# WRH 10 Solutions

5.2: 1, 17, 21, 39, 43
5.3: 2, 15, 27, 37, 57
5.4: 2, 7, 9, 18, 26
5.5: 1, 5, 20, 31

## 5.2

1. $\dfrac{6x^4-2x^3+8x^2+3x+1}{2x^2+2} = \boxed{3x^2-x+1+\dfrac{5x-1}{2x^2+2}}$

$$
\begin{array}{rl}
6x^4-2x^3+8x^2+3x+1 & \big|\ 2x^2+2 \\
-\ \underline{6x^4+6x^2} & \ \ 3x^2-x+1 \\
-2x^3+2x^2+3x & \\
-\ \underline{-2x^3-2x} & \\
2x^2+5x+1 & \\
-\ \underline{2x^2+2} & \\
5x-1 &
\end{array}
$$

17. $\dfrac{2x^3-3ix^2+11x+(1-5i)}{2x-i} = \boxed{x^2-ix+6+\dfrac{1+i}{2x-i}}$

$$
\begin{array}{rl}
2x^3-3ix^2+11x+(1-5i) & \big|\ 2x-i \\
-\ \underline{2x^3-ix^2} & \ \ x^2-ix+6 \\
-2ix^2+11x & \\
-\ \underline{-2ix^2-x} & \\
12x+(1-5i) & \\
-\ \underline{12x-6i} & \\
1+i &
\end{array}
$$

21. $p(x) = 32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x + 2$

$c = 1$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 32 | -80 | 80 | -40 | 10 | 2 |
| | 32 | -48 | 32 | -8 | 2 | 4 |

$c = 1$ is not a root of $p(x)$.

$p(1) = 32 - \cancel{80} + \cancel{80} - 40 + 10 + 2 = \boxed{4}$

39. $p(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$

$c = -2$

| | | | | | |
|---|---|---|---|---|---|
| -2 | 1 | -3 | -3 | 11 | -6 |
| | 1 | -5 | 7 | -3 | 0 |

$\boxed{p(c) = p(-2) = 0.}$

43. $\dfrac{x^8 + x^7 - 3x^3 - 3x^2 + 3}{x+1}$ $c = -1$

| | $x^8$ | $x^7$ | $x^6$ | $x^5$ | $x^4$ | $x^3$ | $x^2$ | $x$ | $x^0$ |
|---|---|---|---|---|---|---|---|---|---|
| -1 | 1 | 1 | 0 | 0 | 0 | -3 | -3 | 0 | 3 |
| | 1 | 0 | 0 | 0 | 0 | 0 | -3 | 0 | 3 |

$$\frac{x^8 + x^7 - 3x^3 - 3x^2 + 3}{x+1} = \boxed{x^7 - 3x^2 + \frac{3}{x+1}}$$

## $\boxed{5.3}$

2. $g(x) = -2x^3 + 11x^2 + x - 30$

$p: -2 \qquad \{\pm 1, \pm 2\}$

$q: -30 \qquad \{\pm 1, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30\}$

$$\frac{p}{q} = \pm\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, 5, 6, \frac{15}{2}, 10, 15, 30\right\}$$

$\underline{c=2}$

| 2 | $-2$ | 11 | 1 | $-30$ |
|---|---|---|---|---|
| | $-2$ | 7 | 15 | 0 |

$$g(x) = (x-2)(-2x^2 + 7x + 15)$$

$-2x^2 + 7x + 15 = 0$

$\Delta = 49 + 120 = 169$

$$x_1 = \frac{-7+13}{-4} = \frac{6}{-4} = -\frac{3}{2}$$

$$x_2 = \frac{-7-13}{-4} = 5$$

$$\boxed{\left\{2, 5, -\frac{3}{2}\right\}}$$

15. $x^3 - 3x^2 + 9x + 13 = 0$

$p: 1 \quad \{\pm 1\}$

$q: 13 \quad \{\pm 13, \pm 1\}$

$\frac{p}{q} : \pm\{1, \frac{1}{13}\}$

$C = -1$

| -1 | 1 | -3 | 9 | 13 |
|---|---|---|---|---|
| | 1 | -4 | 13 | 0 |

$(x+1)(x^2 - 4x + 13) = 0$

$x^2 - 4x + 13 = 0$

$\Delta = 16 - 52 = -36$

$x_1 = \frac{4+6i}{2} = 2+3i$

$x_2 = 2-3i$

$$\boxed{\{-1, 2+3i, 2-3i\}}$$

27. $f(x) = \underbrace{x^3 - 6x^2}_{\text{change}} \underbrace{+3x}_{\text{change}} +10$

$f(-x) = -x^3 - 6x^2 \underbrace{- 3x + 10}_{\text{change}}$

$\boxed{\text{There is 2 or 0 positive real zeros and 1 negative real zero.}}$

37.

$$f(x) = x^3 + 4x^2 + x - 4$$

| 2 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | 6 | 13 | 22 |

| 3 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | 7 | 10 | 10 |

| 1 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | 5 | 6 | 2 |

| 0 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | 4 | 1 | -4 |

Hence, 1 is an upper bound.

| -2 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | 2 | -3 | 2 |

| -5 | 1 | 4 | 1 | -4 |
|---|---|---|---|---|
| | 1 | -1 | 6 | -34 |

Hence, -5 is a lower bound.

We have $\boxed{[-5, 1].}$

57.

$f(x) = 5x^3 - 4x^2 - 31x - 6$

polynomial

$a = -3$
$b = -1$

$f(-3) = 5\cdot(-27) - 4\cdot 9 + 93 - 6 = \boxed{-84 < 0}$
$f(-1) = -5 - 4 + 31 - 6 = \boxed{16 > 0}$

Hence, there exists $c \in (-3, -1)$ such that

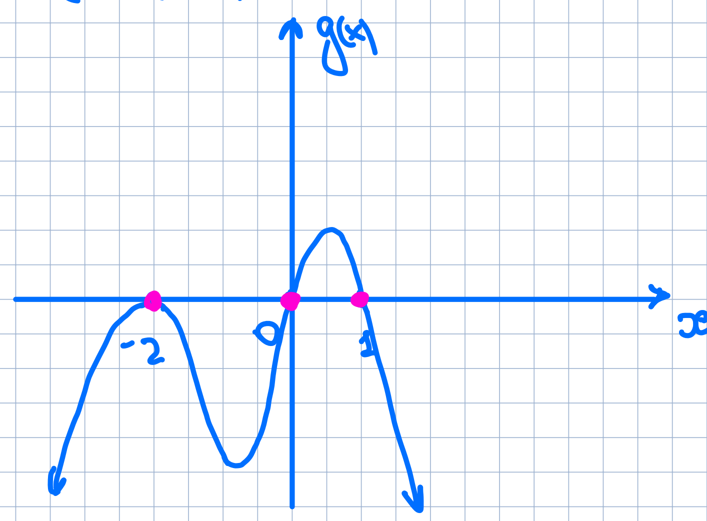
$$\boxed{f(c) = 0.}$$

$\boxed{5.4}$

2. $g(x) = -x^3(x-1)(x+2)^2$

$x=0$
$x=-2$
$x=1$

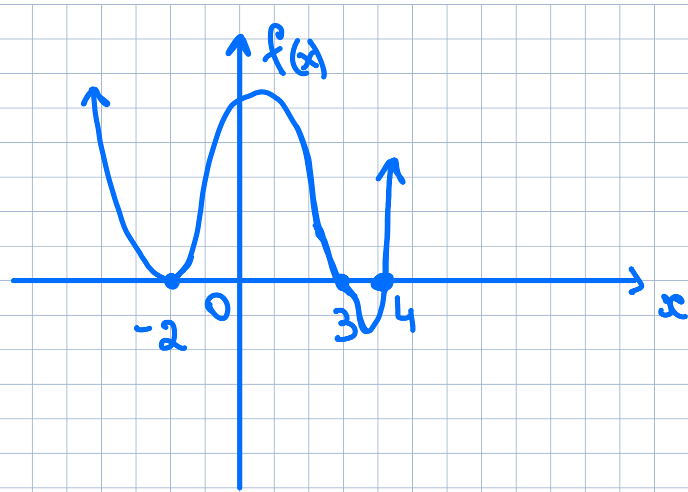
$x \to +\infty$
$g(x) \to -\infty$
$x \to -\infty$
$g(x) \to -\infty$




7. $f(x) = (x-4)(x+2)^2(x-3)^3$

$x \to +\infty$
$f(x) \to +\infty$

$x \to -\infty$
$f(x) \to +\infty$




9. $f(x) = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

$x = 1$

$f(1) = \cancel{1} + \cancel{4} + \cancel{1} - \cancel{10} - \cancel{4} + \cancel{8} = 0$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 1 | -10 | -4 | 8 |
| | 1 | 5 | 6 | -4 | -8 | 0 |

$f(x) = (x-1)(x^4 + 5x^3 + 6x^2 - 4x - 8) =$
$= (x-1)\, p(x)$

$p(1) = \cancel{1} + \cancel{5} + \cancel{6} - \cancel{4} - \cancel{8} = 0$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 5 | 6 | -4 | -8 |
| | 1 | 6 | 12 | 8 | 0 |

$f(x) = (x-1)^2 (x^3 + 6x^2 + 12x + 8) =$
$= \boxed{(x-1)^2 (x+2)^3}$

18.

$$x^4 + 15 = 2x^3 + 8x^2 - 10x$$

$$x^4 - 2x^3 - 8x^2 + 10x + 15 = 0$$

$$\boxed{x = 3}$$

$$81 - 54 - 72 + 30 + 15 = 0$$

| 3 | 1 | -2 | -8 | 10 | 15 |
|---|---|---|---|---|---|
| | 1 | 1 | -5 | -5 | 0 |

$$(x-3)(x^3 + x^2 - 5x - 5) = 0$$

$$\boxed{x = -1}$$

| -1 | 1 | 1 | -5 | -5 |
|---|---|---|---|---|
| | 1 | 0 | -5 | 0 |

$$(x-3)(x+1)(x^2 - 5) = 0$$

$$(x-3)(x+1)(x-\sqrt{5})(x+\sqrt{5}) = 0$$

Answer $\boxed{\{3, -1, \pm\sqrt{5}\}}$.

26. $g(x) = x^3 - (1-i)x^2 - (8-i)x + (12-6i)$

$$x = 2 - i$$

$$g(2-i) = 0$$

| $2-i$ | $1$ | $i-1$ | $i-8$ | $12-6i$ |
|---|---|---|---|---|
| | $1$ | $1$ | $-6$ | $0$ |

$2-i+i-1=1$

$2-i+i-8=-6$

$-12+6i+12-6i=0$

$$g(x)=(2-i)(x^2+x-6)=\boxed{(2-i)(x+3)(x-2)}$$

## $\boxed{5.5}$

1. $f(x)=\dfrac{5}{x-1}$

$x-1=0 \Rightarrow x=1$

Answer: $\boxed{x=1}$ is a vertical asymptote

5. $f(x)=\dfrac{3x^2+1}{x-2}$

Vertical asymptotes: $x-2=0$

$x=2$

Answer: $\boxed{x=2.}$

20. $f(x) = \dfrac{x^2+3}{x+3}$

$p(x) = x^2+3$

$q(x) = x+3$

$a_2 = 2$

$b_1 = 1$

$m = 1$ and $n = 2$

$n > m$

$n = m+1$

$y = g(x)$ is an oblique asymptote.

$g(x) = \text{quotient}\{p(x), q(x)\}$

$$\begin{array}{r|l} x^2 + 3 & x+3 \\ \cline{2-2} -\; x^2 + 3x & x-3 \\ \cline{1-1} -3 - 3x & \\ -\; -3x - 9 & \\ \cline{1-1} 12 & \end{array}$$

$\boxed{y = x-3 \text{ is an oblique asymptote}}$

No vertical \\ horizontal asymptotes

31.

$$f(x) = \frac{x^2 - 81}{x^3 + 7x - 12}$$

$n = 2$

$m = 3$

$\boxed{n < m}$

The horizontal line $y = 0$ is the horizontal asymptote.