THEORETICAL PART: Solutions

Definition:

Let L stand for a given line in the Cartesian plane, and let (x_1, y_1) and (x_2, y_2) be the coordinates of two distinct points on L. The slope of the line L is the ratio

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

which can be described in words as "change in y over change in x" or "rise over run".

Caution.

Correct:

$$\frac{y_2 - y_1}{x_2 - x_1} \quad \textbf{or} \quad \frac{y_1 - y_2}{x_1 - x_2}$$

Incorrect:

$$\frac{y_1 - y_2}{x_2 - x_1} \quad \textbf{or} \quad \frac{y_2 - y_1}{x_1 - x_2}$$

Properties:

- **Horizontal lines**, which can be written in the form $y = c$, have a **slope of** 0.
- **Vertical lines**, which can be written in the form $x = c$, have an **undefined slope**.

Definition (Slope-Intercept Form).

If the equation of the nonvertical line in x and y is solved for y, the result is an equation in **slope-intercept form**:

$$y = mx + b.$$

The constant m is the slope of the line, and the y-intercept of the line is $(0, b)$.

Definition (Point-Slope Form).

The **point-slope form** of the equation for the line passing trough the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1).$$

Note, that m, x_1 and y_1 are all constants.

Definition (Standard Form).

The **standard form** for the line L is the following form:

$$ax + by = c.$$

PRACTICAL PART:

1. Determine the slopes of the lines passing through the following pairs of points in $\mathbb{R}^2$:

(a) $(-4, -3)$ and $(2, -5)$

(b) $\left(\frac{3}{2}, 1\right)$ and $\left(1, -\frac{4}{3}\right)$

(a) $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-5+3}{2+4} = -\frac{2}{6} = \boxed{-\frac{1}{3}}$

(b) $m = \frac{-\frac{4}{3} - 1}{1 - \frac{3}{2}} = \frac{-\frac{7}{3}}{-\frac{1}{2}} = \boxed{\frac{14}{3}}$

2. Determine the slopes of the lines defined by the following equations:

(a) $4x - 3y = 12$

(b) $x = -\frac{3}{4}$

(c) $y = 9$

(a) $4x - 3y = 12$
$-3y = 12 - 4x$
$y = \frac{4}{3}x - 4 \Rightarrow \boxed{m = \frac{4}{3}}$

(b) $x = -\frac{3}{4} \Rightarrow$ $\boxed{\text{m is undefined}}$
vertical line

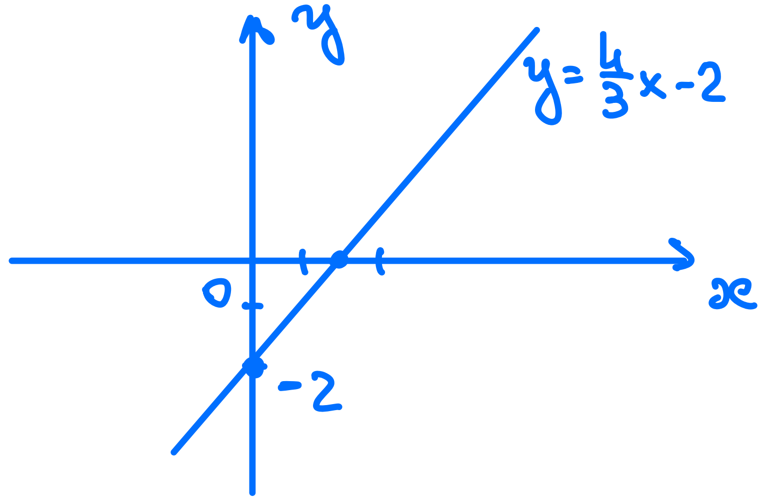
(c) $y = 9 \Rightarrow \boxed{m = 0}$
horizontal line

3. Use the slope-intercept form of the line to graph the equation $4x - 3y = 6$.

$4x - 3y = 6$

$-3y = 6 - 4x$

$y = \frac{4}{3}x - 2$



4. Find the equation of the line that passes through the point $(0, 3)$ and has a slope of $-\frac{3}{5}$. Then graph the line.

$y = mk + b$

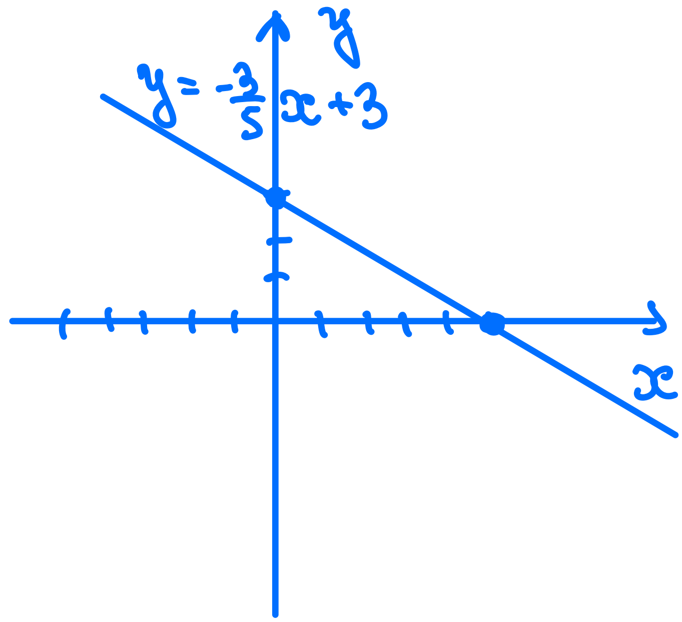
$y = -\frac{3}{5}x + b$

$3 = -\frac{3}{5} \cdot 0 + b \Rightarrow b = 3$

$\boxed{y = -\frac{3}{5}x + 3}$

$y = 0 \Rightarrow -\frac{3}{5}x = -3$

$x = 5$



5. Find the equation, in slope-intercept form, of the line that passes through the point $(-2, 5)$ and has a slope 3.

$y = mx + b$

$m = 3$

$y = 3x + b$

$5 = 3 \cdot (-2) + b \Rightarrow b = 11$

$\boxed{y = 3x + 11}$

6. Find the equation, in slope-intercept form, of the line that passes through the two points $(-3, -2)$ and $(1, 6)$.

$$y = mx + b$$

$$m = \frac{6+2}{1+3} = \frac{8}{4} = 2$$

$$y = 2x + b$$

$$6 = 2 \cdot 1 + b \Rightarrow b = 4$$

Therefore, $\boxed{y = 2x + 4}$