

Name: \_\_\_\_\_

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Assessment 4 Instructions:

- The AS-4 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-4.
- The AS-4 is closed book and closed notes.
- **Calculators are not allowed** on the AS-4.
- Show all your work for full credit and box your final answer.

1. [4 points] For the polynomial function  $k(x) = -(x+2)^3(x-1)$

a. determine the behaviour of  $k(x)$  as  $x \rightarrow \pm\infty$

$$\text{as } x \rightarrow \infty, k(x) \rightarrow -\infty$$

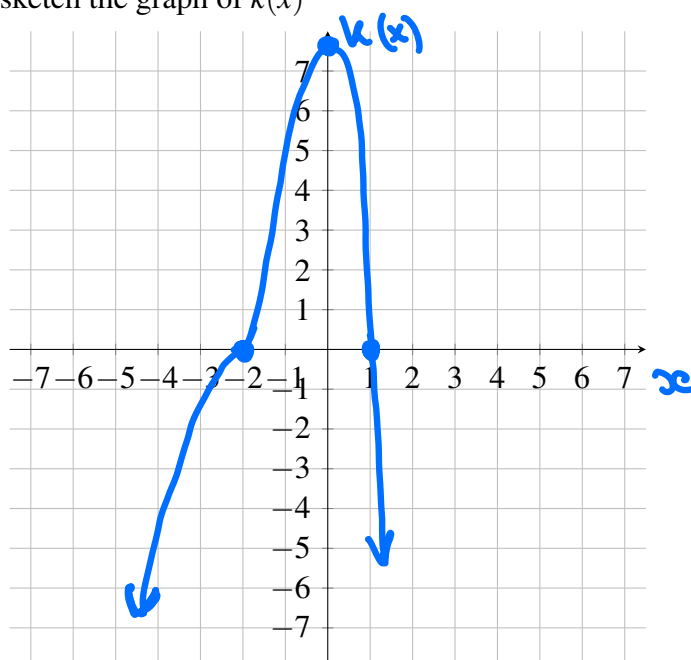
$$\text{as } x \rightarrow -\infty, k(x) \rightarrow -\infty$$

b. identify x- and y-intercepts

$$\text{x-intercept: } k(x) = 0 \Rightarrow x = -2, x = 1 \quad (-2, 0), (1, 0)$$

$$\text{y-intercept: } x = 0 \Rightarrow k(0) = 8 \quad (0, 8)$$

c. sketch the graph of  $k(x)$

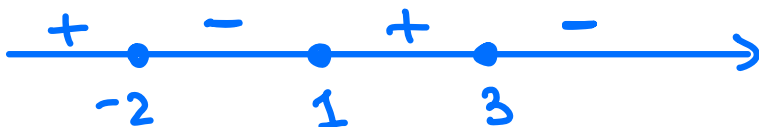


2. [4 points] Solve the following polynomial inequality

$$(x-1)(x+2)(3-x) \leq 0$$

$$(x-1)(x+2)(3-x) = 0$$

$$x = 1 \text{ or } x = -2 \text{ or } x = 3$$



Solution:  
 $[-2, 1] \cup [3, \infty)$

3. [4 points] Use polynomial long division to rewrite

$$\frac{9x^3 + 2x}{3x - 5} = 3x^2 + 5x + 9 + \frac{45}{3x - 5}$$

in the form  $q(x) + \frac{r(x)}{d(x)}$ .

$$\begin{array}{r}
 9x^3 + 2x \overline{) 3x - 5} \\
 \underline{9x^3 - 15x^2} \phantom{+ 9} \\
 15x^2 + 2x \phantom{+ 9} \\
 \underline{15x^2 - 25x} \phantom{+ 9} \\
 27x \phantom{+ 9} \\
 \underline{27x - 45} \\
 45
 \end{array}$$

4. [4 points] Construct the polynomial function with the stated properties:

- third degree
- zeros of  $-3$  with multiplicity 2, and 2 with multiplicity 1
- y-intercept of  $-6$

$$p(x) = a(x+3)^2(x-2)$$

$$p(0) = 9a(-2) = -6$$

$$-18a = -6$$

$$a = \frac{1}{3}$$

$$p(x) = \frac{1}{3}(x+3)^2(x-2)$$

5. [4 points] Using the **Rational Zero Theorem** list **all** possible rational real zeros of the following polynomial function

$$f(x) = 2x^3 - 12x^2 + 26x - 40$$

$$a_0 = -40 \quad \text{factors: } \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$$

$$a_n = 2 \quad \text{factors: } \pm 1, \pm 2$$

$$\text{Answer: } \pm \left\{ 1, \frac{1}{2}, 2, 4, 5, \frac{5}{2}, 8, 10, 20, 40 \right\}$$

6. [4 points] Use the **Intermediate Value Theorem** to show that there exists at least one real zero between the indicated values of the given polynomial function. (Hint: calculate  $f(a)$  and  $f(b)$ )

$$f(x) = x^4 - 9x^2 - 14, \quad a = 1, \quad b = 4$$

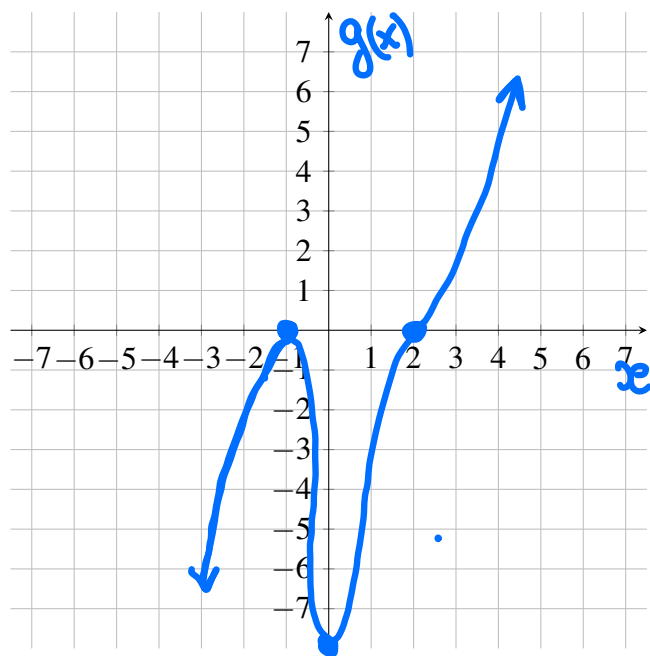
$$f(a) = f(1) = 1 - 9 - 14 = -22 < 0$$

$$f(b) = f(4) = 256 - 9 \cdot 16 - 14 > 0$$

Hence, there is at least one  $1 < c < 4$   
Such that  $f(c) = 0$ .

7. [4 points] Sketch the graph of the factored polynomial function. State **all**  $x$ - and  $y$ -intercept points.

$$g(x) = (x+1)^2(x-2)^3$$



$x$ -intercept:  $(-1, 0)$   
 $(2, 0)$

$y$ -intercept:  $(0, -8)$

8. [4 points] For the given function  $f(x) = \frac{x+2}{x^2-9}$

a. Find the domain of  $f(x)$

$$\text{Dom}(f) = \mathbb{R} \setminus \{\pm 3\}$$

$$\begin{aligned} x^2 - 9 &= 0 \\ x &= \pm 3 \end{aligned}$$

b. Find all *vertical* asymptotes

$$\begin{aligned} x &= 3 \\ x &= -3 \end{aligned}$$

c. Find all *horizontal* asymptotes

$$\begin{aligned} p(x) &= x+2, \quad n=1 \\ q(x) &= x^2-9, \quad m=2 \end{aligned}$$

$$n < m$$

$$y=0 \text{ is a H.A.}$$

d. Does  $f(x)$  have an oblique asymptote? If yes, then state it. If not, then explain why it doesn't have it.

No, Since  $n < m$ .

9. [4 points] Solve the following rational inequality

$$\frac{x-7}{x-3} \geq \frac{x}{x-1}$$

$$\begin{aligned} x &\neq 3 \\ x &\neq 1 \end{aligned} \quad (\text{domain})$$

$$\frac{x-7}{x-3} - \frac{x}{x-1} \geq 0$$

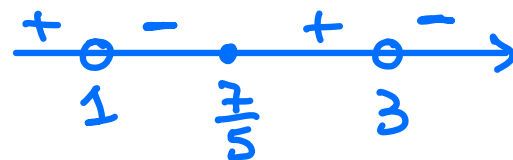
$$\frac{x-7}{x-3} - \frac{x}{x-1} = \frac{(x-7)(x-1) - x(x-3)}{(x-3)(x-1)} \geq 0$$

$$x=3, x=1$$

$$\cancel{x^2} - 8x + 7 - \cancel{x^2} + 3x = 0$$

$$-5x = -7$$

$$x = \frac{7}{5}$$

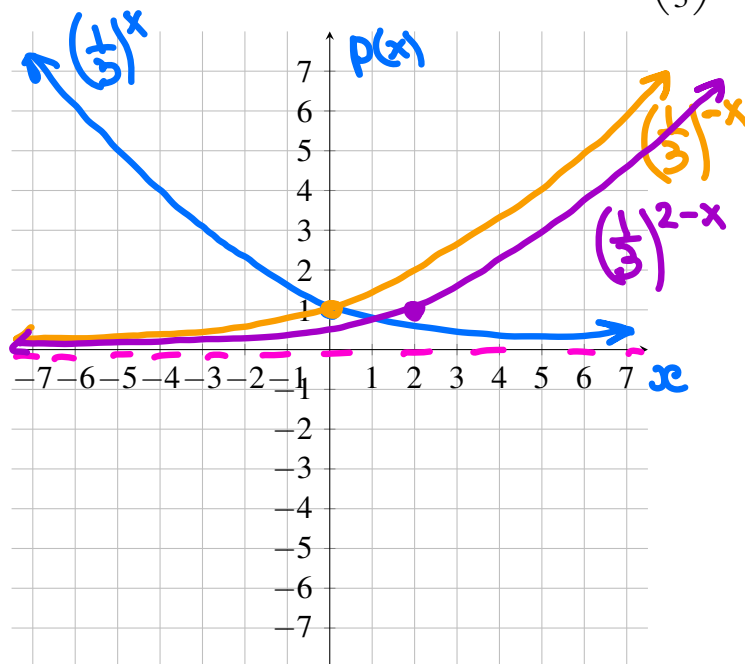


Answer:

$$(-\infty, 1) \cup \left[ \frac{7}{5}, 3 \right)$$

10. [4 points]

- a. Sketch the graph of the following function  $p(x) = \left(\frac{1}{3}\right)^{2-x} = -(x-2)$



•  $p(x) = \left(\frac{1}{3}\right)^{2-x}$

- b. Solve the following exponential equation

$$7^{x^2+3x} = \frac{1}{49}$$

$$7^{x^2+3x} = 7^{-2}$$

$$x^2 + 3x = -2$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2 \quad \text{or} \quad x = -1$$