THEORETICAL PART:

Definition (Addition, Subtraction, Multiplication, and Division of Functions):

Let f and g be two functions. The **sum** $f+g$, **difference** $f-g$, **product** $f \cdot g$, and **quotient** $\frac{f}{g}$ are four new functions defined as follows:

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$
4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$.

Definition (Composing Functions):

Let f and g be two functions. The **composition** of f and g, denoted $f \circ g$, is the function defined by $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ consists of all x in the domain of g for which $g(x)$ is in turn in the domain of f. The function $f \circ g$ is read "f composed with g".

CAUTION:

Note that the order of f and g is **IMPORTANT**.

CAUTION:

When evaluating the composition $(f \circ g)(x)$ at a point x, there are two reasons the value might be undefined:

1. If x is not in the domain of g, then $g(x)$ is undefined and we can't evaluate $f(g(x))$.
2. If $g(x)$ is not in the domain of f, then $f(g(x))$ is undefined and we can't evaluate it.

Definition (Recursion):

Recursion refers to using the output of a function as its input, and repeating the process a certain number of times.

For instance,

$$f^3(x) = f(f(f(x))) = (f \circ f \circ f)(x).$$

PRACTICAL PART:

1. Given that $f(-2) = 5$, $g(-2) = -3$, find:

   (a) $(f - g)(-2) =$

   (b) $\left(\frac{f}{g}\right)(-2) =$

2. Given the two functions $f(x) = 4x^2 - 1$ and $g(x) = \sqrt{x}$, find:

   (a) $(f + g)(x)$

   (b) $(f \cdot g)(x)$

3. Given $f(x) = x^2$ and $g(x) = x - 3$, find the following:

   (a) $(f \circ g)(6)$

   (b) $(f \circ g)(x)$

   (c) $(g \circ f)(6)$

   (d) $(g \circ f)(x)$

4. Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$. Find formulas and state the domains for the following:

   - $f \circ g$
   - $g \circ f$

5. Decompose the function $f(x) = |x^2 - 3| + 2$ into the following:

   a. a composition of two functions

   b. a composition of three functions