

Section 1.4. Rational Expressions

1. Simplifying rational expressions
2. Combining rational expressions
3. Simplifying complex rational expressions.

1.

Def. A rational expression is an expression that can be written as a ratio of two polynomials $\frac{P}{Q}$, $Q \neq 0$

The given rational expression is simplified or reduced when P and Q contain no common factors (other than 1 or -1).

$$\frac{AC}{BC} = \frac{A}{B}, \quad B \neq 0, C \neq 0$$

Example

$$\frac{x^3 - 8}{x^2 - 2x} = \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{x \cancel{(x-2)}} =$$

$$= \frac{x^2 + 2x + 4}{x}, x \neq 0, 2$$

2. Combining rational expressions.

$$\bullet \quad \frac{2x-1}{x^2+x-2} - \frac{2x}{x^2-4} = \frac{(2x-1)(x^2-4) - 2x(x^2+x-2)}{(x^2+x-2)(x^2-4)} =$$

$$= \frac{\cancel{2x^3} - 8x - x^2 + 4 - \cancel{2x^3} - 2x^2 + 4x}{(x^2+x-2)(x^2-4)} = \frac{-3x^2 + 2x + 4}{(x^2+x-2)(x^2-4)}$$

$$\bullet \quad \frac{x^2 + 3x - 10}{x+3} \cdot \frac{x-3}{x^2-x-2} = \frac{(x^2+3x-10)(x-3)}{(x+3)(x^2-x-2)} =$$

$$= \frac{x^3 - \cancel{3x^2} + \cancel{3x^2} - 9x - 10x + 30}{(x+3)(x^2-x-2)} = \frac{x^3 - 19x + 30}{(x+3)(x^2-x-2)}.$$

3. Simplifying complex rational expressions.

Def. A complex rational expression is a fraction in which the numerator or denominator (or both) contains at least one rational expression.

Example

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - x - h}{h \cdot x(x+h)}}{h} =$$

$$= \frac{-1}{x(x+h)}$$