Name: Solutions

/ 40

Assessment 4 Instructions:

- The AS-4 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-4.
- The AS-4 is closed book and closed notes.
- **Calculators are not allowed** on the AS-4.
- Show all your work for full credit and box your final answer.

1. [4 points] For the polynomial function $k(x) = -(x+2)^3(x-1)$

a. determine the behaviour of $k(x)$ as $x \to \pm\infty$

as $x \to \infty$, $k(x) \to -\infty$

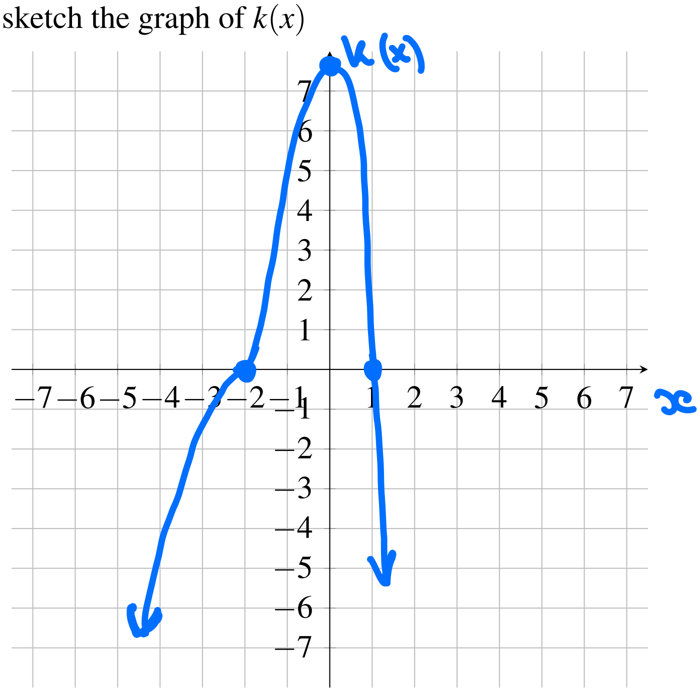
as $x \to -\infty$, $k(x) \to -\infty$

b. identify x- and y-intercepts

x-intercept: $k(x) = 0 \Rightarrow x = -2,\ x = 1$ $(-2,0), (1,0)$

y-intercept: $x = 0 \Rightarrow k(0) = 8$ $(0,8)$

c. sketch the graph of $k(x)$

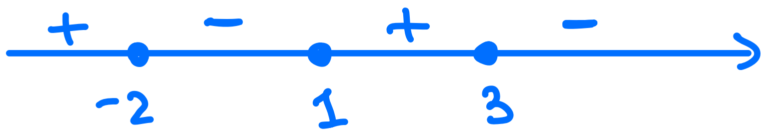


2. [4 points] Solve the following polynomial inequality

$$(x-1)(x+2)(3-x) \leq 0$$

$(x-1)(x+2)(3-x) = 0$

$x = 1$ or $x = -2$ or $x = 3$



$\boxed{\text{Solution: } [-2,1] \cup [3,\infty)}$

3. [4 points] Use polynomial long division to rewrite

$$\boxed{\frac{9x^3+2x}{3x-5} = 3x^2+5x+9+\frac{45}{3x-5}}$$

in the form $q(x)+\dfrac{r(x)}{d(x)}$.

$9x^3 + 2x \;|\; 3x-5$

$- \;\; 9x^3 - 15x^2 \;|\; 3x^2 + 5x + 9$

$15x^2 + 2x$

$- \;\; 15x^2 - 25x$

$27x$

$- \;\; 27x - 45$

45

4. [4 points] Construct the polynomial function with the stated properties:

- third degree
- zeros of -3 with multiplicity 2, and 2 with multiplicity 1
- y-intercept of -6

$p(x) = a(x+3)^2(x-2)$

$p(0) = 9a \cdot (-2) = -6$

$-18a = -6$

$a = \frac{1}{3}$

$$\boxed{p(x) = \frac{1}{3}(x+3)^2(x-2)}$$

5. [4 points] Using the **Rational Zero Theorem** list **all** possible rational real zeros of the following polynomial function

$$f(x) = 2x^3 - 12x^2 + 26x - 40$$

$a_0 = -40$ factors: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

$a_n = 2$ factors: $\pm 1, \pm 2$

Answer: $\pm\{1, \frac{1}{2}, 2, 4, 5, \frac{5}{2}, 8, 10, 20, 40\}$

6. [4 points] Use the **Intermediate Value Theorem** to show that there exists at least one real zero between the indicated values of the given polynomial function. (*Hint: calculate $f(a)$ and $f(b)$*)

$$f(x) = x^4 - 9x^2 - 14, \quad a = 1, \quad b = 4$$

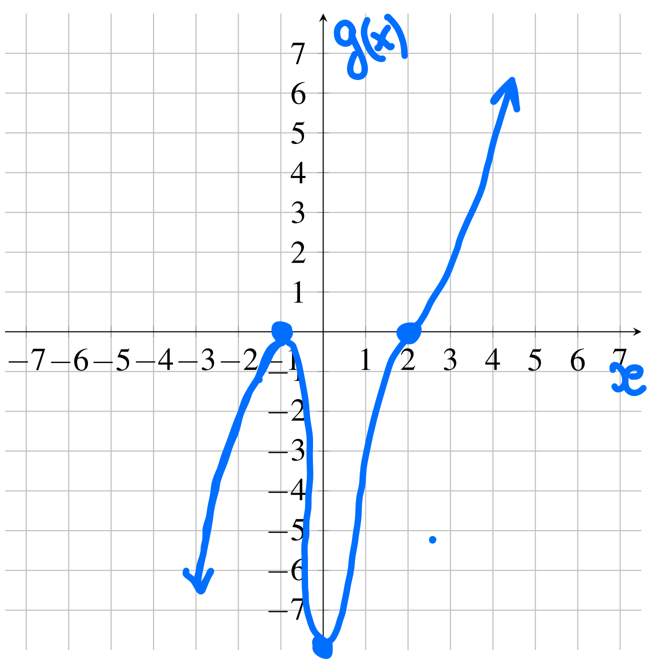
$f(a) = f(1) = 1 - 9 - 14 = -22 < 0$

$f(b) = f(4) = 256 - 9 \cdot 16 - 14 > 0$

Hence, there is at least one $1 < c < 4$ such that $f(c) = 0$.

7. [4 points] Sketch the graph of the factored polynomial function. State **all** x- and y-intercept points.

$$g(x) = (x+1)^2(x-2)^3$$



x-intercept: $(-1, 0)$, $(2, 0)$

y-intercept: $(0, -8)$

8. [4 points] For the given function $f(x) = \dfrac{x+2}{x^2-9}$

a. Find the domain of $f(x)$

$x^2 - 9 = 0$

$x = \pm 3$

$\mathrm{Dom}(f) = \mathbb{R} \setminus \{\pm 3\}$

b. Find all *vertical* asymptotes

$x = 3$

$x = -3$

c. Find all *horizontal* asymptotes

$p(x) = x+2 \quad , n = 1$

$q(x) = x^2 - 9 \quad , m = 2$

$n < m$

$\boxed{y = 0 \text{ is a H.A.}}$

d. Does $f(x)$ have an oblique asymptote? If yes, then state it. If not, then explain why it doesn't have it.

No, since $n < m$.

9. [4 points] Solve the following rational inequality

$$\frac{x-7}{x-3} \geq \frac{x}{x-1}$$

$$\frac{x-7}{x-3} - \frac{x}{x-1} \geq 0$$

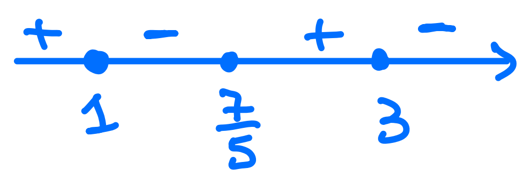
$$\frac{x-7}{x-3} - \frac{x}{x-1} = \frac{(x-7)(x-1) - x(x-3)}{(x-3)(x-1)} = 0$$

$x = 3, \; x = 1$

$\cancel{x^2} - 8x + 7 - \cancel{x^2} + 3x = 0$

$-5x = -7$

$x = \frac{7}{5}$

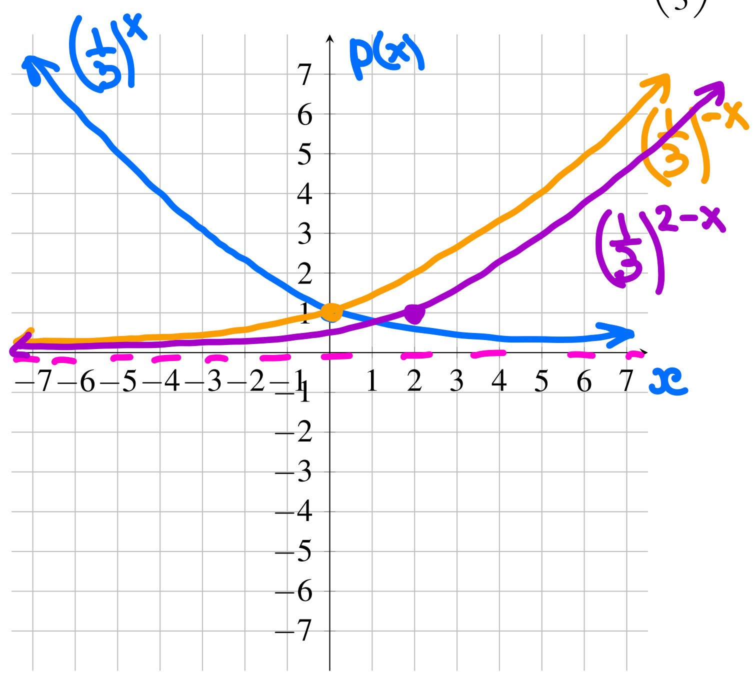


Answer:

$\boxed{(-\infty, 1] \cup [\tfrac{7}{5}, 3]}$

10. [4 points]

a. Sketch the graph of the following function $p(x) = \left(\frac{1}{3}\right)^{2-x}$ $= -(x-2)$



• $p(x) = \left(\frac{1}{3}\right)^{2-x}$

b. Solve the following exponential equation

$$7^{x^2+3x} = \frac{1}{49}$$

$$7^{x^2+3x} = 7^{-2}$$

$$x^2 + 3x = -2$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$\boxed{x = -2 \quad \text{or} \quad x = -1}$$