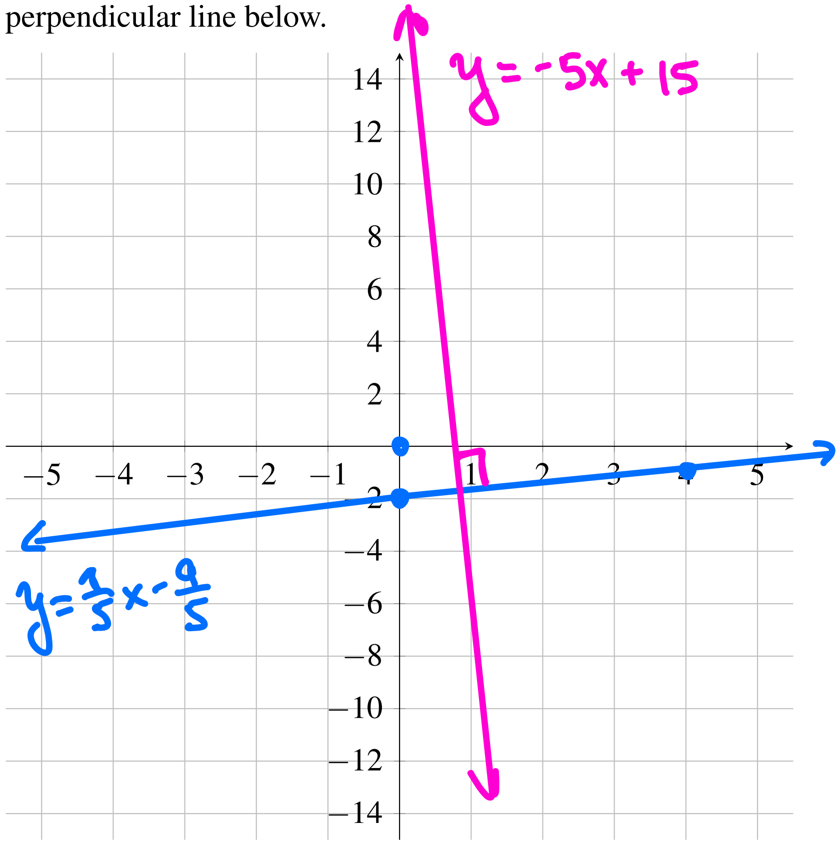
Name: Solutions ______ / 40

Assessment 2 Instructions:

- The AS-2 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-2.
- The AS-2 is closed book and closed notes.
- **Calculators are allowed only for the problem # 10** on the AS-2.
- Show all your work for full credit and box your final answer.

1. [4 points]

Find the equation of the straight line that passes through the point $(2,5)$ and is **perpendicular** to the line $2(y+x)-3(x-y)=-9$. Graph the line $2(y+x)-3(x-y)=-9$ and the obtained perpendicular line below.



$$2(y+x)-3(x-y)=-9$$
$$2y+2x-3x+3y=-9$$
$$5y-x=-9$$
$$y=\frac{1}{5}x-\frac{9}{5}$$
$$m_1=\frac{1}{5}$$
$$m_2=-5$$
$$y=-5x+b$$
$$5=-10+b \Rightarrow b=15$$
$$\boxed{y=-5x+15}$$

2. [4 points] Given the two points $(5,-4)$ and $(4,3)$:

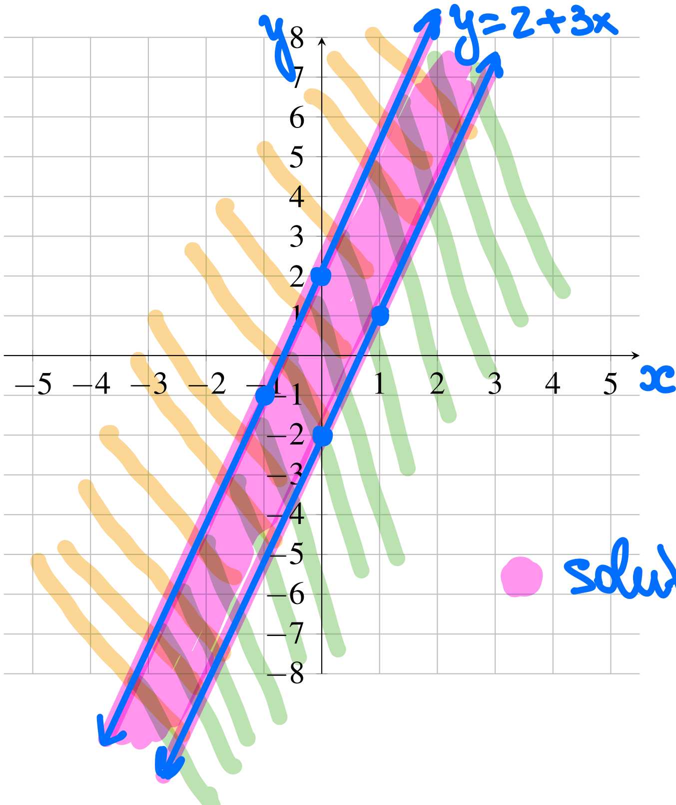
a. find the length of the line segment (distance) between the points

$$d=\sqrt{(4-5)^2+(3+4)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$$

b. find the midpoint of the segment

$$M\left(\frac{5+4}{2},\frac{-4+3}{2}\right)=M\left(\frac{9}{2},-\frac{1}{2}\right)$$

3. [4 points] Solve the linear inequality $|y-3x| \leq 2$ by graphing its solution set



$y-3x \leq 2$ and $y-3x \geq -2$

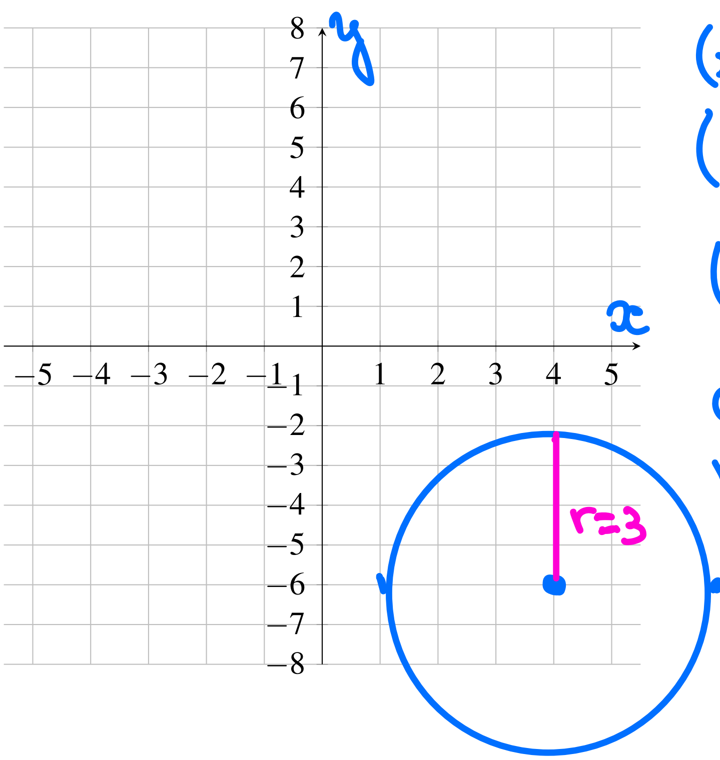
$y \leq 2+3x$ and $y \geq -2+3x$

$y = 2+3x$ $\quad$ $y = -2+3x$

4. [4 points] Find the standard form for the equation of the circle

$$x^2+y^2-8x+12y+43=0$$

Precisely state the coordinates of the center point and the radius. Graph the obtained circle.



$(x^2-8x)+(y^2+12y)+43=0$

$(x-4)^2-16+(y+6)^2-36+43=0$

$(x-4)^2+(y+6)^2=9=3^2$

Center $(4,-6)$

radius $r=3$

5. [4 points] For each of the following relations, determine the **domain** and **range**:

a. $R = \{(3,3), (-4,3), (3,8), (3,-2)\}$

$\text{Dom}(R) = \{3, -4\}$ $\text{Ran}(R) = \{3, 8, -2\}$

b. $y = 7\pi^2$ – horizontal line

$\text{Dom}(y) = \mathbb{R}$ $\text{Ran}(y) = 7\pi^2$

6. [4 points] Determine the implied domain of the following function

$$f(x) = \frac{5}{\sqrt[3]{3 - x^2}}$$

$3 - x^2 \neq 0$

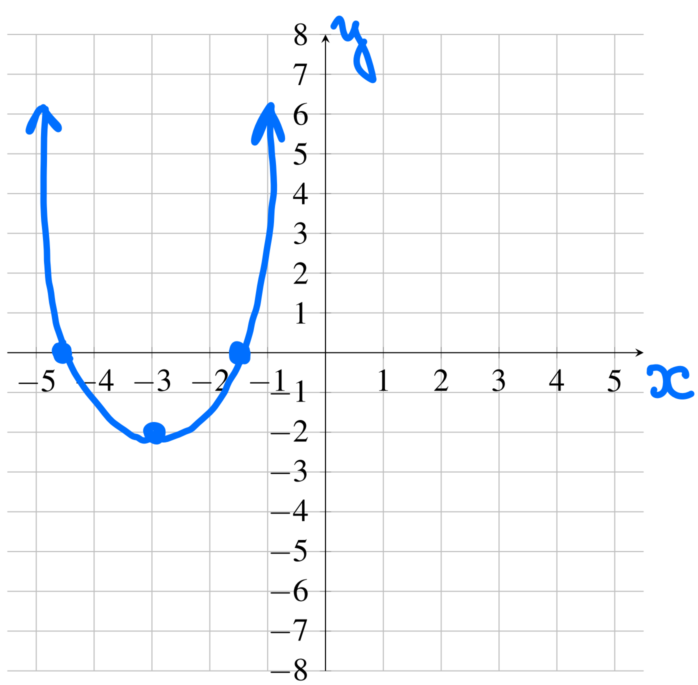
$x^2 \neq 3$

$x \neq \pm\sqrt{3}$

$\text{Dom}(f) = \mathbb{R} \setminus \{\pm\sqrt{3}\}$

7. [4 points] Graph the following quadratic function and **state the coordinates of its vertex**.

$$y = (x + 3)^2 - 2$$



Vertex: $(-3, -2)$

$y = 0$: $(x+3)^2 - 2 = 0$

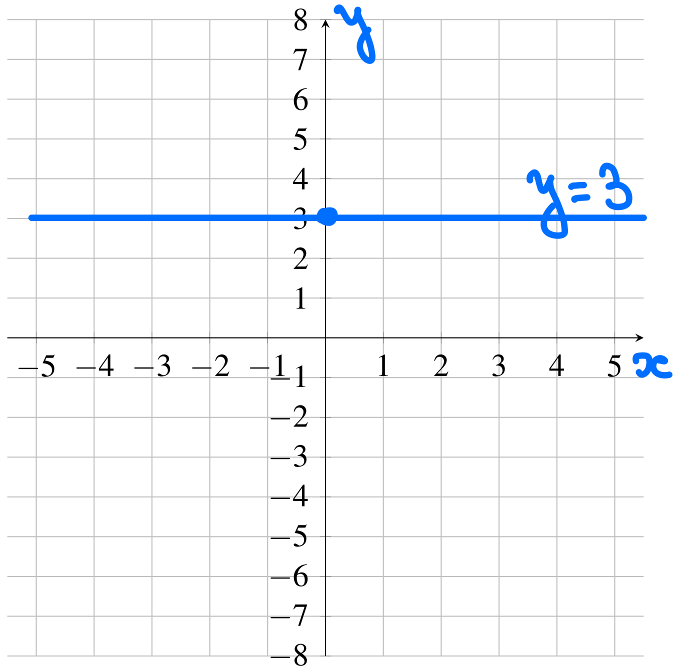
$(x+3)^2 = 2$

$x = \pm\sqrt{2} - 3$

$x \approx -1.6$

$x \approx -4.4$

8. [4 points] Graph the following linear function $f(x) = 3\left(1 - \frac{1}{3}x\right) + x$. **State** precisely its **slope** and y**-intercept point**.



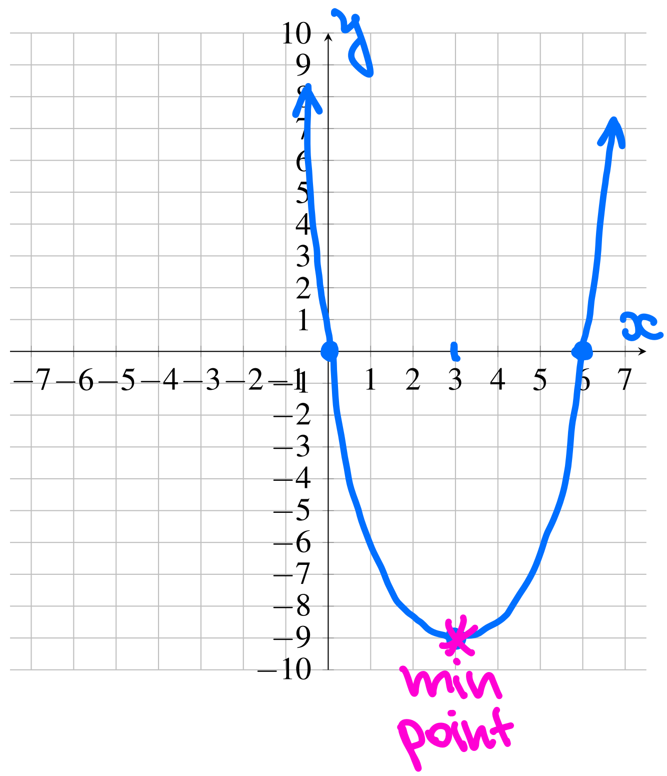
$f(x) = 3 - x + x = 3$

$\boxed{y = 3}$

$m = 0$

y-intercept : $(0, 3)$

9. [4 points] Among all pairs of numbers with a difference of 6, find the pair whose product is minimum. **Write your answer in the form:** $x = \cdots$, $y = \cdots$, $x \cdot y = min\ product$. (***Hint:*** *you need to create a quadratic function, graph it with indicating its vertex coordinates*)



$x - y = 6 \Rightarrow y = x - 6$

$x \cdot y \to \min$

$x(x-6) = x^2 - 6x \to \min$

$f(x) = x(x-6)$

$0 = x(x-6) \Rightarrow x = 0$ or $x = 6$

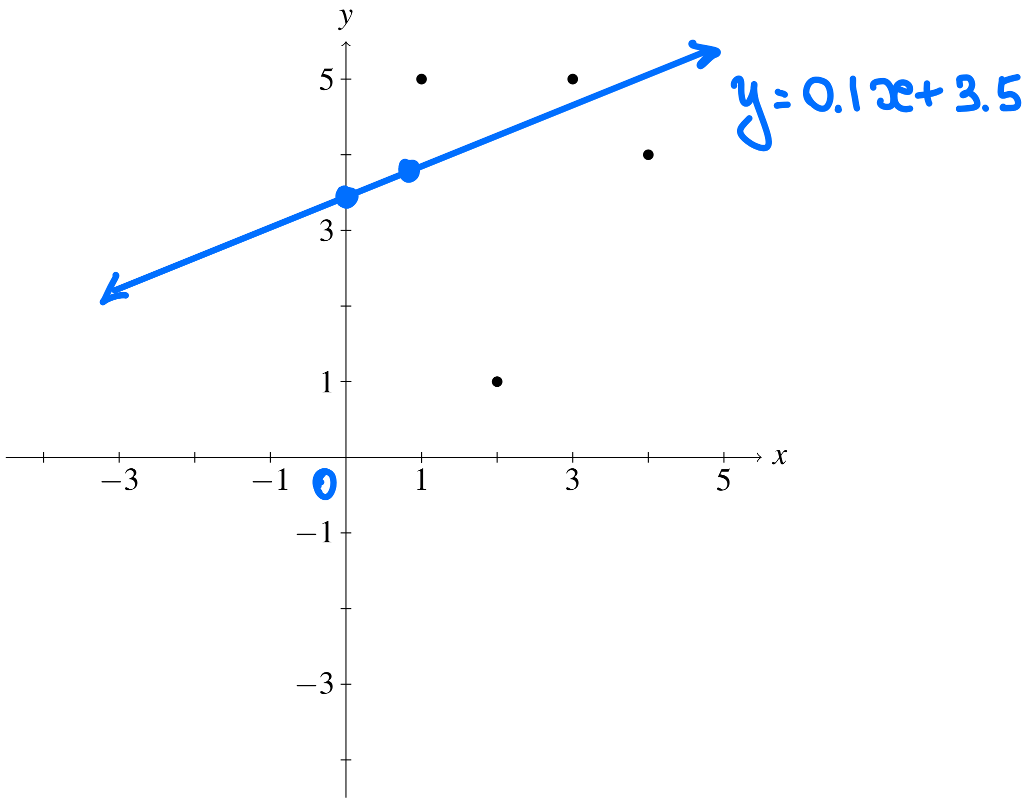
$x = 3$: $f(3) = 3 \cdot (-3) = -9$

$\boxed{x = 3}$ Thus $y = x - 6$ $\boxed{y = -3}$

$\boxed{x \cdot y = -9}$

10. [4 points] The coordinates of the four graphed points are

$$\{(1,5),(2,1),(3,5),(4,4)\}.$$



We also know the following information:

- $\bar{x} = 2.5$, $\bar{y} = 3.75$
- $\sum(\Delta x)^2 = 5$, $\sum \Delta x \Delta y = 0.5$, $\sum(\Delta y)^2 = 10.75$

a. Using the following formulas

$$m = \frac{\sum \Delta x \Delta y}{\sum(\Delta x)^2} \quad \text{and} \quad b = \bar{y} - m\bar{x}$$

find the equation of the line $y = mx + b$ of the best fit. Sketch the obtained straight line on the coordinate plane given above.

$m = \frac{0.5}{5} = 0.1$ $b = 3.75 - 0.1 \cdot 2.5$

$b = 3.5$

Thus $\boxed{y = 0.1x + 3.5}$

b. Using the following formula

$$r = \frac{\sum \Delta x \Delta y}{\sqrt{\sum(\Delta x)^2}\sqrt{\sum(\Delta y)^2}}$$

calculate the Pearson correlation coefficient r. Make a conclusion about the linear dependence of the variable y on the variable x.

$$r = \frac{0.5}{\sqrt{5} \cdot \sqrt{10.75}} \approx \frac{0.5}{7.33} \approx \boxed{0.07}$$

We can conclude that there is almost <u>no</u> linear dependence of ~~x on y~~ y on x.