Lecture Notes – Section 3.6: MATHEMATICAL MODELS

Agenda

- Constructing mathematical models
- Interpolation and extrapolation

Constructing mathematical models

The key steps in mathematical modelling:

1. With a given question or goal in mind, identify and name the relevant variable quantities.
2. Translate verbal, graphical, or other knowledge about the variables into an equation or function. Determine the domain of each independent variable that makes sense in the context of the question.
3. Use the constructed equation or function to answer the given question. Frequently, the model is then also used for predictive purposes and/or the relationship between the variables is analyzed to provide better understanding of the original question.

Example 1. (Modelling the area of the garden plots)

Anne and Kay plan to use 600 feet of fencing to construct two large rectangular garden plots along the river with the plots sharing a common side and no fence along the river. The overall dimensions can vary a bit, but they want to ensure that the total enclosed area is at least 20000 ft^2.

Questions:

(a) Can this be done? If so, what are the bounds on the dimensions of the plots that will guarantee at least this much enclosed area?

(b) What is the maximum possible enclosed area?

Example 2. (Modelling the weight of a person with a given mass)

Consider the weight, or force of gravitational attraction, of a person with a given mass.

Questions:

(a) How does the force of gravitational attraction between a 75 kg person and Earth vary as a function of the person's height above the surface of Earth?

(b) How does the force at sea level compare to the force when the person is in a jet at 30 000 ft?

(c) What if the person were on the International Space Station?

(d) How far above the surface of Earth would the person have to be for the force to be half as much as the force at sea level?

Interpolation and Extrapolation

It's often the case that the basis for the construction of a mathematical model is simply a collection of data points.
Let us consider the following example of the United States population in every census year from 1950 to 2010.

Year	Population
1950	151,325,798
1960	179,323,175
1970	203,302,031
1980	226,542,199
1990	248,709,873
2000	281,421,906
2010	308,745,538

Definition. The acts of estimating values of a function based on given data are called *interpolation* and *extrapolation*. To interpolate a value from a table or graph means to arrive at an estimate for the value of the implied function between two known data points, while to extrapolate means to guess at a value beyond the given points.