THEORETICAL PART:

Identities (Double-angle Identities):

Sine Identity

$$\sin(2u) = 2\sin u \cos u$$

Cosine Identities

$$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

Tangent Identity

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

Identities (Power-Reducing Identities)

Sine Identity

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

Cosine Identity

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

Tangent Identity

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

Identities (Half-Angle Identities)

Sine Identity

$$\sin(x/2) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

Cosine Identity

$$\cos(x/2) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

Tangent Identities

$$\tan(x/2) = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Identities (Product-to-Sum Identities)

$$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$
$$\cos x \sin y = \frac{1}{2}(\sin(x+y) - \sin(x-y))$$
$$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$
$$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

Identities (Sum-to-Product Identities)

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

PRACTICAL PART:

1. Given that $\cos x = -\frac{2}{\sqrt{5}}$ and that $\sin x$ is positive, determine $\cos(2x)$, $\sin(2x)$, and $\tan(2x)$.

2. Prove that $\sin(3x) = 3\sin x - 4\sin^3 x$.

3. Express $\sin^5 x$ in terms containing only first powers of sine and cosine.

4. Determine the exact values of $\sin(\pi/8)$, $\cos(\pi/8)$ and $\tan(\pi/8)$.

5. Express $\sin^5 x$ in terms containing only first powers of sine.

6. Verify the identity
$$\frac{\sin(2x)+\sin(4x)}{\cos(2x)+\cos(4x)} = \tan(3x).$$