Solutions

THEORETICAL PART:

Definition:

A **linear equation in two variables**, say the variables x and y, is an equation that can be written in the form $ax + by = c$, where a, b, c are constants and $a, b \neq 0$. This form of such an equation is called the **standard form**.

Definition.

Given a graph in the Cartesian plane, any point where the graph intersects the x-axis is called an **x-intercept**, and any point where the graph intersects the y-axis is called a **y-intercept**. All x-intercepts are of the form $(c, 0)$, and all y-intercepts are of the form $(0, c)$.

Definition.

We refer to the horizontal number line as the **x-axis**, the vertical number line as the **y-axis**, and the two coordinates of the ordered pair (a, b) as the **x-coordinate** and the **y-coordinate**.

The graph of an equation is a plot in the Cartesian plane of all of the ordered pairs that make up the solution set of the equation.

PRACTICAL PART:

1. Determine if the following equations are linear:
   (a) $3x - (2 - 4y) = x - y + 1$
   (b) $\frac{x+2}{3} - y = \frac{y}{5}$
   (c) $4x^3 - 2y = 5x$
   (d) $x^2 - (x-3)^2 = 3y$

(a) $3x - 2 + 4y = x - y + 1$
$2x + 5y = 3$ ✓ linear

(b) $\frac{x+2}{3} - y = \frac{y}{5}$
$\frac{x}{3} + \frac{2}{3} - y = \frac{y}{5}$
$\frac{x}{3} - \frac{6}{5}y = -\frac{2}{3}$ ✓ linear

(c) $4x^3 - 2y = 5x$
↑ not linear because of x^3

(d) $x^2 - (x-3)^2 = 3y$
$x^2 - x^2 + 6x - 9 = 3y$
$6x - 3y = 9$ ✓ linear

2. Find the x- and y-intercepts of the following equations, and sketch their graphs:

   (a) $3x - 4y = 12$

   (b) $4x - (3 - x) + 2y = 7$

(a) $3x - 4y = 12$

x-intercept: $(?, 0)$

$y = 0 \Rightarrow 3x = 12$

$x = 4$

$\boxed{(4, 0)}$

y-intercept: $(0, ?)$

$x = 0 \Rightarrow y = -3$

$\boxed{(0, -3)}$

(b) $4x - 3 + x + 2y = 7$

$5x + 2y = 10$

x-intercept:

$y = 0$

$x = 2$ $(2, 0)$

y-intercept:

$x = 0$

$y = 5$ $(0, 5)$

3. Graph the following equations (horizontal and vertical lines):

   (a) $x = 5$

   (b) $2x - 2 = 3$

   (c) $3x + 2(x + 7) - 2y = 5x$

(b) $2x - 2 = 3$

$2x = 5$

$x = \frac{5}{2}$

(c) $3x + 2x + 14 - 2y = 5x$

$y = 7$