Section 9.1. The Law of Sines

1. The Law of Sines.
2. Applications of the Law of Sines.
3. The Law of Sines and the area of a triangle.

1.

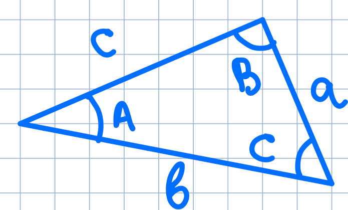


Fig. 1

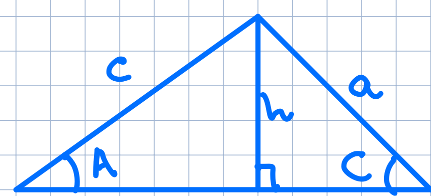
Law of Sines
Two angles and a side (AAS or ASA)
Two sides and a nonincluded angle (SSA)

Theorem (The Law of Sines)

Given a triangle with sides and angles labeled according to the convention in Fig. 1, the following equation represents the Law of Sines:

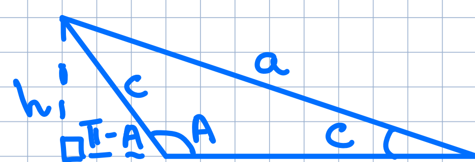
$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

The Law of Sines
(Acute Case)



$c \sin A = h = a \sin C$

The Law of Sines
(Obtuse Case)

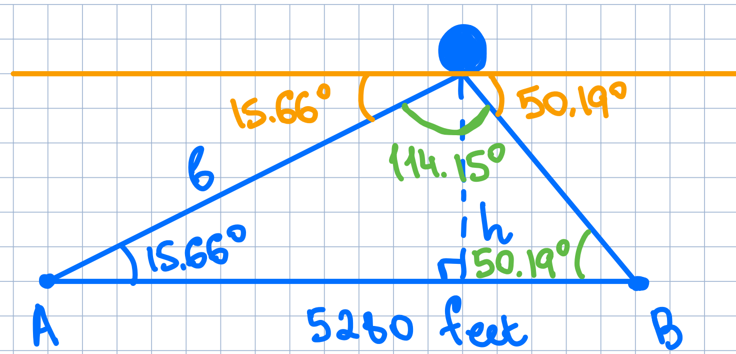


$c \sin A = h = a \sin C$

$\boxed{2.}$ Example 1 (AAS situation)

Sarah is piloting a hot-air ballon, and finds herself becalmed directly above a long straight road. She notices mile markers on the road, and determines the angle of depression to the two markers. How far is she from marker A? What is her altitude?

Solution



$$\frac{b}{\sin(50.19°)} = \frac{5280}{\sin(114.15°)}$$

$$\Downarrow$$

$$b = \sin(50.19°) \cdot \frac{5280}{\sin(114.15°)}$$

$$b \approx 4445 \text{ (feet)}$$

$$\sin(15.66°) = \frac{h}{4445}$$

$$h \approx 1200 \text{ (feet)}$$

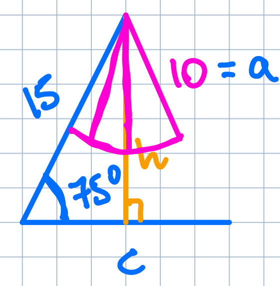
Example (using the Law of Sines in an SSA situation with no solution)

Create a triangle, if possible, for which $A = 75°$, $b = 15$ units, $a = 10$ units.

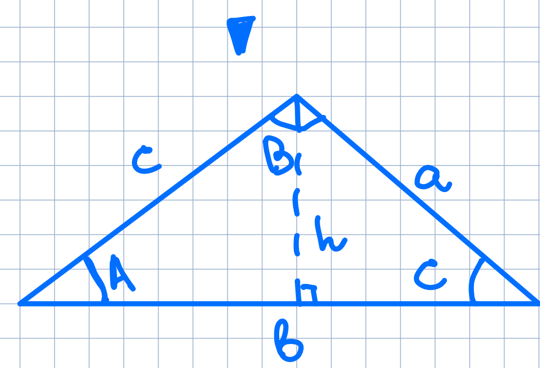
Solution

$h = 15 \sin 75^\circ \approx 14.5$

$a < h$



No triangle satisfies the given conditions.



$\boxed{3.}$

Theorem (Area of a Triangle (sine formula))

The area of a triangle is one-half the product of the lengths of any two sides and the sine of their included angle. That is,

$$\boxed{\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B}$$