THEORETICAL PART:

Solutions

Identities (Already Seen):

Reciprocal Identities

$$\sec(x) = \frac{1}{\cos(x)}, \quad \csc(x) = \frac{1}{\sin(x)}$$

$$\tan(x) = \frac{1}{\cot(x)}, \quad \cot(x) = \frac{1}{\tan(x)}$$

Quotient Identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \quad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Cofunction Identities

$$\sin(x) = \cos\left(\frac{\pi}{2} - x\right), \quad \cos(x) = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan(x) = \cot\left(\frac{\pi}{2} - x\right), \quad \cot(x) = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec(x) = \csc\left(\frac{\pi}{2} - x\right), \quad \csc(x) = \sec\left(\frac{\pi}{2} - x\right)$$

Period Identities

$$\sin(x + 2\pi) = \sin(x), \quad \cos(x + 2\pi) = \cos(x)$$

$$\sec(x + 2\pi) = \sec(x), \quad \csc(x + 2\pi) = \csc(x)$$

$$\tan(x + \pi) = \tan(x), \quad \cot(x + \pi) = \cot(x)$$

Even/Odd Identities

$$\sin(-x) = -\sin(x), \quad \cos(-x) = \cos(x)$$

$$\sec(-x) = \sec(x), \quad \csc(-x) = -\csc(x)$$

$$\tan(-x) = -\tan(x), \quad \cot(-x) = -\cot(x)$$

Pythagorean Identities

$$\cos^2(x) + \sin^2(x) = 1, \quad \tan^2(x) + 1 = \sec^2(x), \quad 1 + \cot^2(x) = \csc^2(x)$$

Procedure for verifying trigonometric identities:

1. Work with one side at a time. Choose one side of the equation to work with and simplify it. The more complicated side is usually the best choice. The goal is to transform it into the other side.
2. Apply trigonometric identities as appropriate. To do so, it will probably be necessary to combine fractions, add or subtract terms, factor expressions, or use other algebraic manipulations.
3. Rewrite in terms of sine or cosine if necessary. If you are stuck, expressing everything in terms of sine and cosine often leads to inspiration.

<u>PRACTICAL PART:</u>

1. Simplify the following expression

$$\cos\theta + \sin\theta\tan\theta$$

$$\cos\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta} = \cos\theta + \frac{\sin^2\theta}{\cos\theta} =$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \boxed{\frac{1}{\cos\theta}} = \sec\theta$$

2. Simplify the expression

$$\cot\alpha + \frac{\sin\alpha}{1+\cos\alpha} = \frac{\frac{\cos\alpha}{\sin\alpha}(1+\cos\alpha) + \sin\alpha}{1+\cos\alpha} =$$

$$= \frac{\frac{\cos\alpha}{\sin\alpha} + \frac{\cos^2\alpha}{\sin\alpha} + \sin\alpha}{1+\cos\alpha} = \frac{\cos\alpha + \cos^2\alpha + \sin^2\alpha}{\sin\alpha(1+\cos\alpha)} =$$

$$= \frac{\cancel{\cos\alpha + 1}}{\sin\alpha\cancel{(1+\cos\alpha)}} = \frac{1}{\sin\alpha} = \boxed{\csc\alpha}$$

3. Verify the identity

$$2\csc^2(x) = \frac{1}{1-\cos(x)} + \frac{1}{1+\cos(x)}$$

$$\boxed{\frac{1}{1-\cos x} + \frac{1}{1+\cos x}} = \frac{1+\cancel{\cos x}+1-\cancel{\cos x}}{(1-\cos x)(1+\cos x)} =$$

$$= \frac{2}{1-\cos^2 x} = \frac{2}{\cancel{\cos^2 x}+\sin^2 x-\cancel{\cos^2 x}} = \frac{2}{\sin^2 x} =$$

$$= \boxed{2\csc^2 x}$$

4. Verify the identity

$$\frac{\tan^2(x)}{1+\sec(x)} = \frac{1-\cos(x)}{\cos(x)}$$

$$\boxed{\frac{\tan^2 x}{1+\sec x}} = \frac{\tan^2 x}{1+\frac{1}{\cos x}} = \frac{\cos x \cdot \frac{\sin^2 x}{\cos^2 x}}{\cos x+1} =$$

$$= \frac{\frac{\sin^2 x}{\cos x}}{1+\cos x} = \frac{1-\cos^2 x}{\cos x(1+\cos x)} =$$

$$= \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cos x\cancel{(1+\cos x)}} = \boxed{\frac{1-\cos x}{\cos x}}$$

5. Verify the identity

$$\frac{\cos\varphi\cot\varphi}{1-\sin\varphi} - 1 = \csc\varphi$$

$$\boxed{\frac{\cos\varphi\cot\varphi}{1-\sin\varphi} - 1} = \frac{\cos^2\varphi}{(1-\sin\varphi)\sin\varphi} - 1 = \frac{1+\sin\varphi}{\sin\varphi} - 1 =$$

$$= \frac{1}{\sin\varphi} + \cancel{1} - \cancel{1} = \frac{1}{\sin\varphi} = \boxed{\csc\varphi}$$

6. Use the substitution $\sin\theta = \frac{x}{2}$ to rewrite $\sqrt{4-x^2}$ as a trigonometric expression. Assume $0 \leq \theta \leq \frac{\pi}{2}$.

$$x = 2\sin\theta$$

$$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta} =$$

$$= 2\sqrt{1-\sin^2\theta} = 2\sqrt{\cos^2\theta} =$$

$$= 2\,|\cos\theta| = \boxed{2\cos\theta} \quad \text{since } 0 \leq \theta \leq \frac{\pi}{2}.$$