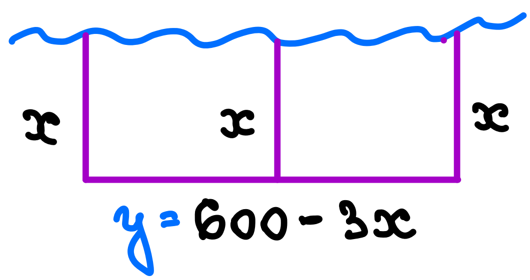
PRACTICAL PART: Solutions

1. (**Modeling the Area of Garden Plots**)

   Anne and Kay plan to use 600 feet of fencing to construct two large rectangular garden plots along a river with the plots sharing a common side and no fence along the river. The overall dimensions can vary a bit, but they want to ensure that the total enclosed area is at least 20000 ft^2.

$3x + (600 - 3x) = 600$



Our goal $A \geq 20000$

$A = h \cdot w$

$A = x \cdot (600 - 3x) \geq 20000$

$600x - 3x^2 \geq 20\,000$

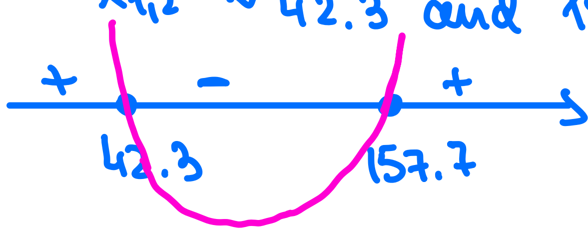
$600x - 3x^2 - 20000 \geq 0$

$3x^2 - 600x + 20000 = 0$

$D = 36000 - 24000 = 12\,000$

$$x_{1,2} = \frac{600 \pm \sqrt{12\,000}}{6}$$

$x_{1,2} \approx 42.3$ and 157.7 (ft)



Hence $A \geq 20000$ for $x \in [42.3, 157.7]$

From this it follows that

$y = 600 - 3x \in [126.9, 473.1]$

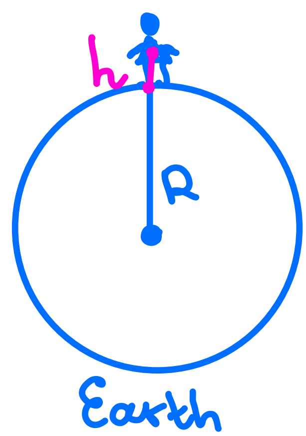
Answer: for $x \in [42.3, 157.7]$, $y \in [126.9, 473.1]$ we have $A \geq 20000$.

2. (**Modeling the Weight of a Person with a Given Mass**)

Consider the weight, or force of gravitational attraction, of a person with a given mass.

a. How does the force of gravitational attraction between a 75 kg person and Earth vary as a function of the person's height above the surface of Earth?
b. How does the force at sea level compare to the force when the person is in a jet at 30000 ft?
c. How far above the surface of Earth would the person have to be for the force to be half as much as the force at sea level?

$$F = G\frac{m_1 m_2}{d^2}$$



G - gravitational constant

$G = 6.67 \cdot 10^{-11}$

m_1 - mass of Earth

m_2 - mass of a person

$d = h + R$

R - radius of Earth

$R = 6.38 \cdot 10^6$

$m_1 = 5.97 \cdot 10^{24}$

Hence,

$$F = 6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^6 + h)^2}$$

(a) $F(h) = 6.67 \cdot 10^{-11} \dfrac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^6 + h)^2}$

(b) $h = 0$ (m)

$F(0) = 6.67 \cdot 10^{-11} \dfrac{5.97 \cdot 10^{24} \cdot 75}{6.38^2 \cdot 10^{12}} \approx 734$ (N)

$h = 30\,000 \text{ ft} = 9144 \text{ m}$

$F(9144) \approx 732 \ (N)$

$F(9144)$ is 99.7% of $F(0)$

(c) $F(h) = \frac{734}{2} = 367$

$$6.67 \cdot 10^{-11} \frac{5.97 \cdot 10^{24} \cdot 75}{(6.38 \cdot 10^{6} + h)^2} = 367$$

$$\frac{6.67 \cdot 10^{-11} \cdot 5.97 \cdot 10^{24} \cdot 75}{367} = (6.38 \cdot 10^{6} + h)^2$$

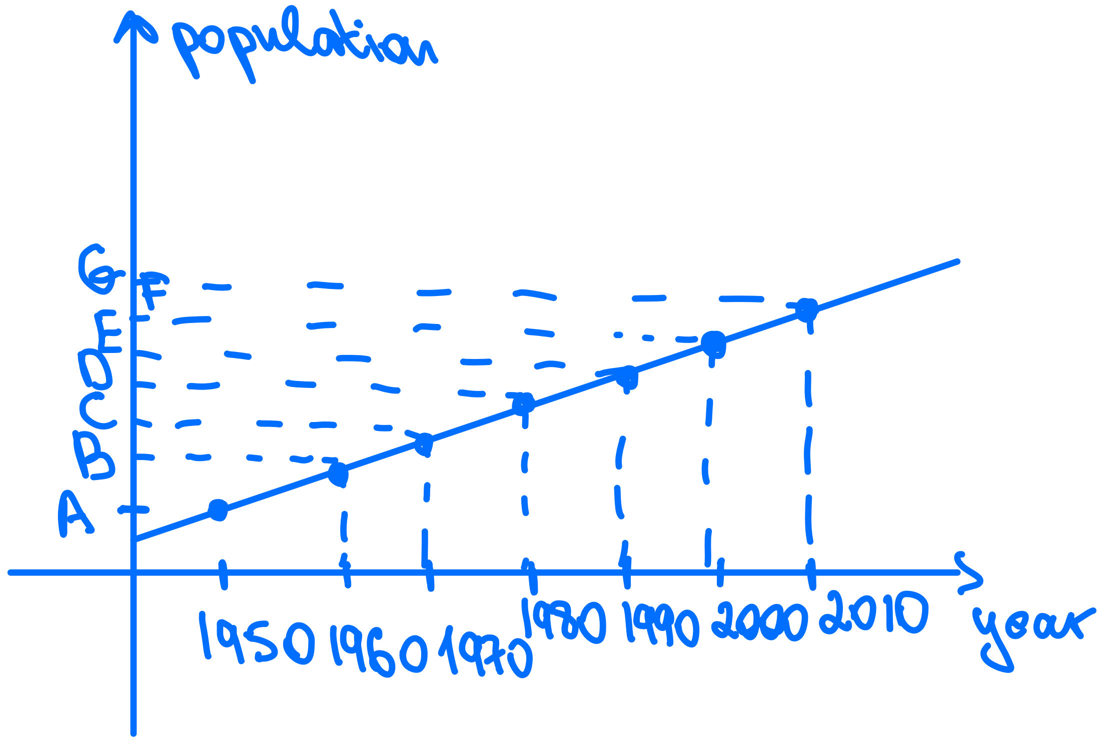
$$\Downarrow$$

$$h \approx 2.64 \cdot 10^{6} \ (m)$$

3. (**Interpolation and Extrapolation**)

Based on the table below, explain the notion of interpolating a value from a table and extrapolating a value from a table.

Year	Population	
1950	151 325 798	A
1960	179 323 175	B
1970	203 302 031	C
1980	226 542 199	D
1990	248 709 873	E
2000	281 421 906	F
2010	308 745 538	G



- To interpolate a value from a table or graph means to arrive at an estimate for the value of the implied function between two known data points

For instance, $P(1985) \approx 240$ million

- To extrapolate means to guess at a value beyond the given points.

For instance, $P(2060) \approx 440$ million.