Section 6.4. Logarithmic Properties and Models

1. Properties of logarithms.
2. The change of base formula.
3. Applications to logarithmic functions.
4. Logarithmic regression.

$\boxed{1.}$

Properties of Logarithms:

Let a (the logarithmic base) be a positive real number not equal to 1, let x and y be positive real numbers, and let r be any real number.

1. $\log_a(xy) = \log_a x + \log_b y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r \log_a x$

Proof. Let $m = \log_a x$
$n = \log_a y$

Then

$$a^m = x$$
$$a^n = y$$

$$x \cdot y = a^n \cdot a^m = a^{n+m}$$
$$a^{n+m} = x \cdot y$$
$$n + m = \log_a (xy)$$

▼

Example

- $\log_4 (64 x^3 \sqrt{y}) = \log_4 64 + \log_4 x^3 + \log_4 \sqrt{y} = \log_4 4^4 + 3 \log_4 x + \frac{1}{2} \log_4 y = 4 + 3 \log_4 x + \frac{1}{2} \log_4 y.$

$\boxed{2.}$ **Change of Base Formula:**

Let $\underline{a}$ and $\underline{b}$ be positive real numbers, neither of them equal to 1, and let x be a positive real number.

Then

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

Example

- $\log_7 15 = \dfrac{\ln 15}{\ln 7} \approx 1.392$

$\boxed{3.}$

Def. (The pH Scale)

The pH of a solution is defined to be $-\log[H_3O^+]$, where $[H_3O^+]$ is the concentration of hydronium ions in units of moles/liter. Solutions with a pH less than 7 are said to be acidic, while those with a pH greater than 7 are basic.

Def.

Earthquake intensity is measured on the Richter scale. In the original formula that follows, I_0 is the intensity of a just-discernible earthquake, I is the intensity of an earthquake being analyzed, and R is its ranking on the Richter scale.

$$R = \log\left(\frac{I}{I_0}\right)$$

By this measure, earthquakes range from a classification of minor $(R < 4)$, to light $(4 \leq R < 5)$, to moderate $(5 \leq R < 6)$, to strong $(6 \leq R < 7)$, to major $(7 \leq R < 8)$, to great $(8 \leq R)$.

Def. (The Dicibel Scale)

In the <u>decibel scale</u>, I_0 is the intensity of a just-discernible sound, I is the intensity of the sound being analyzed, and D is its decibel level.

$$D = 10 \log\left(\frac{I}{I_0}\right).$$

Decibel levels range from 0 for a barely discernible sound, to 60 for the level of normal conversation, to 80 for heavy traffic, to 120 for a loud rock concert, and finally to around 160, at which point the eardrum is likely to rupture.

$\boxed{4.}$

Logarithmic Regression:

$$\boxed{f(x) = a + b \ln x}$$

For a given (x, y) find a, b that fits the curve $f(x)$ in the best way.