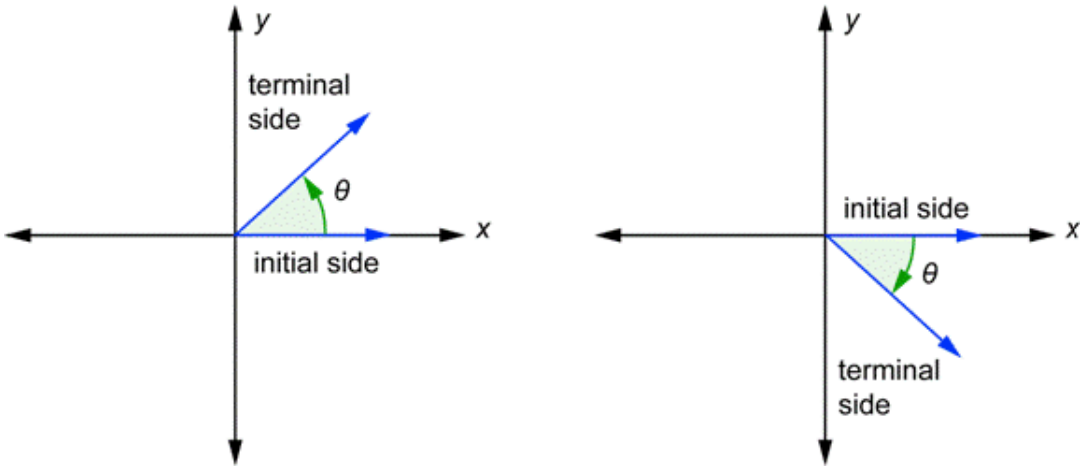
THEORETICAL PART:

Definition (Radian Measure):

Let θ be an angle at the center of a circle of radius 1 (the unit circle). The measure of θ in **radians** is the length of that portion of the circle subtended by *theta*, which is the portion of the circumference.



Formula (Angle Measurement Conversion)

Since $180^\circ = \pi\ rad$, we know that $1^\circ = \frac{\pi}{180}\ rad$ and $\left(\frac{180}{\pi}\right)^\circ = 1\ rad$. Multiplying both sides of these equations by an arbitrary quantity x, we have

1. $x^\circ = x\left(\frac{\pi}{180}\right)\ rad.$
2. $x\ rad = x\left(\frac{180}{\pi}\right)^\circ.$

Formula (Arc Length):

Given a circle of radius r, the length s of the arc subtended by a central angle θ (in radians) is given by the following formula:

$$s = \left(\frac{\theta}{2\pi}\right)(2\pi r) = r\theta$$

Definition (Angular speed and linear speed):

If an object moves along the arc of a circle defined by a central angle θ in time t, the object is said to have an **angular speed** ω given by

$$\omega = \frac{\theta}{t}.$$

If the circle has a radius of r, the distance traveled in time t is the arc length s, and the **linear speed** v is given by

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Formula (Sector Area):

The area A of a sector with a central angle of θ in a circle of radius r is

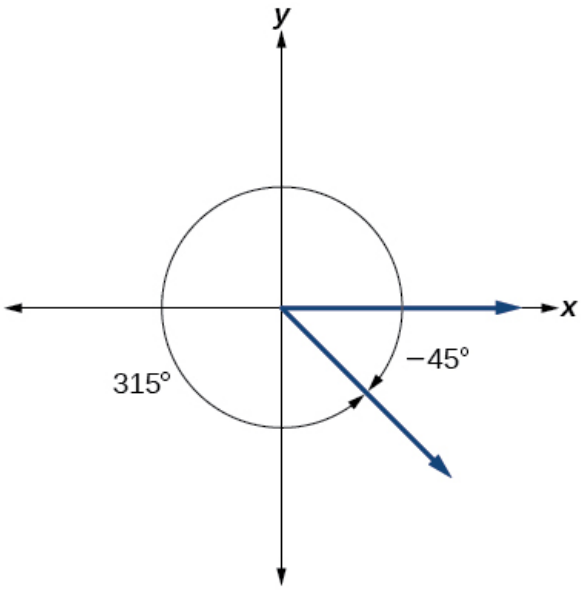
$$A = \left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{r^2\theta}{2}.$$

PRACTICAL PART:

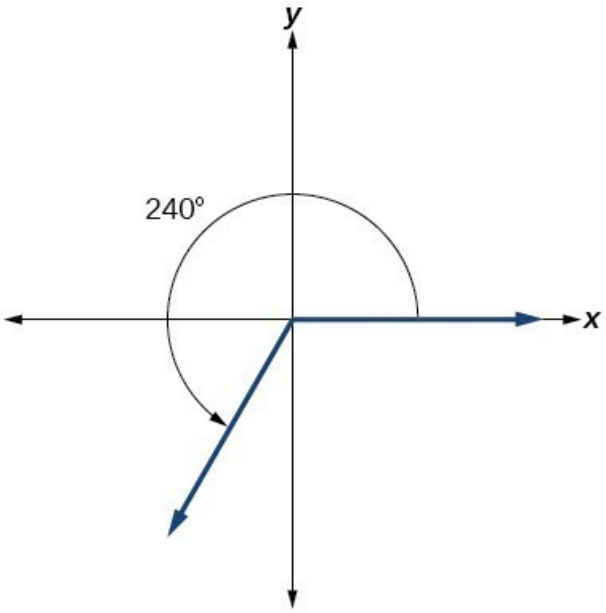
1. Convert the following angle measures as directed.
   a. Express $\frac{\pi}{3}$ *rad* in degrees.
   b. Express 270° in radians.
   c. Express -2 *rad* in degrees.

2. Use the information in each diagram to determine the **radian** measure of the indicated angle.

(a)



(b)



3. Find the length of the arc subtended by the given central angle θ on a circle of radius r.

   a. $r = 4\ in.$, $\theta = 1$.

   b. $r = 16\ ft$, $\theta = \dfrac{\pi}{4}$.

4. Suppose an ant crawls along the rim of a circular glass with radius 2 inches, and traverses the arc in 20 seconds. What are the angular and linear speeds of the ant, and how far does it travel?

5. Determine the areas of the sectors defined by the given radii and angles.

   a. Circle of radius 3 cm, central angle of 52°.

   b. Circle of radius $\frac{1}{2}\ ft$, central angle of $\frac{4\pi}{3}$.