

**THEORETICAL PART:****Definition (Inverse of a Relation):**

Let  $R$  be a relation. The **inverse of  $R$** , denoted  $R^{-1}$ , is the relation defined by switching the first and second coordinates of each ordered pair that is an element of  $R$ .

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

**CAUTION:**

Note that  $f^{-1}$  does not stand for  $\frac{1}{f}$  when  $f$  is a function!

**Theorem (The Horizontal Line Test):**

Let  $f$  be a function. We say that the graph of  $f$  passes the **horizontal line test** if every horizontal line in the plane intersects the graph no more than once. If  $f$  passes the horizontal line test, then  $f^{-1}$  is also a function.

**Definition (One-to-One Function):**

A function  $f$  is **one-to-one** if, for every pair of distinct elements  $x_1$  and  $x_2$  in the domain of  $f$ , we have  $f(x_1) \neq f(x_2)$ .

**Procedure (Finding Formulas of Inverse Functions):**

Let  $f$  be a one-to-one function, and assume that  $f$  is defined by a formula. To find a formula for  $f^{-1}$ , perform the following steps:

1. Replace  $f(x)$  in the definition of  $f$  with the variable  $y$ . The result is an equation in  $x$  and  $y$  that is solved for  $y$  at this point.
2. Solve the equation for  $x$ .
3. Replace the  $x$  in the resulting equation with  $f^{-1}(x)$  and replace each occurrence of  $y$  with  $x$ .

**Theorem (Composition of Functions and Inverses):**

Given a function  $f$  and its inverse  $f^{-1}$ , the following statements are true:

$$f(f^{-1}(x)) = x \quad \text{for all } x \in \text{Dom}(f^{-1})$$

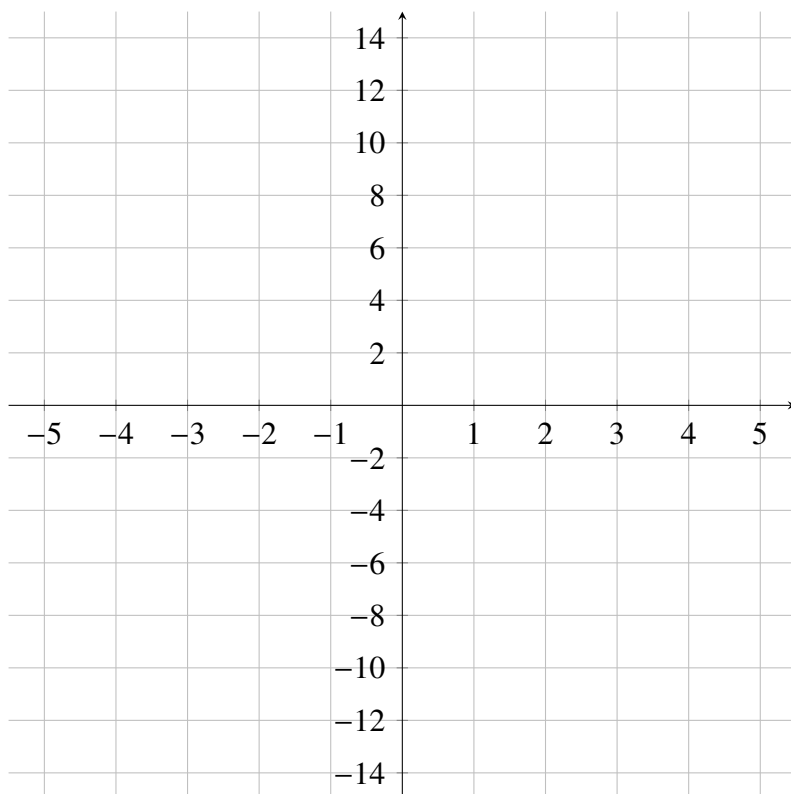
$$f^{-1}(f(x)) = x \quad \text{for all } x \in \text{Dom}(f)$$

**PRACTICAL PART:**

1. Determine the inverse of each of the following relations. Then graph each relation and its inverse, and determine the domain and range of both:

(a)  $R = \{(4, -1), (-3, 2), (0, 5)\}$

(b)  $y = x^2$



2. Determine if the following functions have inverse functions:

(a)  $f(x) = |x|$

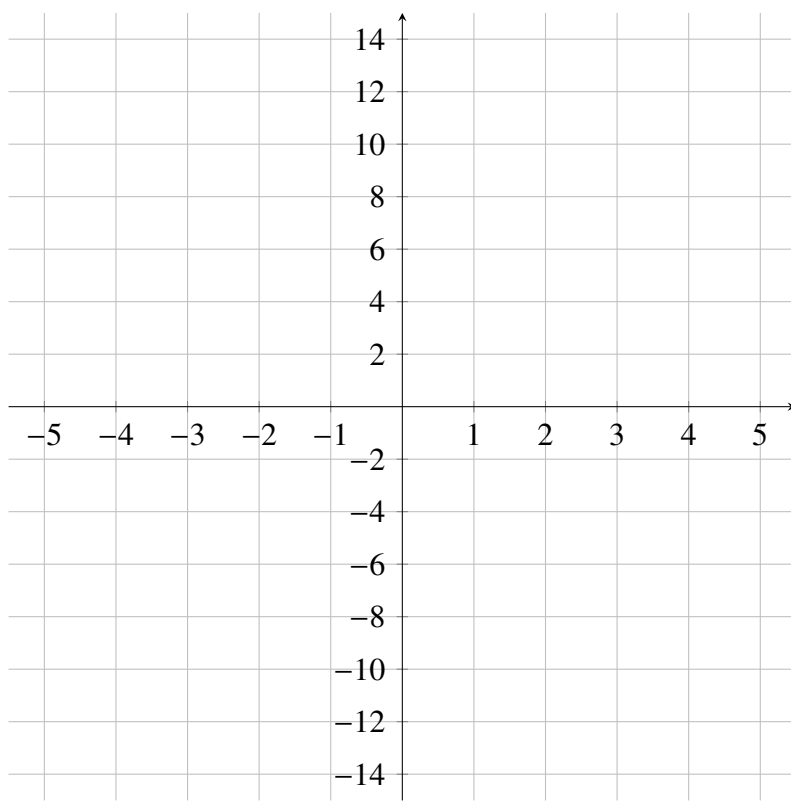
(b)  $g(x) = (x + 2)^3$

3. Find the inverse of each of the following functions:

(a)  $f(x) = (x - 1)^3 + 2$

(b)  $g(x) = \frac{x - 3}{2x + 1}$

4. Find two suitable restrictions of the domain so that the function  $f(x) = (x + 1)^2 - 3$  has an inverse function, then find a formula for the inverse for each restricted function.



5. Use functions  $f(x) = (x-1)^3 + 2$  and  $f^{-1}(x) = (x-2)^{\frac{1}{3}} + 1$  to demonstrate that the composition of a function and its inverse leaves any input unchanged.