Solutions

THEORETICAL PART:

Definition (Extending the domains of the trigonometric functions):

Let s be a real number and let (x, y) be the point on the unit circle associated with s. We define the six trigonometric functions with argument s as follows:

$$\sin(s) = y, \quad \cos(s) = x, \quad \tan(s) = \frac{y}{x},\ x \neq 0,$$

$$\csc(s) = \frac{1}{y},\ y \neq 0, \quad \sec(s) = \frac{1}{x},\ x \neq 0, \quad \cot(s) = \frac{x}{y},\ y \neq 0$$

Definition (Trigonometric functions defined for an arbitrary angle):

Let θ be an angle in standard position, let (x, y) be any point (other than the origin) on the terminal side of the angle θ, and let $r = \sqrt{x^2 + y^2}$. We define the six trigonometric functions with argument θ as follows:

$$(\theta) = \frac{y}{r}, \quad \cos(\theta) = \frac{x}{r}, \quad \tan(\theta) = \frac{y}{x},\ x \neq 0,$$

$$\csc(\theta) = \frac{r}{y},\ y \neq 0, \quad \sec(\theta) = \frac{r}{x},\ x \neq 0, \quad \cot(\theta) = \frac{x}{y},\ y \neq 0$$

Definition (Reference Angles):

Given an angle θ in standard position, the **reference angle** θ' associated with it is the angle formed by the x-axis and the terminal side of θ. Reference angles are always greater than or equal to 0 and less than or equal to $\frac{\pi}{2}$ radians. That is, $0 \leq \theta' \leq \frac{\pi}{2}$.

Identities (Cofunction Identities):

Given an angle (measured in radians), $\frac{\pi}{2} - \theta$ is the measure of its complement, so

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right), \quad \csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right), \quad \cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$$

Identities (Reciprocal Identities):

For a given angle θ for which both sides of the equation are defined,

$$\csc(\theta) = \frac{1}{\sin(\theta)}, \quad \sec(\theta) = \frac{1}{\cos(\theta)}, \quad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Identities (Quotient Identities):

For a given angle θ for which both sides of the equation are defined,

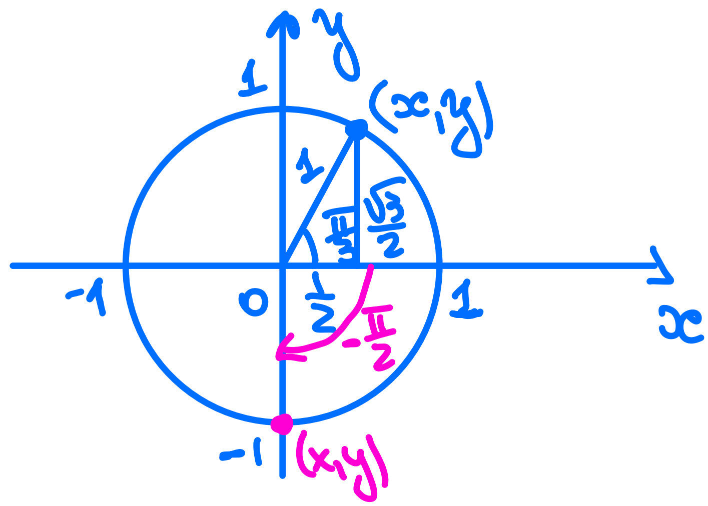
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}, \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

PRACTICAL PART:

1. Determine the point (x, y) on the unit circle associated with each real number s.

   a. $s = \frac{\pi}{3}$

   b. $s = -\frac{5\pi}{2}$ $= -2\pi - \frac{\pi}{2}$



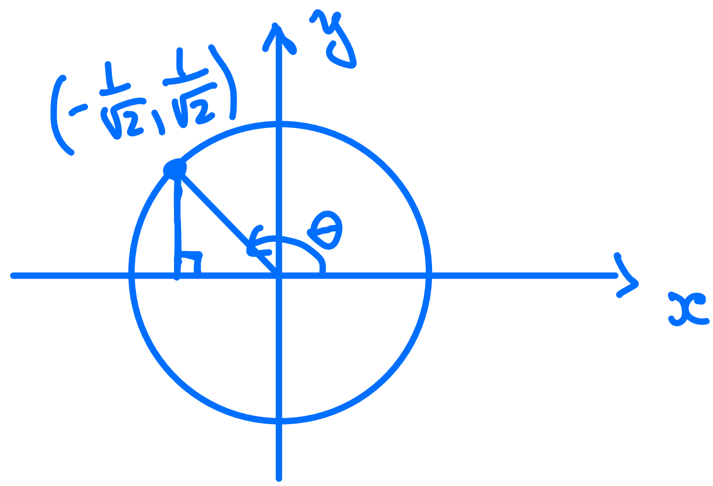
(a) $(x,y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(b) $(x,y) = (0,-1)$

2. Determine all real numbers s associated with the point $(x, y) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ on the unit circle.

$S = \frac{\pi}{2} + \frac{\pi}{4} + 2\pi n =$

$= \frac{3\pi}{4} + 2\pi n$



3. Determine the values of six trigonometric functions of each angle θ.

   a. $\theta = -\frac{5\pi}{2}$

   b. $\theta = 210^\circ$

(a) $\sin\left(-\frac{5\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$

$\cos(\theta) = 0$

$\sec(\theta)$ – DNE

$\csc(\theta) = -1$

$\tan(\theta)$ — DNE

$\cot(\theta) = 0$

(b) $\theta = 210° = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$\tan(\theta) = \frac{1}{\sqrt{3}}$

$\cot(\theta) = \sqrt{3}$

$\sec(\theta) = -\frac{2}{\sqrt{3}}$

$\csc(\theta) = -2$

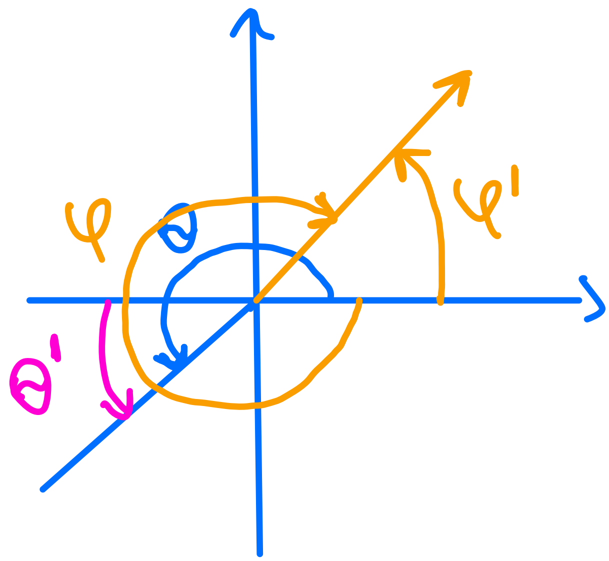
4. Find the reference angle associated with each of the following angles.

   a. $\theta = \frac{9\pi}{8} = \pi + \frac{\pi}{8}$

   b. $\phi = -655° = 65°$

(a) $\theta' = \frac{\pi}{8}$

(b) $\varphi' = 65°$



5. Evaluate the following:

   a. $\cos\left(\frac{4\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

   b. $\tan(-225°) = \tan(-270° + 45°) = -\tan(45°) = -1$

6. Express each of the following in terms of the appropriate cofunction, and verify the equivalence of the two expressions.

   a. $\cos\left(-\frac{5\pi}{11}\right)$

   b. $\cot(195^\circ)$

(a) $\cos\left(-\frac{5\pi}{11}\right) = \sin\left(\frac{\pi}{2} - \left(-\frac{5\pi}{11}\right)\right) =$

$= \sin\left(\frac{\pi}{2} + \frac{5\pi}{11}\right) = \boxed{\sin\left(\frac{21\pi}{22}\right)}$

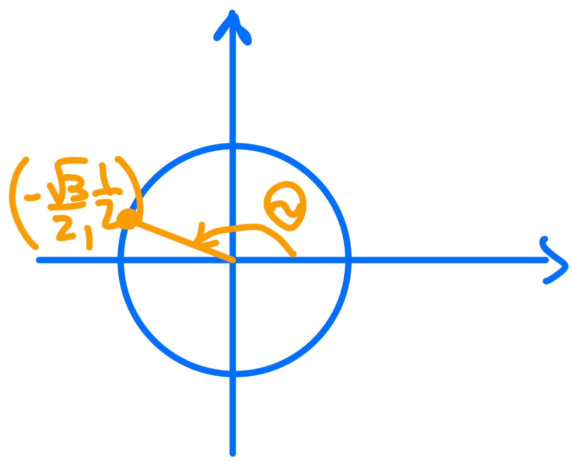
(b) $\cot(195^\circ) = \tan(90^\circ - 195^\circ) = \tan(-105^\circ) =$

$= \boxed{-\tan(105^\circ)}$

7. Given that $\cos(\theta) = -\frac{\sqrt{3}}{2}$ and $\tan(\theta)$ is negative, determine θ and $\tan(\theta)$.

$\tan(\theta) < 0$ in II
IV

$\cos(\theta) = -\frac{\sqrt{3}}{2}$ in II



$\theta = \frac{\pi}{2} + \frac{\pi}{3} = \boxed{\frac{5\pi}{6}}$

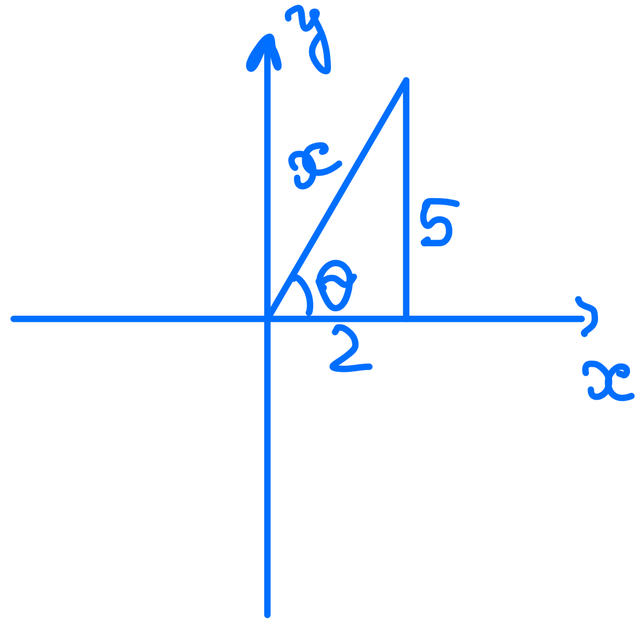
$\tan(\theta) = \tan\left(\frac{5\pi}{6}\right) = \tan\left(\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right) =$

$= \cot\left(-\frac{\pi}{3}\right) = -\cot\left(\frac{\pi}{3}\right) = -\frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)} =$

$= \boxed{-\frac{1}{\sqrt{3}}}$

8. Given that $\cot(\theta) = 0.4$ and θ lies in the first quadrant, determine $\sin(\theta)$.

$$\cot(\theta) = 0.4 = \frac{4}{10} = \frac{2}{5}$$



$$\frac{\cos\theta}{\sin\theta} = \frac{2}{5}$$

$$\cos\theta = \frac{2}{5} \cdot \sin\theta$$

$$x = \sqrt{4+25} = \sqrt{29}$$

$$\boxed{\sin\theta = \frac{5}{\sqrt{29}}}$$