

THEORETICAL PART:

Definition:

A **rational equation** is an equation that contains at least one rational expression, while any nonrational expressions are polynomials.

General approach to solve such equations: multiply each term in the equation by the LCD of all the rational expressions.

Applications of Rational Equations:

- The rate of work is the reciprocal of the time needed to complete the task: if a given job can be done by a worker in x units of time, the worker works at a rate of $\frac{1}{x}$ jobs per unit of time.
- Rates of work are additive.

Definition.

A **radical equation** is an equation that has at least one radical expression containing a variable, while any nonradical expressions are polynomial terms.

Procedure of solving radical equations:

- Step 1. Begin by isolating the radical expression on one side of the equation. If there is more than one radical expression, choose one to isolate on one side.
- Step 2. Raise both sides of the equation by the power necessary to "undo" the isolated radical. That is, if the radical is an *n*-th root, raise both sides to the *n*-th power.
- Step 3. If any radical expressions remain, simplify the equation if possible and then repeat steps 1 and 2 until the result is a polynomial equation. When a polynomial equation has been obtained, solve the equation using polynomial methods.
- Step 4. Check your solutions in the original equation! Any extraneous solutions must be discarded.

Definition.

Meaning of $a^{\frac{m}{n}}$: If m and n are natural numbers with $n \neq 0$, if m and n have no common factors greater than 1, and if $\sqrt[n]{a}$ is a real number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.

PRACTICAL PART:

1. Solve the following rational equation:

•
$$\frac{x^3+3x^2}{x^2-2x-15} = \frac{4x+5}{x-5}$$

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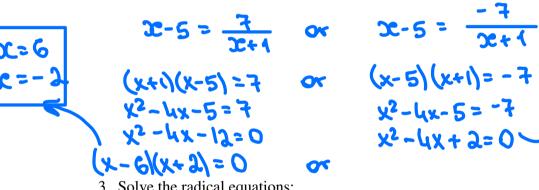
• $\frac{x^3}{x^2-2x-15} = \frac{4x+5}{x-5}$

• $\frac{4x+5}{x^2-2x-15} = \frac{4x+5}{x-5}$

$$x^{2} = 4x + 5$$
 $x^{2} - 4x - 5 = 0$
 $(x - 5)(x + 1) = 0$
 $x = 5$ or $x = -1$

2. Solve the equation:

•
$$|x - 5| = \frac{7}{x+1}$$



D = 16 - 8 = 8

3. Solve the radical equations:

(b) •
$$\sqrt{1-x} - 1 = x$$

(b) • $\sqrt{x+1} + \sqrt{x+2} = 1$

$$(a) \frac{1}{\sqrt{4-x}} = 4 + x^{2}$$

$$x^{2} + 3x = 0$$

$$x(x+3) = 0$$

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(b)
$$(\sqrt{x+1})^2 (1-\sqrt{x+2})^2$$

 $x+x=x-2\sqrt{x+2}+x+2$
 $x+x=x$
 $(\sqrt{x+2})^2 (1)^2$
 $x+2=x$
 $x=-x$

4. Solve the following equation with rational exponents:

$$(32x^2 - 32x + 17)^{\frac{1}{4}} = 3$$

(a)
$$X^{\frac{2}{3}} = 9$$

 $(^{3}\sqrt{x^{2}})^{\frac{3}{2}} = 9^{3}$
 $X^{2} = 9^{3} = 729$
 $X = \frac{1}{2}\sqrt{729}$

(b)
$$\sqrt{32x^2 - 32x + 17} = 3$$

 $32x^2 - 32x + 17 = 3^4 = 61$
 $32x^2 - 32x - 64 = 0$
 $x^2 - x - 2 = 0$
 $(x-2)(x+1) = 0$

$$v_e = \sqrt{\frac{2GM}{r}}$$

Solve for r.