Section 6.2. Exponential models

1. Models of population growth.
2. Models of radioactive decay.
3. Compound interest and the number e.
4. Exponential regression.

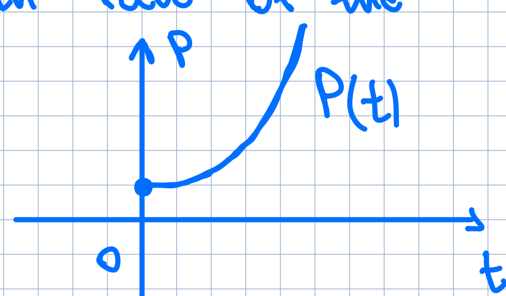
1.

In any situation where the rate of growth of a population is proportional to the size of the population, the population will grow exponentially,

$$\boxed{P(t) = P_0 a^t}$$

- $P(t)$ – population at time t
- At $t=0$: $P(0) = P_0$ – initial population
- $a > 1$ is the growth rate of the population

$$\boxed{P(t) = P_0 a^t}$$

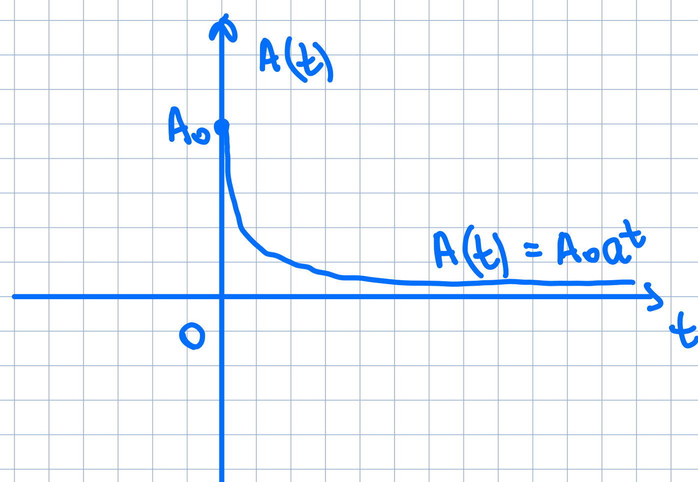


$\boxed{2.}$ Radioactive decay is modeled by:

$$A(t) = A_0 a^t,$$

where

- $A(t)$ – amount of a given substance at time t
- A_0 – amount at time $t=0$
- $0 < a < 1$



$\boxed{3.}$

Formula: (Compound Interest Formula)

An investment of P dollars, compounded n times per year at an annual interest rate of r, has a value after t years of

$$\boxed{A(t) = P\left(1 + \frac{r}{n}\right)^{nt}}$$

Def. (The Number e)

The number e is defined as the value of $\left(1+\frac{1}{m}\right)^m$ as $m \to \infty$.

$$e \approx 2.718281828\,46$$

Formula: (Continuous Compounding Formula)

An investment of P dollars, compounded continuously at an annual interest rate of r, has a value after t years of

$$\boxed{A(t) = P e^{rt}}$$

$\boxed{4.}$ Exponential regression can be used to fit an exponential curve to points that we suspect exhibit exponential behaviour.

Exponential regression is the process of determining constants a and b so that the graph of the function $f(x) = ab^x$ models the given data well.

Logistic curves are a family of curves

based on exponential functions that are designed to model behaviour often seen in biology, ecology, ...

The graph of a logistic function, known as a <u>sigmoid curve</u>, is S-shaped.

Logistic Function:

$$\boxed{f(x) = \frac{c}{1 + a(e^{-bx})}},$$

where $a, b, c > 0$.