

PRACTICAL PART:Solutions**1. (Modeling the Area of Garden Plots)**

Anne and Kay plan to use 600 feet of fencing to construct two large rectangular garden plots along a river with the plots sharing a common side and no fence along the river. The overall dimensions can vary a bit, but they want to ensure that the total enclosed area is at least 20000 ft^2 .

$$3x + (600 - 3x) = 600$$

$$\text{Our goal } A \geq 20000$$

$$A = h \cdot w$$

$$A = x \cdot (600 - 3x) \geq 20000$$

$$600x - 3x^2 \geq 20000$$

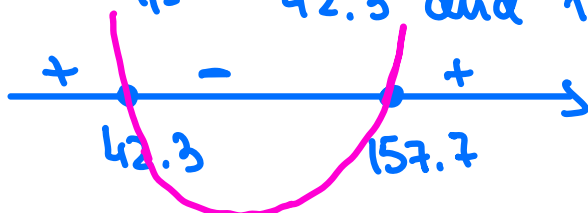
$$600x - 3x^2 - 20000 \geq 0$$

$$3x^2 - 600x + 20000 = 0$$

$$D = 36000 - 24000 = 12000$$

$$x_{1,2} = \frac{600 \pm \sqrt{12000}}{6}$$

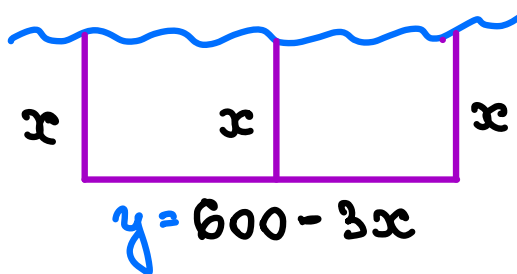
$$x_{1,2} \approx 42.3 \text{ and } 157.7 \text{ (ft)}$$



Hence $A \geq 20000$ for $x \in [42.3, 157.7]$
From this it follows that

$$y = 600 - 3x \in [126.9, 473.1]$$

Answer: for $x \in [42.3, 157.7]$ we have $A \geq 20000$.



2. (Modeling the Weight of a Person with a Given Mass)

Consider the weight, or force of gravitational attraction, of a person with a given mass.

- How does the force of gravitational attraction between a 75 kg person and Earth vary as a function of the person's height above the surface of Earth?
- How does the force at sea level compare to the force when the person is in a jet at 30000 ft?
- How far above the surface of Earth would the person have to be for the force to be half as much as the force at sea level?

$$F = G \frac{m_1 m_2}{d^2}$$

G - gravitational constant

$$G = 6.67 \cdot 10^{-11}$$

m_1 - mass of Earth

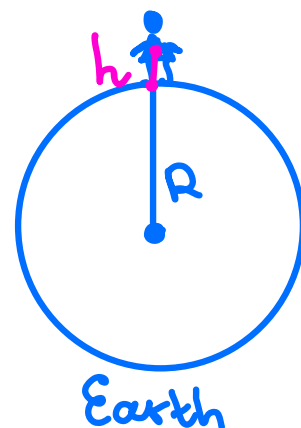
m_2 - mass of a person

$$d = h + R$$

R - radius of Earth

$$R = 6.38 \cdot 10^6$$

$$m_1 = 5.97 \cdot 10^{24}$$



Hence,

$$F = 6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^6 + h)^2}$$

$$(a) \quad F(h) = 6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^6 + h)^2}$$

$$(b) \quad h = 0 \text{ (m)}$$

$$F(0) = 6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{6.38^2 \cdot 10^{12}} \approx 734 \text{ (N)}$$

$$h = 30\,000 \text{ ft} = 9144 \text{ m}$$

$$F(9144) \approx 732 \text{ (N)}$$

$$F(9144) \text{ is } 99.7\% \text{ of } F(0)$$

$$(c) \quad F(h) = \frac{734}{2} = 367$$

$$6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^6 + h)^2} = 367$$

$$\frac{6.67 \cdot 10^{-11} \cdot 5.97 \cdot 10^{24} \cdot 75}{367} = (6.38 \cdot 10^6 + h)^2$$

\Downarrow

$$h \approx 2.64 \cdot 10^6 \text{ (m)}$$



3. (Interpolation and Extrapolation)

Based on the table below, explain the notion of interpolating a value from a table and extrapolating a value from a table.

Year	Population
1950	151 325 798
1960	179 323 175
1970	203 302 031
1980	226 542 199
1990	248 709 873
2000	281 421 906
2010	308 745 538