

## Section 1.6. Linear equations in one variable

1. Equations and the meaning of solutions
2. Solving linear equations
3. Solving linear absolute value equations.
4. Solving linear equations for one variable.
5. Applications of linear equations.

1.

Identities:

$$x^{\frac{1}{2}}(x+1) = x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

$$5+3=8$$

Contradiction:

$$t+3=t$$

Conditional:

$$x^2 = 9$$

Solution Set :

$$\{-3, 3\}$$

↑      ↑  
solution      solution

Def. Two equations that have the same solution set are called equivalent

equations.

2.

Def. A linear equation in one variable  $x$  is an equation that can be transformed into the form  $ax + b = 0$ ,  $a, b \in \mathbb{R}$ ,  $a \neq 0$ .

↑ first-degree equations

A linear equation is an example of a polynomial equation.

Example

$$3(x-2) + 7x = 1 - 2\left(x + \frac{1}{2}\right)$$

$$3x - 6 + 7x = 1 - 2x - 1$$

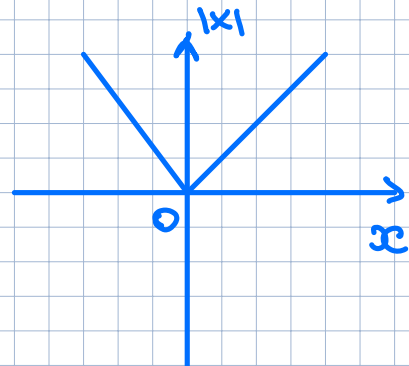
$$3x + 7x + 2x = 6$$

$$\frac{12x}{12} = \frac{6}{12}$$

$$x = \frac{1}{2}$$

3.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Then

$$|ax+b| = c$$

$$|ax+b| = \begin{cases} ax+b = c \\ -(ax+b) = c \end{cases}$$

### Example

1)  $|3x-2| = 1$

$$3x-2 = 1 \text{ or } -(3x-2) = 1$$

$$3x = 3$$

$$x = 1$$

$$-3x = -1$$

$$x = \frac{1}{3}$$

2)  $|x-4| = |2x+1|$

$$|x-4| = |2x+1| = \begin{cases} -(x-4) = 2x+1 \\ x-4 = 2x+1 \\ x-4 = -(2x+1) \\ -(x-4) = -(2x+1) \end{cases}$$

4.

Solving for a variable means to transform the equation into an equivalent one in which the specified variable is isolated on one side of the equation.

### Example

$$S = 2\pi r^2 + 2\pi r h, \quad \text{solve for } (h)$$

$$2\pi r h = S - 2\pi r^2$$

$$h = \frac{S - 2\pi r^2}{2\pi r}$$

5.

### Applications of Linear Equations

Distance:

$$d = v \cdot t$$

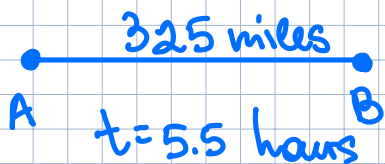
Simple Interest:

$$I = P r t$$

↑      ↑      ↑  
principal   rate   time  
invested

### Example

Average Speed:



$$d = r \cdot t$$

$$r = \frac{325}{5.5} = \frac{325}{11} \cdot 2$$

$$r \approx 59.1 \text{ m/h.}$$

### Example 7 (p. 82)

$$14\% - 500(\text{ml})$$

$$10\% - x$$

$$20\% - y$$

$$0.14 \cdot 500 = 0.1x + 0.2y$$

$$x + y = 500$$

$$y = 500 - x$$

$$0.14 \cdot 500 = 0.1x + 0.2(500 - x)$$

↓

$$x = 300 (\text{ml})$$

$$y = 200 (\text{ml})$$

