Section 1.2. Properties of exponents and radicals

1. Natural number and integer exponents
2. Properties of exponents
3. Scientific notation
4. ~~Working with geometric formulas~~
5. Radical notation
6. Simplifying and combining radical expressions
7. Rational number exponents.

$\boxed{1.}$ **Def.** If $a \in \mathbb{R}$, $n \in \mathbb{N}$, then

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$

a is the base

n is an exponent

Properties

$$a^n \cdot a^m = \underbrace{a \cdot \ldots \cdot a}_{n} \cdot \underbrace{a \cdot \ldots \cdot a}_{m} = a^{n+m}$$

$$a^0 \cdot a^m = a^{0+m} = a^m$$

$$\boxed{a^0 = 1}$$

For any $a \in \mathbb{R}$ s.t. $a \neq 0$:

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

2. Properties of Exponents:

1. $a^n \cdot a^m = a^{n+m}$

2. $\dfrac{a^n}{a^m} = a^{n-m}$

3. $a^{-n} = \dfrac{1}{a^n}$

4. $(a^n)^m = a^{nm}$

5. $(ab)^n = a^n b^n$

6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

3. Scientific Notation

$$a \cdot 10^n,$$

where $1 \leq |a| < 10$, $n \in \mathbb{Z}$.

5. **Def.**

Case 1: n is an even natural number:

$a \geq 0$: $\sqrt[n]{a} = b$, $b \geq 0$ and $b \in \mathbb{R}$

s.t. $b^n = a$

$\sqrt[n]{a} = b$ iff $a = b^n$

Case 2: n is an odd natural number:

$a \in \mathbb{R}$: $\sqrt[n]{a} = b$, $b \in \mathbb{R}$ s.t.

$b^n = a$.

Def. A perfect square is an integer equal to the square of another integer.

A perfect cube is an integer equal to the cube of another integer.

Example

$$\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$$

$$\sqrt[4]{-16} = \sqrt[4]{-2^4} \quad \emptyset$$

$$\sqrt{0} = 0$$

$\sqrt[n]{1} = 1$, when n is an even

$\sqrt[n]{-1} = -1$, when n is an odd

Properties of Radicals

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Simplifying the denominator:

Denom. is a single term containing a root

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

Denom. consists of two terms, one or both of which are square roots

$$\frac{1}{\sqrt{a}+\sqrt{b}} = \frac{1}{(\sqrt{a}+\sqrt{b})} \frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}-\sqrt{b})} =$$

$$= \frac{\sqrt{a}-\sqrt{b}}{a-b}$$

$\boxed{7.}$ Def.

$a^{\frac{1}{n}}$: if $n \in \mathbb{N}$ and if $\sqrt[n]{a}$ is a real number, then $a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{n}{m}}$: if m and $n \in \mathbb{N}$, $m \neq 0$, if m and n have no common factors greater than 1, and if $\sqrt[m]{a} \in \mathbb{R}$, then $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.

$$a^{-\frac{n}{m}} = \frac{1}{a^{\frac{n}{m}}}.$$