THEORETICAL PART:

Solutions

Theorem: The Law of Cosines

Given a triangle ABC, with sides labeled conventionally, the following equations are all true. These equations represent the **Law of Cosines**

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

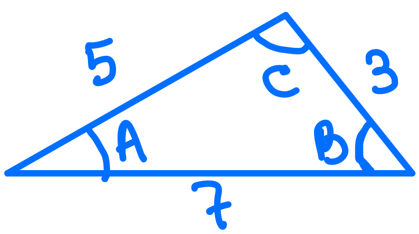
Theorem (Area of a Triangle (Heron's Formula))

Given a triangle with sides a, b, and c, let $s = \frac{a+b+c}{2}$. Then the following is true

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

PRACTICAL PART:

1. Determine the three angles for a triangle in which $a = 3$ inches, $b = 5$ inches, and $c = 7$ inches.



$$7^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cdot \cos C$$

$$\cos C = -\frac{1}{2}$$

$$\angle C = \arccos\left(-\frac{1}{2}\right) = \boxed{120^\circ}$$

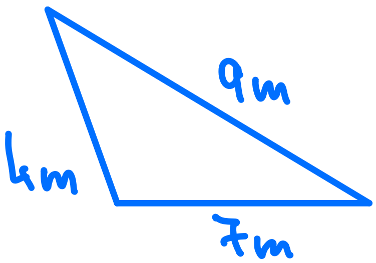
$$\frac{\sin A}{3} = \frac{\sin 120^\circ}{7}$$

$$\sin A \approx 0.37$$

$$\angle A \approx \boxed{21.79^\circ}$$

$$\angle B = 180^\circ - \angle A - \angle C \approx \boxed{38.21^\circ}$$

2. A set designer is putting together a backdrop for a play, and one element of the scene is a large triangular piece of wood. The edges of the triangle are of lengths 4 meters, 7 meters, and 9 meters. She wants to know the square area of the triangle in order to estimate the amount of paint needed to cover it.



By Heron's formula:

$$S = \frac{4+7+9}{2} = 10$$

So

$$\text{Area} = \sqrt{10(10-4)(10-7)(10-9)} =$$

$$= 6\sqrt{5}\ (m^2).$$