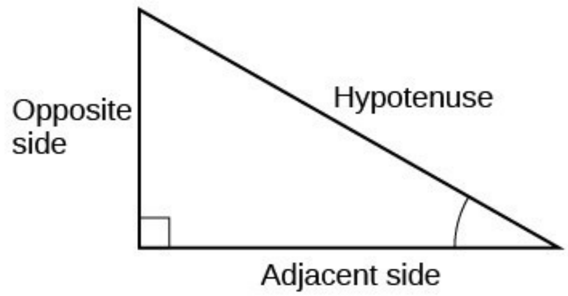
Solutions

THEORETICAL PART:

Definition (Sine, Cosine, and tangent):

Assume θ is one of the acute (less than a right angle) angles in a right triangle, and let adj and opp stand for, respectively, the lengths of the legs adjacent to and opposite θ. Let hyp stand for the length of the hypotenuse of the right triangle. Then the **sine**, **cosine**, and **tangent** of θ, are the ratios

$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}, \quad \tan\theta = \frac{opp}{adj}.$$



Definition (Cosecant, Secant, and Cotangent)

Assume θ is one of the acute angles in a right triangle. Then the **cosecant**, **secant**, and **cotangent** of θ, are the reciprocals of $\sin\theta$, $\cos\theta$, and $\tan\theta$. That is,

$$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}, \quad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}.$$

Definition (Degree, Minute, and Second Notation):

In the context of angle measure,

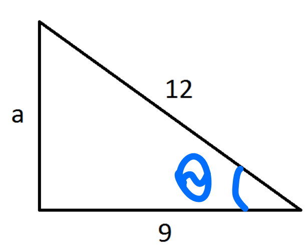
$$1' = \textit{one minute} = \left(\frac{1}{60}\right)(1^\circ)$$

and

$$1'' = \textit{one second} = \left(\frac{1}{60}\right)(1') = \left(\frac{1}{3600}\right)(1^\circ).$$

PRACTICAL PART:

1. Use the information contained in the two figures to determine the values of the six trigonometric functions of θ.



$9^2 + a^2 = 144$

$a^2 = 144 - 81$

$a^2 = 63$

$a = \sqrt{63}$

$$\sin\theta = \frac{a}{12} = \frac{\sqrt{63}}{12}$$

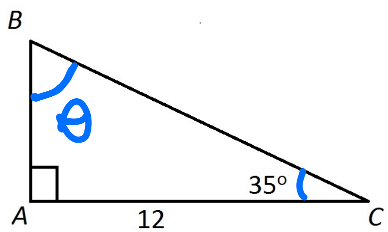
$$\cos\theta = \frac{9}{12} = \frac{3}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{63}}{12} \cdot \frac{4}{3} = \frac{\sqrt{63}}{9}$$

$$\cot\theta = \frac{9}{\sqrt{63}}$$

$$\sec\theta = \frac{4}{3}$$

$$\csc\theta = \frac{12}{\sqrt{63}}$$



$$\tan 35^\circ = \frac{AB}{12}$$

$$AB = \tan(35^\circ) \cdot 12$$

$$BC = \sqrt{144 + 144 \cdot \tan(35^\circ)}$$

$$\sin\theta = \frac{12}{BC}$$

$$\cos\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{12}{AB}$$

$$\cot\theta = \frac{AB}{12}$$

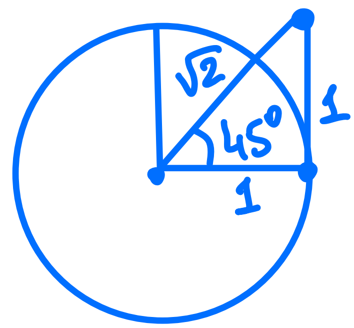
$$\sec\theta = \frac{BC}{AB}$$

$$\csc\theta = \frac{BC}{12}$$

2. Evaluate the tangent and secant of $\theta = \frac{\pi}{4}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = \sqrt{2}$$



3. Use a calculator to evaluate the following expressions.

   a. $\sin(56.4°)$

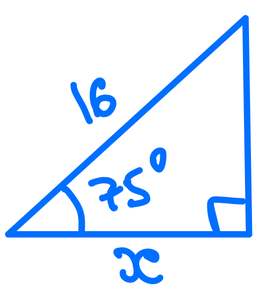
   b. $\cot(5\pi/11)$

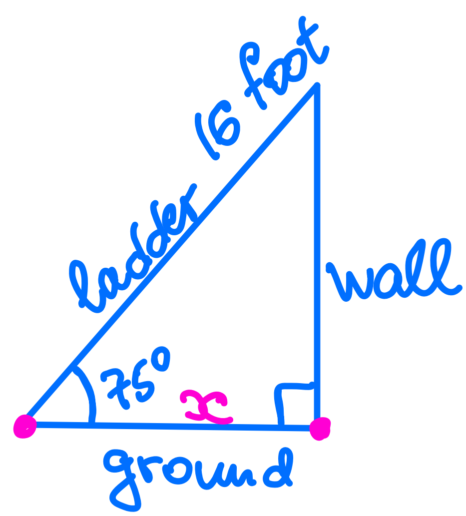
(a) $\sin(56.4°) \approx 0.8329$

(b) $\cot\left(\frac{5\pi}{11}\right) \approx 0.1438$

4. The manufacturer of a certain brand of 16-foot ladder recommends that, when in use, the angle between the ground and the ladder should equal 75°. What distance should the foot of the ladder be from the base of the wall it is leaning against?

x - distance ?





$$\cos 75° = \frac{x}{16}$$

$$x = 16 \cdot \cos 75° \approx \boxed{4.14 \text{ (ft)}}$$

Example

(a) $61^\circ\, 55'\, 39'' = 61^\circ + 55 \cdot \frac{1}{60} + 39 \cdot \frac{1}{3600} =$

$= 61.9275^\circ.$