

**Assessment Instructions:****Solutions**

- The Assessment 2 is 10 problems and is worth 40 points. Each numbered problem will earn you a score of 1-4 based on your set up of the function, your use of course methods to solve and prove your solution and your statement of the solution.
- You will have 1 hour to complete AS-2.
- The AS-2 is closed book and closed notes.
- Calculators are not allowed on the Assessment.

1. Find the equation of the line that:

- (a) passes through  $(5, 0)$  and is perpendicular to  $-5x + 2y = 1$
- (b) passes through  $(2, -1)$  and is parallel to  $y = -3x + 4$
- (c) is parallel to  $y$ -axis and has an  $x$ -intercept at  $(2, 0)$
- (d) is perpendicular to the  $x$ -axis and passes through  $(-2, -3)$
- (e) passes through the point  $(3, -4)$  and  $(2, 9)$
- (f) satisfies  $f(1) = -5$  and  $f(9) = 4$

2. Given the two points, find the length of the line segment between the points and the midpoint of the segment:

- (a)  $(3, 2)$  and  $(5, 1)$
- (b)  $(2, 2)$  and  $(8, 3)$
- (c)  $(5, 4)$  and  $(4, 3)$

3. Graph the linear inequality:

- (a)  $x + y > 0$
- (b)  $-(y - x) > -\frac{5}{2} - y$
- (c)  $-2y \leq -x + 4$
- (d)  $y > -2$  and  $2y > -3x - 4$
- (e)  $-2y < -3x - 6$  or  $-3y \geq -6x - 18$

4. Graph the absolute value linear inequality:

(a)  $|3y - 1| \leq 2$

(b)  $|x + y| \geq 1$

(c)  $|x - 3| > 2$

5. Find the standard form for the equation of the circle:

(a)  $9x^2 + 9y^2 - 18x + 36y + 44 = 0$

(b) center  $(12, -4)$  passing through  $(-9, 5)$

(c) endpoints of the diameter are  $(-8, 6)$  and  $(1, 11)$

(d)  $5x^2 + 5y^2 + 50x + 40y = -185$

(d)  $x^2 + y^2 - 18x - 8y + 48 = 0$

6. Sketch the graph:

(a)  $(x + 2)^2 + y^2 = 169$

(b)  $(x + 3)^2 + (y - 7)^2 = 64$

(c)  $x^2 + y^2 + 8x = 9$

7. For each of the following relations, determine the domain and range:

(a)  $R = \{(0, 0), (-5, 2), (3, 3), (5, 3)\}$

(b)  $3x - 4y = 17$

(c)  $y = x^2$

(d)  $x = 4x$

8. Rewrite each of the following relations as a function of  $x$  and evaluate it at  $x = -1$ :

(a)  $6x^2 - x + 3y = x + 2y$

(b)  $\frac{9y + 2}{6} = \frac{3x - 1}{2}$

9. Identify the domain, the codomain, and the range of the following functions:

- (a)  $f : \mathbb{N} \rightarrow \mathbb{N}$  and  $f(x) = x + 5$
- (b)  $h : [0, \infty) \rightarrow \mathbb{R}$  and  $h(x) = \sqrt{x}$

10. Determine the implied domain of the following functions:

- (a)  $g(x) = \frac{2x}{1 - 3x}$
- (b)  $h(x) = \sqrt{3 - x}$
- (c)  $f(x) = \frac{5}{\sqrt{3 - x^2}}$

11. Graph the following linear functions:

- (a)  $y = -2$
- (b)  $f(x) = 3 - 2x$
- (c)  $g(x) = \frac{2x - 8}{4}$

12. For the given points use linear regression to find and graph the line of best fit along with the points and find the Pearson correlation coefficient  $r$

- (a)  $\{(1, 5), (2, -1), (3, 5), (4, 0), (5, 4)\}$

13. Graph the following quadratic functions (parabolas) and state the coordinates of its vertex:

- (a)  $y = (x - 2)^2 + 3$
- (b)  $f(x) = -3x^2 - 1$
- (c)  $g(x) = 4x^2 - 6$
- (d)  $f(x) = x^2 + 2x + 4$

14. Among all the pairs of numbers with a sum of 10, find the pair whose product is maximum.

15. The total revenue for Thompson's Studio Apartments is given by the function

$$R(x) = 100x - 0.1x^2,$$

where  $x$  is the number of rooms rented.

What number of rooms rented produces the maximum revenue?

1. (a)

$$A(5,0)$$

$$l \perp -5x + 2y = 1$$

$$m_1 = -\frac{1}{m_2}$$

$$2y = 1 + 5x$$

$$y = \frac{5}{2}x + \frac{1}{5}$$

$$m_2 = \frac{5}{2}$$

$$m_1 = -\frac{1}{\frac{5}{2}} = -\frac{2}{5}$$

$$y = -\frac{2}{5}x + b$$

$$A(5,0) \rightarrow 0 = -\frac{2}{5} \cdot 5 + b \Rightarrow b = 2$$

Hence,

$$y = -\frac{2}{5}x + 2$$

(b)

$$A(2,-1) \quad l \parallel y = -3x + 4$$

$$m_1 = m_2$$

$$m_2 = -3 = m_1$$

$$y = -3x + b$$

$$-1 = -3 \cdot 2 + b$$

$$-1 = -6 + b$$

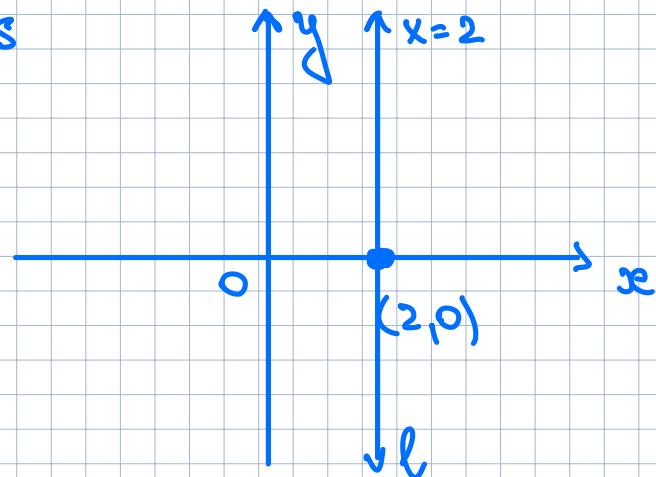
$$b = 5$$

Hence,

$$y = -3x + 5$$

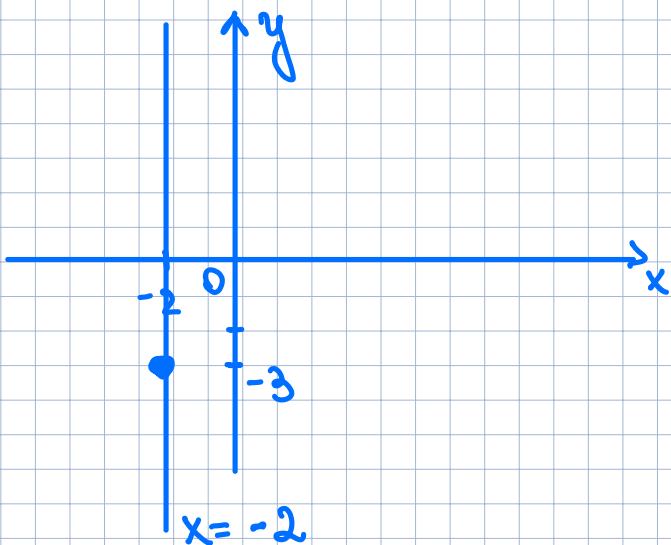
(c)  $l \parallel y\text{-axis}$

$$x = 2$$



(d)  $l \perp x\text{-axis}$

$$x = -2$$



(e) A (3, -4)

B (2, 9)

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9+4}{2-3} = -13$$

$$y = -13x + b$$

$$9 = -13 \cdot 2 + b \Rightarrow b = 9 + 26 = 35$$

Hence,

$$y = -13x + 35$$

$$(f) \quad f(1) = -5 \Leftrightarrow A(1, -5)$$

$$f(9) = 4 \Leftrightarrow B(9, 4)$$

See problem (e).

2.

(a)  $(3, 2)$  and  $(5, 1)$

$$d = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M\left(\frac{3+5}{2}, \frac{2+1}{2}\right) = M\left(\frac{8}{2}, \frac{3}{2}\right) = M\left(4, \frac{3}{2}\right)$$

(b) See (a)

(c) See (a)

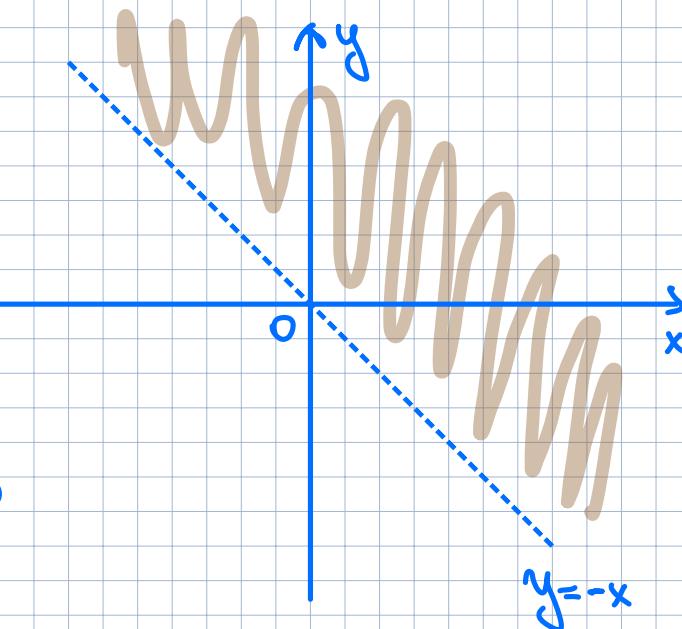
3.

(a)  $x+y > 0$

$$x+y=0$$

$$y=-x$$

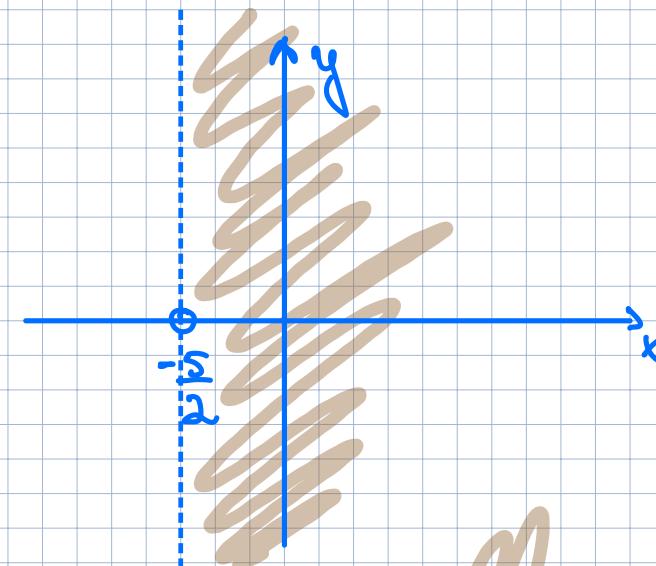
$$(2,0): 2+0>0$$



(b)  $-(y-x) > -\frac{5}{2} - y$

$$-y+x > -\frac{5}{2} - y$$

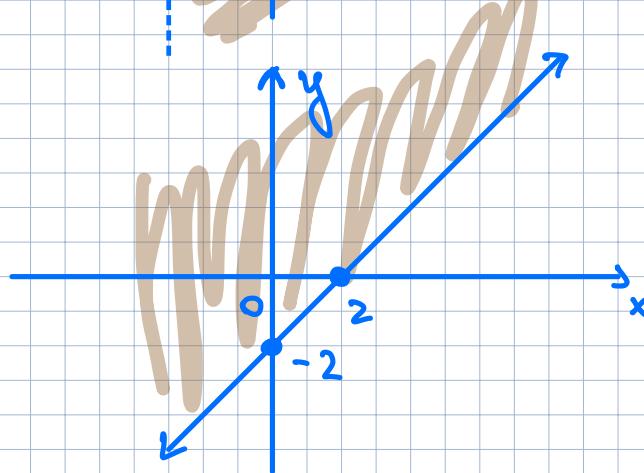
$$x > -\frac{5}{2}$$



(c)  $-2y \leq -x+4$

$$y \geq \frac{x}{2} - 2$$

$$y = \frac{x}{2} - 2$$

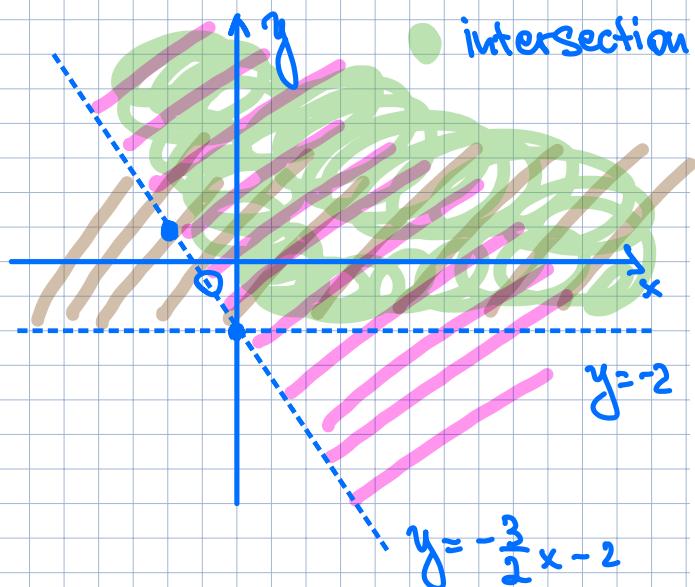


$$(0,0) : 0 \geq -2 \vee$$

(d)

$$y > -2 \text{ and } 2y > -3x - 4$$

$$y > -2 \text{ and } y > -\frac{3}{2}x - 2$$



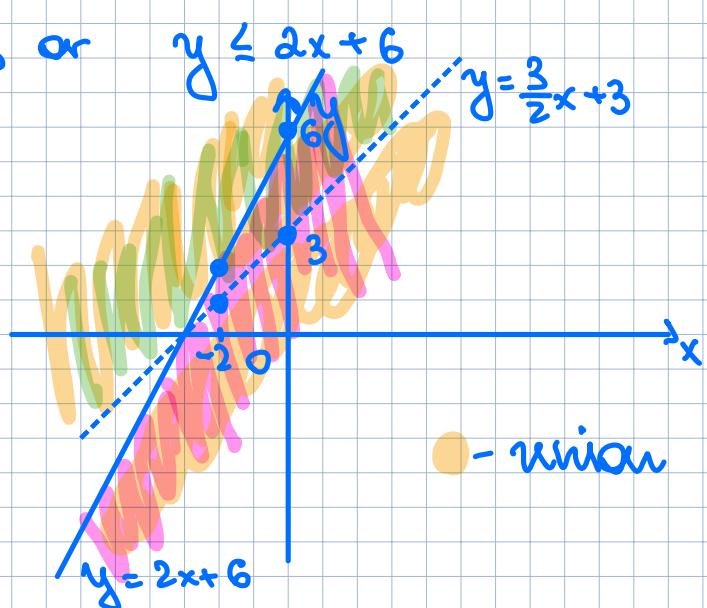
(e)

$$-2y < -3x - 6 \quad \text{or} \quad -3y \geq -6x - 18$$

$$y > \frac{3}{2}x + 3 \quad \text{or} \quad y \leq 2x + 6$$

$$y = \frac{3}{2}x + 3$$

$$y = 2x + 6$$



4.

$$|3y-1| \leq 2$$

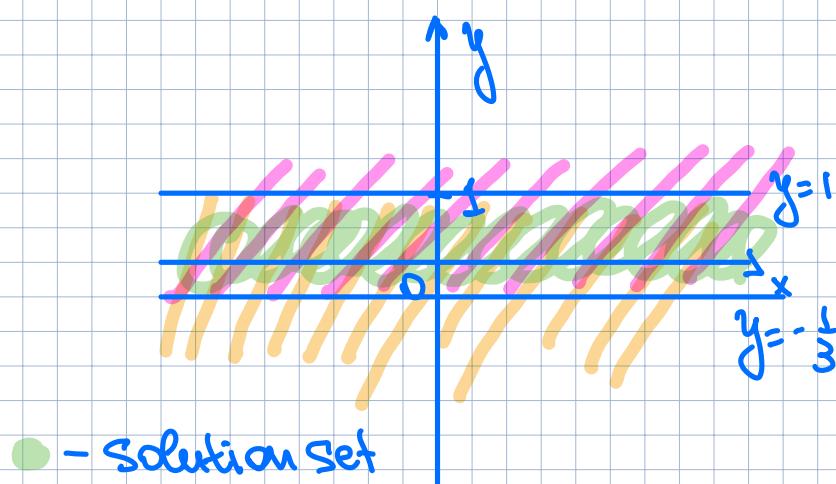
$$3y-1 \leq 2 \text{ and } 3y-1 \geq -2$$

$$3y \leq 3$$

$$y \leq 1$$

$$3y \geq -1$$

$$y \geq -\frac{1}{3}$$

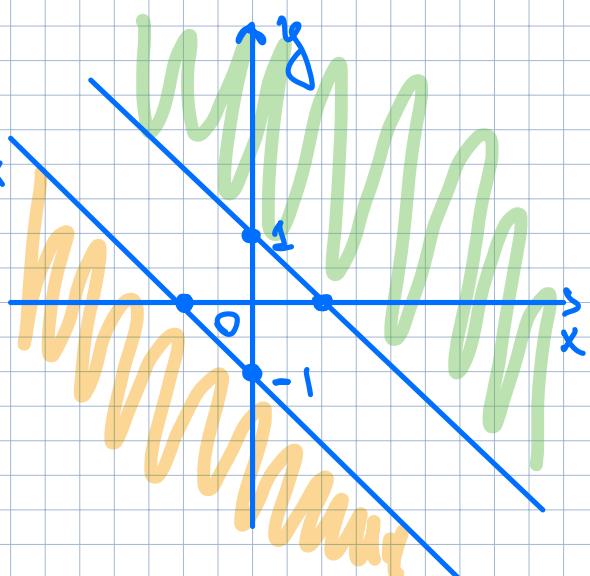


(b)  $|x+y| \geq 1$

$$x+y \geq 1 \text{ or } x+y \leq -1$$

$$y \geq 1-x \text{ or } y \leq -1-x$$

● U ● is a Solution Set



5.

(a)

$$9x^2 + 9y^2 - 18x + 36y + 44 = 0$$

$$(3x)^2 + (3y)^2 - 18x + 36y + 44 = 0$$

$$(3x-3)^2 + (3y+6)^2 - 9 - 36 + 44 = 0$$

$$(3x-3)^2 + (3y+6)^2 = 1$$

$$9(x-1)^2 + 9(y+2)^2 = 1$$

$$(x-1)^2 + (y+2)^2 = \frac{1}{9} = \left(\frac{1}{3}\right)^2$$

(b)

$$(12, -4)$$

$$(-9, 5)$$

$$(x-12)^2 + (y+4)^2 = r^2$$

$$r = ?$$

$$(-9-12)^2 + (5+4)^2 = r^2$$

$$21^2 + 9^2 = r^2$$

$$r^2 = 522$$

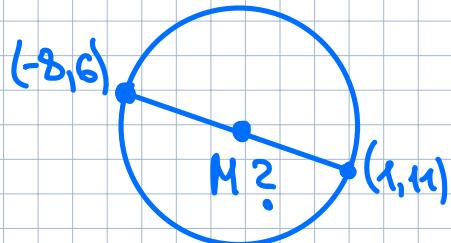
Hence,

$$(x-12)^2 + (y+4)^2 = 522$$

(c)

$$(-3, 6)$$

$$(1, 11)$$



$$d = \sqrt{(1+8)^2 + (11-6)^2} = \sqrt{81 + 25} = \sqrt{106}$$

$$r = \frac{\sqrt{106}}{2}$$

$$M\left(\frac{1-9}{2}, \frac{6+11}{2}\right) = M\left(-\frac{7}{2}, \frac{17}{2}\right)$$

$$(x + \frac{7}{2})^2 + (y - \frac{17}{2})^2 = \frac{106}{4}$$

(d) Will not appear on AS-2

$$(e) x^2 + y^2 - 18x - 8y + 49 = 0$$

$$(x-9)^2 + (y-4)^2 - 81 - 16 + 49 = 0$$

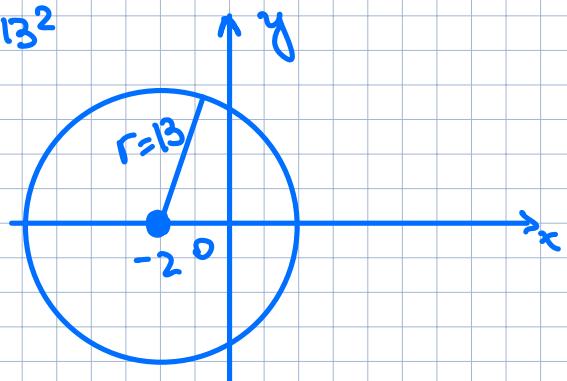
$$(x-9)^2 + (y-4)^2 = 49 = r^2$$

6.

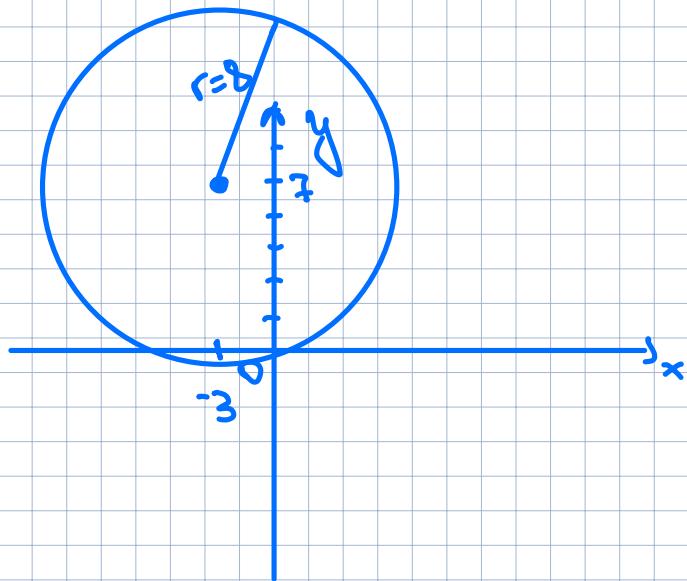
$$(a) (x+2)^2 + y^2 = 169 = 13^2$$

center (-2, 0)

$$r = 13$$



$$(b) (x+3)^2 + (y-7)^2 = 64 = 8^2$$

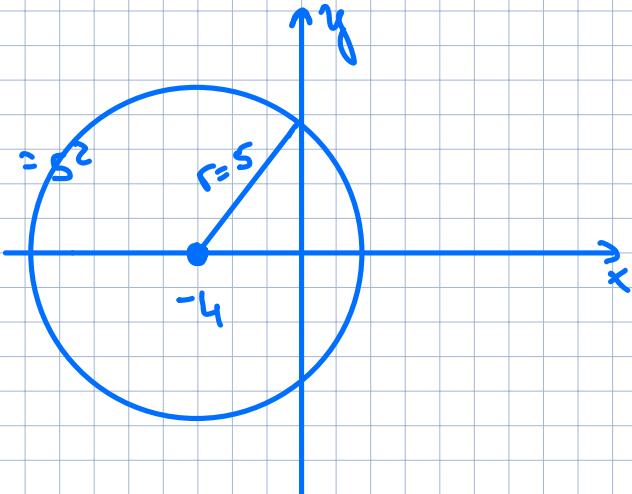


$$(c) x^2 + y^2 + 8x = 9$$

$$(x+4)^2 + y^2 = 25 = 5^2$$

center  $(-4, 0)$

$$r = 5$$



7.

$$(a) R = \{(0,0), (-5,2), (3,3), (5,3)\}$$

$$\text{Dom}(R) = \{0, -5, 3, 5\}$$

$$\text{Ran}(R) = \{0, 2, 3\}$$

$$(b) \quad 3x - 4y = 12$$

$$-4y = 12 - 3x$$

$$y = \frac{3}{4}x - \frac{12}{4} \quad - \text{ straight nonvertical line}$$

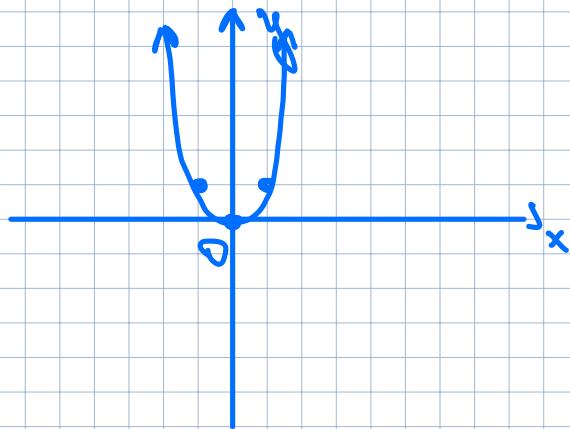
$$\text{Dom}(f) = \mathbb{R}$$

$$\text{Ran}(f) = \mathbb{R}$$

$$(c) \quad y = x^2$$

$$\text{Dom}(y) = \mathbb{R}$$

$$\text{Ran}(y) = [0, \infty)$$



$$(d) \quad x = 4x$$

$$3x = 0$$

$$x = 0$$

$$\text{Dom}(R) = \{0\}$$

$$\text{Ran}(R) = \emptyset$$

8.

$$(a) \quad 6x^2 - x + 3y = x + 2y$$

$$y = x + x - 6x^2$$

$$y = 2x - 6x^2$$

$$y(-1) = -2 - 6 \cdot 1 = -8$$

(b)

$$\frac{9y+2}{6} = \frac{3x-1}{2}$$

$$9y+2 = 6 \frac{3x-1}{2} = 3(3x-1)$$

$$9y = 9x - 3 - 2 = 9x - 5$$

$$y = x - \frac{5}{9}$$

$$y(-1) = -1 - \frac{5}{9} = -\frac{14}{9}$$

9.

$$(a) \quad f: N \rightarrow N \quad f(x) = x + 5$$

$$\text{Dom}(f) = N$$

$$\text{Codomain}(f) = N$$

$$\text{Ran}(f) = \{6, 7, 8, 9, 10, 11, \dots\}$$

$$(b) \ h: [0, \infty) \rightarrow \mathbb{R} \quad h(x) = \sqrt{x}$$

$$\text{Dom}(h) = [0, \infty)$$

$$\text{Codomain}(h) = \mathbb{R}$$

$$\text{Ran}(h) = [0, \infty)$$

10.

$$(a) \ g(x) = \frac{2x}{1-3x}$$

$$1-3x \neq 0$$

$$1 \neq 3x$$

$$x \neq \frac{1}{3}$$

$$\text{Dom}(g) = \mathbb{R} \setminus \left\{ \frac{1}{3} \right\}$$

$$(b) \ h(x) = \sqrt{3-x}$$

$$3-x \geq 0$$

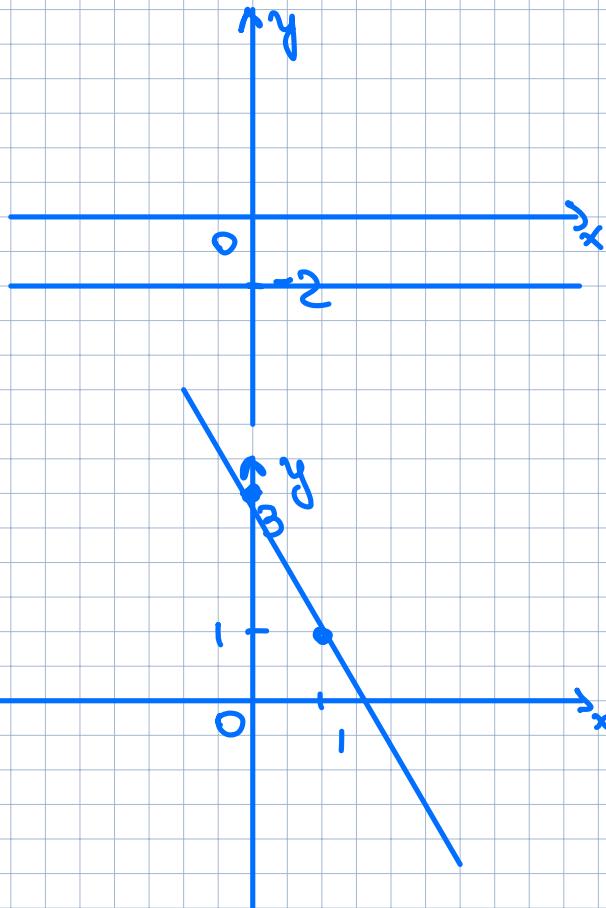
$$-x \geq -3$$

$$x \leq 3$$

$$\text{Dom}(h) = (-\infty, 3]$$

11.

$$(a) \ y = -2$$

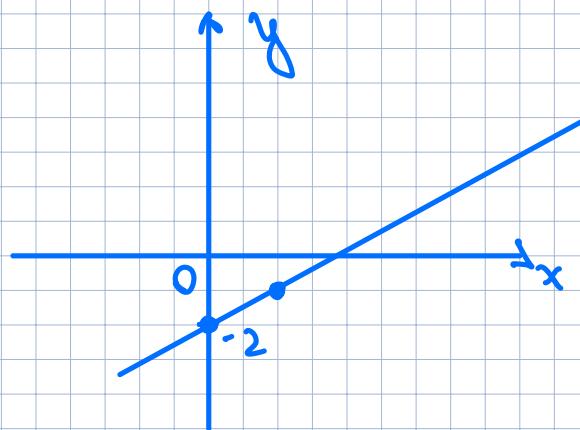


$$(b) f = 3 - 2x$$

x	y
0	3
1	1

$$(c) g(x) = \frac{2x-8}{4} = \frac{x}{2} - 2$$

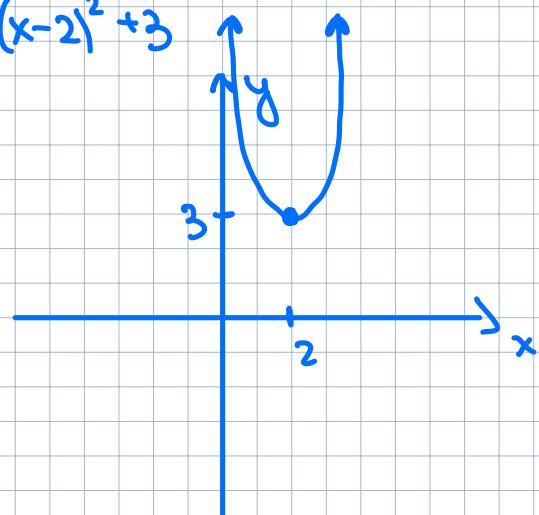
x	y
0	-2
2	-1



13.

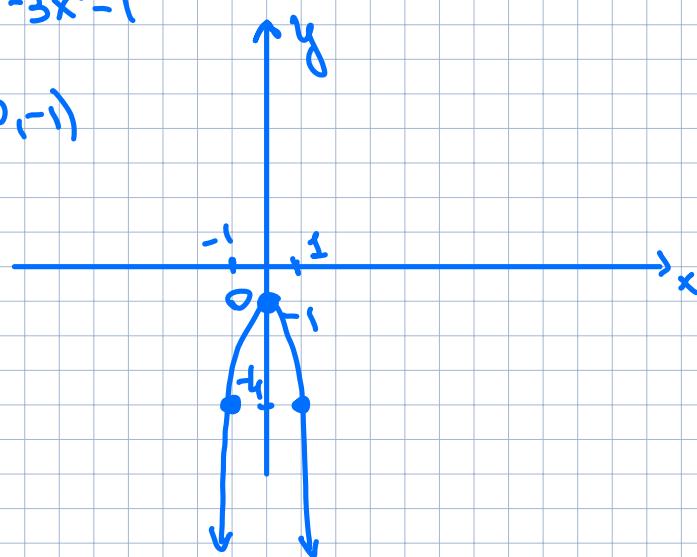
(a)  $y = (x-2)^2 + 3$

vertex  $(2, 3)$



(b)  $f(x) = -3x^2 - 1$

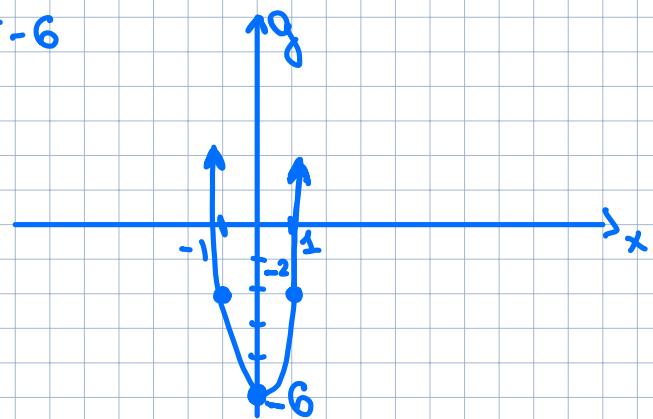
vertex  $(0, -1)$



(c)  $g(x) = 4x^2 - 6$

vertex:

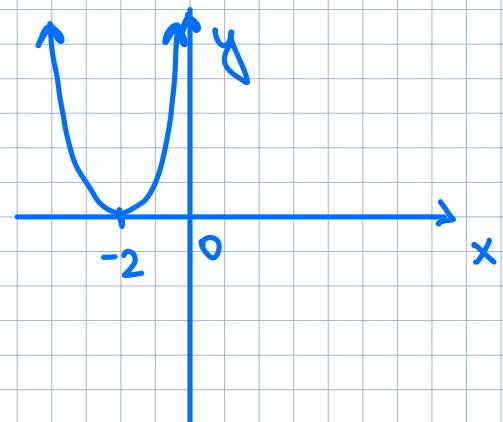
$(0, -6)$



(d)  $f(x) = x^2 + 2x + 4$

$$f(x) = (x+2)^2$$

vertex  $(-2, 0)$



14.

$$x+y=10$$

$$x \cdot y \rightarrow \max$$

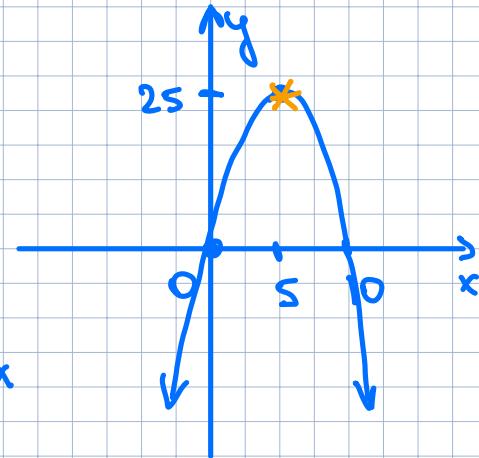
$$y = 10 - x$$

$$x(10-x) = f(x) \rightarrow \max$$

$$10x - x^2 = f(x)$$

↑ parabola open  
downward

$$x=5 \Rightarrow f(5)=25$$



Thus  $x=5$  and  $y=10-x$

$$y = 10-5 = 5$$

$x=5$  and  $y=5$ , therefore

$x \cdot y = 25$

15.

Will not appear on AS- 2.