Solutions

THEORETICAL PART:

Definitions:

- A polynomial in the variable x of degree n can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_n, \cdots, a_0 \in \mathbb{R}$, $a_n \neq 0$, and n is nonnegative integer.

- **Basic operations with polynomials:** addition, subtraction, multiplication, division (will be considered later)

- **Special Product Formulas:** Let A and B be algebraic expressions. Then

$$(A - B)(A + B) = A^2 - B^2$$

$$(A + B)^2 = A^2 + 2AB + B^2$$
$$(A - B)^2 = A^2 - 2AB + B^2$$

- The polynomial is **factorable** if it can be written as a product of two or more polynomials with integer coefficients. If it cannot be done, the polynomial is **irreducible** or **prime**.

- The **greatest common factor (GCF)** among all the terms is the product of all the factors common to each.

Factoring Special Binomials:

In the following equations, A and B are algebraic expressions.

- **Difference of two squares:**
$$A^2 - B^2 = (A - B)(A + B)$$

- **Difference of two cubes:**

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

- **Sum of two cubes:**
$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

Factoring a Trinomial by Grouping:

To factor the trinomial $ax^2 + bx + c$, perform the following steps:

- Multiply a and c.

- Factor ac into two integers whose sum is b. If no such factors exist, the trinomial is irreducible over the integers.

- Rewrite b in the trinomial with the sum found in step 2, and distribute. The resulting polynomial of four terms may now be factored by grouping.

Perfect Square Trinomials:

Let A and B be algebraic expressions. Then

$$A^2 + 2AB + B^2 = (A + B)^2$$

$$A^2 - 2AB + B^2 = (A - B)^2$$

<u>PRACTICAL PART:</u>

1. Classify each of the following expressions as either polynomial or not a polynomial. For those that are polynomials, identify the degree of the polynomial and the number of terms.

   (a) $17x^2y^5 + 2z^3 - 4$ polynomial
   degree 7, trinomial

   (b) $3n^4m^{-3} + n^2m$ not a polynomial

   (c) $3x^{\frac{3}{2}} - 2x$ Not a polynomial

2. Add and subtract the following polynomials:

   (a) $(x^2y - xy - 6y) + (xy^2 + xy + 6x) =$ $x^2y + xy^2 - 6y + 6x$

   (b) $(-8x^4 + 13 - 9x^2) - (8 - 2x^4) =$ $-6x^4 + 5 - 9x^2$

3. Multiply the polynomials:

   (a) $(x^2 - 2y)(x^2 + y) =$ $x^4 + x^2y - 2yx^2 - 2y^2$

(b) $(2xy^2 + 4y - 6x)(x^2y - 5xy) =$ $2x^3y^3 - 10x^2y^3 + 4x^2y^2 - 20xy^2 -$
$- 6x^3y + 30x^2y$

4. Use a special product formula to perform the indicated operations:

(a) $(x - 3y)^2 =$ $x^2 - 6xy + 9y^2$

(b) $\left(\frac{1}{x} - y\right)\left(\frac{1}{x} + y\right) =$ $\frac{1}{x^2} - y^2$

5. Factor each polynomial by factoring out the greatest common factor:

(a) $12x^5 - 4x^2 + 8x^3z^3 =$ $4x^2(3x^3 - 1 + 2xz^3)$

(b) $(x^2 + y)^3 + 3(x^2 + y)^2 =$ $(x^2+y)^2(x^2+y+3)$

6. Factor the following polynomials by grouping:

(a) $ax - ay - bx + by =$ $x(a-b) - y(a-b) = (x-y)(a-b)$

(b) $4x - 2x^2 - 2x^3 + x^4 =$ $2x(2-x) - x^3(2-x) =$

$= (2-x)(2x-x^3) = x(2-x)(2-x^2) =$

$= x(2-x)(\sqrt{2}-x)(\sqrt{2}+x)$

7. Use the special factoring patterns to factor the following binomials:

(a) $49a^2 - 144b^2 =$ $(7a)^2 - (12b)^2 = (7a-12b)(7a+12b)$

(b) $27a^9 + 8b^{12} =$ $(3a^3)^3 + (2b^4)^3 = (3a^3+2b^4)((3a^3)^2 -$

$- 3a^3 \cdot 2b^4 + (2b^4)^2)$

(c) $343y^9 - 27x^3z^6 =$ $(7y^3)^3 - (3xz^2)^3 =$

$= (7y^3 - 3xz^2)((7y^3)^2 + 7y^3 \cdot (3xz^2) + (3xz^2)^2)$

8. Factor the following trinomial by grouping:

$$6x^2 - x - 12 =$$

$= 6x^2 - 9x + 8x - 12 = 3x(2x-3) + 4(2x-3) =$

$= (2x-3)(3x+4)$

9. Factor the algebraic expressions:

(a) $x^2 - 4x + 4 =$ $(x-2)^2 = (x-2)(x-2)$

(b) $25y^2 + 10y + 1 =$ $(5y)^2 + 2 \cdot 5y + 1 = (5y+1)^2$

(c) $x^2 + 6x + 9 =$ $x^2 + 2 \cdot 3x + 3^2 = (x+3)^2$

10. Factor the following expressions with noninteger rational exponents:

(a) $2x^{-2} + 3x^{-1} =$ $x^{-2}\left(2 + 3\frac{x^{-1}}{x^{-2}}\right) = \frac{1}{x^2}(2+3x)$

(b) $(5x+7)^{\frac{7}{3}} - (5x+7)^{\frac{4}{3}} =$ $(5x+7)^{\frac{4}{3}}(5x+7-1) =$

$= (5x+7)^{\frac{4}{3}}(5x+6)$

(c) $5x^{-4} - 4x^{-5}y =$ $x^{-5}(5x-4y) = \frac{1}{x^5}(5x-4y)$