

Solutions

THEORETICAL PART:

Theorem (Horizontal Shifting/Translation):

Let $f(x)$ be a function, and let h be a fixed real number. If we replace x with $x - h$, we obtain a new function $g(x) = f(x - h)$. The graph of g has the same shape as the graph of f , but shifted h units to the right if $h > 0$ and shifted h units to the left if $h < 0$.

Theorem (Vertical Shifting/Translation):

Let $f(x)$ be a function whose graph is known, and let k be a fixed real number. The graph of the function $g(x) = f(x) + k$ is the same shape as the graph of f , but shifted k units up if $k > 0$ and k units down if $k < 0$.

Theorem (Reflecting with Respect to the Axes):

Given a function $f(x)$,

1. the graph of the function $g(x) = -f(x)$ is the reflection of the graph of f with respect to the x -axis;
2. the graph of the function $g(x) = f(-x)$ is the reflection of the graph of f with respect to the y -axis.

Theorem (Vertical Stretching and Compressing):

Let $f(x)$ be a function and let a be a positive real number.

1. The graph of the function $g(x) = af(x)$ is stretched vertically compared to the graph of f by a factor of a if $a > 1$.
2. The graph of the function $g(x) = af(x)$ is compressed vertically compared to the graph of f by a factor of a if $0 < a < 1$.

Theorem (Horizontal Stretching and Compressing):

Let $f(x)$ be a function and let a be a positive real number.

1. The graph of the function $g(x) = f(ax)$ is stretched horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $0 < a < 1$.
2. The graph of the function $g(x) = f(ax)$ is compressed horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $a > 1$.

Order of Transformations:

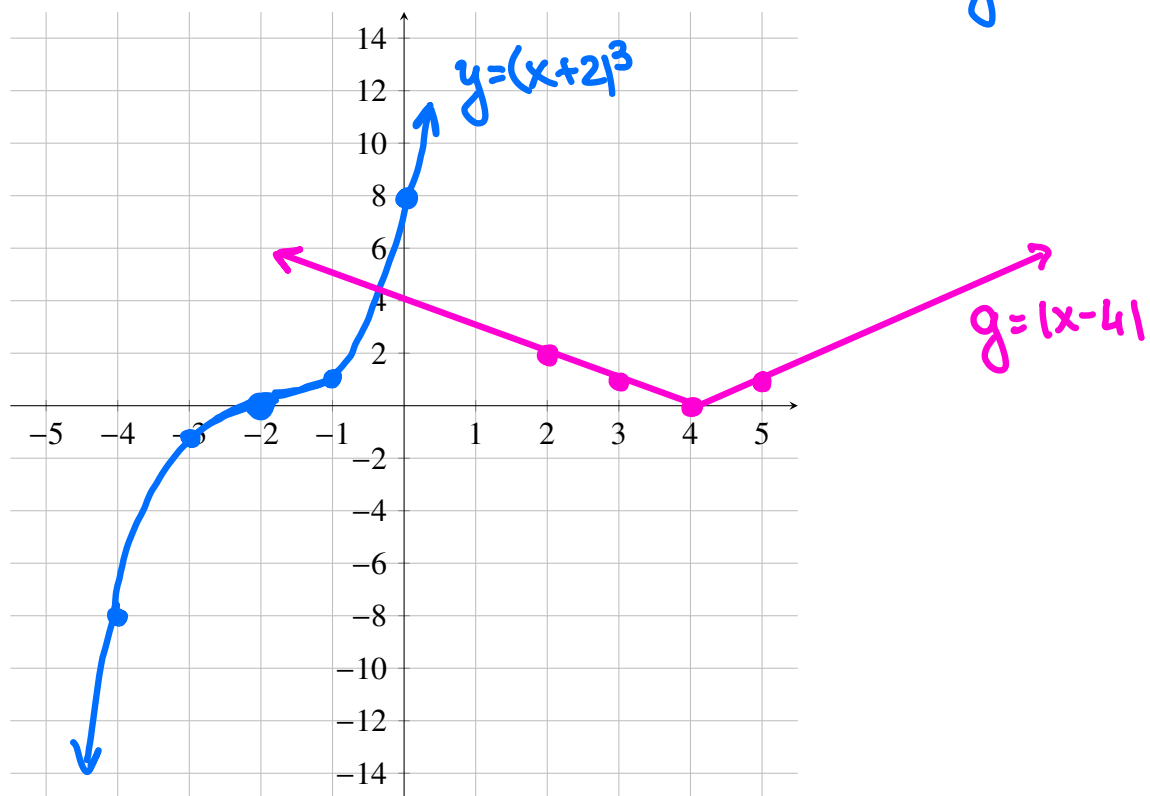
1. Horizontal shifts.
2. Horizontal and vertical stretching and compressing.
3. Reflections.
4. Vertical shifts.

PRACTICAL PART:

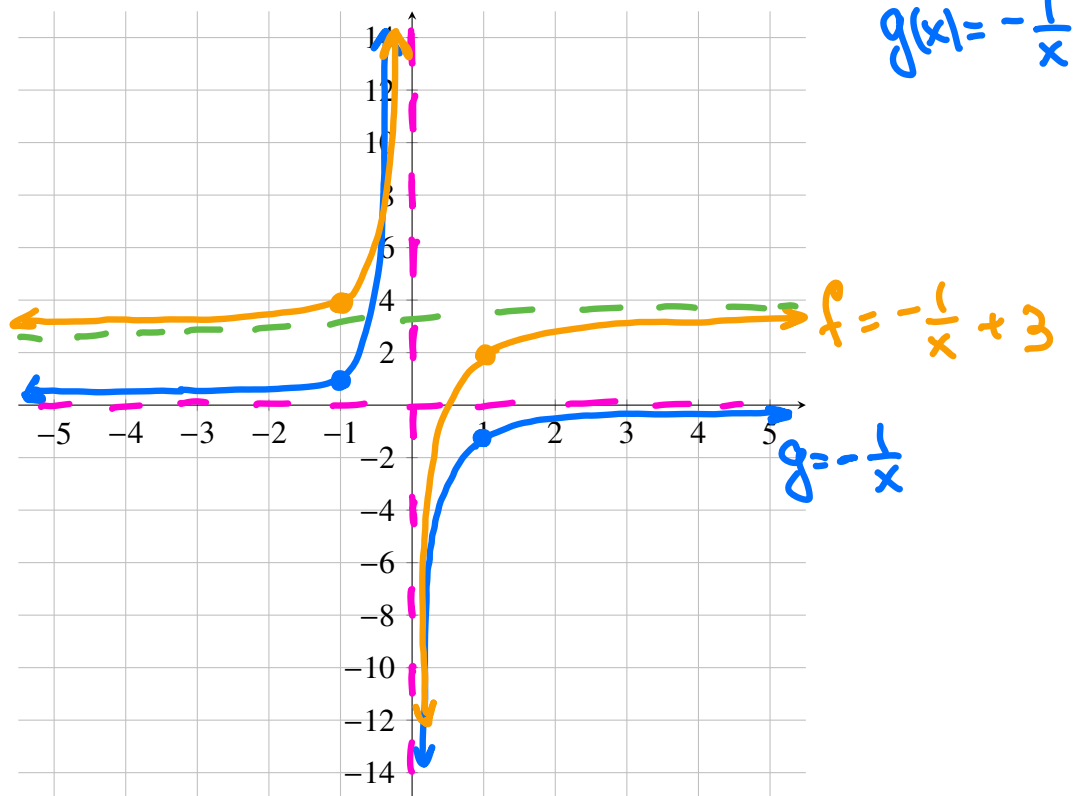
1. Sketch the graphs of the following functions:

(a) $f(x) = (x + 2)^3$ - horizontal shift to the left

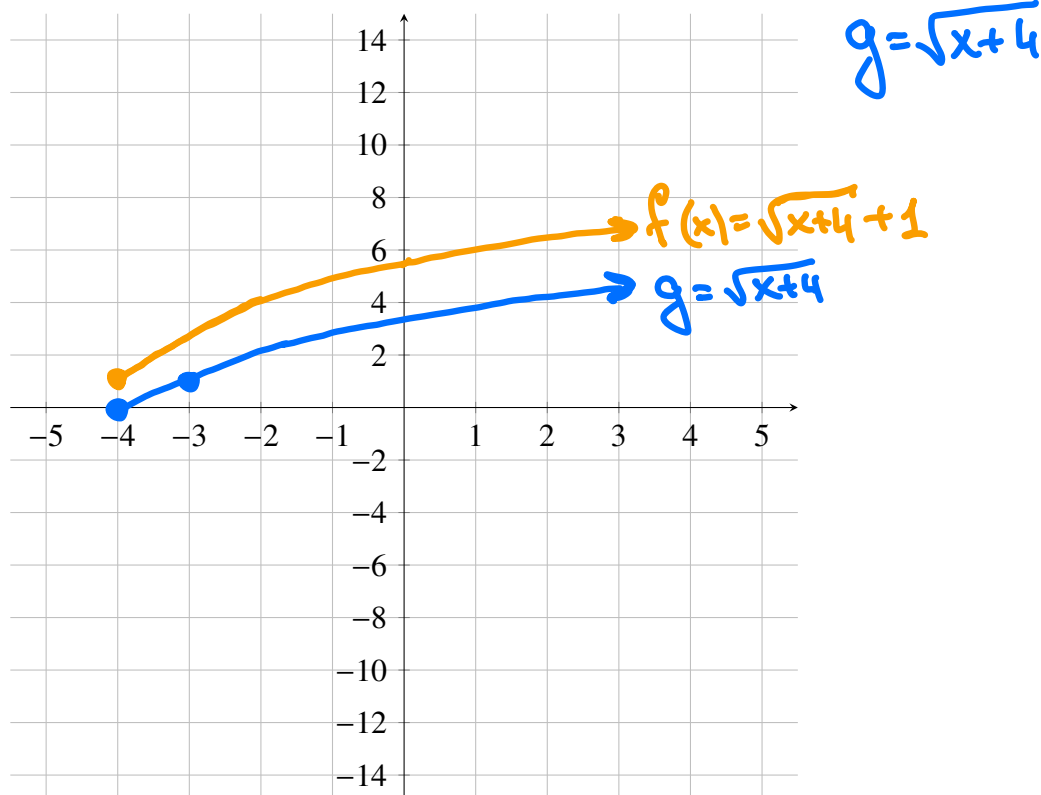
(b) $g(x) = |x - 4|$ - horizontal shift to the right



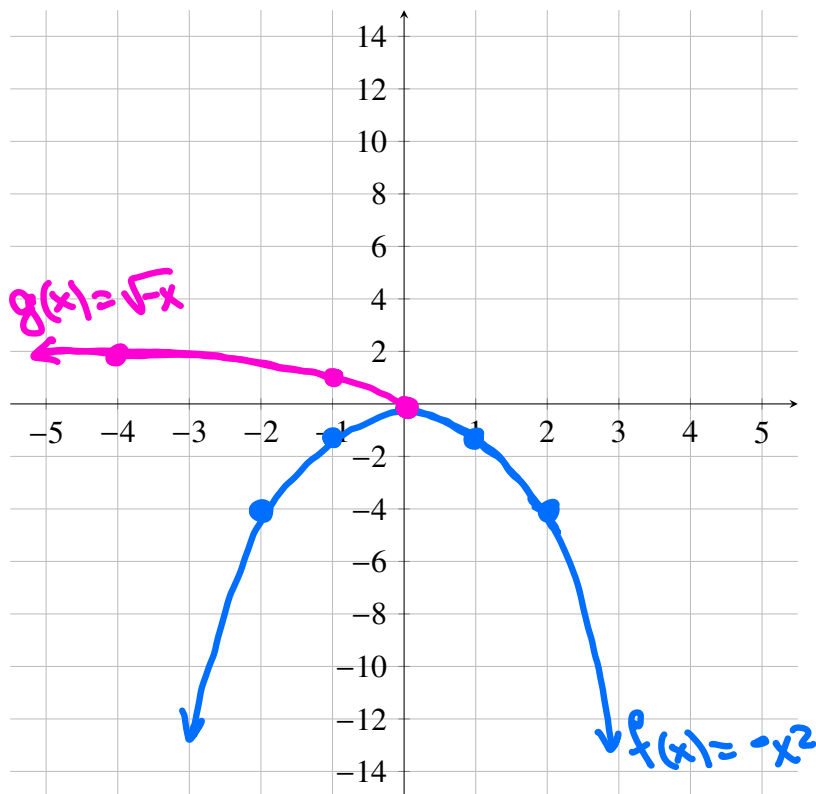
2. Sketch the graph of the function $f(x) = -\frac{1}{x} + 3$.



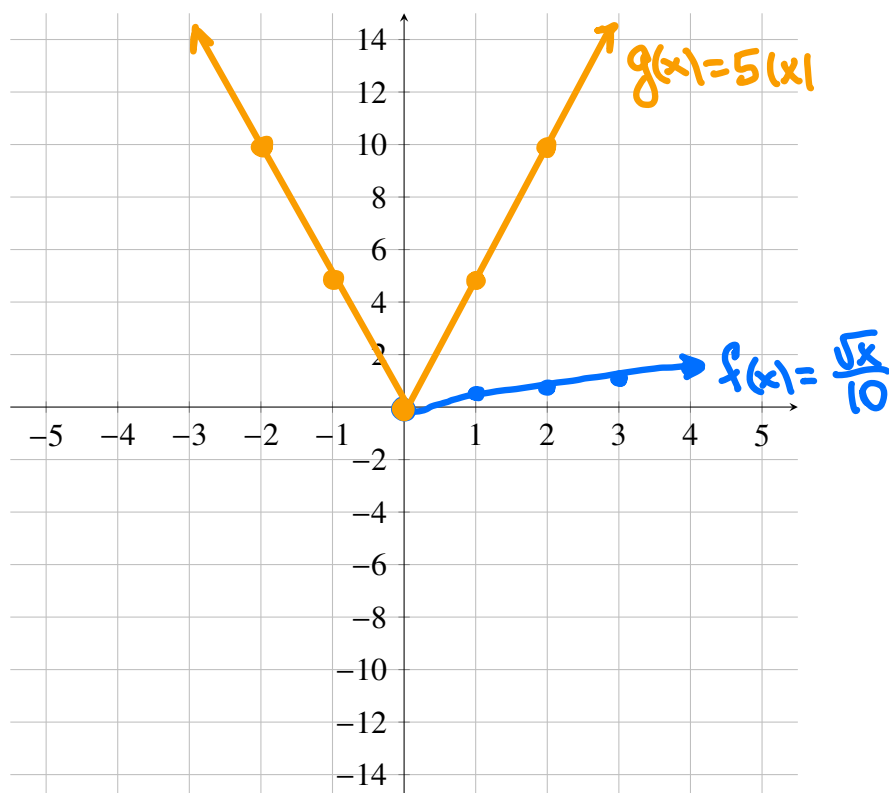
3. Sketch the graph of the function $f(x) = \sqrt{x+4} + 1$.



4. Sketch the graphs of the following functions $f(x) = -x^2$, $g(x) = \sqrt{-x}$.



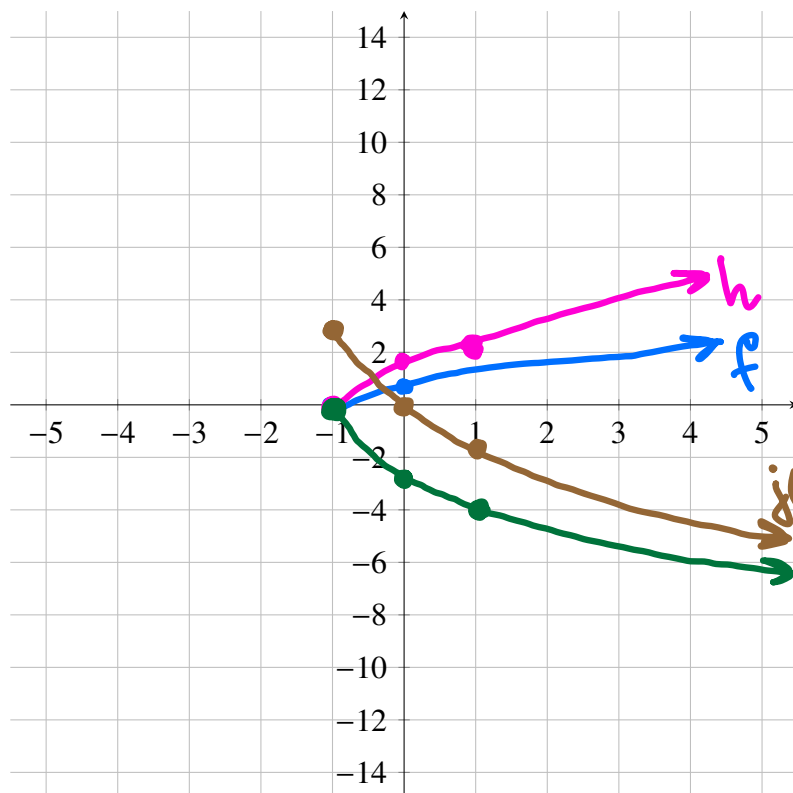
5. Sketch the graphs of the following functions $f(x) = \frac{\sqrt{x}}{10}$, $g(x) = 5|x|$.



6. Describe the transformations needed to construct the function

$$g(x) = -2\sqrt{2x+2} + 3$$

from the basic square root function using the order of transformations.



$$\begin{aligned} f(x) &= \sqrt{x+1} \\ h(x) &= \sqrt{2(x+1)} \\ k(x) &= -2\sqrt{2(x+1)} \\ j(x) &= -2\sqrt{2(x+1)} + 3 \end{aligned}$$