THEORETICAL PART:

Definition:

Let a be a fixed positive real number not equal to 1. The **logarithmic function with base** a is defined to be the inverse of the exponential function with base a, and is denoted $\log_a x$. In symbols, if $f(x) = a^x$, then $f^{-1}(x) = \log_a x$.

In equation form, the definition of logarithm means that the equations

$$x = a^y \quad \text{and} \quad y = \log_a x$$

are equivalent. Note that a is the base in both equations: either the base of the exponential function or the base of the logarithmic function.

Properties:

1. $\log_a 1 = 0$, because $a^0 = 1$
2. $\log_a a = 1$, because $a^1 = a$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$

Definition:

- The function $\log_{10} x$ is called the **common logarithm** and is usually written $\log x$.
- The function $\log_e x$ is called the **natural logarithm** and is usually written $\ln x$.

Properties of Natural Logarithms:

$$\ln x = y \Leftrightarrow e^y = x$$

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

PRACTICAL PART:

1. Use the definition of logarithmic functions to rewrite the following exponential equations as logarithmic equations:

   (a) $8 = 2^3$

   (b) $5^4 = 625$

   (c) $7^x = z$

2. Rewrite the following logarithmic equations as exponential equations:

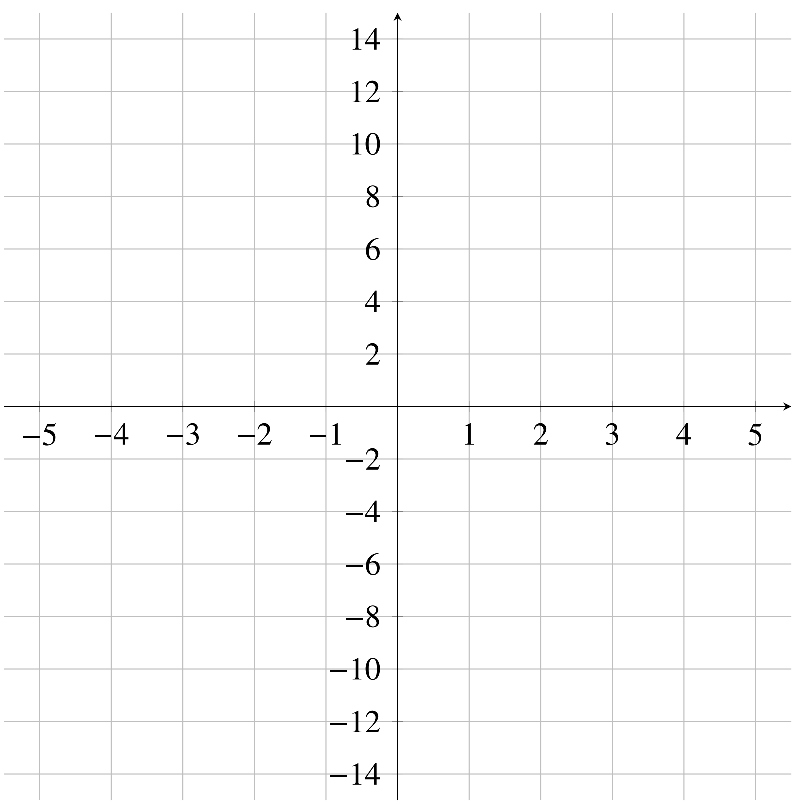
   (a) $\log_3 9 = 2$

   (b) $3 = \log_8 512$

3. Sketch the graphs of the following logarithmic functions:

   (a) $f(x) = \log_3 x$

   (b) $g(x) = \log_{\frac{1}{2}} x$

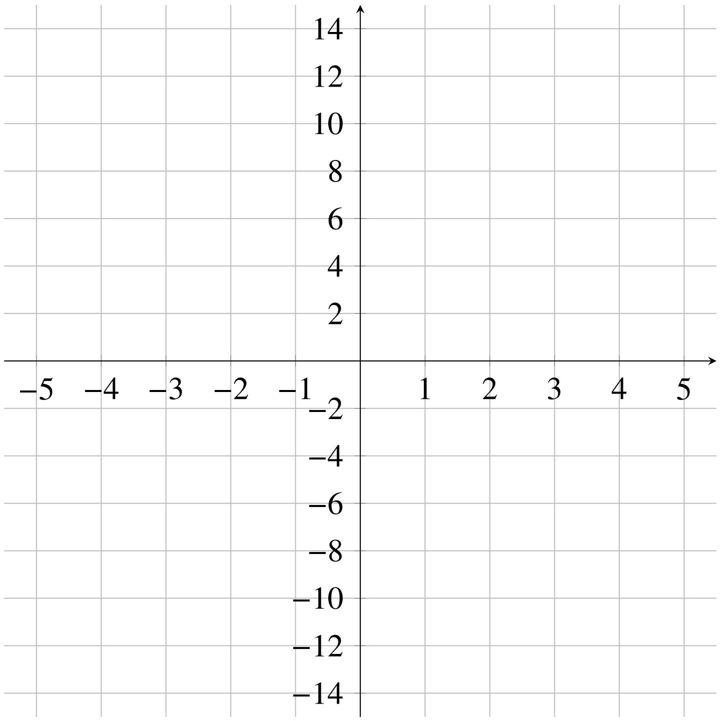


4. Sketch the graph of the following functions. State their domain and range.

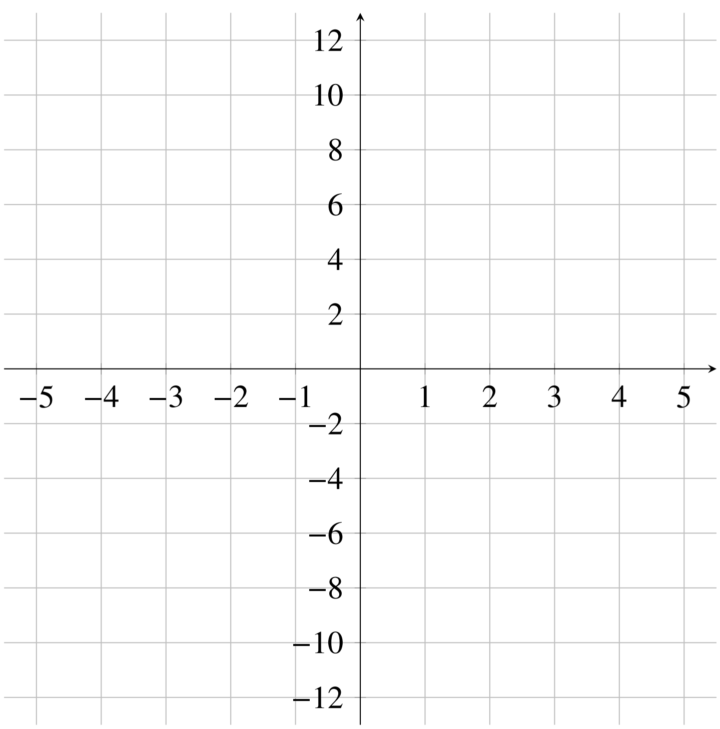
   (a) $f(x) = \log_3(x+2) + 1$

   (b) $g(x) = \log_{\frac{1}{2}} x - 2$

(a)



(b)



5. Evaluate the following logarithmic equations:

    (a) $\log_5 25 =$

    (b) $\log_{\frac{1}{2}} 2 =$

    (c) $\log_{16} 4 =$

    (d) $\log_{10} \dfrac{1}{100} =$

6. Use elementary properties of exponents and logarithms to solve the following equations.

    (a) $\log_6 (2x) = -1$

    (b) $3^{\log_{3x} 2} = 2$

    (c) $\log_2 8^x = 5$

7. Evaluate the following logarithmic expressions.

   (a) $\ln(\sqrt[3]{e}) =$

   (b) $\log 1000 =$

   (c) $\ln(4.78) =$