THEORETICAL PART:

Properties of logarithms:

Let a (the logarithmic base) be a positive real number not equal to 1, let x and y be positive real numbers, and let r be any real number.

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r\log_a x$

Formula (Change of Base Formula):

Let a and b be positive real numbers, neither of them equal to 1, and let x be a positive real number. Then

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Definition (The pH Scale):

The **pH** of a solution is defined to be $-\log[H_3O^+]$, where $[H_3O^+]$ is the concentration of hydronium ions in units of moles/liter. Solutions with a pH less than 7 are said to be acidic, while those with a pH greater than 7 are basic.

Definition (The Richter Scale):

Earthquake intensity is measured on the **Richter Scale**:

$$R = \log\left(\frac{I}{I_0}\right),$$

where I_0 is the intensity of a just-discernible earthquake, I is the intensity of an earthquake being analyzed, and R is its ranking on the Richter scale.

Scale: $R < 4$ – minor, $4 \le R < 5$ – light, $5 \le R < 6$ – moderate, $6 \le R < 7$ – strong, $7 \le R < 8$ – major, $8 \le R$ – great.

Definition (The Decibel Scale):

$$D = 10\log\left(\frac{I}{I_0}\right),$$

where I_0 is the intensity of a just-discernible sound, I is the intensity of the sound being analyzed, and D is its decibel level.

Scale: $0 < D < 60$ – normal conversation, $60 < D < 80$ – heavy traffic, $80 < D < 120$ – loud rock concert, $120 < D < 160$ – eardrum is likely to rupture.

PRACTICAL PART:

1. Use properties of logarithms to expand the following expressions as much as possible.

   (a) $\log_4 (64x^3 \sqrt{y})$

   (b) $\log_a \left(\sqrt[3]{\frac{xy^2}{z^4}} \right)$

   (c) $\log \left(\frac{2.7 \times 10^4}{x^{-2}} \right)$

2. Use the properties of logarithms to condense the following expressions as much as possible.

   (a) $2\log_3\left(\frac{x}{3}\right) - \log_3\left(\frac{1}{y}\right)$

   (b) $\ln(x^2) - \frac{1}{2}\ln y + \ln 2$

   (c) $\log_b 5 + 2\log_b(x^{-1})$

3. Evaluate the following logarithmic expressions, using the base of your choice.

   (a) $\log_7 15$

   (b) $\log_{\frac{1}{2}} 3$

   (c) $\log_\pi 5$

4. If a sample of orange juice is determined to have a $[H_3O^+]$ concentration of 1.58×10^{-4} moles/liter, what is its pH?

5. Given that $I_0 = 10^{-12}$ $watts/meter^2$, what is the decibel level of jet airliner's engines at a distance of 45 meters, for which the sound intensity is 50 $watts/meter^2$?