Section 2.4. Slope and forms of linear equations
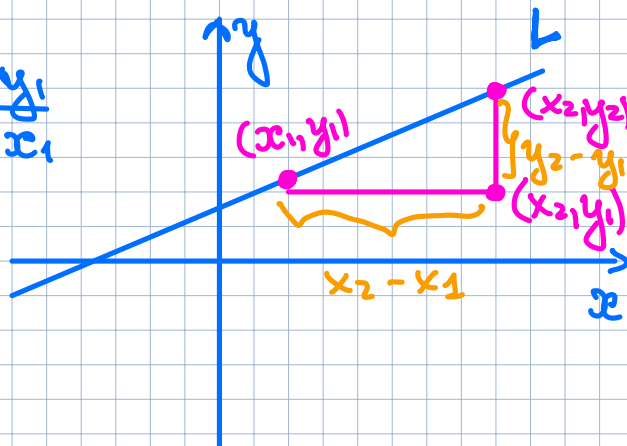
1. The slope of a line.

2. The slope-intercept form of the equation of a line

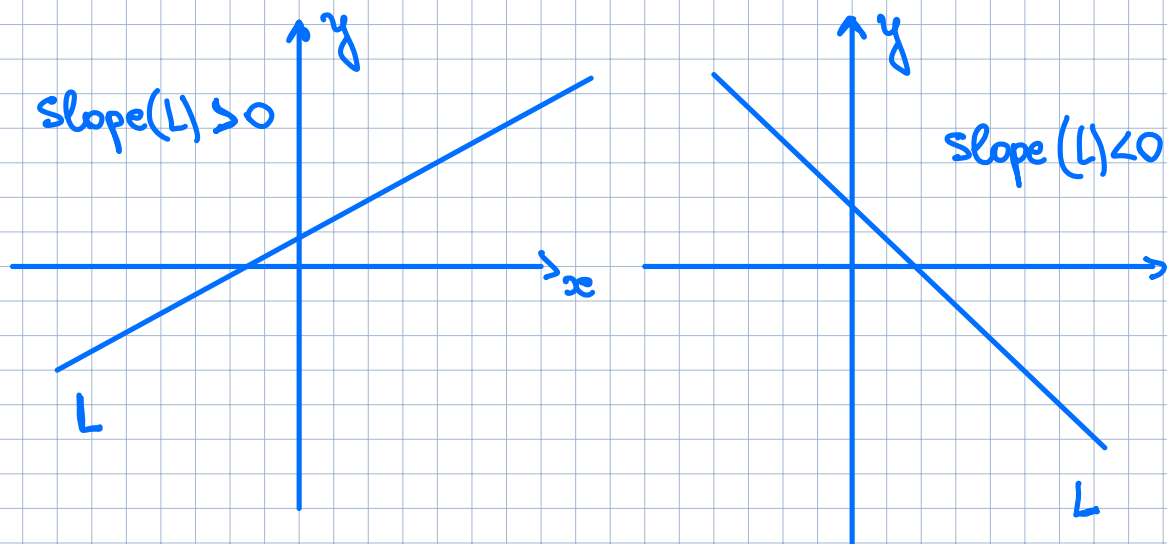3. The point-slope form of the equation of a line.

1.

<u>Def.</u>

Let L stand for a given line in $\mathbb{R}^2$, and let (x_1, y_1) and (x_2, y_2) be the coordinates of any two distinct points on L. Then

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope(L) > 0

Slope (L) < 0

Caution:

Correct: $\dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{y_1 - y_2}{x_1 - x_2}$

Incorrect: $\dfrac{y_1 - y_2}{x_2 - x_1}$ or $\dfrac{y_2 - y_1}{x_1 - x_2}$

Properties: Slopes of horizontal and vertical lines

Horizontal line : $y = c$ has a <u>Slope $= 0$</u>

Vertical line : $x = c$ has an <u>undefined slope</u>

<u>Example</u>

- $4x - 3y = 12$

OX intercept : $y = 0 \Rightarrow 4x = 12 \Rightarrow x = 3$ $(3, 0)$

OY intercept : $x = 0 \Rightarrow -3y = 12 \Rightarrow y = -4$ $(0, -4)$

Slope $= m = \dfrac{-4 - 0}{0 - 3} = \boxed{\dfrac{4}{3}}$

- $y = 9$ (horizontal line)

  Slope $= m = 0$

- $x = -\frac{3}{4}$ (vertical line)

  Slope — undefined

2.

<u>Def.</u> (Slope-intercept form of a line)

If the equation of a nonvertical line in x and y is solved for y, the result is an equation in <u>Slope-intercept</u> form.

$$ y = mx + b $$

$\boxed{m}$ is a slope of the line

$\boxed{(0, b)}$ is the y-intercept

If x does not appear in the equation, the slope is 0 and the equation is simply of the form $y = b$. Thus, its graph is a horizontal line.

1) $4x - 3y = 6$

$-3y = 6 - 4x$

$y = \dfrac{6}{-3} + \dfrac{4}{3}x$

$$y = \dfrac{4}{3}x - 2$$

$m = \dfrac{4}{3}$

$(0, -2)$ — y-intercept

3.   Def. (Point-Slope form of a line)

The point-slope form of the equation for the line passing through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1)$$

x_1, y_1, m are all constants
x, y are variables

Example

Find the equation, in slope-intercept form, of the line that passes through the point $(-2, 5)$ with $m = 3$.

$y_1 = 5, \quad x_1 = -2, \quad m = 3$

Hence,

$$y - 5 = 3(x + 2)$$

$$y - 5 = 3x + 6$$

$$\boxed{y = 3x + 11}$$

Summary:

Standard Form: $ax + by = c$

Slope-Intercept Form: $y = \bigcirc\!\!\!m x + \bigcirc\!\!\!b$

Point-Slope Form: $y - \bigcirc\!\!\!y_1 = \bigcirc\!\!\!m (x - \bigcirc\!\!\!x_1)$