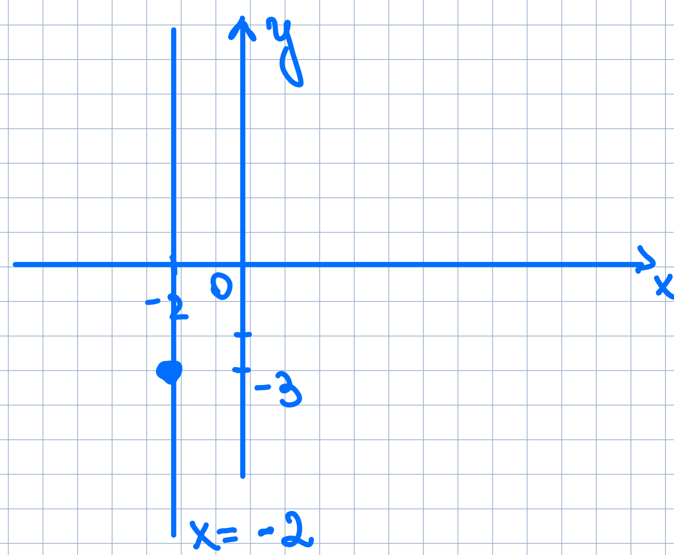
(c) $\ell \parallel$ y-axis

$\boxed{x = 2}$



(d) $\ell \perp$ x-axis

$\boxed{x = -2}$



(e) A(3,-4)
B(2,9)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9+4}{2-3} = -13$$

$y = -13x + b$

$9 = -13 \cdot 2 + b \Rightarrow b = 9 + 26 = 35$

Hence,

$$\boxed{y = -13x + 35}$$

(f) $f(1) = -5 \leftrightarrow A(1, -5)$

$f(9) = 4 \leftrightarrow B(9, 4)$

See problem (e).

$\boxed{2.}$ (a) $(3,2)$ and $(5,1)$

$$d = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{3+5}{2}, \frac{2+1}{2}\right) = M\left(\frac{8}{2}, \frac{3}{2}\right) = M\left(4, \frac{3}{2}\right)$$

(b) See (a)

(c) See (a)

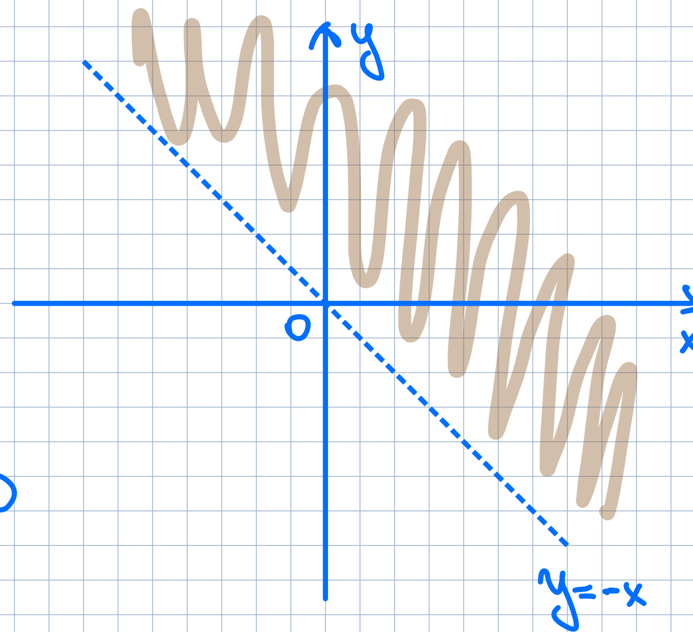
$\boxed{3.}$

(a) $x+y>0$

$x+y=0$

$y=-x$

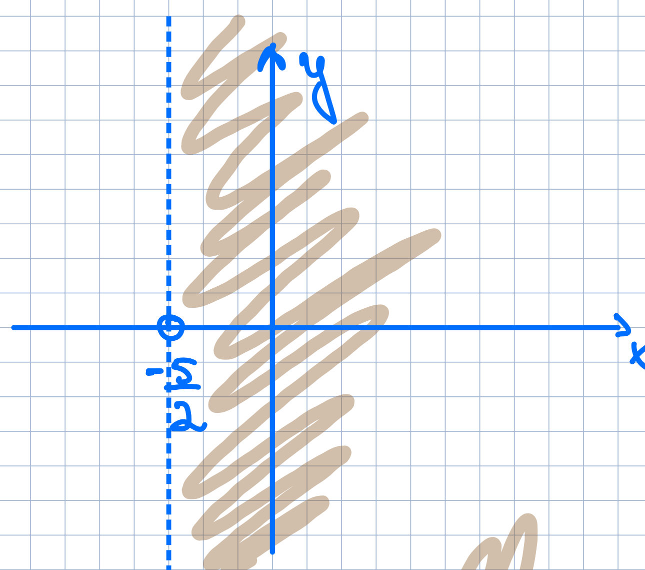
$(2,0)$: $2+0>0$



(b) $-(y-x)>-\frac{5}{2}-y$

$-\cancel{y}+x>-\frac{5}{2}-\cancel{y}$

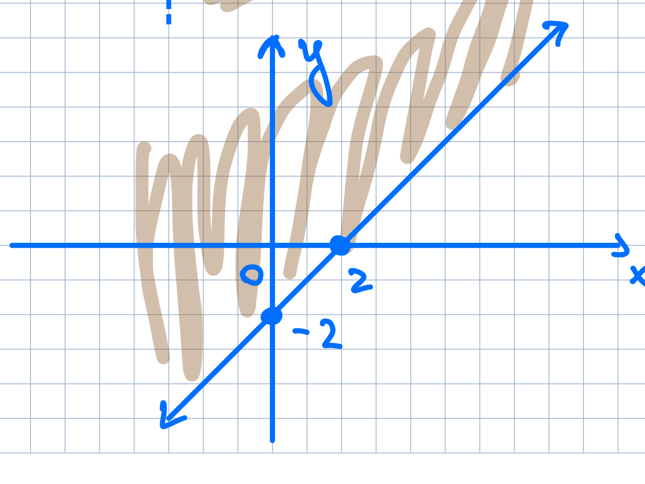
$x>-\frac{5}{2}$



(c) $-2y\leq -x+4$

$y\geq \frac{x}{2}-2$

$y=\frac{x}{2}-2$

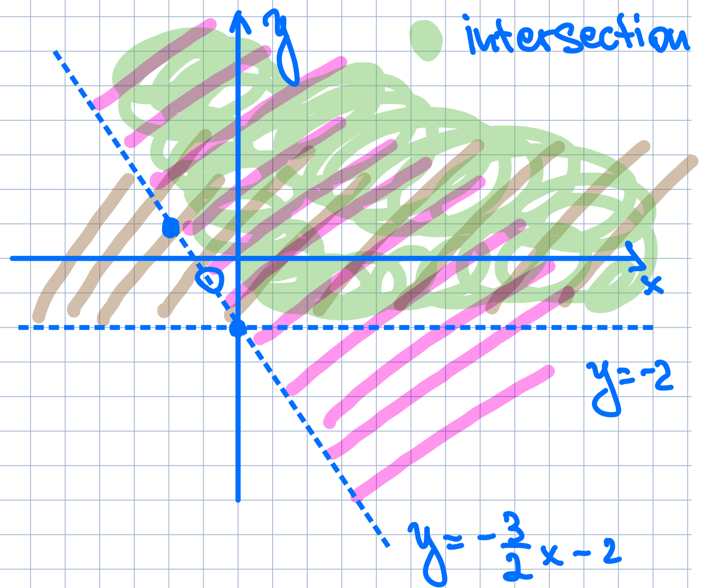


$(0,0)$: $0 \geq -2$ ✓

(d) $\cap$

$y > -2$ and $2y > -3x - 4$

$y > -2$ and $y > -\frac{3}{2}x - 2$

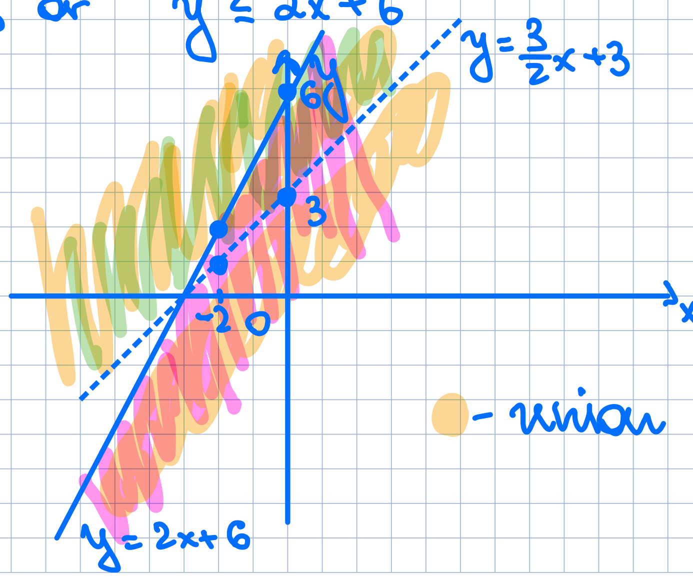


(e) $\cup$ $-2y < -3x - 6$ or $-3y \geq -6x - 18$

$y > \frac{3}{2}x + 3$ or $y \leq 2x + 6$

$y = \frac{3}{2}x + 3$

$y = 2x + 6$



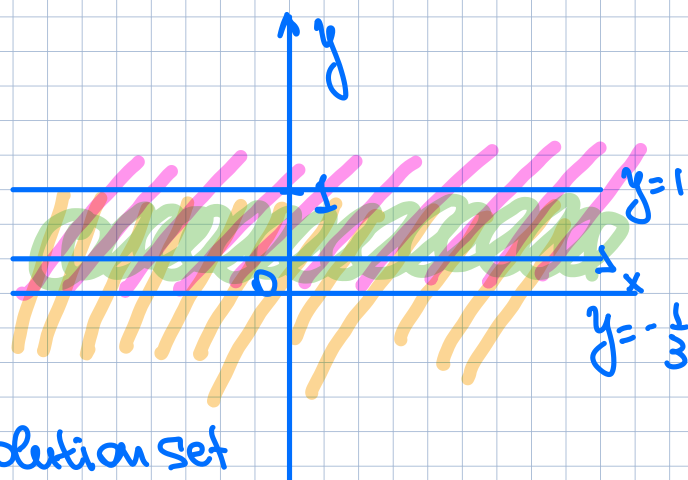
$\boxed{4.}$

$|3y-1| \le 2$

$3y-1 \le 2$ and $3y-1 \ge -2$

$3y \le 3$ $\quad$ $3y \ge -1$

$y \le 1$ $\quad$ $y \ge -\frac{1}{3}$

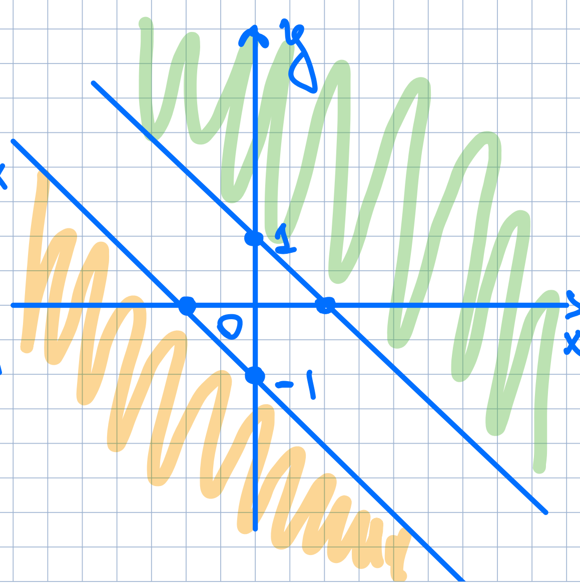


– solution set

(b) $|x+y| \ge 1$

$x+y \ge 1$ or $x+y \le -1$

$y \ge 1-x$ or $y \le -1-x$



$\cup$ is a solution set

5. (a)

$$9x^2 + 9y^2 - 18x + 36y + 44 = 0$$

$$(3x)^2 + (3y)^2 - 18x + 36y + 44 = 0$$

$$(3x-3)^2 + (3y+6)^2 - 9 - 36 + 44 = 0$$

$$(3x-3)^2 + (3y+6)^2 = 1$$

$$9(x-1)^2 + 9(y+2)^2 = 1$$

$$\boxed{(x-1)^2 + (y+2)^2 = \frac{1}{9} = \left(\frac{1}{3}\right)^2}$$

(b) $(12,-4)$

$(-9,5)$

$$(x-12)^2 + (y+4)^2 = r^2$$

r - ?

$$(-9-12)^2 + (5+4)^2 = r^2$$

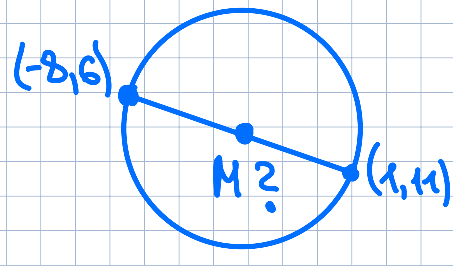
$$21^2 + 9^2 = r^2$$

$$r^2 = 522$$

Hence, $\boxed{(x-12)^2 + (y+4)^2 = 522}$

(c) $(-8,6)$

$(1,11)$



$$d = \sqrt{(1+8)^2 + (11-6)^2} = \sqrt{81 + 25} = \sqrt{106}$$

$$r = \frac{\sqrt{106}}{2}$$

$$M\left(\frac{1-8}{2}, \frac{6+11}{2}\right) = M\left(-\frac{7}{2}, \frac{17}{2}\right)$$

$$\boxed{\left(x+\frac{7}{2}\right)^2 + \left(y-\frac{17}{2}\right)^2 = \frac{106}{4}}$$

(d) Will not appear on AS-2

(e) $x^2 + y^2 - 18x - 8y + 48 = 0$

$$(x-9)^2 + (y-4)^2 - 81 - 16 + 48 = 0$$

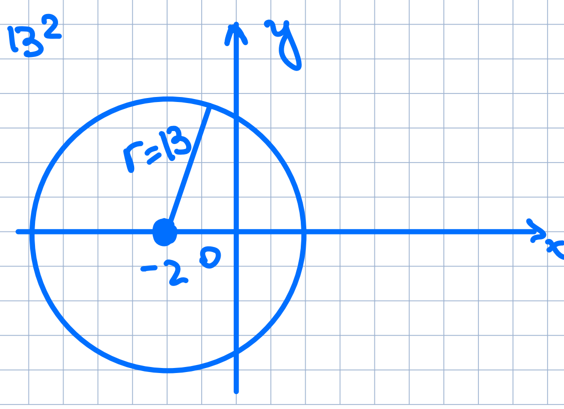
$$\boxed{(x-9)^2 + (y-4)^2 = 49 = 7^2}$$

$\boxed{6.}$

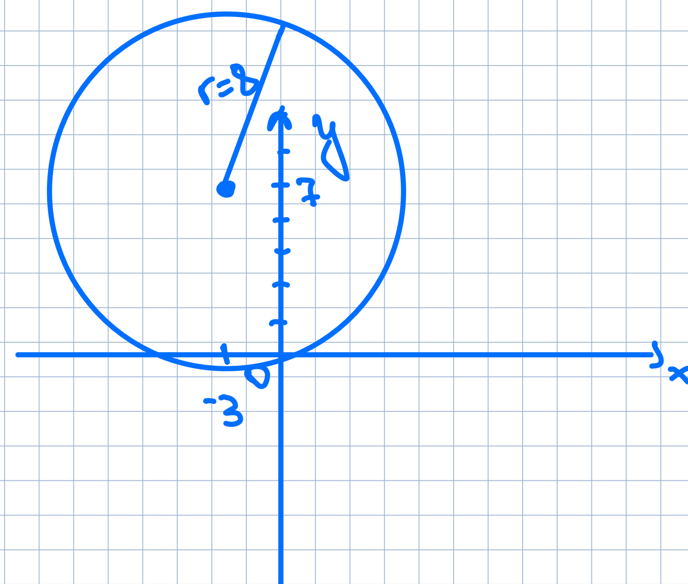
(a) $(x+2)^2 + y^2 = 169 = 13^2$

center $(-2, 0)$

$r = 13$



(b) $(x+3)^2+(y-7)^2=64=8^2$

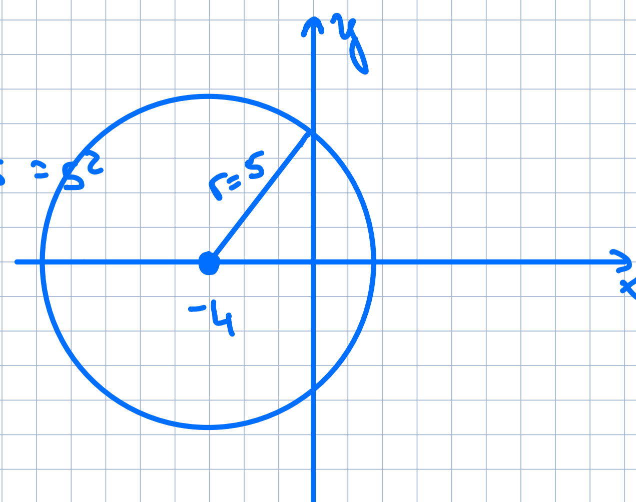


(c) $x^2+y^2+8x=9$

$(x+4)^2+y^2=25=5^2$

center $(-4,0)$

$r=5$



$\boxed{7.}$

(a) $R=\{(0,0),(-5,2),(3,3),(5,3)\}$

$Dom(R)=\{0,-5,3,5\}$

$Ran(R) = \{0, 2, 3\}$

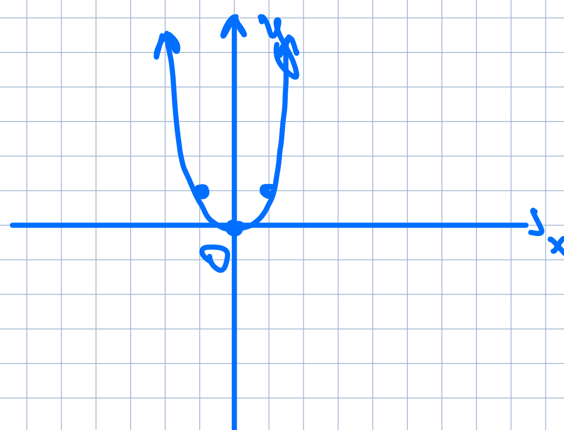
(b) $3x - 4y = 17$

$-4y = 17 - 3x$

$y = \frac{3}{4}x - \frac{17}{4}$ — straight nonvertical line

$Dom(f) = \mathbb{R}$
$Ran(f) = \mathbb{R}$

(c) $y = x^2$



$Dom(y) = \mathbb{R}$

$Ran(y) = [0, \infty)$

(d) $x = 4x$

$3x = 0$

$x = 0$

$Dom(R) = \{0\}$
$Ran(R) = \emptyset$

8. (a) $6x^2 - x + 3y = x + 2y$

$y = x + x - 6x^2$

$$\boxed{y = 2x - 6x^2}$$

$y(-1) = -2 - 6 \cdot 1 = -8$

(b) $\frac{9y+2}{6} = \frac{3x-1}{2}$

$9y + 2 = 6\,\frac{3x-1}{2} = 3(3x-1)$

$9y = 9x - 3 - 2 = 9x - 5$

$y = x - \frac{5}{9}$

$y(-1) = -1 - \frac{5}{9} = -\frac{14}{9}$

9. (a) $f: \mathbb{N} \to \mathbb{N}$ $\quad f(x) = x + 5$

Dom $(f) = \mathbb{N}$

Codomain $(f) = \mathbb{N}$

Ran$(f) = \{6, 7, 8, 9, 10, 11, \ldots\}$

(b) $h: [0,\infty) \to \mathbb{R}$ $\quad h(x) = \sqrt{x}$

$\mathrm{Dom}(h) = [0,\infty)$

$\mathrm{Codomain}(h) = \mathbb{R}$

$\mathrm{Ran}(h) = [0,\infty)$

$\boxed{10.}$

(a) $g(x) = \dfrac{2x}{1-3x}$

$1-3x \neq 0$

$1 \neq 3x$

$x \neq \frac{1}{3}$

$\mathrm{Dom}(g) = \mathbb{R} \setminus \{\frac{1}{3}\}$

(b) $h(x) = \sqrt{3-x}$

$3-x \geq 0$

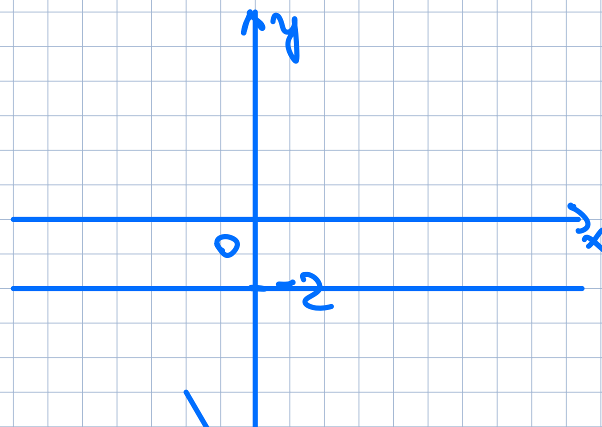
$-x \geq -3$

$x \leq 3$

$\mathrm{Dom}(h) = (-\infty, 3]$

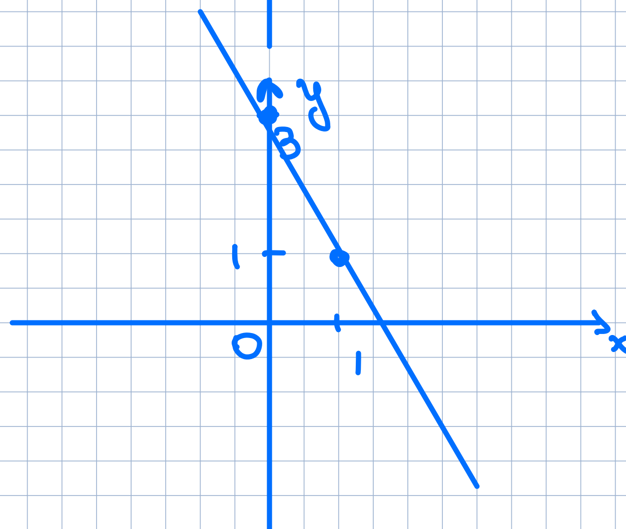
$\boxed{11.}$

(a) $y = -2$



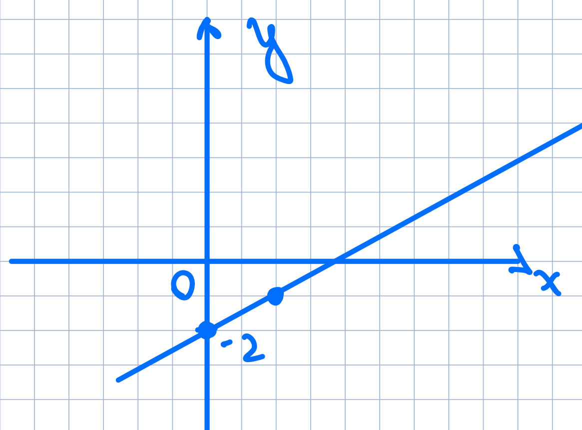
(b) $f = 3 - 2x$

x	y
0	3
1	1



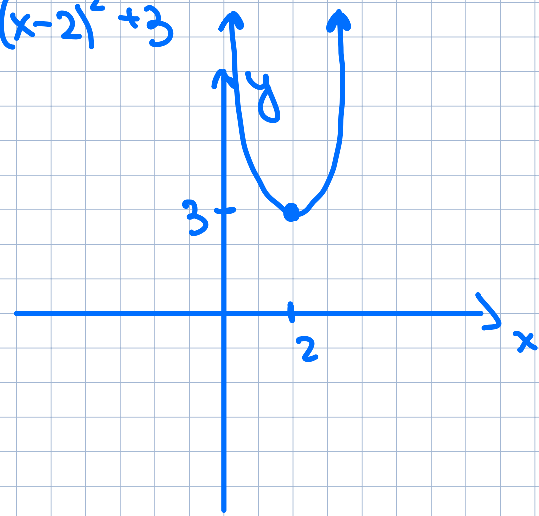
(c) $g(x) = \frac{2x-8}{4} = \frac{x}{2} - 2$

x	y
0	-2
2	-1



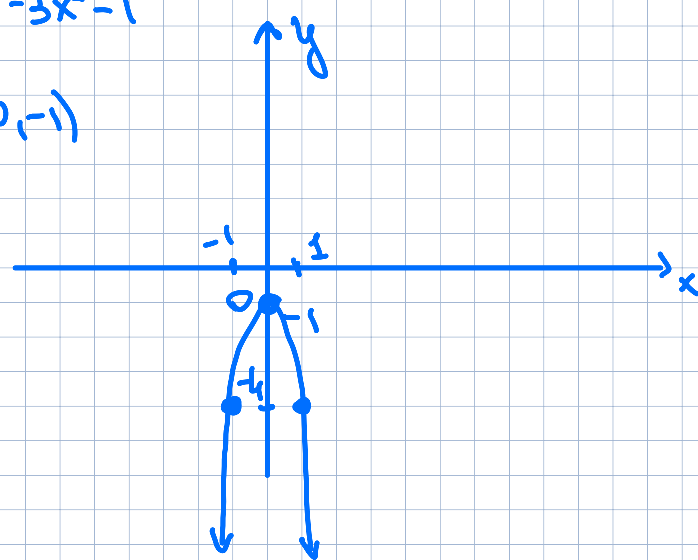
13. (a) $y = (x-2)^2 + 3$

vertex $(2,3)$



(b) $f(x) = -3x^2 - 1$

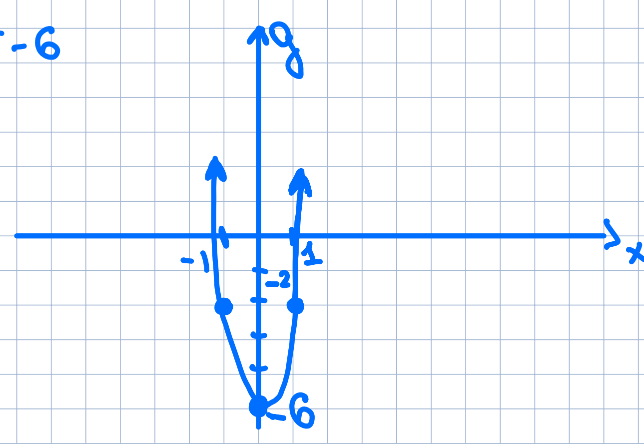
Vertex $(0,-1)$



(c) $g(x) = 4x^2 - 6$

Vertex:

$(0,-6)$

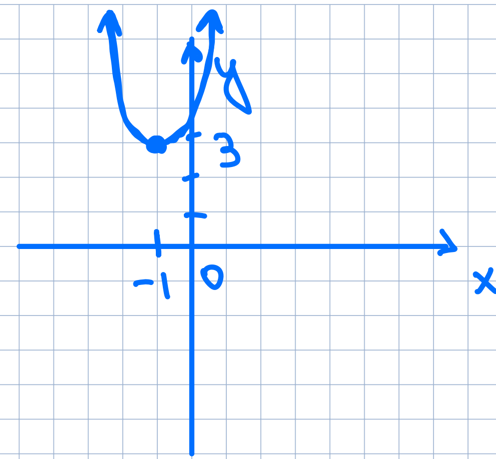


(d) $f(x) = x^2 + 2x + 4$

$f(x) = (x^2 + 2x + 1) - 1 + 4 =$

$= (x+1)^2 + 3$

vertex: $(-1, 3)$



$\boxed{14.}$

$x + y = 10$

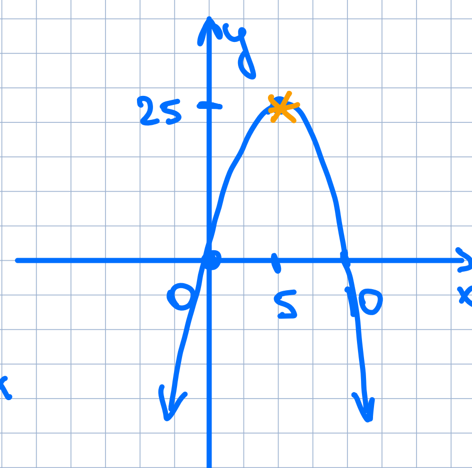
$x \cdot y \to \max$

$y = 10 - x$

$x(10 - x) = f(x) \to \max$

$10x - x^2 = f(x)$

↑ parabola open downward



$x = 5 \Rightarrow f(5) = 25$

Thus $x = 5$ and $y = 10 - x$

$y = 10 - 5 = 5$

$x = 5$ and $y = 5$, therefore

$\boxed{x \cdot y = 25}$

$\boxed{15.}$ Will not appear on AS-2.