Solutions

THEORETICAL PART:

Definition (Exponential Functions):

Let a be a fixed, positive real number not equal to 1. The **exponential function with base** a is the function

$$f(x) = a^x.$$

PROPERTIES (Behaviour of Exponential Functions):

Given a positive real number a not equal to 1, the function $f(x) = a^x$ is

- a **decreasing function** if $0 < a < 1$, with $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$, and is
- an **increasing function** if $a > 1$, with $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

In either case, the point $(0, 1)$ lies on the graph of f, the domain of f is the set of real numbers, and the range of f is the set of positive real numbers.

PROCEDURE (Solving Elementary Exponential Equations):

To solve an elementary exponential equation perform the following steps:

Step 1. Isolate the exponential. Move the exponential containing x to one side of the equation and any constants or other variables in the expression to the other side. Simplify, if necessary.

Step 2. Find a base that can be used to rewrite both sides of the equation.

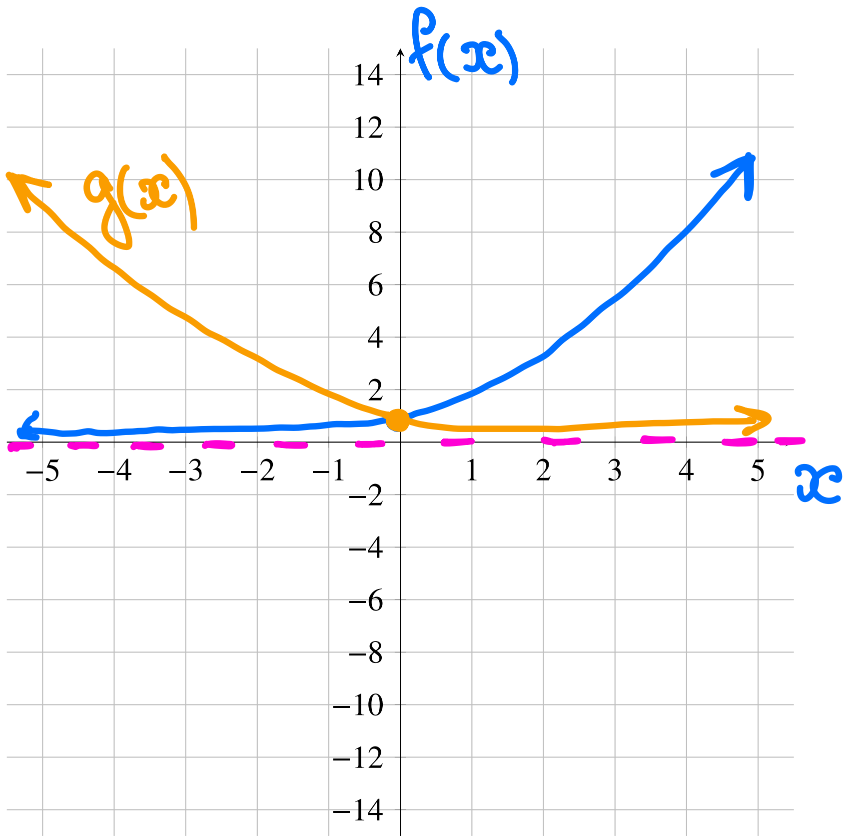
Step 3. Equate the powers, and solve the resulting equation.

PRACTICAL PART:

1. Sketch the graphs of the following exponential functions:

   (a) $f(x) = 3^x$

   (b) $g(x) = \left(\dfrac{1}{2}\right)^x$

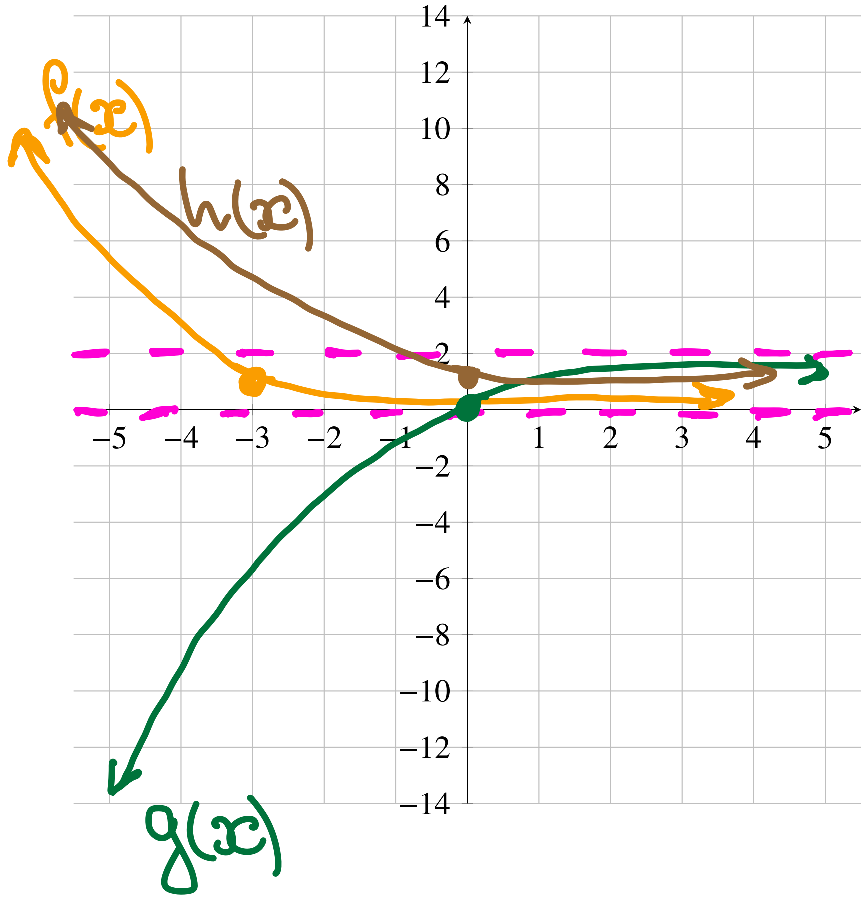


2. Sketch the graphs of each of the following functions. State their domain and range.

(a) $f(x) = \left(\frac{1}{2}\right)^{x+3}$ shift left per 3 units

(b) $g(x) = -3^x + 1$ reflect across x-axis and shift up per 1 unit

(c) $h(x) = 2^{-x}$ $= \left(\frac{1}{2}\right)^x$



3. Solve the following exponential equations.

(a) $25^x - 125 = 0$

(b) $8^{y-1} = \frac{1}{2}$

(c) $\left(\frac{2}{3}\right)^x = \frac{9}{4}$

(a) $25^x = 125$

$(5^2)^x = 5^3$

$5^{2x} = 5^3$

$2x = 3 \Rightarrow \boxed{x = \frac{3}{2}}$

(b) $8^{y-1} = \frac{1}{2}$

$(2^3)^{y-1} = 2^{-1}$

$2^{3y-3} = 2^{-1}$

$3y - 3 = -1$

$3y = 2$

$\boxed{y = \frac{2}{3}}$

(c) $\left(\frac{2}{3}\right)^x = \frac{9}{4}$

$\left(\frac{2}{3}\right)^x = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2 = \left(\frac{2}{3}\right)^{-2}$

$\boxed{x = -2}$