

Section 6.3. Logarithmic functions and their graphs

1. Characteristics of logarithmic functions.
2. Graphing logarithmic functions.
3. Evaluating elementary logarithmic expressions.
4. Solving elementary logarithmic equations.
5. Common and natural logarithms.

1.

Def.

Let a be a fixed positive real number not equal to 1. The logarithmic function with base a is defined to be the inverse of the exponential function with base a , and is denoted $\log_a x$. In symbols, if $f(x) = a^x$, then $f^{-1}(x) = \log_a x$.

In equation form, the definition of

logarithm means that the equations

$$x = a^y \text{ and } y = \log_a x$$

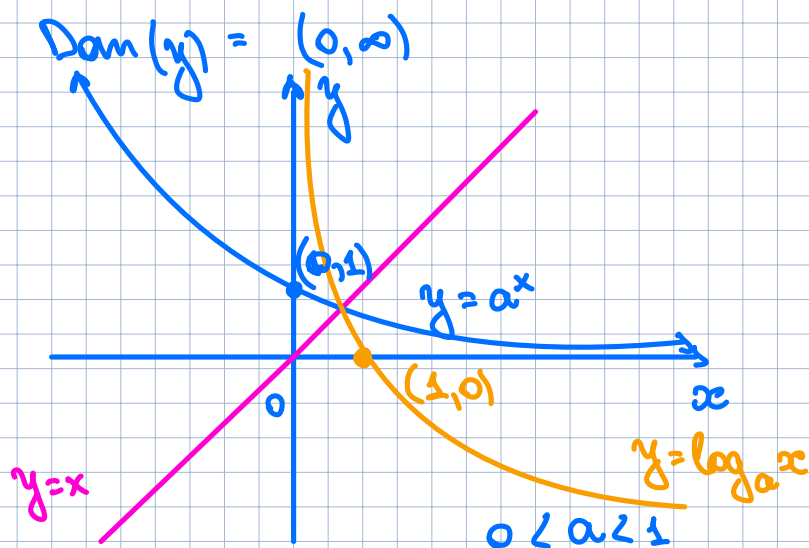
are equivalent.

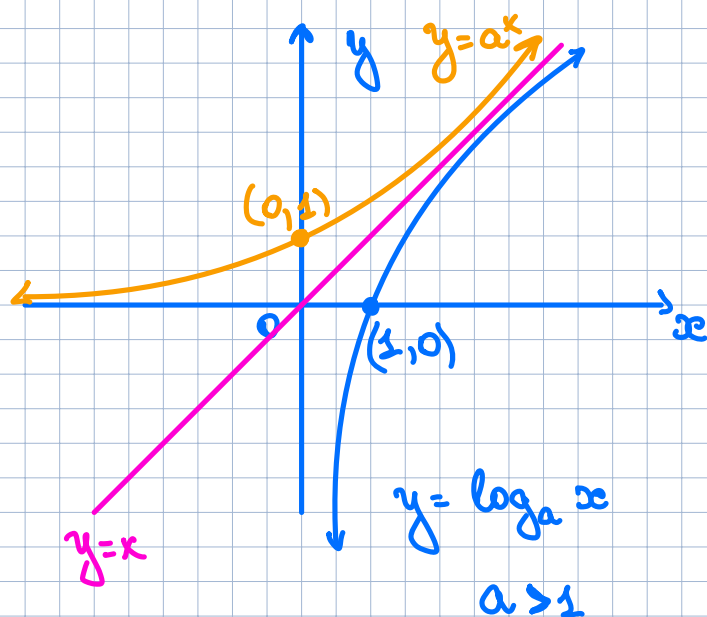
Example

- $8 = 2^3$
 $3 = \log_2 8$
- $7^x = z$
 $x = \log_7 z$

2.

$$y = \log_a x$$
$$x = a^y$$





3.

Properties:

$$\log_a 1 = 0, \quad a^0 = 1$$

$$\log_a a = 1, \quad a^1 = a$$

$$\log_a a^x = x \quad \text{and} \quad a^{\log_a x} = x$$

4.

Example (Solving logarithmic equations)

- $\log_6(2x) = -1$

$$2x = 6^{-1}$$

$$2x = \frac{1}{6} \Rightarrow x = \frac{1}{12}$$

- $3^{\log_{3x} 2} = 2$

$$\log_{3x} 2 = \log_3 2$$

$$3x = 3$$

$$\boxed{x = 1}$$

- $\log_2 8^x = 5$

$$8^x = 2^5$$

$$2^{3x} = 2^5$$

$$3x = 5$$

$$\boxed{x = \frac{5}{3}}$$

5.

Def. (Common and Natural Logarithms)

- The function $\log_{10} x$ is called the common logarithm and is usually written $\log x$.

- The function $\log_e x$ is called the natural logarithm and is usually written $\ln x$.

Properties (Natural logarithms)

$$\ln x = y \Leftrightarrow e^y = x$$

1. $\ln 1 = 0$

$$e^0 = 1$$

2. $\ln e = 1$

$$e^1 = e$$

3. $\ln e^x = x$

$$e^x$$

4. $e^{\ln x} = x$

Example

- $\ln(\sqrt[3]{e}) = \ln(e^{\frac{1}{3}}) = \frac{1}{3} \ln e = \frac{1}{3}$

- $\log 1000 = \log_{10} 10^3 = 3 \log_{10} 10 = 3$