WRH 8 Solutions

4.1: 1, 5, 6, 14, 28, 41, 46, 56
4.2: 1, 3, 20, 25, 29, 35, 38, 43
4.3: 2, 11, 16

$\boxed{4.1}$

1. $f(x) = -(1-x)^2 + 2$

$f(x) = -(x-1)^2 + 2$

Basic function: $\boxed{g(x) = x^2}$

5. $f(x) = \sqrt{x+2} - 5$

Basic function: $\boxed{g(x) = \sqrt{x}}$

6. $f(x) = [\![-2-x]\!]$

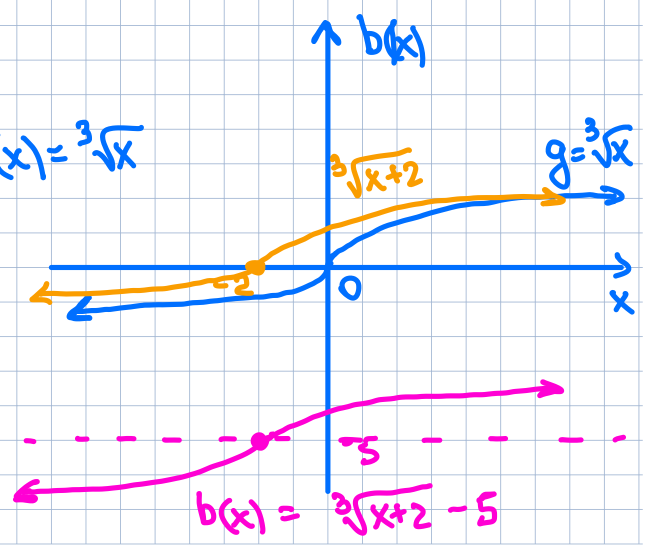
Basic function: $\boxed{g(x) = [\![x]\!]}$

14. Basic function: $\boxed{g(x) = |x|}$

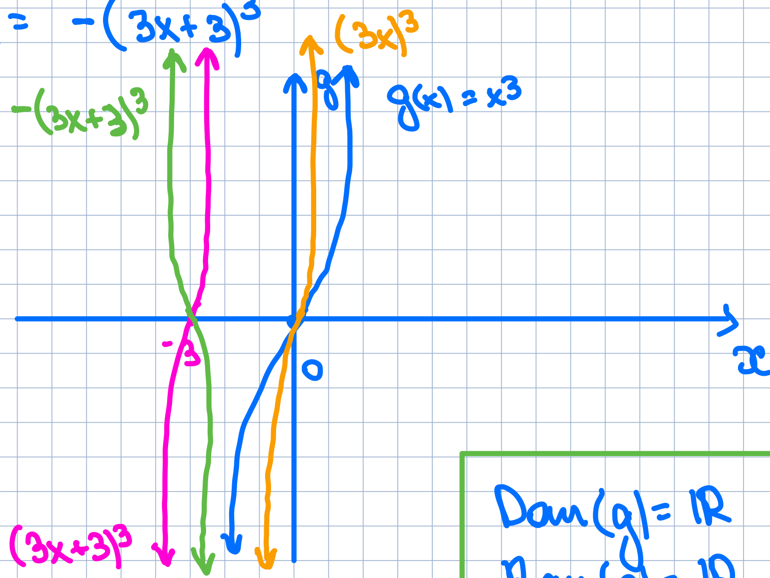
28. $b(x) = \sqrt[3]{x+2} - 5$

Basic function: $g(x) = \sqrt[3]{x}$

$$\boxed{\begin{array}{l} Dom(b) = \mathbb{R} \\ Ran(b) = \mathbb{R} \end{array}}$$



41. $g(x) = -(3x+3)^3$



$\boxed{\begin{array}{l} Dom(g) = \mathbb{R} \\ Ran(g) = \mathbb{R} \end{array}}$

46. $g(x) = x^2$

1) $g_1(x) = (x+3)^2$

2) $\boxed{g_2(x) = (x+3)^2 - 4}$

56. $g(x) = |x|$

1) $g_1(x) = |x+7|$

2) $g_2(x) = -|x+7|$

3) $\boxed{g_3(x) = -|-x+7|}$

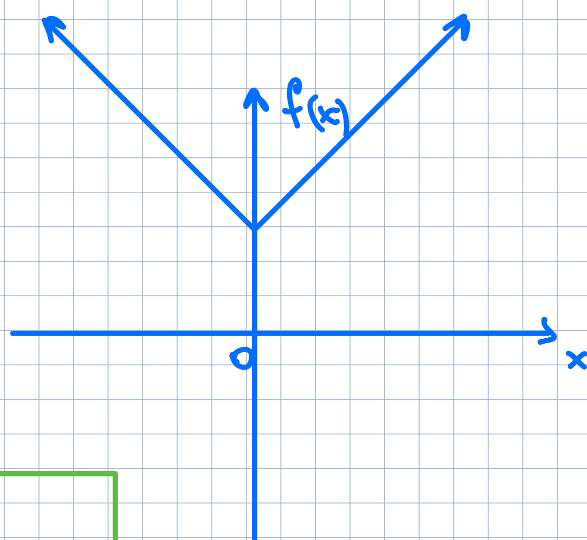
$\boxed{4.2}$

1. $f(x) = |x| + 3$

f is a function

$f(-x) = |-x| + 3 =$

$= |x| + 3$



$\boxed{\text{f is an even} \\ \text{f has y-symmetry}}$

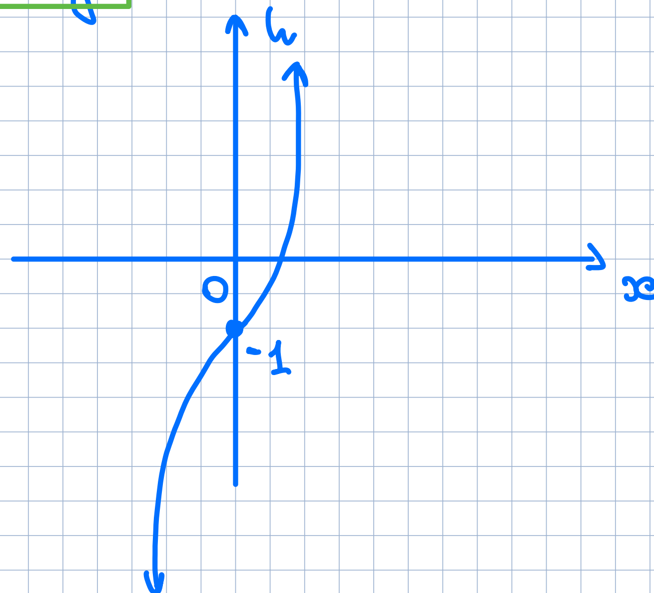
3. $h(x) = x^3 - 1$

h is a function

$h(-x) = (-x)^3 - 1 = -x^3 - 1$

$\boxed{\text{h is neither odd nor even}}$

$\boxed{h \text{ has no symmetry}}$



20. $G(x) = \sqrt{x+1}$

G is $\nearrow$ if : for any $x_1 < x_2 \Rightarrow$
$f(x_1) < f(x_2)$

$Dom(G) = [-1, \infty)$

Let $x_1, x_2 \in Dom(G)$. Then

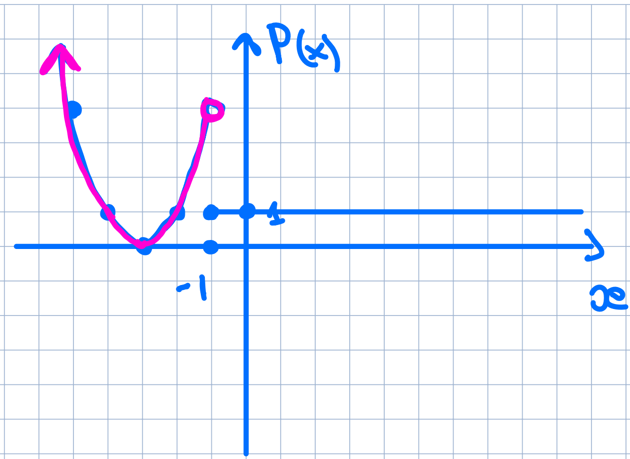
$$G(x_1) = \sqrt{x_1+1} < \sqrt{x_2+1} = G(x_2)$$

Therefore, $\boxed{G \text{ is } \nearrow \text{ for all } x \in (-1, \infty).}$

25.
$$P(x) = \begin{cases} (x+3)^2, & x < -1 \\ 1, & x \geq -1 \end{cases}$$

We see that

- $P(x)$ is constant for $x \in (-1, \infty)$
- $P(x)$ is $\uparrow$ for $x \in (-3, -1)$
- $P(x)$ is $\searrow$ for $x \in (-\infty, -3)$

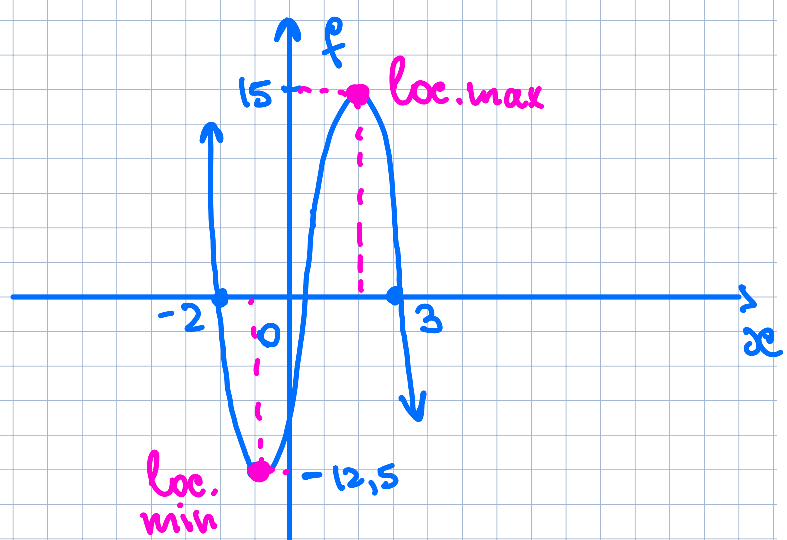


29. $f(x) = -2x^3 + 3x^2 + 12x - 5$

(a) + (b)

At $x = -1$ we have a loc. min. $f(-1) = -12.5$

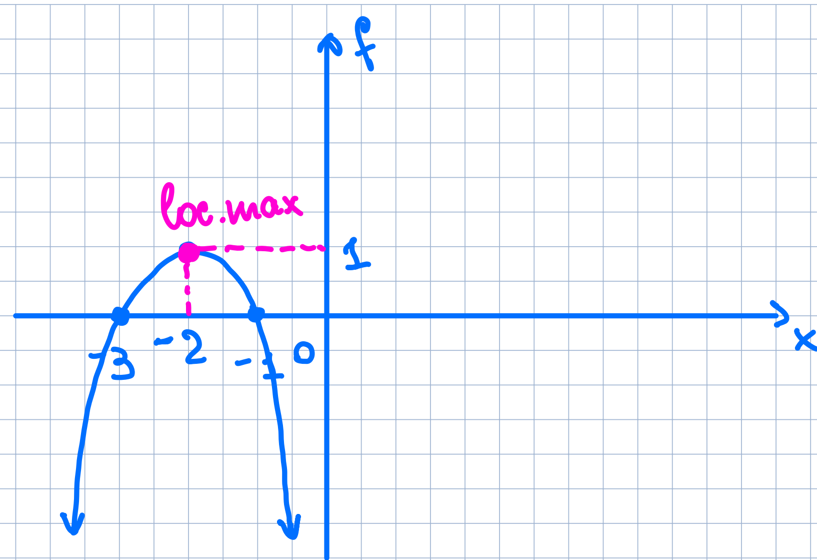
At $x = 2$ we have a loc. max. $f(2) = 15$.



35. $f(x) = -x^2 - 4x - 3 = -(x^2 + 4x + 3) =$

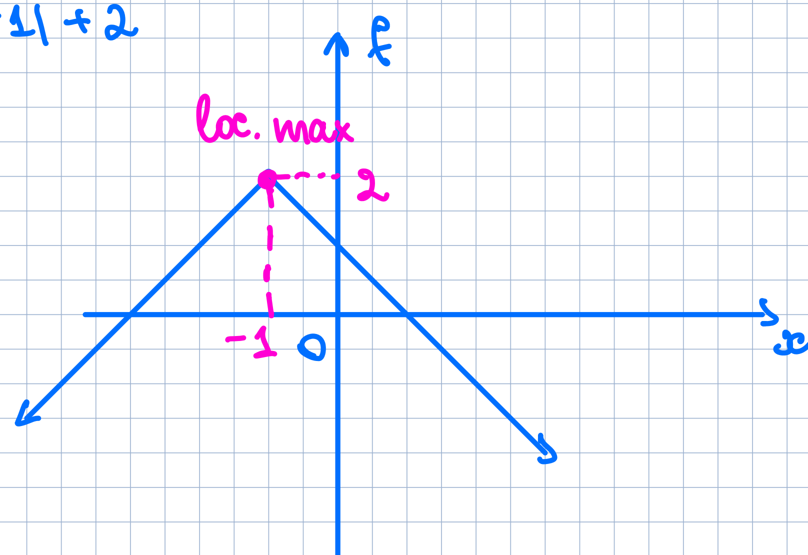
$= -(x+1)(x+3)$

We have a loc. max at $x = -2$.
$f(-2) = 1$.



38. $f(x) = -|x+1| + 2$

We have loc. max at $x = -1$
$f(-1) = 2$.



43. $f(x) = \sqrt{x}$; $[2,4]$

$\Delta x = x_2 - x_1 = 4 - 2 = 2$

$\Delta f = f(x_2) - f(x_1) = f(4) - f(2)$

$$\frac{\Delta f}{\Delta x} = \frac{\sqrt{4} - \sqrt{2}}{2} = \frac{2 - \sqrt{2}}{2} = \boxed{1 - \frac{1}{\sqrt{2}}.}$$

$\boxed{4.3}$

2. $f(-1)=0$

$g(-1)=-1$

(a) $(f+g)(-1) = f(-1)+g(-1) = 0-1 = \boxed{-1}$

(b) $(f-g)(-1) = f(-1)-g(-1) = 0+1 = \boxed{1}$

(c) $(fg)(-1) = f(-1)\cdot g(-1) = 0\cdot(-1) = \boxed{0}$

(d) $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{0}{-1} = \boxed{0.}$

11. $f(-1)=2$

$g(-1)=3$

(a) $(f+g)(-1) = f(-1)+g(-1) = 2+3 = \boxed{5}$

(b) $(f-g)(-1) = 2-3 = \boxed{-1}$

(c) $(f\cdot g)(-1) = 2\cdot 3 = \boxed{6}$

(d) $\left(\frac{f}{g}\right)(-1) = \boxed{\frac{2}{3}}$

16. $f(x) = x^2 - 1$

$g(x) = \sqrt[3]{x}$

(a) $(f+g)(x) = \boxed{x^2 - 1 + \sqrt[3]{x}}$

$\text{Dom}(f+g) = \boxed{\mathbb{R}}$

(b) $\frac{f}{g}(x) = \boxed{\frac{x^2-1}{\sqrt[3]{x}}}$

$\text{Dom}\left(\frac{f}{g}\right) = \boxed{\mathbb{R} \setminus \{0\}}$