THEORETICAL PART:

Solutions

The Law of Sines

Given a triangle with sides and angles, the following equation represents the **Law of Sines**:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

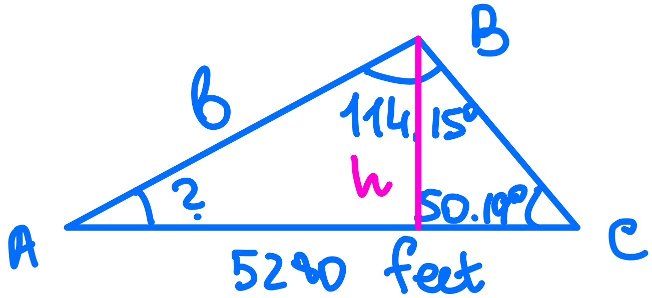
Theorem (Area of a Triangle (Sine Formula))

The area of a triangle is one-half the product of the lengths of any two sides and the sine of their included angle. That is,

$$Area = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

PRACTICAL PART:

1. Sarah is piloting a hot-air balloon, and finds herself becalmed directly above a long straight road. She notices mile markers on the road, and determines the angle of depression to the two markers. How far is she from marker A? What is her altitude?



$$\frac{b}{\sin(50.19°)} = \frac{5280}{\sin(114.15°)}$$

$b \approx 4445$ feet

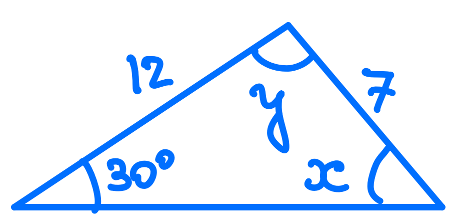
$\angle A = 15.66°$

$$\sin(15.66°) = \frac{h}{4445}$$

$h \approx 1200$ feet

2. Create a triangle, if possible, for which $A = \frac{\pi}{6}$, $b = 12$ cm, and $a = 7$ cm.

(1) $\frac{7}{\sin 30°} = \frac{12}{\sin x}$



$\sin x = \sin 30° \cdot 12 : 7$

$$\sin x = \frac{1}{2} \cdot 12 : 7 = \frac{6}{7}$$

$$x = \arcsin\left(\frac{6}{7}\right)$$

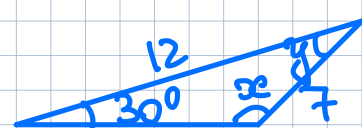
$$y = 180^\circ - 30^\circ - \arcsin\left(\frac{6}{7}\right)$$

So, by the Law of Sines

$$\frac{c}{\sin(1.59)} = \frac{7}{\sin(30^\circ)}$$

$c \approx 14.0$ cm.

(2)



$$\sin(x) = \sin(\pi - x) = \frac{6}{7}$$

$$\pi - x \approx 1.03$$

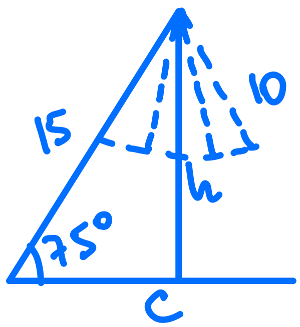
$$x \approx 2.11$$

$$y = \pi - 30^\circ - 2.11 = 0.51$$

$$\frac{c}{\sin(0.51)} = \frac{7}{\sin 30^\circ}$$

$c \approx 6.8$ cm.

3. Create a triangle, if possible, for which $A = 75°$, $b = 15$ units, and $a = 10$ units.

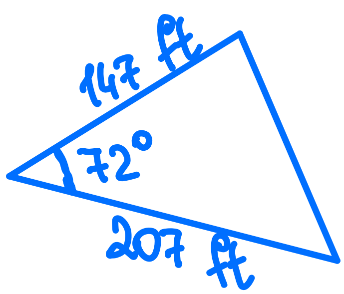


No such triangle is possible.

$h = 15 \cdot \sin 75° \approx 14.5$ and $a < h$.

a is too short to reach c.

4. A businessman has an opportunity to buy a triangular plot of land in a part of town known colloquially as "Five Points", as five roads intersect there to form five equal angles. The property has 147 feet of frontage on one road, and 207 feet of frontage on the other. What is the square footage of the property?



$$\frac{360°}{5} = 72°.$$

$$\text{Area} = \frac{1}{2}(147)(207)(\sin 72°) \approx$$

$$\approx 14.470\,(ft^2)$$