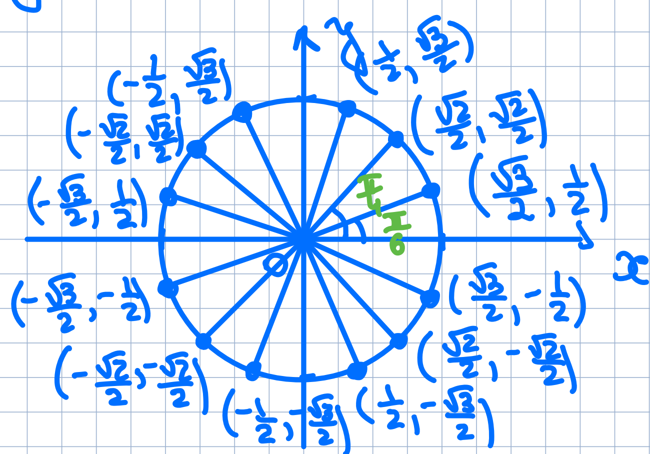
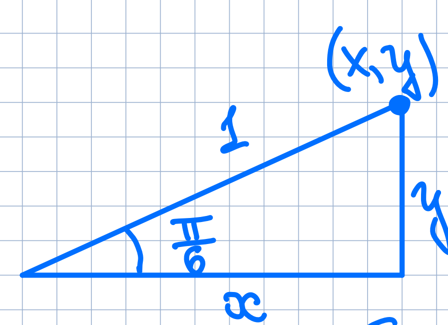
Section 7.4. Graphs of sine and cosine functions

1. Graphing sine and cosine functions.
2. Periodicity and symmetry.
3. Amplitude, frequency, and phase shifts.
4. Simple harmonic motion.
5. Damped harmonic motion.

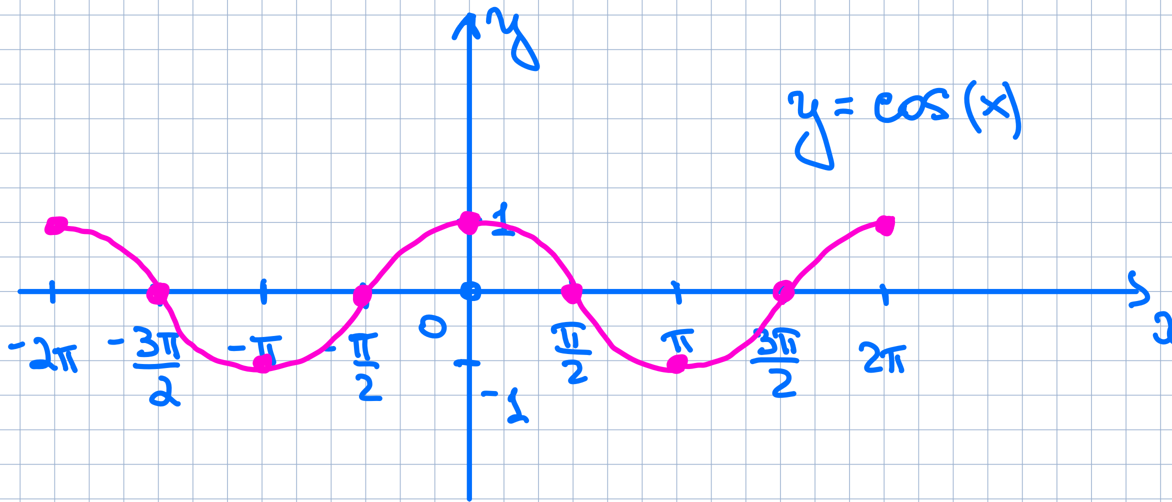
1.

$y = \cos(x)$



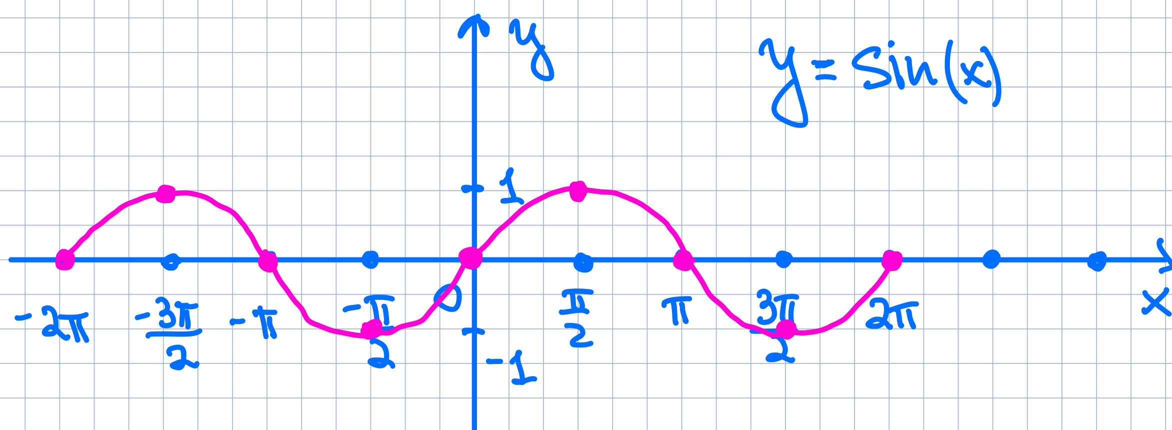


$$\cos\left(\frac{\pi}{6}\right) = \frac{x}{1} \Rightarrow x = \frac{\sqrt{3}}{2}$$



Dom(y) = $\mathbb{R}$

Ran(y) = $[-1, 1]$



Dom(sin(x)) = $\mathbb{R}$

Ran(sin(x)) = $[-1, 1]$.

$\boxed{2.}$

Def. (Period of a function)

A function f is said to be

periodic if there is a positive number p such that

$$f(x+p) = f(x)$$

for all x in the domain of f.
The smallest such number p is called the period of f.

$$\sin(x+2\pi) = \sin(x)$$

$$\cos(x+2\pi) = \cos(x)$$

$$\sin(x+2\pi n) = \sin(x)$$
$$\cos(x+2\pi n) = \cos(x)$$

$$\sec(x+2\pi) = \frac{1}{\cos(x+2\pi)} = \frac{1}{\cos(x)} = \sec(x)$$

$$\tan(x+\pi) = \tan(x)$$

<u>Identities</u> (Even/Odd identities)

$\sin(-x) = -\sin(x)$ $\qquad$ $\cos(-x) = \cos(x)$

$\csc(-x) = -\csc(x)$ $\qquad$ $\sec(-x) = \sec(x)$

$\tan(-x) = -\tan(x)$ $\qquad$ $\cot(-x) = -\cot(x)$

Example

Prove: $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$ for all x.

Sol.

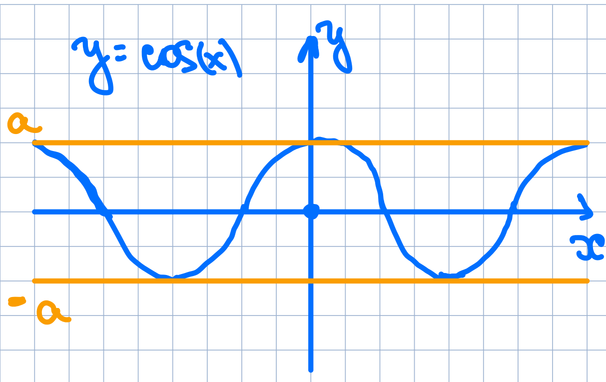
$$\cos\left(x - \frac{\pi}{2}\right) = \cos\left(-\left(\frac{\pi}{2} - x\right)\right) =$$

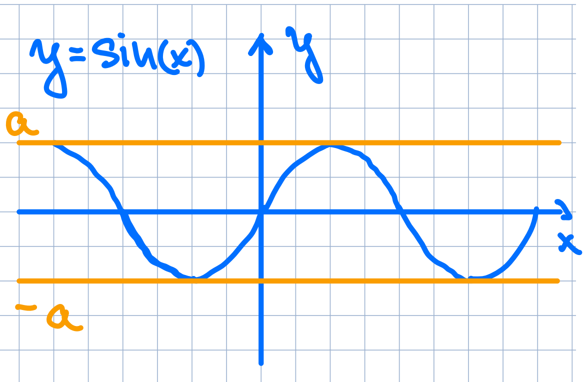
$$= \cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

$\boxed{3.}$ **Def.** (Amplitudes of Sine and Cosine Curves)

Given a fixed real number a, the amplitude of the function $f(x) = a\sin(x)$ or the function $g(x) = a\cos(x)$ is the value $|a|$.

As we know, the multiplication of $\sin x$ or $\cos x$ by a stretches (or compresses, if $-1 < a < 1$) the graph vertically by a factor $|a|$, so the amplitude represents the distance between the x-axis and the maximum value of the function.





Def. (Frequency of $y=\sin(x)$, $y=\cos(x)$)

Given a fixed real number b, the <u>frequency</u> of the function $f(x)=\sin(bx)$ or the function $g(x)=\cos(bx)$ is the number $\frac{b}{2\pi}$. When the independent variable represents time, measured in seconds, the measurement of frequency is stated in terms of <u>cycles per second</u>, or <u>hertz (Hz)</u>.

Example

Find an amplitude and frequency of

$$f(x)=3\sin\left(\frac{x}{2}\right).$$

$a=|3|=3$

$b=\frac{1}{2}/2\pi=\frac{1}{4\pi}.$

Def. (Period Revisited)

Given a fixed real number b, the <u>period</u> of the function $f(x) = \sin(bx)$ or the function $g(x) = \cos(bx)$ is the number $\frac{2\pi}{b}$.

The period and frequency of a sinusoidal function are reciprocals of one another.

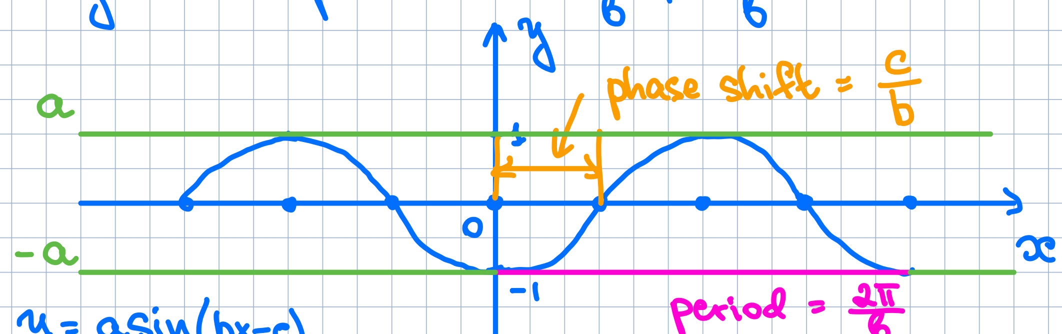
Def. (Amplitude, period and phase shift combined)

Given constants a, b ($b > 0$), and c, the functions

$$f(x) = a\sin(bx - c) \quad \text{and} \quad g(x) = a\cos(bx - c)$$

have <u>amplitude</u> $|a|$, <u>period</u> $\frac{2\pi}{b}$, and a <u>phase shift</u> of $\frac{c}{b}$.

The left endpoint of one cycle of either function is $\frac{c}{b}$ and the right endpoint is $\frac{c}{b} + \frac{2\pi}{b}$.



$y = a\sin(bx - c)$

$\boxed{4.}$ **Def.** (Simple Harmonic Motion)

If an object is oscillating and its displacement from some midpoint at time t can be described by either

$$f(t) = a\sin(bt) \quad \text{or} \quad g(t) = a\cos(bt),$$

the object is said to be in <u>simple harmonic motion</u> (SHM). In both cases, the maximum displacement of the object from its midpoint is the amplitude $|a|$ and its frequency of oscillation is $\frac{b}{2\pi}$.

$\boxed{5.}$

Example (Modeling Damped Harmonic Motion)

$$f(t) = -4e^{-t}\cos(6\pi t)$$