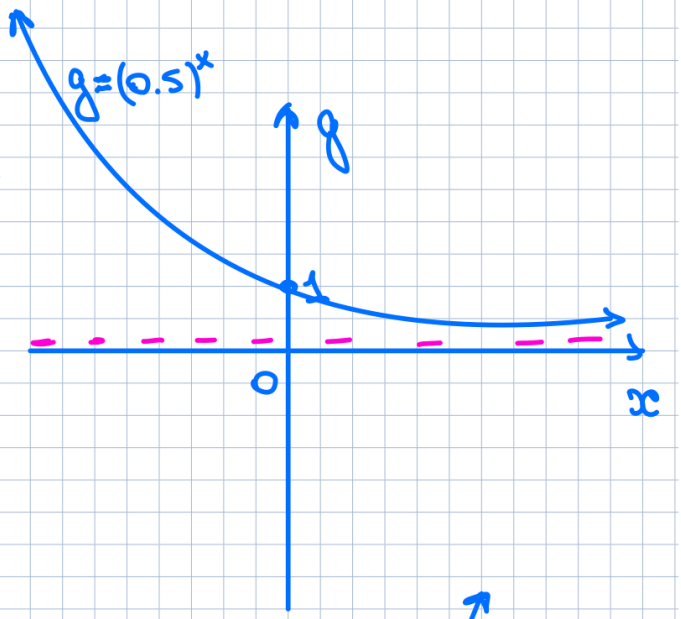
WRH 11 Solutions

6.1

2. $g(x) = (0.5)^x$

$a = \frac{1}{2}$

$0 < a < 1$



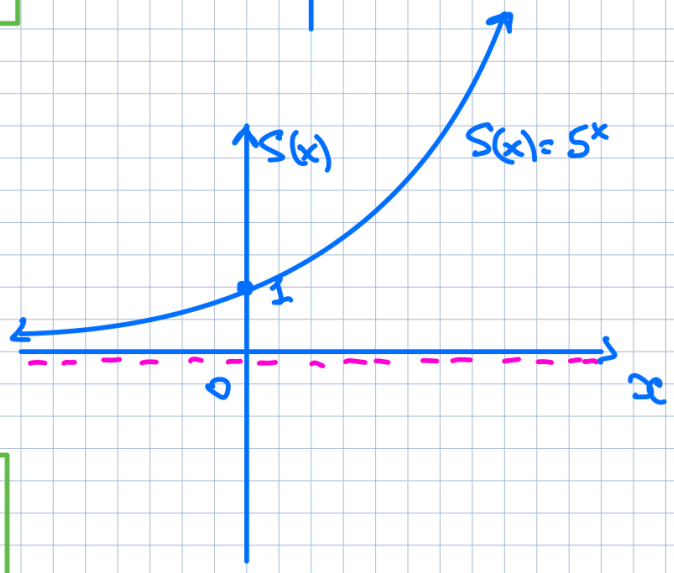
$Dom(g) = \mathbb{R}$

$Ran(g) = (0, \infty)$

4. $S(x) = (0.2)^{-x}$

$S(x) = \left(\frac{1}{5}\right)^{-x} =$

$= \frac{1}{\left(\frac{1}{5}\right)^x} = 5^x$



$Dom(S) = \mathbb{R}$

$Ran(S) = (0, \infty)$

24. $9^{2x-5} = 27^{x-2}$

$3^{2(2x-5)} = 3^{3(x-2)}$

$4x - 10 = 3x - 6$

$\boxed{x = 4}$

31. $4^{3x+2} = \left(\frac{1}{4}\right)^{-2x}$

$4^{3x+2} = 4^{-(-2x)}$

$3x + 2 = 2x$

$\boxed{x = -2}$

41. $-10^{x} = -0.001$

$-10^{x} = -10^{-3}$

$10^{x} = 10^{-3}$

$\boxed{x = -3}$

49. $g(x) = 2(4^{x-1}) = 2 \cdot 2^{2(x-1)} = 2 \cdot 2^{2x-2} =$

$= 2^{2x-2+1} = 2^{2x-1}$

$2x - 1 = 0$

$x = \frac{1}{2}$ then $g\left(\frac{1}{2}\right) = 1$

If $x \to +\infty$, then $g(x) \to +\infty$

(i)

55. $h(x) = 3^{\frac{1}{2}x}$

$x = 0 \Rightarrow h(0) = 1$

$x \to +\infty$, then $h(x) \to +\infty$

(h)

6.2

1. $V = P(1 - e^{-0.18d})$

$d = 5$

$P = 300$

$V = 300(1 - e^{-0.18 \cdot 5})$

$V \approx 178$ (people)

7. $A(t) = A_0 a^t$

$A(1600) = \frac{A_0}{2}$

$$A_0\, a^{1600} = \frac{A_0}{2}$$

$$a^{1600} = \frac{1}{2}$$

$$a = \left(\frac{1}{2}\right)^{\frac{1}{1600}} \approx 0.9996$$

Therefore,

(a) $\boxed{A(t) = A_0 \cdot (0.9996)^t}$

(b) at $t = 0$: $A_0 = 1$ gram

$A(100) = 1 \cdot 0.9996^{100} = \boxed{0.9608}$ (gram)

(c) $A(1000) = 1 \cdot 0.9996^{1000} = \boxed{0.6703}$ (gram)

19. $N(t) = \dfrac{10\,000}{1 + 999e^{-t}}$

(a) $t = 0$: $N(0) = \boxed{10\,000}$ (people)

(b) $t = 8$: $N(8) = \dfrac{10000}{1 + 999 \cdot e^{-8}} \approx \boxed{7490}$ (people)

(c) If $t \to \infty$, then $N(t) \to \boxed{10\,000}$.

$\dfrac{10\,000}{1 + 999e^{-t}} \to 0$ as $t \to \infty$

6.3

2. $216 = 6^3$

$\log_6 216 = 3$

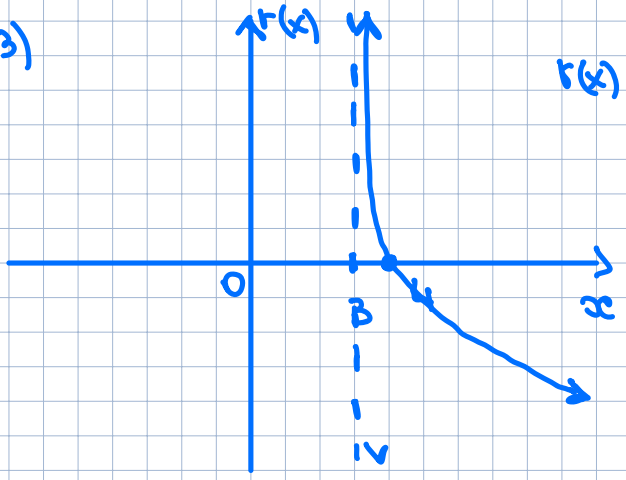
13. $\log_3 81 = 4$

$3^4 = 81$

27. $r(x) = \log_{\frac{1}{2}}(x-3)$

$\text{Dom}(r) = (3, +\infty)$

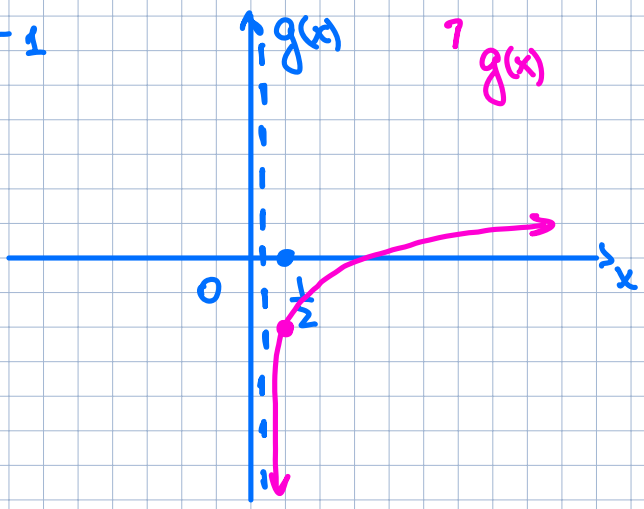
$\text{Ran}(r) = \mathbb{R}$



36. $g(x) = \log_5(2x) - 1$

$\text{Dom}(g) = (0, \infty)$

$\text{Ran}(g) = \mathbb{R}$



38. $f(x) = \log_2(2-x)$

(g)

46. $\log_7(\sqrt{7}) = \log_7 7^{\frac{1}{2}} = \frac{1}{2}\log_7 7 = \boxed{\frac{1}{2}}$

63. $\log_8 4^{\log 1000} = \log 1000 \cdot \log_8 4 =$

$= \log_{10} 10^3 \cdot \log_8 4 = 3 \cdot \log_8 4 = 3 \frac{\ln 4}{\ln 8} =$

$= 3 \frac{2 \cdot \ln 2}{3 \cdot \ln 2} = \boxed{2}$

64. $\log_{16} x = \frac{3}{4}$

$16^{\frac{3}{4}} = x$

$x = \sqrt[4]{16^3} = (\sqrt[4]{16})^3 = 2^3 = 8$

$\boxed{x = 8}$

77. $\log(x^2) = -2$

$\log_{10} x^2 = -2$

$$10^{-2} = x^2$$

$$\frac{1}{100} = x^2$$

$$\boxed{x = \pm \frac{1}{10}}$$

81.

$$\ln(\ln(x^2)) = 0$$

$$\ln(x^2) = 1$$

$$e = x^2$$

$$\boxed{x = \pm \sqrt{e}}$$

6.4

1. $\log_5(125x^3) = \log_5 125 + \log_5 x^3 =$

$= \log_5 5^3 + 3\log_5 x = \boxed{3 + 3\log_5 x}$

14. $\log(\log(100\,000^{2x})) =$

$= \log(2x \cdot \log_{10} 10^5) = \log(10x) =$

$= \log 10 + \log x = \boxed{1 + \log x}$