THEORETICAL PART:

Definition (Period of a function):

A function f is said to be **periodic** if there is a positive number p such that

$$f(x+p) = f(x)$$

for all x in the domain of f. The smallest such number p is called the **period** of f.

Identities (Even/Odd Identities):

$$\sin(-x) = -\sin(x), \quad \cos(-x) = \cos(x), \quad \tan(-x) = -\tan(x),$$
$$\csc(-x) = -\csc(x), \quad \sec(-x) = \sec(x), \quad \cot(-x) = -\cot(x)$$

Definition (Amplitude of Sine and Cosine curves):

Given a fixed real number a, the **amplitude** of the function $f(x) = a\sin(x)$ or the function $g(x) = a\cos(x)$ is the value $|a|$. As we know, the multiplication of $\sin(x)$ or $\cos(x)$ by a stretches (or compresses, if $-1 < a < 1$) the graph vertically by a factor of $|a|$, so the amplitude represents the distance between the x-axis and the maximum value of the function.

Definition (Frequency of Sine and Cosine curves):

Given a fixed real number b, the **frequency** of the function $f(x) = \sin(bx)$ or the function $g(x) = \cos(bx)$ is the number $b/2\pi$. When the independent variable represents time, measured in seconds, the measurement of frequency is stated in terms of **cycles per second**, or **hertz (Hz)**.

Definition (Period Revisited):

Given a fixed real number b, the **period** of the function $f(x) = \sin(bx)$ or the function $g(x) = \cos(bx)$ is the number $2\pi/2$. The period and frequency of a sinusoidal function are reciprocals of one another.

Definition (Amplitude, Period, and Phase Shift Combined):

Given constants a, b (such that $b > 0$), and c, the functions

$$f(x) = a\sin(bx - c), \quad g(x) = a\cos(bx - c)$$

have **amplitude** $|a|$, **period** $2\pi/2$, and a **phase shift** of c/b. The left endpoint of one cycle of either function is c/b and the right endpoint is $c/b + 2\pi/b$.

Definition (Simple Harmonic Motion):

If an object is oscillating and its displacement from some midpoint at time t can be described by either $f(t) = a\sin(bt)$ or $g(t) = a\cos(bt)$, the object is said to be in **simple harmonic motion (SHM)**. In both cases, the maximum displacement of the object from its midpoint is the amplitude $|a|$ and its frequency of oscillation is $b/2\pi$.

PRACTICAL PART:

1. Based on graphs of sine and cosine functions and the values of sine and cosine, construct a transformation of cosine that is equal to sine.

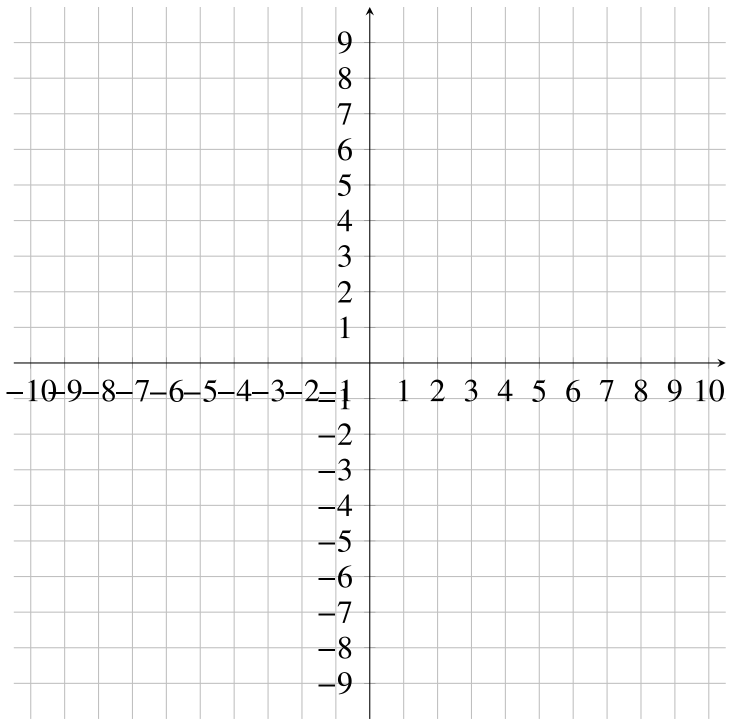
2. Determine the periods of the secant, cosecant, tangent, and cotangent functions.

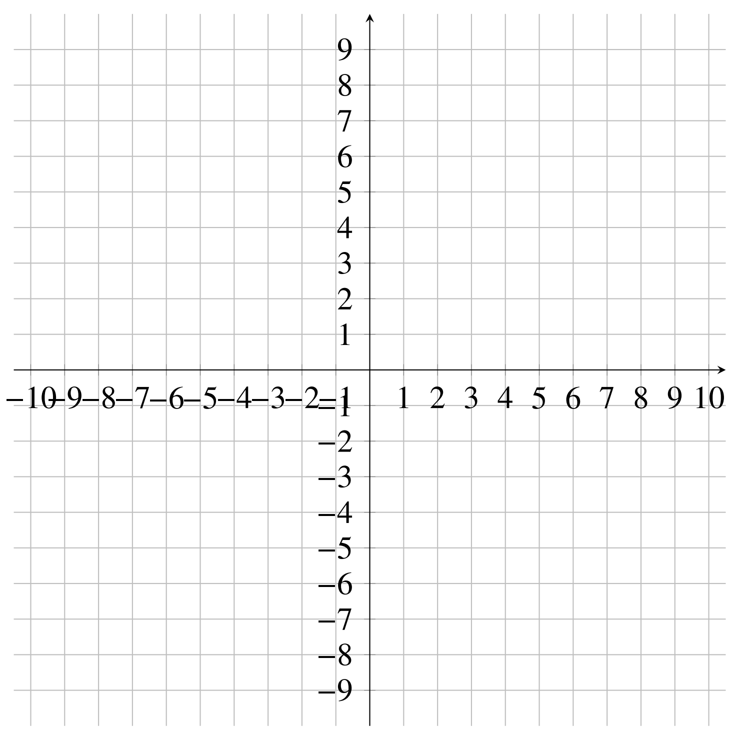
3. Use a cofunction identity and an even/odd identity to prove the transformation statement $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ for all x.

4. Determine the amplitude and frequency of each of the following functions. Then use your results to sketch the graph of one complete cycle of each function starting at $x = 0$.

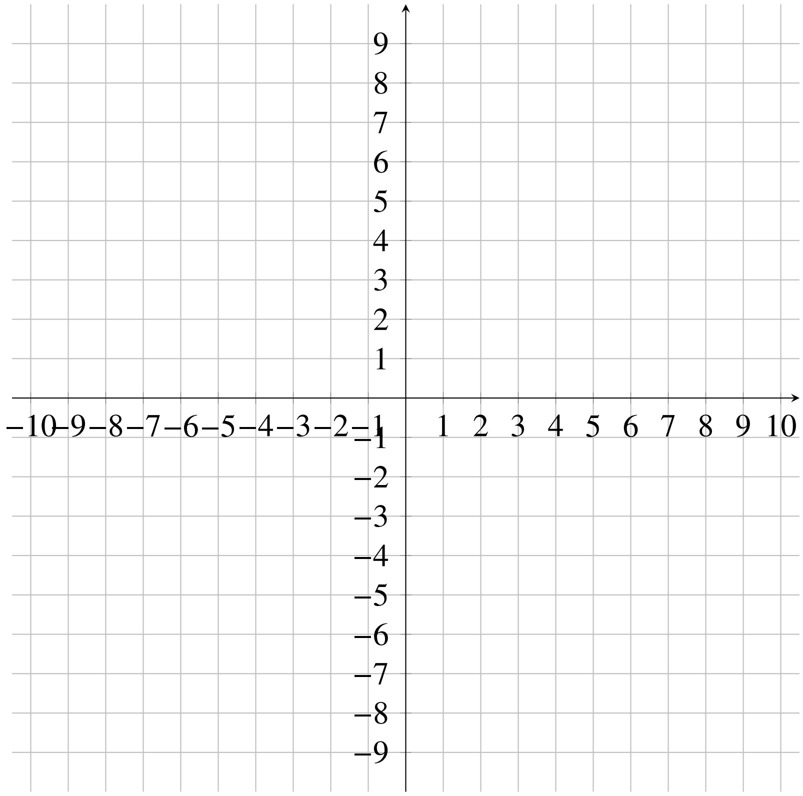
   a. $f(x) = 3\sin(x/2)$

   b. $g(x) = -\frac{1}{2}\cos(2\pi x)$





5. Sketch the graph of $f(x) = -2\cos(\pi x - \pi)$.



6. A heart rate of 1200 beats per minute (bpm) is typical for a hummingbird. What is the length of the period, in seconds, of such a heart rate?

7. Sketch the graph of $f(t) = -4e^{-t}\cos(6\pi t)$.