THEORETICAL PART: Solutions

Definitions:

- An equation is a statement that two expressions are equal.
- If the statement is always true for any allowable value(s) of the variable(s), then the equation is identity.
- If the statement is never true, it is a contradiction.
- If the equation is true for some values of the variables and false for others, then the equation is called **conditional**.
- Two equations that have the same solution set are called **equivalent equations**.

Definition: A **linear equation in one variable x** is an equation that can be transformed into the form $ax + b = 0$, where a and b are real numbers and $a \neq 0$. Such equations are also called **first-degree equations**, as x appears to the first power.

Remark. Solving absolute value equations:

$$|ax + b| = c \quad \text{means} \quad ax + b = c \quad \text{or} \quad -(ax + b) = c$$

Caution: Absolute value equations require to check your final answer in the original equation. An apparent solution that does not solve the original problem is called an **extraneous solution**.

Remark. Solving for a variable means to transform the equation into an equivalent one in which the specified variable is isolated on one side of the equation.

Important formulas:

- **Distance:** $d = rt$, where d is the distance traveled at rate r for time t.
- **Simple interest:** $I = Prt$, where I is the interest earned on principal P invested at rate r for time t.

PRACTICAL PART:

1. Identify types of the following equations:

   (a) $x^{\frac{1}{2}}(x + 1) = x^{\frac{3}{2}} + x^{\frac{1}{2}}$ identity

   $x^{3/2} + x^{1/2} = x^{3/2} + x^{1/2}$

(b) $t + 3 = t$ Contradiction

$t + 3 \neq t$

(c) $x^2 = 9$ Conditional

$x = \{+3, -3\}$

2. Solve the following equations:

(a)

$$3(x-2) + 7x = 1 - 2\left(x + \frac{1}{2}\right) \neq$$

$3x - 6 + 7x = \cancel{1} - 2x - \cancel{1}$

$12x = 6 \Rightarrow x = \frac{6}{12} = \frac{1}{2}$

(b)

$$5x + 12 = 5(x+3) - 3$$

$\cancel{5x} + 12 = \cancel{5x} + 15 - 3$

$12 = 12$ identity

$x \in \mathbb{R}$

3. Solve the absolute value equations:

(a)

$$|3x - 2| = 1$$

$3x - 2 = 1$ or $-(3x-2) = 1$

$3x = 3$

$x = 1$ or $-3x = 1 - 2 = -1$

$x = \frac{1}{3}$

(b)

$$|x - 4| = |2x + 1|$$

$$\begin{cases} x - 4 = 2x + 1 \\ -(x-4) = 2x + 1 \\ x - 4 = -(2x+1) \\ -(x-4) = -(2x+1) \end{cases} \Rightarrow \begin{cases} x = 5 \\ -3x = -3 \Rightarrow x = 1 \\ x = 1 \\ x = -5 \end{cases}$$

(c)

$$|6x - 7| + 5 = 3$$

$$|6x-7| = -2$$

$$\begin{cases} 6x-7=-2 \\ -(6x-7)=-2 \end{cases} \Rightarrow \begin{cases} 6x = 5 \\ -6x = -9 \end{cases} \Rightarrow \begin{cases} x = \frac{5}{6} \\ x = \frac{9}{6} = \frac{3}{2} \end{cases}$$

4. Solve the following equations for the specified variable:

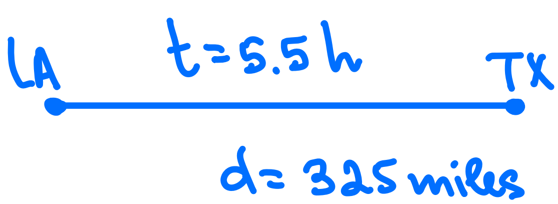
   (a) $P = 2l + 2w$; solve for w

   $$P - 2l = 2w \Rightarrow w = \frac{1}{2}(P - 2l)$$

   (b) $A = P\left(1 + \frac{r}{m}\right)^{mt}$; solve for P

   $$P = A \cdot \left(1 + \frac{r}{m}\right)^{-mt}$$

5. The distance from Shreveport, LA to Austin, TX by one route is 325 miles. If Kevin made the trip in five and half hours, what was his average speed?



r - average speed
t - time
d - distance

$$r = \frac{d}{t} \Rightarrow d = r \cdot t$$

$$r = \frac{325}{5.5} = 325 \cdot \frac{11}{2} = 1787.5 \ (\text{miles}/h)$$