Name: Solutions ______ / 76

Final Assessment Instructions:

- The Assessment is 19 problems and is worth 76 points.
- You will have 2 hours to complete the Assessment.
- The Assessment is closed book and closed notes.
- **Calculators are not allowed** on the Assessment.
- Show all your work for full credit and box your final answer.

1. [4 points] Simplify the following expressions:

a. $\left((3^{-1}x^{-1}y)(x^2y)^{-1}\right)^{-3} = \left(3^{-1}\cdot x^{-1}y\cdot x^{-2}y^{-1}\right)^{-3} =$

$= \left(3^{-1}x^{-3}y^0\right)^{-3} = \left(3^{-1}x^{-3}\right)^{-3} = 3^3x^9 = \boxed{27x^9}$

b. $\dfrac{3}{\sqrt{6}-\sqrt{3}} = \dfrac{3(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})} = \dfrac{3(\sqrt{6}+\sqrt{3})}{6-3} = \boxed{\sqrt{6}+\sqrt{3}}$

2. [4 points]

a. Multiply the polynomials, as indicated:
$(x+xy+y)(x-y) = x^2 - \cancel{xy} + x^2y - xy^2 + \cancel{xy} - y^2 =$
$= x^2 + x^2y - xy^2 - y^2$

b. Factor the polynomial by factoring out the greatest common factor:
$6xy^3 + 9y^3 - 12xy^4 = 3y^3(2x+3-4xy)$

3. [4 points] Simplify the following expressions:

a. $i^{13} = i^{12} \cdot i = (i^4)^3 \cdot i = 1 \cdot i = \boxed{i}$

b. $\frac{10}{3-i} = \frac{10(3+i)}{(3-i)(3+i)} = \frac{10(3+i)}{9+1} = \boxed{3+i}$

4. [4 points]

a. Solve the following absolute value equation:

$|4x+15| = 3$

$4x+15=3$ or $4x+15=-3$

$4x=-12$ or $4x=-18$

$\boxed{x=-3}$ or $x=-\frac{9}{2}$

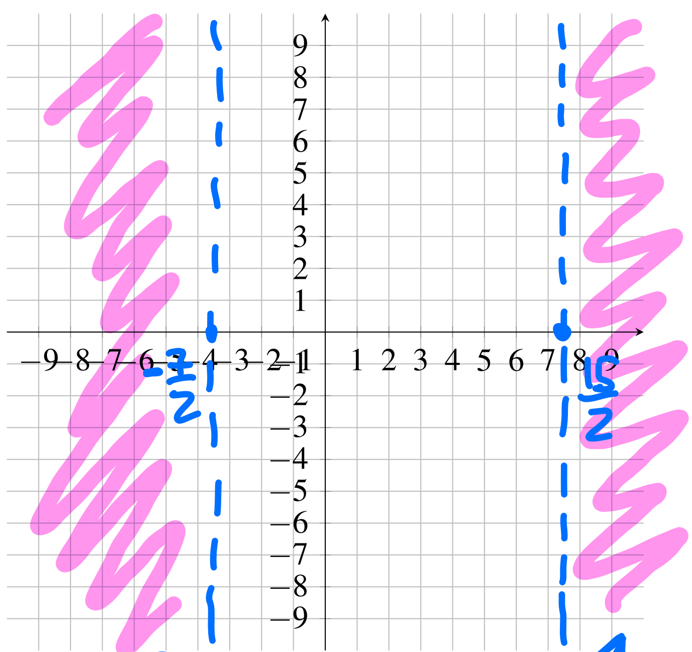
b. Solve the following absolute value inequality by graphing the solution set:

$|4-2x| > 11$

$4-2x>11$ or $4-2x<-11$

$-2x>7$ $\quad$ $-2x<-15$

$x<-\frac{7}{2}$ or $x>\frac{15}{2}$



solution set

5. [4 points] Solve the following polynomial equation by factoring

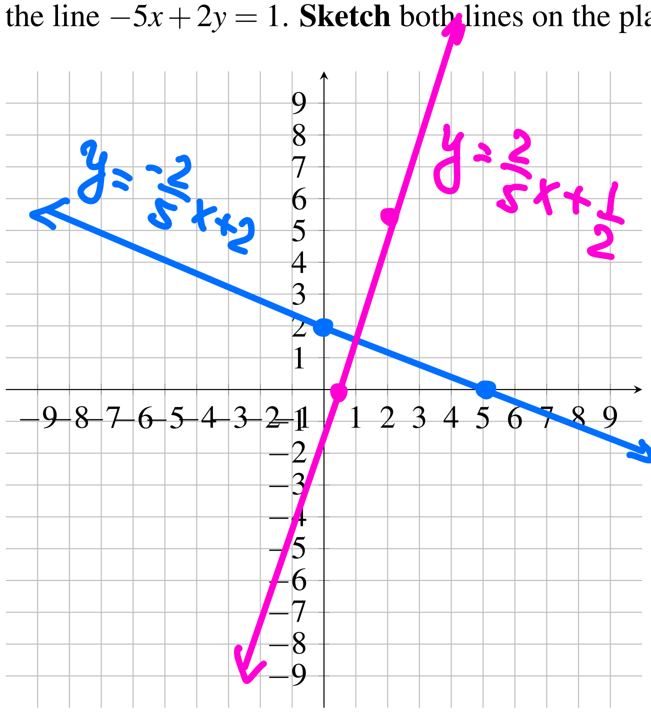
$$a^3 - 3a^2 = a - 3$$

$a^2(a-3) = a-3$

$a^2(a-3) - (a-3) = 0$

$(a^2-1)(a-3) = 0 \Rightarrow \boxed{a = \pm 1 \text{ or } a = 3}$

6. [4 points] Find the equation of the line that passes through the point $(5,0)$ and is perpendicular to the line $-5x + 2y = 1$. **Sketch** both lines on the plane below.



$2y = 1 + 5x$

$y = \frac{5}{2}x + \frac{1}{2}$ $m = \frac{5}{2}$

$m_1 \cdot m = -1$

$m_1 \cdot \frac{5}{2} = -1 \Rightarrow m_1 = -\frac{2}{5}$

$y = -\frac{2}{5}x + b$

$(5,0)$: $0 = -\frac{2}{5} \cdot 5 + b$

$b = 2$

$\boxed{y = -\frac{2}{5}x + 2}$

7. [4 points] Find the standard form for the equation of the circle

$$x^2 + y^2 - 4x + 8y - 16 = 0$$

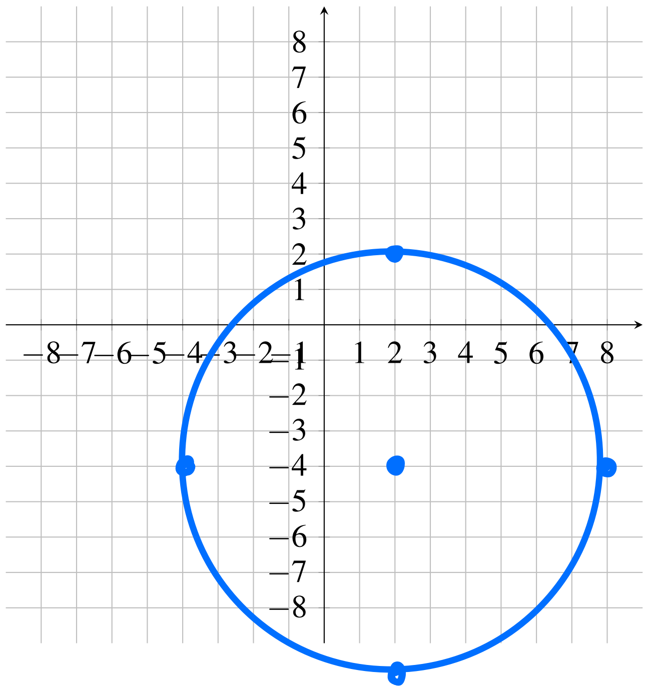
Sketch the obtained circle on the plane below.

$(x-2)^2 + (y+4)^2 - 4 - 16 - 16 = 0$

$(x-2)^2 + (y+4)^2 = 36 = 6^2$

$r = 6$

$(2,-4)$ - center



8. [4 points] Determine the implied domain of the following function

$$f(x) = \frac{5}{\sqrt{3-x^2}}$$

$3 - x^2 > 0$

$(\sqrt{3}-x)(\sqrt{3}+x) > 0$

$-$ $+$ $-$

$-\sqrt{3}$ $\sqrt{3}$

$\mathrm{Dom}(f) = (-\sqrt{3}, \sqrt{3})$

9. [4 points] Graph the following function with stating precisely **all** transformations.

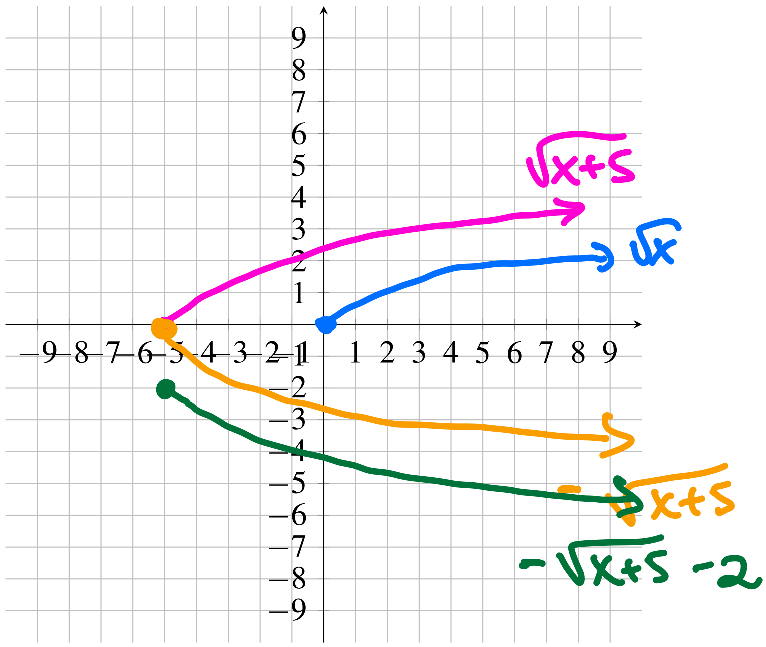
$$g(x) = -\sqrt{x+5} - 2$$

$g(x) = \sqrt{x}$ – basic function

$g_1(x) = \sqrt{x+5}$ – shift left horiz. per 5

$g_2(x) = -\sqrt{x+5}$ – reflection via x-axis

$g_3(x) = -\sqrt{x+5} - 2$ – shift down vertically per 2



10. [4 points] For the given function

$$h(x) = \frac{5}{x+3}$$

a. Determine $Dom(h)$

$$Dom(h) = \mathbb{R} \setminus \{-3\}$$

b. Evaluate $\frac{h(x-3)-h(x)}{x} = \frac{\frac{5}{x} - \frac{5}{x+3}}{x} =$

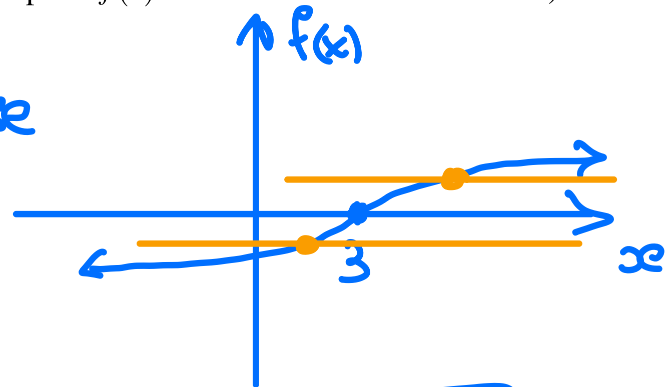
$$= \frac{\frac{5(x+3)-5x}{x(x+3)}}{x} = \frac{\cancel{5x} + 15 - \cancel{5x}}{x^2(x+3)} = \boxed{\frac{15}{x^2(x+3)}}$$

11. [4 points] For the given function

$$f(x) = \sqrt[3]{x-3}$$

a. determine if it has an inverse (Hint: sketch the graph of $f(x)$ and use a Horizontal Line Test).

f(x) has an inverse



b. if it has an inverse, then find a formula for it.

$y = \sqrt[3]{x-3}$

$y^3 = x-3$

$x = y^3 + 3$

$\boxed{f^{-1}(x) = x^3 + 3}$

12. [4 points] Construct a polynomial function with the stated properties:

- second-degree
- zeros of -4 and 3
- and goes to $-\infty$ as $x \to -\infty$

$P(x) = a(x+4)(x-3)$

$\boxed{a = -1}$

$P(x) = -(x+4)(x-3)$

13. [4 points] Find equations for the vertical asymptotes, if any, for the following rational function

$$f(x) = \frac{x^2+5}{(x+3)(x-4)(x^2-1)}$$

VA: $\boxed{\begin{array}{l} x = -3 \\ x = 4 \\ x = \pm 1 \end{array}}$

14. [4 points]

a. Solve the following exponential equation: $27^{y^2} = 3^{18y-27}$

$$3^{3y^2} = 3^{18y-27}$$

$$3y^2 - 18y + 27 = 0$$
$$3(y^2 - 6y + 9) = 0$$

$$y^2 - 6y + 9 = 0$$
$$(y-3)^2 = 0 \Rightarrow \boxed{y = 3}$$

b. Solve the following logarithmic equation: $\log_4(x-3) + \log_4 2 = 3$

$$\log_4(2(x-3)) = 3$$
$$4^3 = 2(x-3)$$
$$x - 3 = \frac{64}{2} = 32$$

$$\boxed{x = 35}$$

15. [4 points] Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations. Find the **complete solution set**.

a. $\sqrt{2} - 2\cos x = 0$

$$\cos x = \frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4} + 2\pi n, \quad x = -\frac{\pi}{4} + 2\pi n, \quad n \in \mathbb{Z}$$

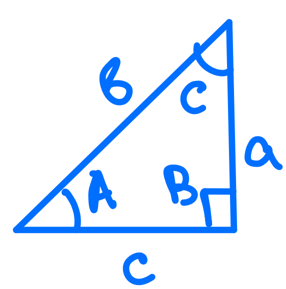
b. $\cos^2 x - 3 = -2\cos x$

$$\cos^2 x + 2\cos x - 3 = 0$$
$$(t+3)(t-1) = 0$$
$$\cancel{\cos x = -3} \quad \text{or} \quad \cos x = 1$$

$$x = 2\pi n, \quad n \in \mathbb{Z}$$

16. [4 points] Solve for the remaining angle and sides of the triangle (**apply Law of Sines**):

$$A = 45^\circ, \quad B = 90^\circ, \quad a = 3$$



$A = 45^\circ$, $B = 90^\circ$

$\underline{C = 45^\circ}$

$$\frac{b}{\sin 90^\circ} = \frac{a}{\sin 45^\circ} \Rightarrow \frac{b}{1} = \frac{3}{\sqrt{2}/2}$$

$$b = \frac{6}{\sqrt{2}} = \underline{3\sqrt{2}}$$

$$\underline{c = 3}$$

17. [4 points]

a. Find $\sin\theta$ if $\csc\theta = -7/5$

$\sin\theta = \frac{1}{\csc\theta} = -\frac{5}{7}$

b. Determine the values of the **six** trigonometric functions of the given angle $\theta = \frac{\pi}{6}$.

$\sin\frac{\pi}{6} = \frac{1}{2}$ $\cot\frac{\pi}{6} = \sqrt{3}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $\csc\frac{\pi}{6} = 2$

18. [4 points] Sketch the graph of the following trigonometric function

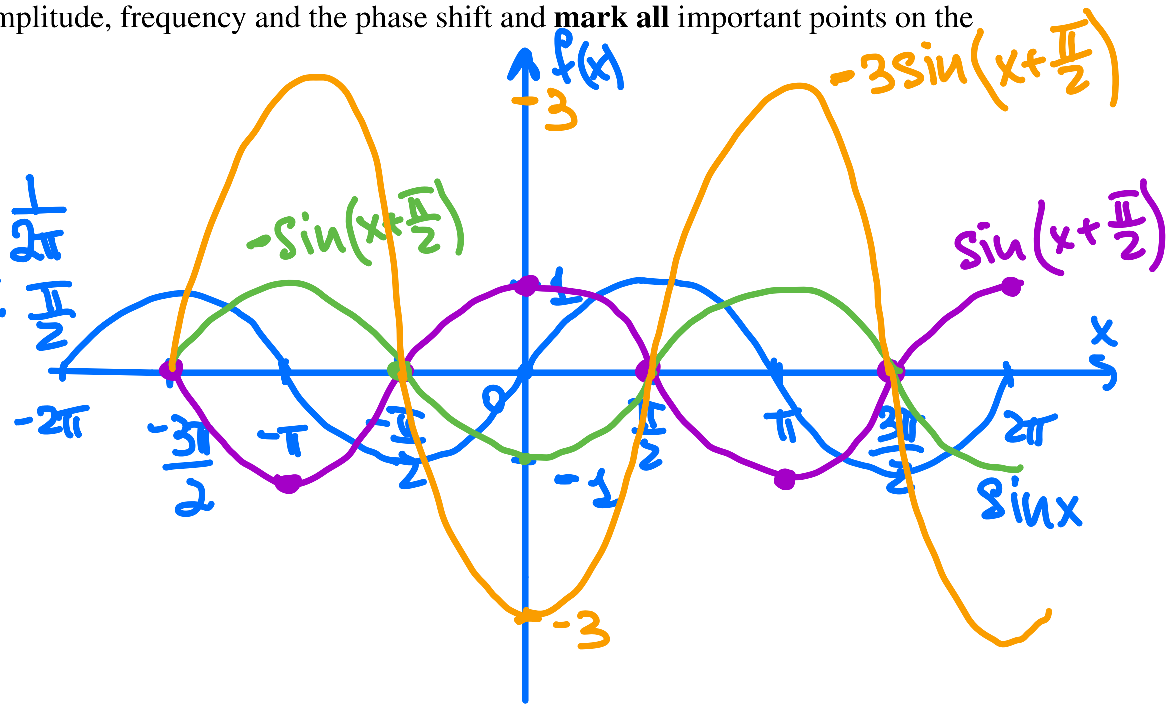
$$f(x) = -3\sin\left(x + \frac{\pi}{2}\right)$$

State precisely the amplitude, frequency and the phase shift and **mark all** important points on the x- and y-axis.

$|a| = 3$

frequency: $\frac{1}{2\pi}$

phase shift: $\frac{\pi}{2}$



19. [4 points]

a. Evaluate $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

b. Use trigonometric identities to simplify the expression

$\sin t(\csc t - \sin t) = \sin t\left(\frac{1}{\sin t} - \sin t\right) = 1 - \sin^2 t = \boxed{\cos^2 t}$