Section 2.3. Linear equations in two variables

1. Recognizing linear equations in two variables
2. Intercepts of the coordinate axes.
3. Horizontal and vertical lines.

1.

Def. A linear equation in two variables, say x and y, is an equation that can be written in the form

$$\boxed{ax + by = c,} \quad \text{standard form}$$

$$a, b, c \in \mathbb{R} \quad \text{and} \quad a, b \neq 0.$$

Example

- $3x - (2-4y) = x - y + 1$

  $3x - 2 + 4y - x + y - 1 = 0$

  $2x + 5y - 3 = 0$

  $2x + 5y = 3$ — linear equation

- $x^2 - (x-3)^2 = 3y$

  $\cancel{x^2} - \cancel{x^2} + 6x - 9 = 3y$

  $6x - 3y = 9$ – linear equation

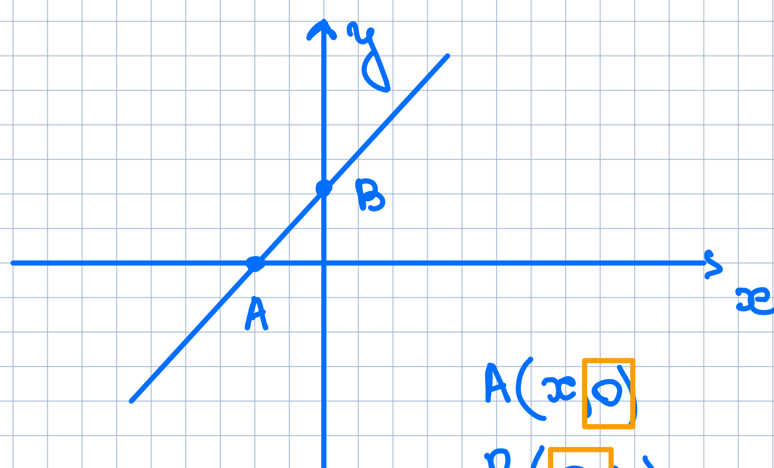
- $4x^3 - 2y = 5x$ – not a linear equation

$\boxed{2.}$

Suppose we have the linear equation in two variables:

$$ax + by = c$$

If the line crosses x and y axes, then



$A(x, \boxed{0})$

$B(\boxed{0,} y)$

<u>Def.</u> Given a graph in the Cartesian

plane, any point where the graph intersects the x-axis is called an <u>x-intercept</u>, and any point where the graph intersects the y-axis is called a <u>y-intercept</u>.

All x-intercepts are of the form $(c, 0)$ and all y-intercepts are of the form $(0, c)$.

<u>Example</u> (Finding x and y intercepts)

- $3x - 4y = 12$

X-interept: $y = 0 \Rightarrow 3x = 12$

$x = 4$

$(4, 0)$

Y - intercept: $x = 0 \Rightarrow -4y = 12$

$y = -3$

$(0, -3)$

$\boxed{3.}$ Horizontal and vertical lines.

Let us consider the linear equation:

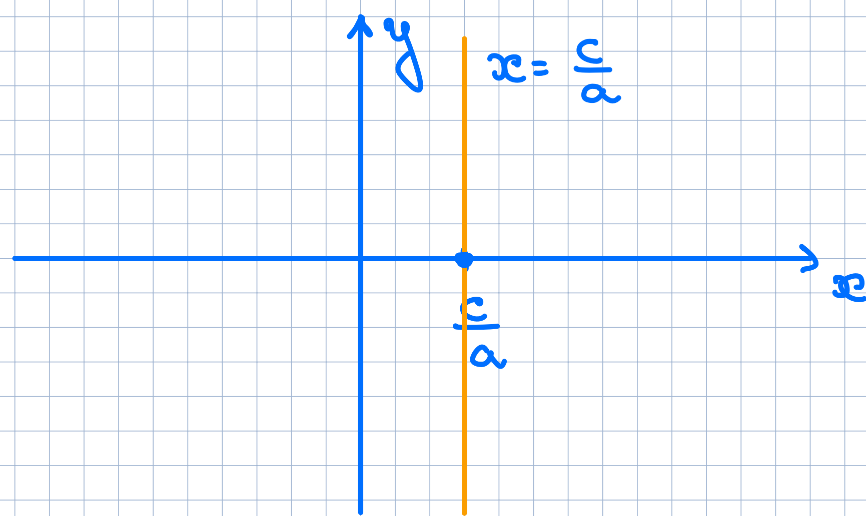
$$ax + by = c$$

Let $b = 0$. Then

$ax = c \Rightarrow x = \frac{c}{a}, \ a \neq 0$

$\boxed{x = \frac{c}{a}}$ y is arbitrary

vertical line



Now, let $a = 0$ and $b \neq 0$. Then

$by = c \Rightarrow \boxed{y = \frac{c}{b}}$ x is arbitrary

horizontal line