

Name: \_\_\_\_\_

Solutions

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No aids (calculator, notes, text, etc.) are permitted. Show all work for full credit and box your final answer.

## 1. [2 points]

- a. Give a **full** definition of the absolute value of a real number  $a$ .

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

- b. Simplify the following set expression:  $\mathbb{Z} \cup \mathbb{R}$ .

$$\mathbb{Z} \cup \mathbb{R} = \boxed{\mathbb{R}}$$



## 2. [2 points] Order each of the following expressions:

a.  $3^5 \cdot 3^3 < 3^{15}$

$$3^5 \cdot 3^3 = 3^{5+3} = 3^8 < 3^{15}$$

b.  $3^3 \cdot 5^3 = 15^3$

$$3^3 \cdot 5^3 = (3 \cdot 5)^3 = 15^3$$

3. [2 points] Evaluate the expression for  $x = -4$  and  $y = 3$ :

$$|x - 7y| + 5x + 3y = | -4 - 7 \cdot 3 | - 20 + 9 = |-25| - 20 + 9 = 5 + 9 = \boxed{14}$$

4. [2 points] Simplify the following expression and write a final answer **only with positive exponents**:

$$\frac{12m^{-7}n^9p^{-6}}{3n^{-2}p^4} = \frac{12}{3} \cdot m^{-7} \cdot n^{11} p^{-10} = \boxed{\frac{4n^{11}}{m^7 p^{10}}}$$

## 5. [2 points] Rationalize the denominator and simplify:

$$\frac{7}{2\sqrt{3}+4} = \frac{7 \cdot (2\sqrt{3}-4)}{(2\sqrt{3}+4)(2\sqrt{3}-4)} = \frac{14\sqrt{3}-28}{4 \cdot 3 - 16} = \frac{14\sqrt{3}-28}{-4} = \boxed{\frac{7\sqrt{3}}{-2} + 7}$$

6. [2 points] Simplify each expression:

a.  $\sqrt{242x^5t^3} = \sqrt{2 \cdot 11^2 x^4 \cdot x \cdot t^2 \cdot t} = 11x^2t\sqrt{2xt}$

b.  $(3.2 \times 10^4)(5 \times 10^{-6}) = (3.2 \cdot 5) \cdot 10^{-2} = 16 \cdot 10^{-2} = 0.16$

7. [4 points]

a. Factor the following expression **completely**:

$$64x^7y^2 - 100xy^2 = 4xy^2(16x^6 - 25) = 4xy^2((4x^3)^2 - 5^2) = 4xy^2(4x^3 - 5)(4x^3 + 5)$$

b. Factor the following algebraic expression:

$$(8x+6)^{-\frac{7}{2}} - (8x+6)^{-\frac{1}{2}} = (8x+6)^{-\frac{7}{2}}(1 - (8x+6)^3) = (8x+6)^{-\frac{7}{2}}(1 - (8x+6))(1 + (8x+6) + (8x+6)^2) = (8x+6)^{-\frac{7}{2}}(-8x-5)(43 + 104x + 64x^2)$$

8. [4 points]

a. Simplify the following rational expression, indicating which real values of the variable **must be excluded**:

$$\frac{x^4 - x^3}{x^2 - 3x + 2} =$$

b. Simplify the following complex rational expression:

$$\frac{1 + xy}{x^{-2} - y^2} =$$