THEORETICAL PART: Solutions

Definition:

A **relation** is a set of ordered pairs. Any set of ordered pairs automatically relates the set of first coordinates to the set of second coordinates, and these sets have spacial names. The **domain** of a relation is the set of all the first coordinates, and the **range** of a relation is the set of all second coordinates.

Definition:

A **function** is a relation in which every element of the domain is paired with exactly one element of the range. Equivalently, a function is a relation in which no two distinct ordered pairs have the same first coordinate.

Notation:

$$y = f(x),$$

where y is a dependent variable and x is an independent variable. Also, we call x as an argument of the function f.

Theorem (The vertical Line Test):

If a relation can be graphed in the Cartesian plane, the relation is a function if and only if no vertical line passes through the graph more than once. If even one vertical line intersects the graph of the relation two or more times, the relation fails to be a function.

Definition (Domain and Codomain Notation):

The notation $f : A \to B$ (read "f defined from A to B) implies that f is a function from the set A to the set B. The symbols indicate that the domain of f is the set A, and that the range of f is a subset of the set B. In this context, the set B is often called the **codomain** of f.

PRACTICAL PART:

1. Determine the domain and the range for the following relations:

   (a) $R = \{(-4, 2), (6, -1), (0, 0), (-4, 0)\}$

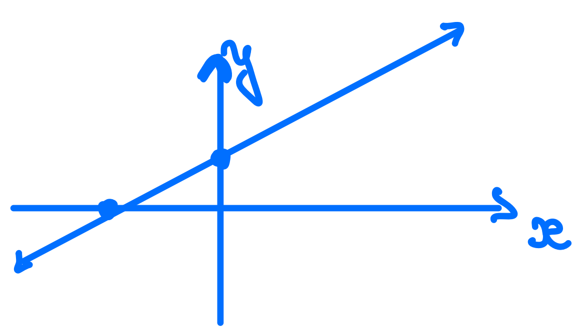
   (b) $-3x + 7y = 13$

(a) $Dom(R) = \{-4, 6, 0, -4\}$

$Ran(R) = \{2, -1, 0\}$

(b) $Dom(R) = \mathbb{R}$

$Ran(R) = \mathbb{R}$

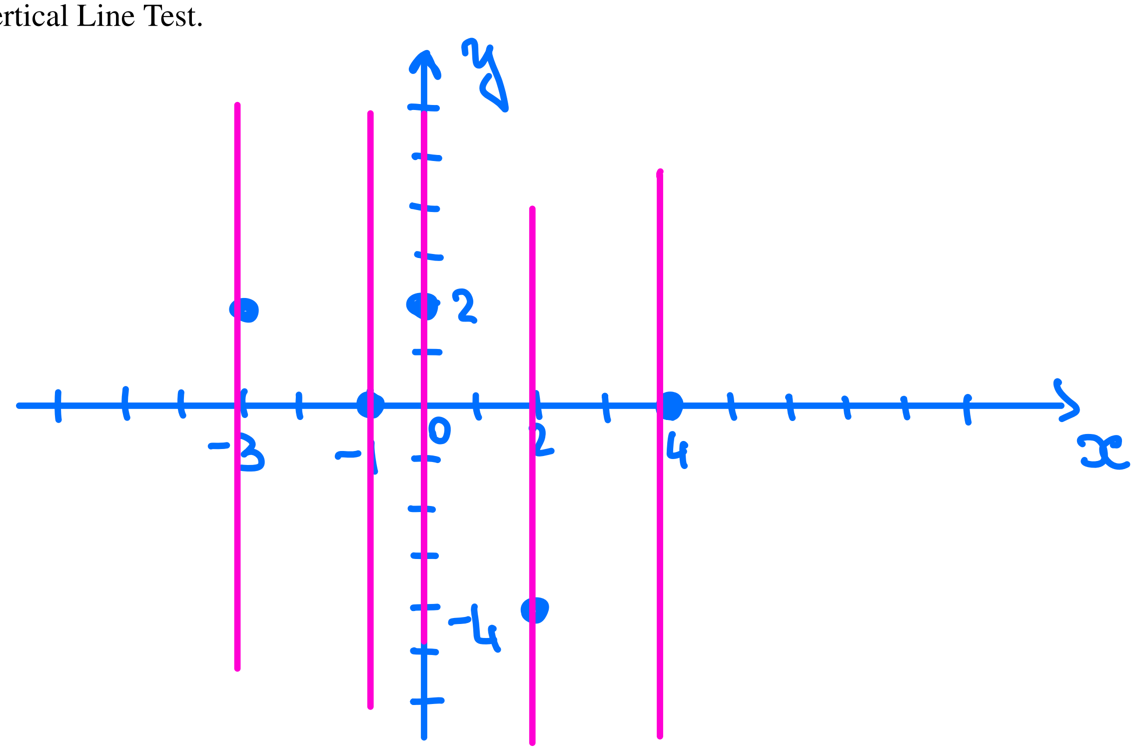


2. For each relation in Problem 1, identify whether the relation is also a function.

(a) R is not a function since for every
$(-4,0)$ $(-4,2)$ x there must be
exactly one y value

(b) straight line is a function.

3. The relation $R = \{(-3, 2), (-1, 0), (0, 2), (2, -4), (4, 0)\}$ is a function. Check it by applying the Vertical Line Test.



4. Give an example of the relation which is not a function.

$R = \{(-1,0), (-1,3), (0,2), (3,1)\}$

5. Each of the following equations in x and y represents a function. Rewrite each one using function notation, and then evaluate each function at $x = -3$.

   (a) $y = \dfrac{3}{x} + 2$

   (b) $y - 5 = x^2$

   (c) $\sqrt{1-x} - 2y = 6$

(a) $y = \dfrac{3}{x} + 2$ $\qquad y(-3) = \dfrac{3}{-3} + 2 = 1$

(b) $y - 5 = x^2$

$y = 5 + x^2$

$y(-3) = 5 + (-3)^2 = 5 + 9 = 14$

(c) $\sqrt{1-x} - 2y = 6$

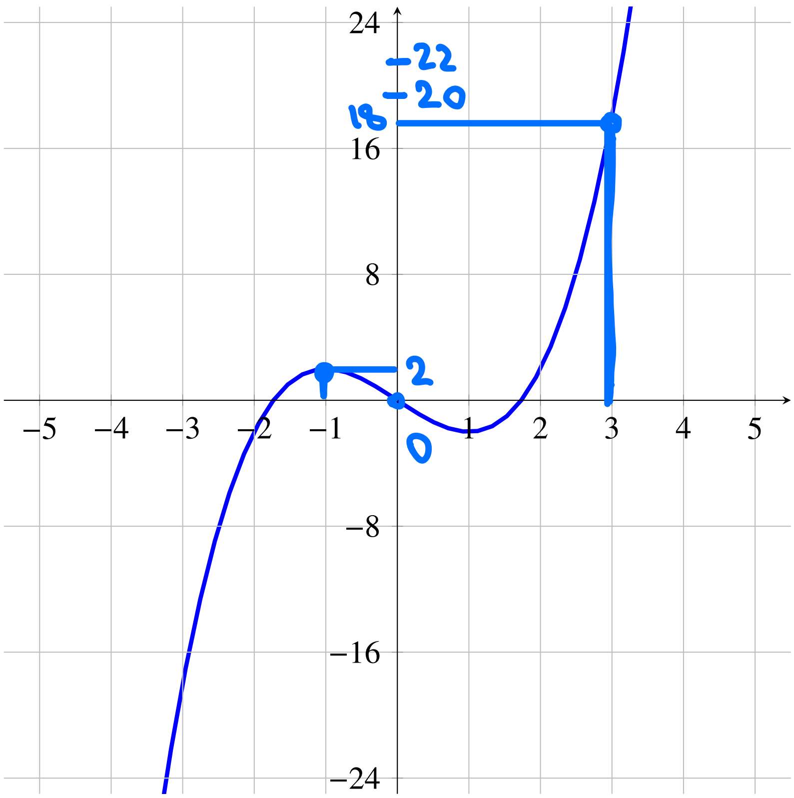
$2y = \sqrt{1-x} - 6$

$y = \frac{1}{2}\sqrt{1-x} - 3$

$y(-3) = \frac{1}{2}\sqrt{1+3} - 2 = 1 - 2 = -1$

6. Use the graph of the function f to answer the following questions:

- $f(0) =$ 0
- $f(3) =$ 18
- $f(-1) =$ 2



7. Given the function $f(x) = 3x^2 - 2$, evaluate the following:

(a) $f(a) =$ $3a^2 - 2$

(b) $f(x+h) =$ $3(x+h)^2 - 2 = 3(x^2 + 2xh + h^2) - 2$

(c) $\dfrac{f(x+h) - f(x)}{h} =$

$$= \frac{3(x^2 + 2xh + h^2) - 2 - 3x^2 + 2}{h} =$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-2} \cancel{-3x^2} \cancel{+2}}{h} = \frac{\cancel{h}(6x + 3h)}{\cancel{h}} =$$

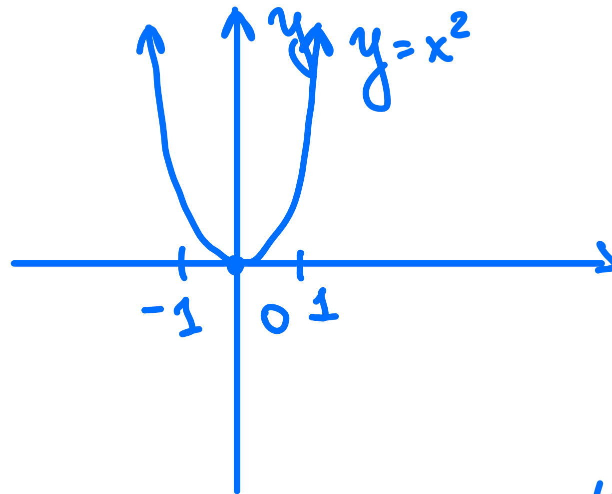
$$= \boxed{6x + 3h}$$

8. Identify the domain, the codomain, and the range of each of the following functions:

   (a) $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$

   (b) $g : \mathbb{N} \to \mathbb{R}$ by $g(x) = x^2$

   (c) $f : \mathbb{R} \to [0, \infty)$ by $f(x) = x^2$



(a) $Dom(f) = \mathbb{R}$
$Codomain(f) = \mathbb{R}$
$Ran(f) = [0, \infty)$

(b) $Dom(f) = \mathbb{N}$
$Codomain(f) = \mathbb{R}$
$Ran(f) = \{1, 4, 9, 16, 25, \ldots\}$

(c) $Dom(f) = \mathbb{R}$
$Codomain(f) = [0, \infty)$
$Ran(f) = [0, \infty)$

9. Determine the implied domain of the following functions:

   (a) $f(x) = 5x - \sqrt{3 - x}$

   (b) $g(x) = \dfrac{x - 3}{x^2 - 1}$

(a) Domain: $3 - x \geq 0$
$-x \geq -3$
$x \leq 3$

$Dom(f) = (-\infty, 3]$

(b) Domain: $x^2 - 1 \neq 0$
$(x-1)(x+1) \neq 0$
$x \neq 1$ or $x \neq -1$

$Dom(f) = \mathbb{R} \setminus \{+1, -1\}$