Section 8.1. Fundamental Trigonometric Identities

1. Fundamental trigonometric identities.
2. Simplifying trigonometric expressions.
3. Verifying trigonometric identities.
4. Trigonometric substitutions.

1.

Identities (Identities already seen)

Reciprocal Identities:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Cofunction Identities:

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\csc x = \sec\left(\frac{\pi}{2} - x\right)$$

$$\cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\sec x = \csc\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right)$$

Period Identities:

$$\sin(x + 2\pi) = \sin x \qquad \cos(x + 2\pi) = \cos x$$

$$\csc(x + 2\pi) = \csc x \qquad \sec(x + 2\pi) = \sec x$$

$$\tan(x + \pi) = \tan x \qquad \cot(x + \pi) = \cot x$$

Even / Odd Identities:

$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$$

$$\csc(-x) = -\csc(x) \qquad \sec(-x) = \sec(x)$$

$$\tan(-x) = -\tan(x)$$

$$\cot(-x) = -\cot(x)$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

2. Example (Simplifying trigonometric expressions)

$$\cos\theta + \sin\theta \tan\theta = \cos\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta} =$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta.$$

▼

3. Procedure: (Verifying Trig. identities)

1. Work with one side at a time
2. Apply trigonometric identities as appropriate.
3. Rewrite in terms of sine and cosine if necessary.

Example

Verify the identity

$$2\csc^2 x = \frac{1}{1-\cos x} + \frac{1}{1+\cos x}$$

Solution

$$\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = \frac{2}{(1-\cos x)(1+\cos x)} =$$

$$= \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x} = 2\cdot \csc^2 x$$

▼

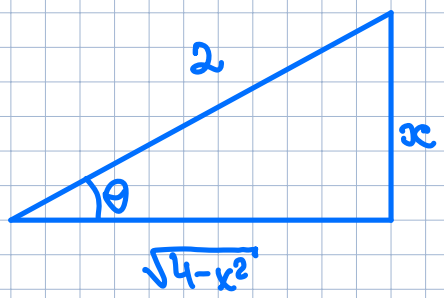
4.

Example (Using trig. substitutions)

Use $\sin\theta = \frac{x}{2}$ to write $\sqrt{4-x^2}$ as a trig. expression. Assume $0 \le \theta \le \frac{\pi}{2}$.

Solution

$$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} =$$



$$= 2\sqrt{1-\sin^2\theta} =$$

$$= 2\sqrt{\cos^2\theta} = 2|\cos\theta| = 2\cos\theta.$$

▼