

Assessment Instructions:

- The Assessment 4 is 10 problems and is worth 40 points. Each numbered problem will earn you a score of 1-4 based on your set up of the function, your use of course methods to solve and prove your solution and your statement of the solution.
- You will have 1 hour to complete AS-4.
- The AS-4 is closed book and closed notes.
- **Calculators are not allowed** on the Assessment.

5.1

1. Solve the following polynomial equations by factoring and/or using the quadratic formula, making sure to identify all the solutions.

(a) $x^3 - x^2 = 72x$

(b) $x^4 - 8x^3 + 25x^2 = 0$

2. For each of the following polynomial functions, determine the behaviour of its graph as $x \rightarrow \pm\infty$ and identify the x - and y -intercepts. Use this information to sketch the graph of each polynomial.

(a) $r(x) = x^2 - 2x - 3$

(b) $f(x) = (3 - x)(x + 2)(x + 4)$

3. Solve the following polynomial inequalities.

(a) $-x^3 - x^2 + 30x > 0$

(b) $(x^2 - 1)(x - 4)(x + 5) \leq 0$

5.2

4. Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$.

(a) $\frac{x^3 + 2x^2 - 4x - 8}{x - 3}$

(b) $\frac{2x^3 - 3ix^2 + 11x + (1 - 5i)}{2x - i}$

5. Use synthetic division to determine if the given value for c is a zero of the corresponding polynomial. If not, determine $p(c)$.

(a) $p(x) = 12x^4 - 7x^3 - 32x^2 - 7x + 6, c = 1$

(b) $p(x) = 2x^2 - (3 - 5i)x + (3 - 9i), c = -3i$

6. Construct a polynomial function with the stated properties.

(a) Third-degree, zeros of -2, 1, and 3, and y-intercept of -12.

(b) Second-degree, zeros of -4 and 3, and goes to $-\infty$ as $x \rightarrow -\infty$.

5.3

7. List all of the potential rational zeros of the following polynomials.

(a) $q(x) = x^3 - 10x^2 + 23x - 14$

(b) $k(x) = x^4 - 10x^2 + 24$

8. Use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of the following polynomials.

(a) $f(x) = x^4 - 25$

(b) $g(x) = x^3 + 6x^2 + 11x + 6$

9. Use the Intermediate Value Theorem to show that the following polynomial has a real zero between the indicated values.

(a) $f(x) = 5x^3 - 4x^2 - 31x - 6; -3$ and -1

5.4

10. Sketch the graph of each factored polynomial.

(a) $f(x) = (x + 1)^2(x - 2)^3$

(b) $k(x) = -x^3(x - 2)(x + 1)^2$

11. Use all available methods to solve each polynomial equation.

(a) $x^4 + 15 = 2x^3 + 8x^2 - 10x$

(b) $x^3 - 5 = 5x^2 - 9x$

5.5

12. Find equations for the vertical asymptotes, if any, for each of the following rational functions.

(a) $f(x) = \frac{x^2 + 5}{x^3 - 27}$

(b) $g(x) = \frac{x^2 - 1}{x^2 - 8x + 7}$

13. Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions.

(a) $f(x) = \frac{5}{x - 1}$

(b) $g(x) = \frac{2x^2 - 5x + 6}{x - 3}$

(c) $k(x) = \frac{-3x + 5}{x - 2}$

14. Sketch the graphs of the following rational functions.

(a) $f(x) = \frac{x^2 - 4}{2x - x^2}$

(b) $g(x) = \frac{5}{x - 1}$

15. Solve the following rational inequalities.

(a) $\frac{5}{x - 2} > \frac{3}{x + 2}$

(b) $\frac{x}{x^2 - x - 6} \leq \frac{-1}{x^2 - x - 6}$

6.1

16. Sketch the graphs of the following functions. State their domain and range.

(a) $f(x) = (0.5)^x$

(b) $g(x) = 10^x$

(c) $h(x) = 2^{-x}$

(d) $k(x) = 2 - 4^{2-x}$

17. Solve the following exponential equations.

(a) $3^{2x-1} = 27$

(b) $3^{x^2+4x} = 81^{-1}$

(c) $10^x = 0.01$

6.2

18. Assuming a current world population of 7.75 billion people, an annual growth rate of 1.9% per year, and a worst-case scenario of exponential growth, what will the world population be in 10 years (in 50 years)?
19. Problem 19, page 455 (textbook).