Section 3.4. Other Common Functions

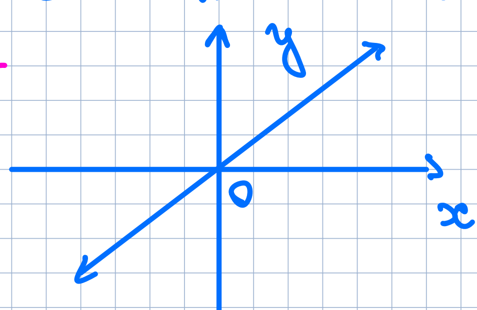
1. Power functions of the form ax^n
2. Power functions of the form ax^{-n}
3. Power functions of the form $ax^{\frac{1}{n}}$
4. The absolute value function
5. The greatest integer function
6. Piecewise-defined functions.

$\boxed{1.}$ **Def.** A power function is a function of the form $f(x) = ax^r$, where $a, r \in \mathbb{R}$.

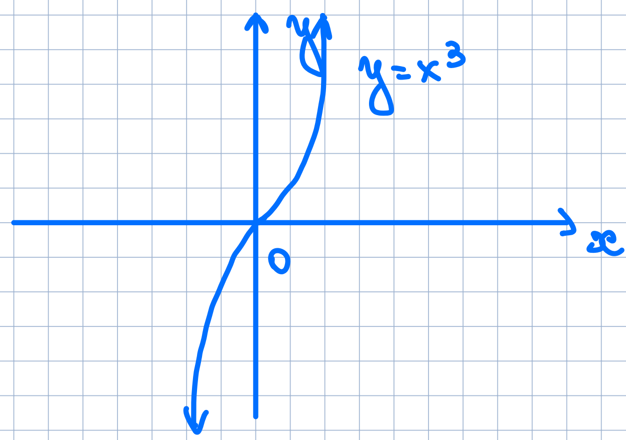
Let us consider the case when $r \in \mathbb{N}$.

- $r \in \mathbb{N}$ and r is an odd number, $a = 1$.

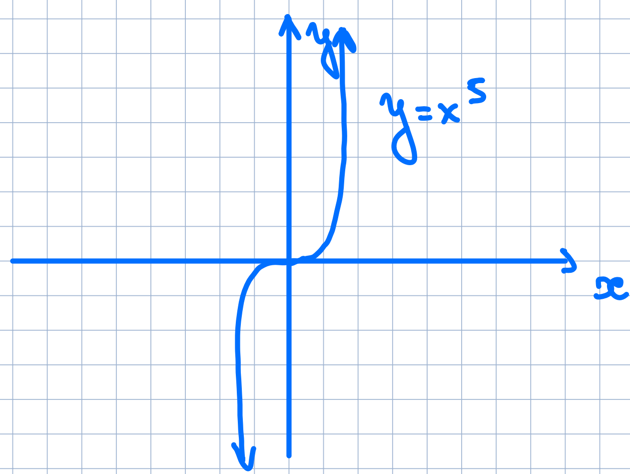
$r = 1$



$r=3$

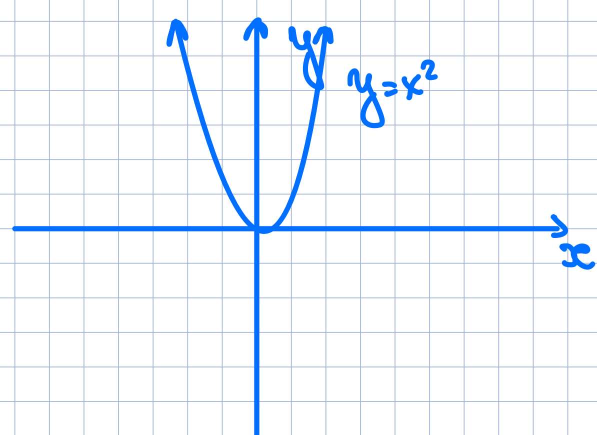


$r=5$

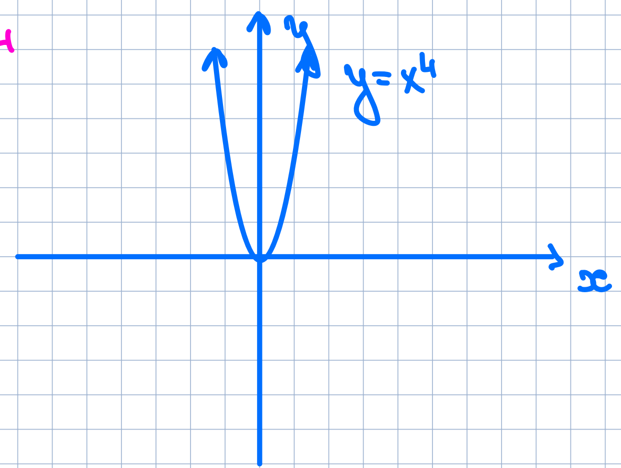


- $r \in \mathbb{N}$ and r is an even number, $a=1$.

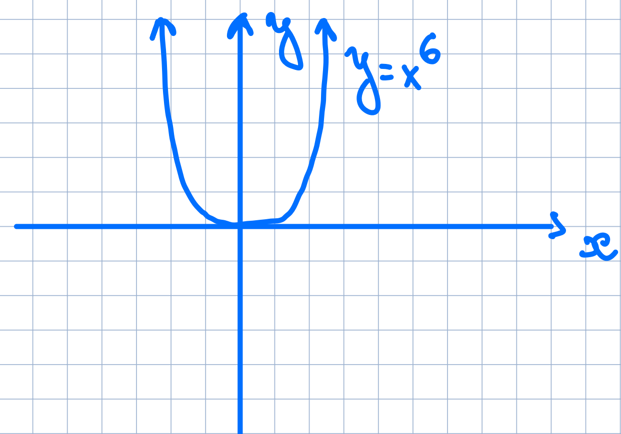
$r=2$



$r = 4$



$r = 6$



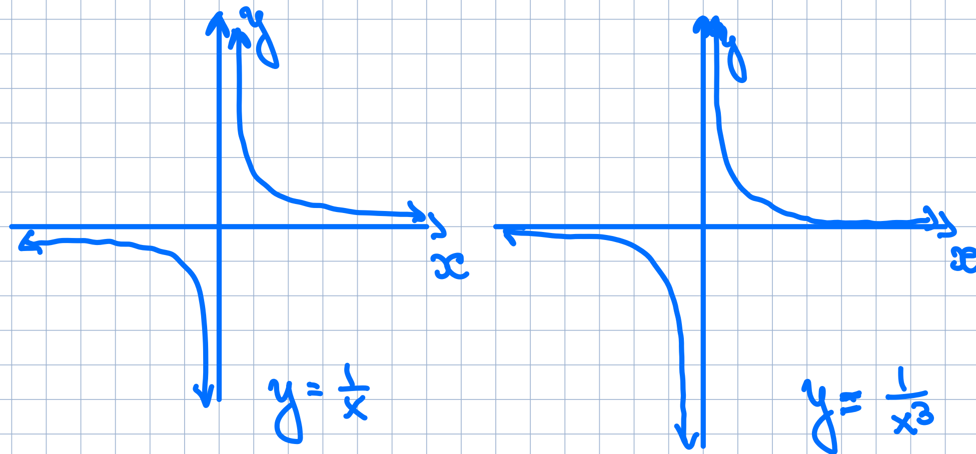
$f(x) = a x^n$.

- $|a| > 1$, then $f(x)$ is stretched vertically
- $0 < |a| < 1$, then $f(x)$ is compressed vertically
- $a < 0$, then $f(x)$ is reflected with respect to the x-axis.

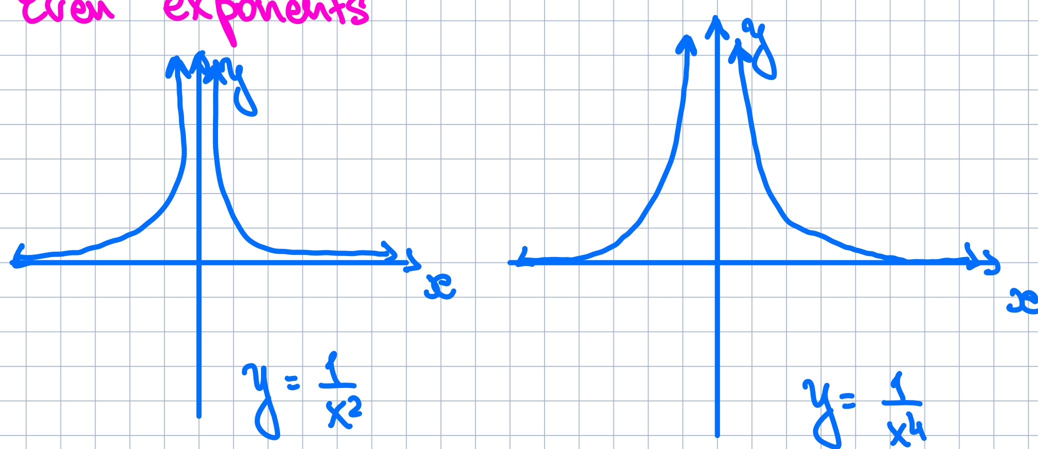
$\boxed{2.}$ Power functions of the form $a x^{-n}$

$$f(x) = \frac{a}{x^n}, \quad a \in \mathbb{R}, \; n \in \mathbb{N}$$

- Odd exponents



- Even exponents



$\text{Dom}(f(x)) = (-\infty, 0) \cup (0, \infty)$.

If n is odd, then $\text{Ran}(f(x)) = (-\infty, 0) \cup (0, \infty)$

If n is even, then $\text{Ran}(f(x)) = (0, \infty)$.

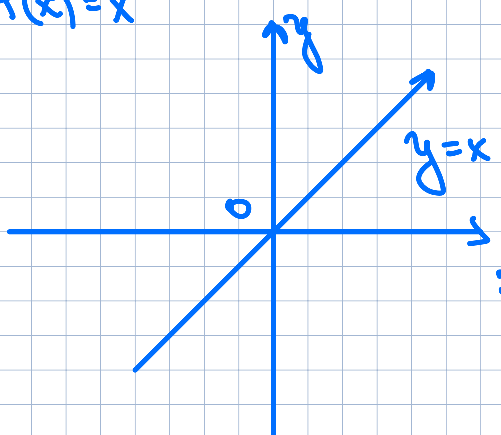
$\boxed{3.}$ Power functions of the form $ax^{\frac{1}{n}}$.

$f(x) = a\sqrt[n]{x}$

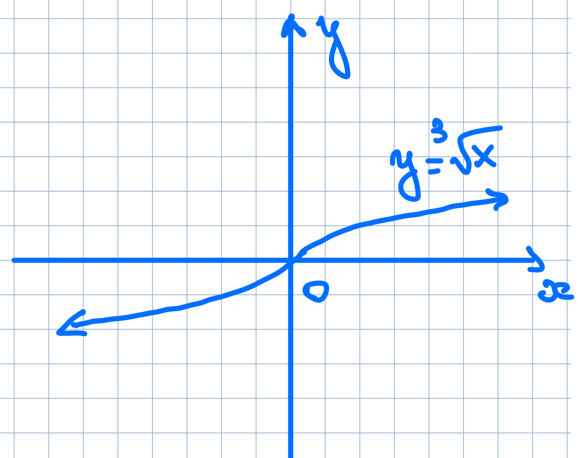
- n is odd $\Rightarrow$ $\operatorname{dom}(f(x)) = \mathbb{R}$
  $\operatorname{Ran}(f(x)) = \mathbb{R}$
- n is even $\Rightarrow$ $\operatorname{dom}(f(x)) = [0, \infty)$
  $\operatorname{Ran}(f(x)) = [0, \infty)$

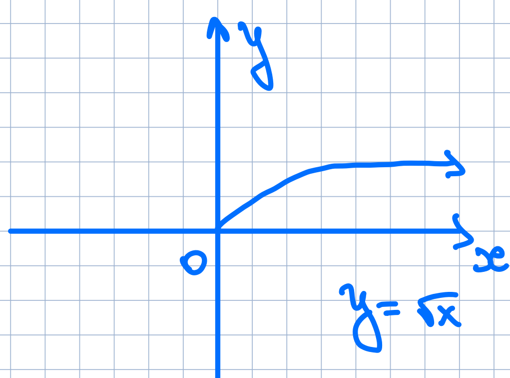
1) $n = 1$, $a = 1$

$f(x) = x$



2) $n = 3$, $a = 1$

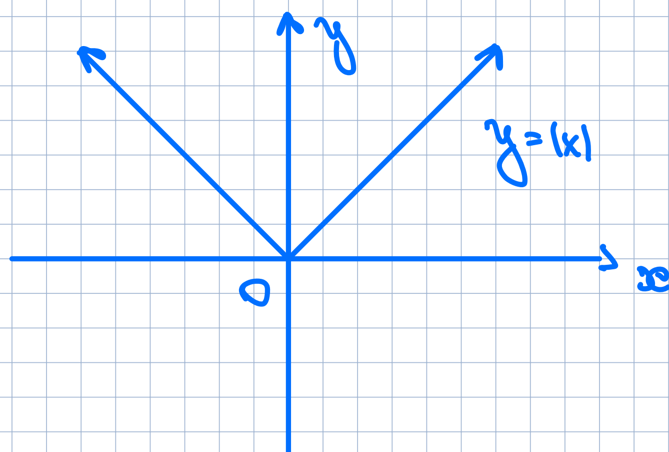




$\boxed{4.}$ The absolute value function.

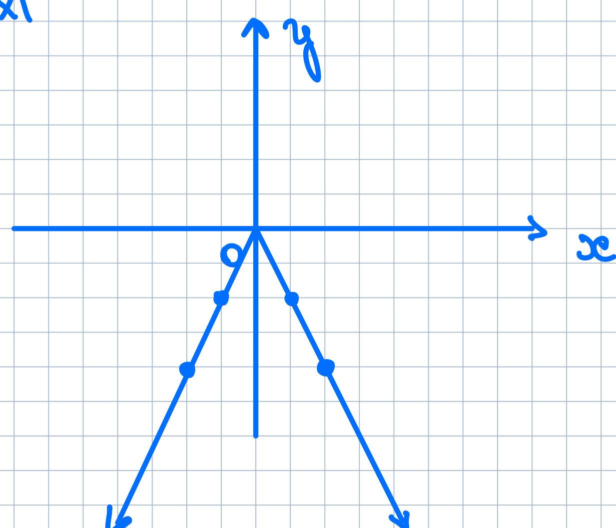
The basic absolute value function is

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Example

$y = -2|x|$



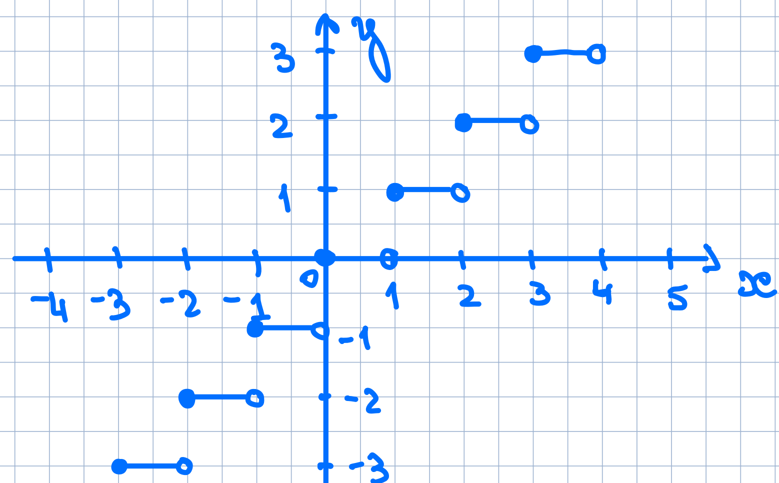
$\boxed{5.}$ Def. (The Greatest Integer Function)

The greatest integer function,

$f(x) = [x]$ is defined as follows:

the <u>greatest integer</u> of x is the largest integer less than or equal to x.

For instance, $[4.3] = 4$ and $[-2.9] = -3$.



$\boxed{6.}$ Piecewise-defined Functions.

<u>Def.</u>

A piecewise-defined function is a function defined in terms of two or more formulas, each valid for its own unique portion of the real number line.

<u>Example</u>

$$f(x) = \begin{cases} -2x-2, & x \le -1 \\ x^2, & x > -1 \end{cases}$$

$g(x) = -2x - 2$

$h(x) = x^2$