Section 8.4. Trigonometric Equations

1. Solving trigonometric equations using algebraic techniques.
2. Solving trigonometric equations using inverse trigonometric functions.

1.

Example 1

Solve the equation $3-6\cos x=0$

Solution

$$3 = 6\cos(x)$$

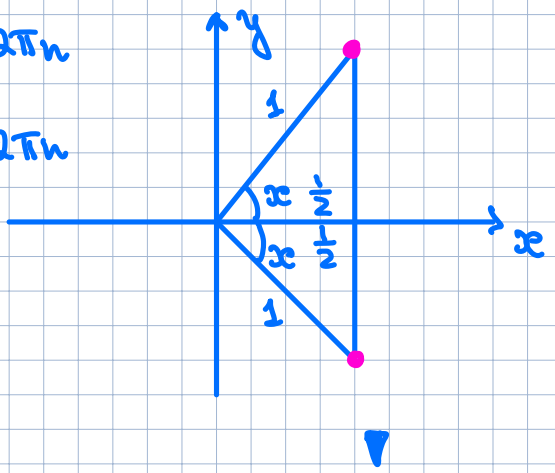
$$\cos(x)=\frac{1}{2}$$

$$x=\frac{\pi}{3}+2\pi n$$

or

$$x=-\frac{\pi}{3}+2\pi n$$

$n\in\mathbb{Z}$



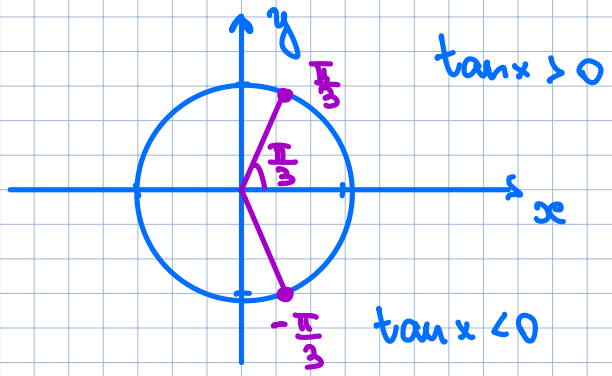
Example 2 (Solving equations by isolating the trig. function)

$$\tan^2 x - 1 = 2$$

Solution

$$\tan^2 x = 3$$

$$\tan(x) = \pm\sqrt{3}$$



$$x = \frac{\pi}{3} + \pi n$$

or

$$x = -\frac{\pi}{3} + \pi n, \quad n \in \mathbb{Z}$$

▼

Example 3 (Solving trig. equations by factoring)

$$\sin^2 x - \sin x = \sin x + 3$$

Solution

$$\sin^2 x - 2\sin x - 3 = 0$$

$$(\sin x - 3)(\sin x + 1) = 0$$

$\sin x = 3$ or $\sin x = -1$

$\emptyset$ $\qquad x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$

Answer: $x = \frac{3\pi}{2} + 2\pi n, n \in \mathbb{Z}$.

▼

Example 4 (Solving equations using trig. identities)

$-2\cos^2 x + 1 = \sin x$

Solution

$-2(1-\sin^2 x) + 1 = \sin x$

$2\sin^2 x - 2 + 1 - \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$\sin x = t$

$2t^2 - t - 1 = 0$

$D = 1 + 8 = 9$

$t_1 = \frac{1+3}{4} = 1$ $\qquad t_2 = \frac{1-3}{4} = -\frac{1}{2}$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$

$\sin x = -\frac{1}{2}$

$x = -\frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$

▼

Example 5 (Solving equations using trig. identities)

$\sin x - 1 = \cos x$ on $[0, 2\pi)$

Solution

$$(\sin x - 1)^2 = \cos^2 x$$

$$\sin^2 x - 2\sin x + 1 = \cos^2 x$$

$$\sin^2 x - 2\sin x + 1 = 1 - \sin^2 x$$

$$2\sin^2 x - 2\sin x = 0$$

$$\sin x(\sin x - 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \sin x = 1$$

$$x = 0, \pi \quad \text{or} \quad x = \frac{\pi}{2}$$

0: $\quad -1 = 1$ ✗

π: $\quad -1 = -1$ ✓

$\frac{\pi}{2}$: $\quad 0 = 0$ ✓

Answer: $x = \pi, \frac{\pi}{2}$.

▼

2.

Example 6 (Solving equations using inverse trig. functions)

$\tan^2 x + 2\tan x = 3$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Solution

$$\tan x = t$$

$$t^2 + 2t - 3 = 0$$

$$(t+3)(t-1) = 0$$

$$\tan x = -3 \quad \text{or} \quad \tan x = 1$$

$$x = \tan^{-1}(-3) \quad \text{or} \quad x = \tan^{-1}(1)$$

$$x = \frac{\pi}{4}$$

▼

Example 7 (Solving equations using inverse trig. functions)

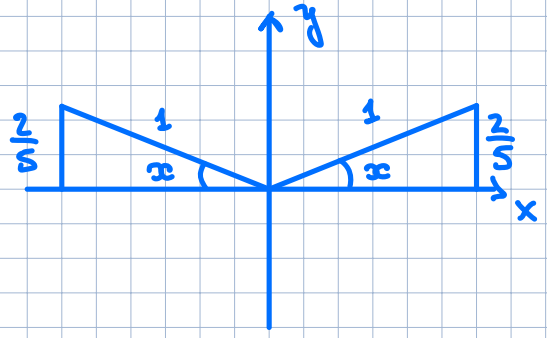
$$6\sin x - 2 = \sin x \quad \text{on} \quad [0, 2\pi)$$

Solution

$$6\sin x - \sin x = 2$$

$$\sin x = \frac{2}{5}$$

$$x = \sin^{-1}\left(\frac{2}{5}\right) \quad \text{or} \quad x = \pi - \sin^{-1}\left(\frac{2}{5}\right)$$



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