

Name: \_\_\_\_\_

Solutions

\_\_\_\_\_ / 76

Final Assessment Instructions:

- The Assessment is 19 problems and is worth 76 points.
- You will have 2 hours to complete the Assessment.
- The Assessment is closed book and closed notes.
- **Calculators are not allowed** on the Assessment.
- Show all your work for full credit and box your final answer.

1. [4 points] Simplify the following expressions:

$$\begin{aligned} \text{a. } \left( (3^{-1}x^{-1}y)(x^2y)^{-1} \right)^{-3} &= (3^{-1} \cdot x^{-1}y \cdot x^{-2}y^{-1})^{-3} = \\ &= (3^{-1}x^{-3}y^0)^{-3} = (3^{-1}x^{-3})^{-3} = 3^3x^9 = \boxed{27x^9} \end{aligned}$$

$$\text{b. } \frac{3}{\sqrt{6}-\sqrt{3}} = \frac{3(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} = \frac{3(\sqrt{6}+\sqrt{3})}{6-3} = \boxed{\sqrt{6}+\sqrt{3}}$$

2. [4 points]

a. Multiply the polynomials, as indicated:

$$\begin{aligned} (x+xy+y)(x-y) &= x^2 - \cancel{xy} + x^2y - xy^2 + \cancel{y} - y^2 = \\ &= x^2 + x^2y - xy^2 - y^2 \end{aligned}$$

b. Factor the polynomial by factoring out the greatest common factor:

$$6xy^3 + 9y^3 - 12xy^4 = 3y^3(2x + 3 - 4xy)$$

3. [4 points] Simplify the following expressions:

a.  $i^{13} = i^{12} \cdot i = (i^4)^3 \cdot i = 1 \cdot i = \boxed{i}$

b.  $\frac{10}{3-i} = \frac{10(3+i)}{(3-i)(3+i)} = \frac{10(3+i)}{9+1} = \boxed{3+i}$

4. [4 points]

a. Solve the following absolute value equation:

$$|4x + 15| = 3$$

$$4x + 15 = 3 \quad \text{or} \quad 4x + 15 = -3$$

$$4x = -12$$

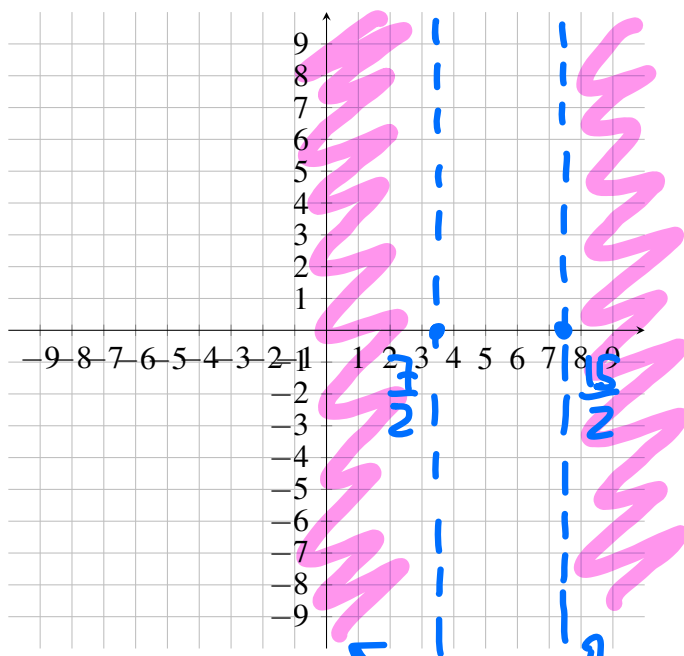
$$x = -3$$

$$4x = -18$$

$$x = -\frac{9}{2}$$

b. Solve the following absolute value inequality by graphing the solution set:

$$|4 - 2x| > 11$$



$$4 - 2x > 11 \quad \text{or} \quad 4 - 2x < -11$$

$$-2x > 7$$

$$-2x < -15$$

$$x < \frac{7}{2}$$

$$\text{or} \quad x > \frac{15}{2}$$

solution set

5. [4 points] Solve the following polynomial equation by factoring

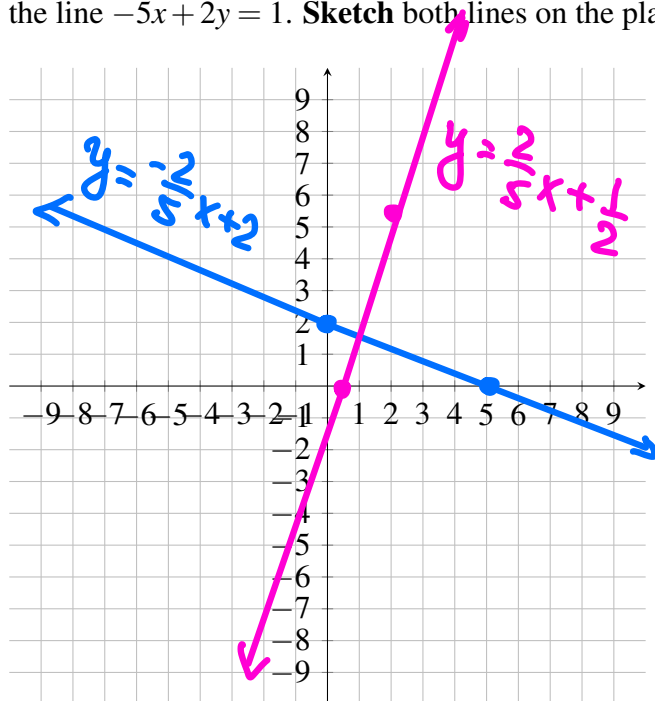
$$a^3 - 3a^2 = a - 3$$

$$a^2(a-3) = a-3$$

$$a^2(a-3) - (a-3) = 0$$

$$(a^2-1)(a-3) = 0 \Rightarrow \boxed{a = \pm 1 \text{ or } a = 3}$$

6. [4 points] Find the equation of the line that passes through the point (5,0) and is perpendicular to the line  $-5x + 2y = 1$ . **Sketch** both lines on the plane below.



$$2y = 1 + 5x$$

$$y = \frac{5}{2}x + \frac{1}{2} \quad m = \frac{5}{2}$$

$$m_1 \cdot m = -1$$

$$m_1 \cdot \frac{5}{2} = -1 \Rightarrow m_1 = -\frac{2}{5}$$

$$y = -\frac{2}{5}x + b$$

$$(5,0): 0 = -\frac{2}{5} \cdot 5 + b$$

$$b = 2$$

$$\boxed{y = -\frac{2}{5}x + 2}$$

7. [4 points] Find the standard form for the equation of the circle

$$x^2 + y^2 - 4x + 8y - 16 = 0$$

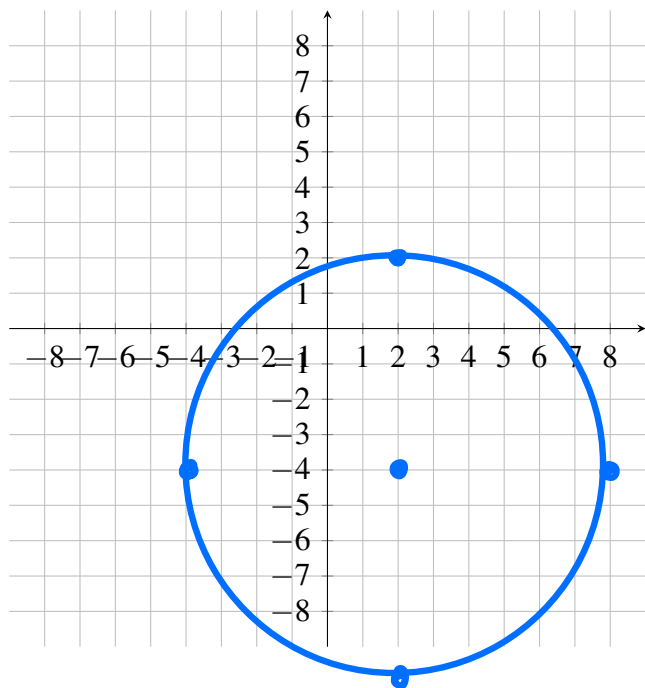
**Sketch** the obtained circle on the plane below.

$$(x-2)^2 + (y+4)^2 - 4 - 16 - 16 = 0$$

$$(x-2)^2 + (y+4)^2 = 36 = 6^2$$

$$r = 6$$

$$(2, -4) - \text{center}$$

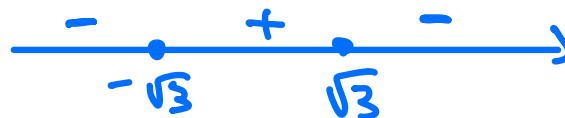


8. [4 points] Determine the implied domain of the following function

$$f(x) = \frac{5}{\sqrt{3-x^2}}$$

$$3-x^2 > 0$$

$$(\sqrt{3}-x)(\sqrt{3}+x) > 0$$



$$\text{Dom}(f) = (-\sqrt{3}, \sqrt{3})$$

9. [4 points] Graph the following function with stating precisely **all** transformations.

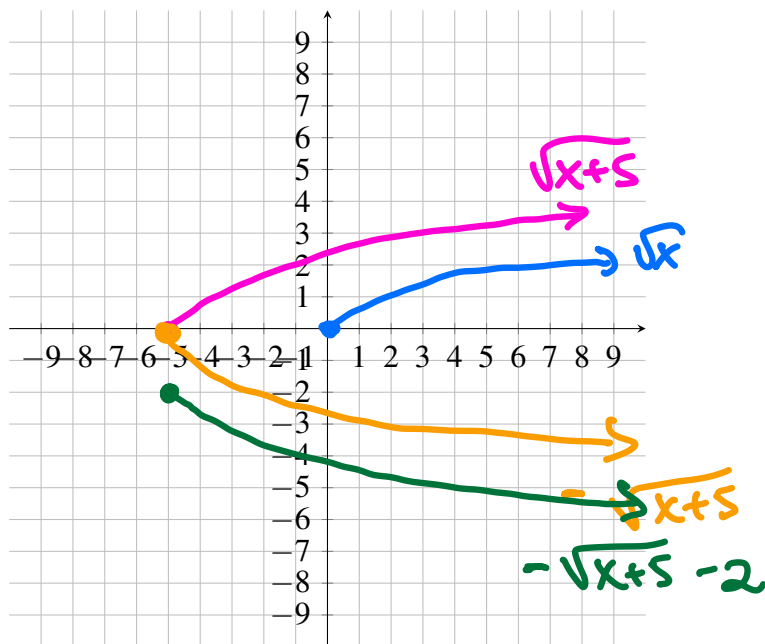
$$g(x) = -\sqrt{x+5} - 2$$

$$g_1(x) = \sqrt{x} \quad - \text{basic function}$$

$$g_2(x) = \sqrt{x+5} \quad - \text{shift left horiz. per } \textcircled{5}$$

$$g_3(x) = -\sqrt{x+5} \quad - \text{reflection via x-axis}$$

$$g_4(x) = -\sqrt{x+5} - 2 \quad - \text{shift down vertically per } \textcircled{2}$$



10. [4 points] For the given function

$$h(x) = \frac{5}{x+3}$$

a. Determine  $\text{Dom}(h)$

$$\text{Dom}(h) = \mathbb{R} \setminus \{-3\}$$

b. Evaluate  $\frac{h(x-3) - h(x)}{x} = \frac{\frac{5}{x} - \frac{5}{x+3}}{x} =$

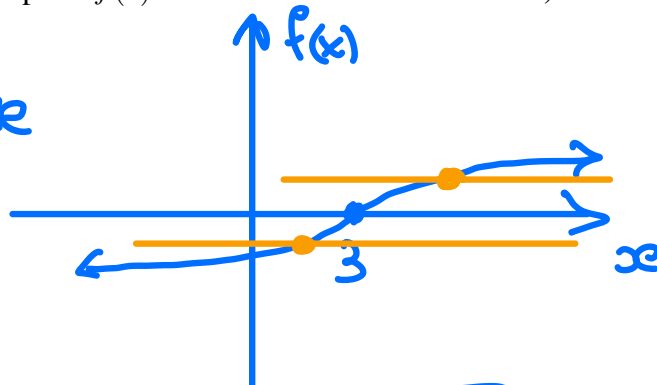
$$= \frac{\frac{5(x+3) - 5x}{x(x+3)}}{x} = \frac{\cancel{5x} + 15 - \cancel{5x}}{x^2(x+3)} = \boxed{\frac{15}{x^2(x+3)}}$$

11. [4 points] For the given function

$$f(x) = \sqrt[3]{x-3}$$

a. determine if it has an inverse (Hint: sketch the graph of  $f(x)$  and use a Horizontal Line Test).

$f(x)$  has an inverse



b. if it has an inverse, then find a formula for it.

$$\begin{aligned} y &= \sqrt[3]{x-3} \\ y^3 &= x-3 \\ x &= y^3 + 3 \end{aligned}$$

$$f^{-1}(x) = x^3 + 3$$

12. [4 points] Construct a polynomial function with the stated properties:

- second-degree
- zeros of  $-4$  and  $3$
- and goes to  $-\infty$  as  $x \rightarrow -\infty$

$$P(x) = a(x+4)(x-3)$$

$$a = -1$$

$$P(x) = -(x+4)(x-3)$$

13. [4 points] Find equations for the vertical asymptotes, if any, for the following rational function

$$f(x) = \frac{x^2 + 5}{(x+3)(x-4)(x^2 - 1)}$$

VA:

$$\begin{aligned} x &= -3 \\ x &= 4 \\ x &= \pm 1 \end{aligned}$$

## 14. [4 points]

- a. Solve the following exponential equation:
- $27^{y^2} = 3^{18y-27}$

$$3^{3y^2} = 3^{18y-27}$$

$$3y^2 - 18y + 27 = 0$$

$$3(y^2 - 6y + 9) = 0$$

$$y^2 - 6y + 9 = 0$$

$$(y-3)^2 = 0 \Rightarrow \boxed{y=3}$$

- b. Solve the following logarithmic equation:
- $\log_4(x-3) + \log_4 2 = 3$

$$\log_4(2(x-3)) = 3$$

$$4^3 = 2(x-3)$$

$$x-3 = \frac{64}{2} = 32$$

$$\boxed{x=35}$$

15. [4 points] Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations. Find the **complete solution set**.

- a.
- $\sqrt{2} - 2\cos x = 0$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4} + 2\pi n, \quad x = -\frac{\pi}{4} + 2\pi n, \quad n \in \mathbb{Z}$$

- b.
- $\cos^2 x - 3 = -2\cos x$

$$\cos^2 x + 2\cos x - 3 = 0$$

$$(t-3)(t+1) = 0$$

$$\cancel{\cos x = 3} \quad \text{or} \quad \cos x = -1 \Rightarrow x = \pi n, \quad n \in \mathbb{Z}$$

## 16. [4 points] Solve for the remaining angle and sides of the triangle (apply Law of Sines):

$$A = 45^\circ, \quad B = 90^\circ, \quad a = 3$$

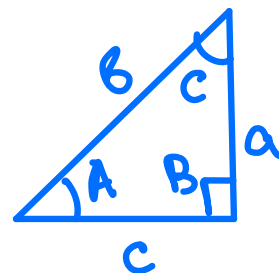
$$A = 45^\circ, \quad B = 90^\circ$$

$$C = 45^\circ$$

$$\frac{b}{\sin 90^\circ} = \frac{a}{\sin 45^\circ} \Rightarrow \frac{b}{1} = \frac{3}{\frac{\sqrt{2}}{2}}$$

$$b = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

$$c = 3\sqrt{2}$$



17. [4 points]

- a. Find
- $\sin \theta$
- if
- $\csc \theta = -7/5$

$$\sin \theta = \frac{1}{\csc \theta} = -\frac{5}{7}$$

- b. Determine the values of the
- six**
- trigonometric functions of the given angle
- $\theta = \frac{\pi}{6}$
- .

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cot \frac{\pi}{6} = \sqrt{3}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$$

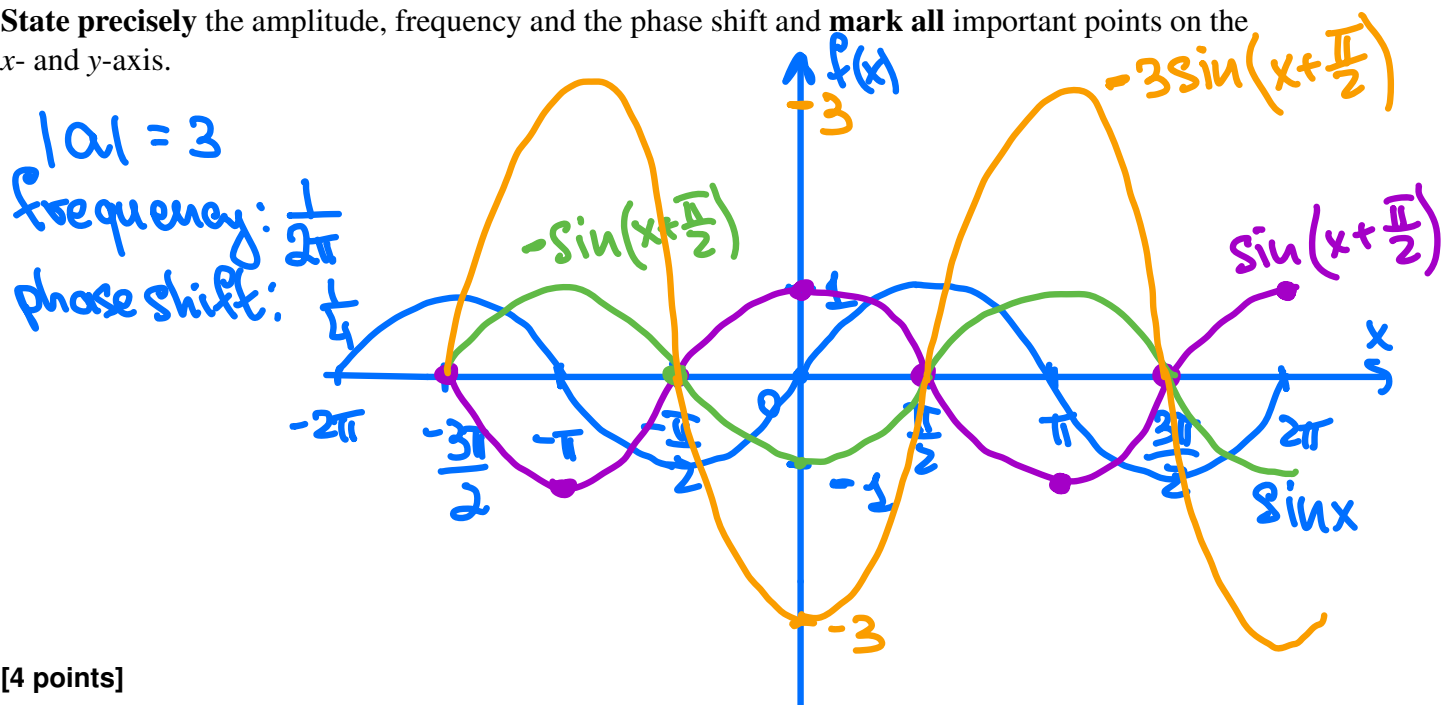
$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\csc \frac{\pi}{6} = 2$$

18. [4 points] Sketch the graph of the following trigonometric function

$$f(x) = -3 \sin \left( x + \frac{\pi}{2} \right)$$

State **precisely** the amplitude, frequency and the phase shift and **mark all** important points on the  $x$ - and  $y$ -axis.



19. [4 points]

a. Evaluate  $\arccos \left( -\frac{\sqrt{3}}{2} \right) = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

- b. Use trigonometric identities to simplify the expression

$$\sin t (\csc t - \sin t) = \sin t \left( \frac{1}{\sin t} - \sin t \right) = 1 - \sin^2 t = \boxed{\cos^2 t}$$