Section 6.1. Exponential functions and their graphs

1. Characteristics of exponential functions.
2. Graphing exponential functions.
3. Solving elementary exponential equations.

1.

Def. (Exponential functions)

Let a be a fixed, positive real number not equal to 1. The <u>exponential function with base a</u> is the function

$$\boxed{f(x) = a^x}$$

Properties (Behaviour of exponential functions)

Given a positive real number $a \neq 1$, the function $f(x) = a^x$ is

- <u>a decreasing function</u> if $0 < a < 1$, with $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to 0$ as $x \to \infty$

- <u>an increasing function</u> if $a > 1$, with $f(x) \to 0$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

$dom(f) = \mathbb{R}$
$Ran(f) = (0, \infty)$.

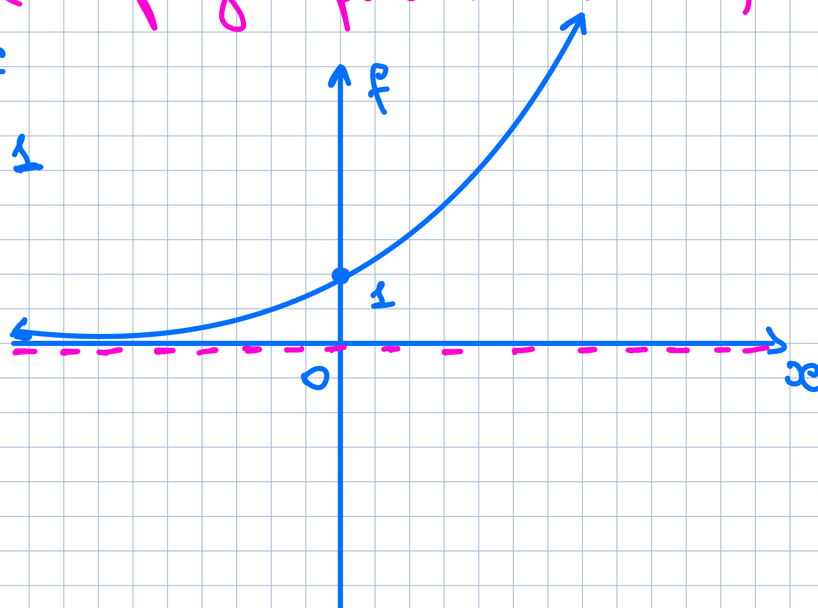
$\boxed{2.}$

<u>Example</u> (Graphing exponential functions)

(a) $f(x) = 3^x$

$a = 3 > 1$

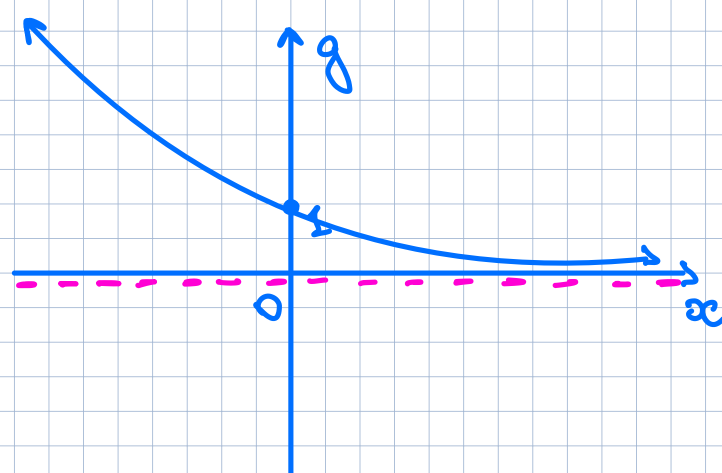
f is ↑



(b) $g(x) = \left(\frac{1}{2}\right)^x$

$0 < a = \frac{1}{2} < 1$

g is ↓

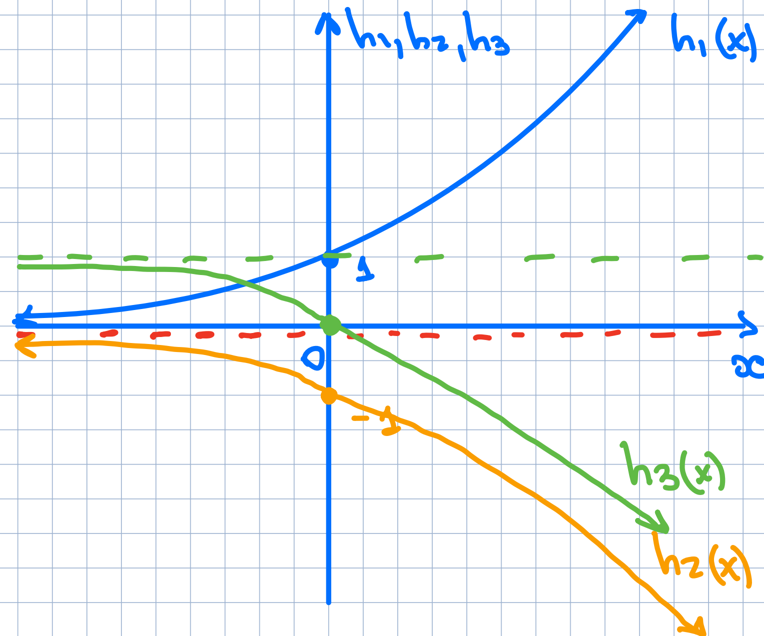


(c) $h(x) = -3^x + 1$

$h_1(x) = 3^x$

$h_2(x) = -3^x$

$h_3(x) = -3^x + 1$



$\boxed{3.}$

Def. An equation in which the variable appears as an exponent is called an <u>exponential equation</u>.

$2^x = 5$

Procedure (Solving elementary exponential equations)

To solve an elementary exponential equation perform the following steps:

1. Isolate the exponential. Move the exponential containing x to one side of the equation and any constants

or other variables in the expression to the other side. Simplify, if necessary.

2. Find a base that can be used to rewrite both sides of the equation.

3. Equate the powers, and solve the resulting equation.

Example

- $25^x - 125 = 0$

$$25^x = 125$$
$$25^x = 5^3$$
$$(5)^{2x} = 5^3$$
$$2x = 3$$
$$\boxed{x = \frac{3}{2}}$$