Solutions

THEORETICAL PART:

Definition:

A linear inequality in the two variables x and y is an inequality that can be written in the form:

$$ax + by < c, \quad ax + by > c, \quad ax + by \leq c, \quad ax + by \geq c,$$

where a, b, c are constants and a and b are not both 0.

Procedure (Solving linear inequalities in two variables):

- Graph the line that results from replacing the inequality symbol with =.
- Make the line solid if the inequality sign is $\leq$ or $\geq$ and dashed if the symbol is $<$ or $>$. A solid line indicates that points on the line are included in the solution set while a dashed line indicates that points on the line are excluded from the solution set.
- Determine which of the half-planes defined by the boundary line solves the inequality by substituting a **test point** from one of the two half-planes into the inequality. If the resulting numerical statement is true, all the points in the same half-plane as the test point solve the inequality. Otherwise, the points in the other half-plane solve the inequality. Shade in the half-plane that solves the inequality.

Definition: Absolute value inequality meaning:

$$|x| < a \equiv x < a \quad \text{and} \quad x > -a$$

$$|x| > a \equiv x > a \quad \text{or} \quad x < -a$$

PRACTICAL PART:

1. Solve the following linear inequalities by graphing their solution sets:

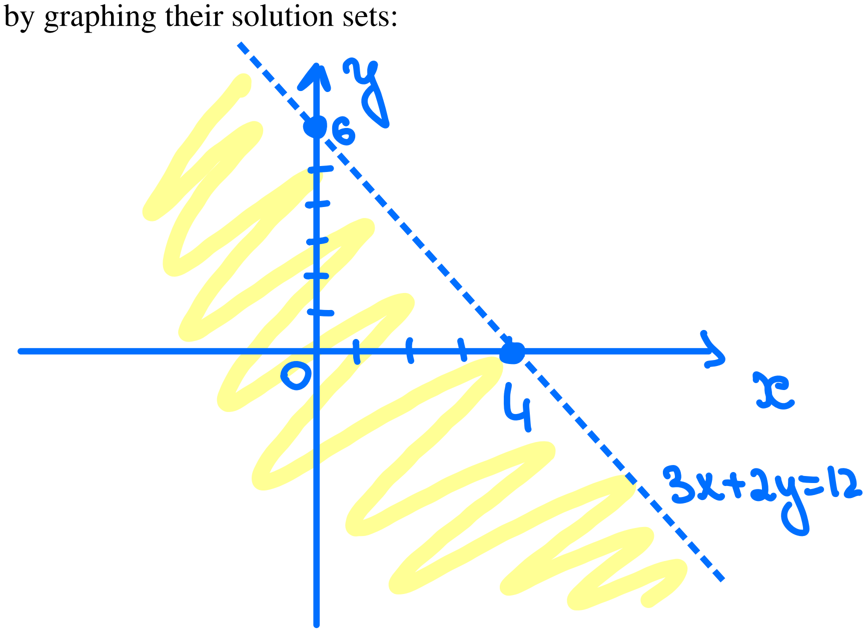
   (a) $3x + 2y < 12$

   (b) $x - y \leq 0$

(a) $3x + 2y < 12$

$3x + 2y = 12$

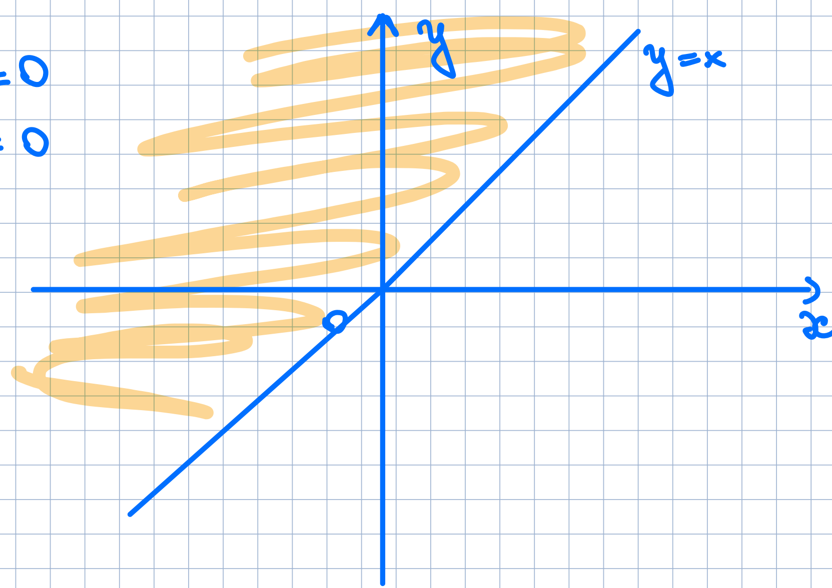
x	y
0	6
4	0



(b) $x - y \leq 0$

$x - y = 0$

$x = y$



2. Graph the solution sets that satisfy the following inequalities:
   - $5x - 2y < 10$ and $y \leq x$.
   - $x + y < 4$ or $x \geq 4$.

(a) $5x-2y<10$ "and" $y \leq x$

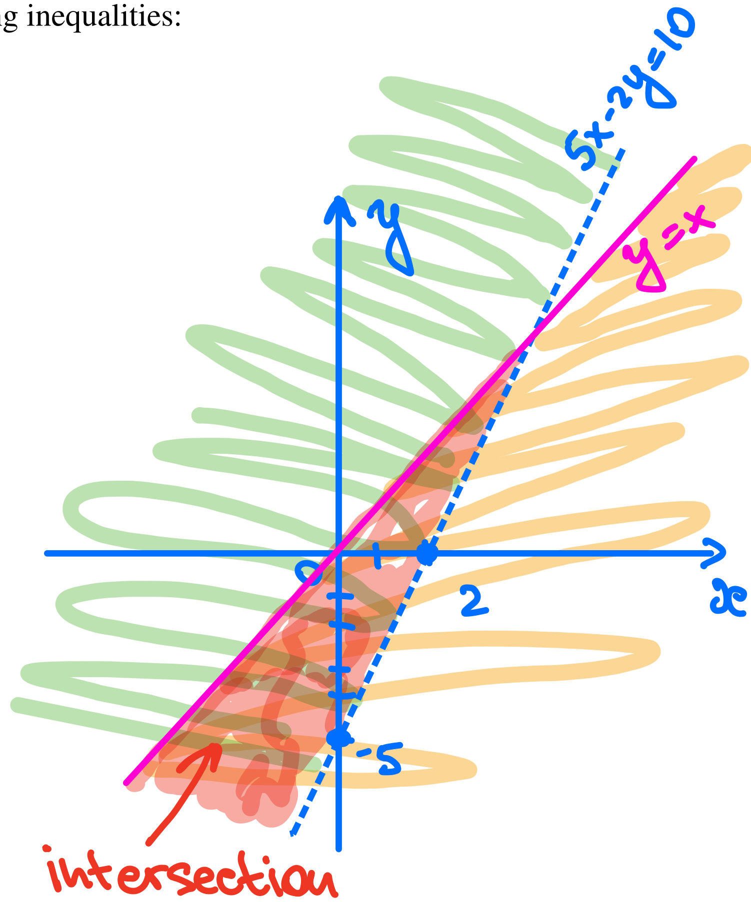
↑ intersection

- $5x-2y=10$

x	y
0	-5
2	0

- $y \leq x$

$y=x$

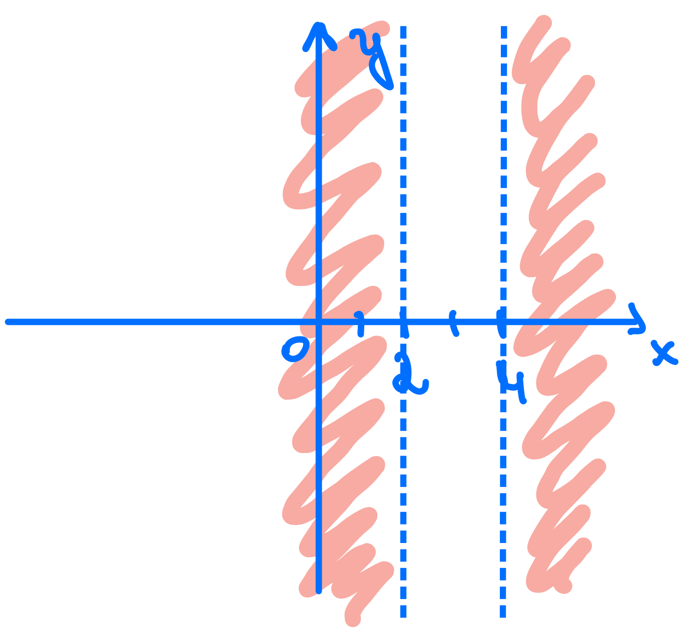


3. Graph the solution set in $\mathbb{R}^2$ that satisfies the joint conditions $|x-3| > 1$ and $|y-2| \leq 3$.

$|x-3|>1$ and $|y-2| \leq 3$

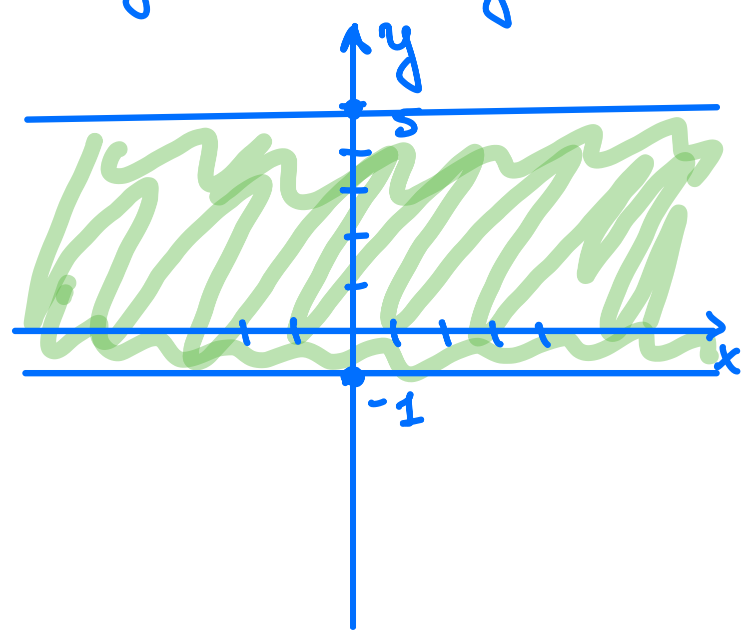
$x-3>1$ or $x-3<-1$

$x>4$ or $x<2$



$y-2 \leq 3$ and $y-2 \geq -3$

$y \leq 5$ and $y \geq -1$



Now we need to sketch the intersection of the two above pictures.