THEORETICAL PART:

Solutions

CAUTION:

When using the notation $\sin^{-1}(x)$, remember that

$$\sin^{-1}(x) \neq \frac{1}{\sin(x)}$$

Definition (Arcsine)

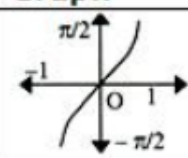
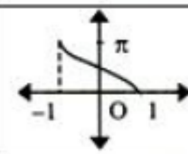
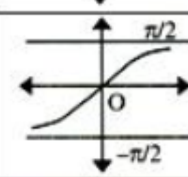
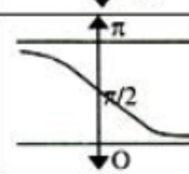
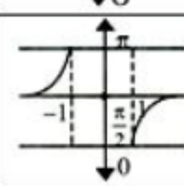
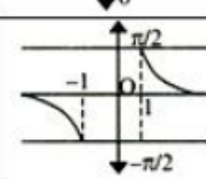
Given $x \in [-1, 1]$, **arcsine** is defined by either of the following:

$$\arcsin(x) = y \Leftrightarrow x = \sin y$$

or

$$\sin^{-1}(x) = y \Leftrightarrow x = \sin y$$

In words, x is the angle whose sine is x; that is, $\sin(\arcsin x) = x$. Since the restricted domain of sine is $[-\pi/2, \pi/2]$ and its range is $[-1, 1]$, the domain of arcsine is $[-1, 1]$ and its range is $[-\pi/2, \pi/2]$.

S.No.	Inverse Cir. Fn.	Domain	Range	Graph
1.	$\sin^{-1} x = \theta$ iff $\sin\theta = x,\ -\pi/2 \leq \theta \leq \pi/2$	$[-1, 1]$	$[-\pi/2, \pi/2]$	
2.	$\cos^{-1} x = \theta$ iff $\cos\theta = x,\ 0 \leq \theta \leq \pi$	$[-1, 1]$	$[0, \pi]$	
3.	$\tan^{-1} x = \theta$ iff $\tan\theta = x,\ \frac{\pi}{2} < \theta < \frac{\pi}{2}$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$	
4.	$\cot^{-1} x = \theta$ iff $\cot\theta = x,\ 0 \leq \theta \leq \pi$	$(-\infty, \infty)$	$(0, \pi)$	
5.	$\sec^{-1} x = \theta$ iff $\sec\theta = x,\ 0 \leq \theta \leq \pi$ and $\theta \neq \frac{\pi}{2}$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi]$ $\theta \neq \frac{\pi}{2}$	
6.	$\operatorname{cosec}^{-1} x = \theta$ iff $\operatorname{cosec}\theta = x$ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2},\ \theta \neq 0$	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, \pi/2]$ $\theta \neq 0$	

PRACTICAL PART:

1. Evaluate the following expressions:

   a. $\arctan(-1)$

   b. $\csc^{-1} 2$

   c. $\sin^{-1}(2.3)$

(a) $\arctan(-1) = -\frac{\pi}{4}$

(b) $\csc^{-1}(2) = \frac{\pi}{6}$

(c) $\sin^{-1}(2.3)$ DNE!!!

2. Evaluate the following expressions is possible.

   a. $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

   b. $\cos(\cos^{-1}(-0.2))$

   c. $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right)$

$\arcsin x = y \iff x = \sin y$

(a) $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

(b) $\cos(\cos^{-1}(-0.2)) = \boxed{-0.2}$

$-0.2 \in \mathrm{Dom}(\cos^{-1} x)$

(c) $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$

$\tan\left(\frac{7\pi}{6}\right) = \tan\left(\pi + \frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

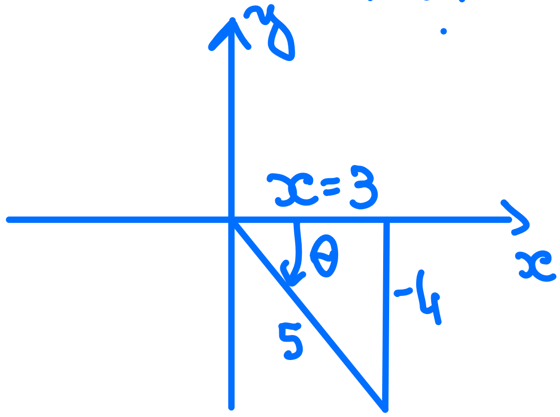
3. Evaluate the following expressions:

   a. $\tan\left(\arcsin\left(-\frac{4}{5}\right)\right)$

   b. $\cos(\arctan(0.4))$

(a) $\tan\left(\arcsin\left(-\frac{4}{5}\right)\right) =$

$\arcsin\left(-\frac{4}{5}\right) = \theta \Rightarrow \sin\theta = -\frac{4}{5}$



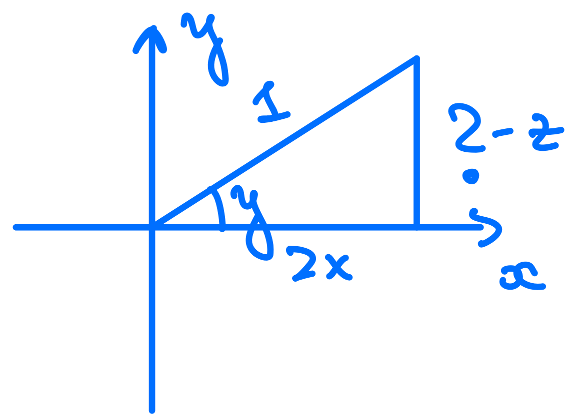
$\boxed{\tan\theta = -\frac{4}{3}}$

4. Express $\sin(\cos^{-1}(2x))$ as an algebraic function of x, assuming that $-1/2 \le x \le 1/2$.

$\cos^{-1}(2x) = y$

$2x = \cos y$

$x = \frac{1}{2}\cos y$

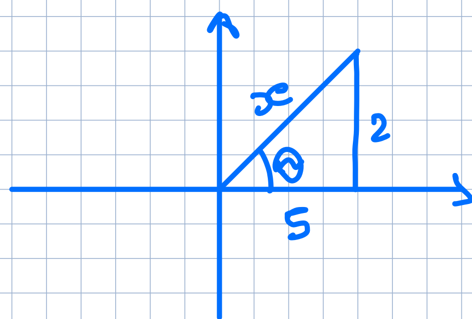


$z = \sqrt{1-4x^2}$

$\boxed{\sin y = \sqrt{1-4x^2}}$

3. (b) $\cos(\arctan(0.4))$

$\arctan\left(\frac{2}{5}\right) = \theta \Rightarrow \tan\theta = \frac{2}{5}$



$x = \sqrt{29}$

$$\boxed{\cos\theta = \frac{5}{\sqrt{29}}}$$