THEORETICAL PART:

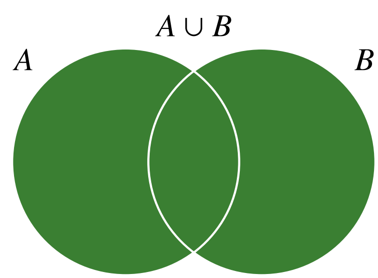
Definitions:

- The natural numbers set: $\mathbb{N} = \{1, 2, 3, 4, 5, \cdots\}$
- The whole numbers set: $\{0, 1, 2, 3, 4, \cdots\}$
- The integers numbers set: $\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$
- The rational numbers set: $\mathbb{Q} = \left\{\frac{p}{q} \mid p \in \mathbb{Z},\ q \in \mathbb{Z},\ q \neq 0\right\}$
- The irrational numbers set: $\mathbb{I} = \mathbb{R} \backslash \mathbb{Q}$
- The real numbers set: $\mathbb{R}$
- Empty set or the null set notation: $\emptyset$, $\{\ \}$
- The notation $\{x \mid x \text{ has property } P\}$ is used to describe a set of real numbers, all of which have the property P

Basic Set Operations and Venn Diagrams:

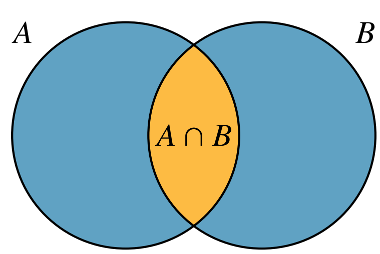
- The **union** of two sets A and B:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



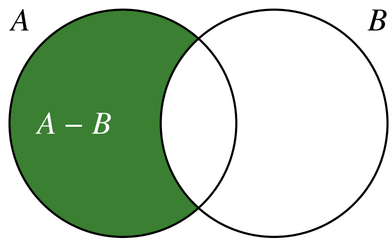
- The **intersection** of two sets A and B:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



- The **difference** of two sets A and B:

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$



Definitions:

- The absolute value of a real number a "$|a|$" is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

- **Properties of Absolute Value:**

  $|a| \geq 0$

  $|-a| = a$

  $a \leq |a|$

  $|ab| = |a||b|$

  $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

  $|a + b| \leq |a| + |b|$ (triangle inequality)

- The given two real numbers a and b, the **distance** between them is defined to be $|a - b|$.

- **Field Properties:**

  Closure:

  additive: $a + b$ is a real number

  multiplicative: ab is a real number

  Commutative:

  additive: $a + b = b + a$

  multiplicative: $ab = ba$

  Associative:

  additive: $a + (b + c) = (a + b) + c$

  multiplicative: $a(bc) = (ab)c$

  Identity:

  additive: $a + 0 = 0 + a = a$

  multiplicative: $a \cdot 1 = 1 \cdot a = a$

  Inverse:

  additive: $a + (-a) = 0$

  multiplicative: $a \cdot \frac{1}{a} = 1, a \neq 0$

  Distributive:

  $a(b + c) = ab + ac$

- **Cancellation Properties:** Let A, B and C be algebraic expressions. We have

$$A = B \Leftrightarrow A + C = B + C \quad \text{(Additive cancellation)}$$

$$A = C \Leftrightarrow A \cdot C = B \cdot C, \quad \text{where} \quad C \neq 0 \quad \text{(Multiplicative cancellation)}$$

- **Zero-Factor Property:** Let A, B be algebraic expressions. Then we have

$$AB = 0 \Rightarrow A = 0 \quad \text{or} \quad B = 0.$$

PRACTICAL PART:

1. Which elements of the following set $\left\{5\sqrt{7}, 4\pi, -1, \frac{22}{7}, |-8|, 3.\bar{3}\right\}$ are
   - natural numbers
   - whole numbers
   - integers
   - rational numbers
   - irrational numbers
   - real numbers?

2. Which set the following intervals do represent?
   (a) $(2, 8)$
   (b) $[-3, 10)$
   (c) $(-\infty, \infty)$

3. Write the following sets as an interval using interval notation:
   (a) $A = \left\{x \,\middle|\, -3 \leq x < 19\right\}$
   (b) $B = \left\{\text{The nonnegative real numbers}\right\}$

4. Using absolute value properties simplify the following expressions:
   (a) $|(-3)(5)| =$

(b) $\left|\frac{-3}{7}\right| =$

5. Simplify the following set expressions:

   (a) $\mathbb{N} \cap \mathbb{Z} \cap \mathbb{Q}$

   (b) $(5, 10) \cup \mathbb{Z}$

   (c) $(-2, 4] \cap [0, 9]$

6. Evaluate the following algebraic expressions for the given values of the variables:

   (a) for $x = 8$
   $$\sqrt{2x} + \frac{3x}{4},$$

   (b) for $x = 2$, $y = -1$, $z = 3$
   $$\frac{x^2y^3}{8z} - \frac{|2xy|}{8z}$$

7. Identify the property that justifies each of the following statements.

   (a)
   $$4(y - 3) = 4y - 12,$$

   (b)
   $$25x^3 = 10y \Leftrightarrow 5x^3 = 2y,$$

   (c)
   $$x^2z = 0 \Rightarrow x^2 = 0 \quad \text{or} \quad z = 0.$$

   (d)
   $$y + 12 = 18$$