

Section 7.1. Radian and Degree Measure

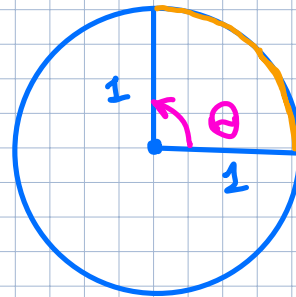
1. The unit circle and angle measure.
2. Converting between degrees and radians.
3. The Pythagorean Theorem and commonly encountered angles.
4. Arc length and angular speed.
5. Area of a circular sector.

1.

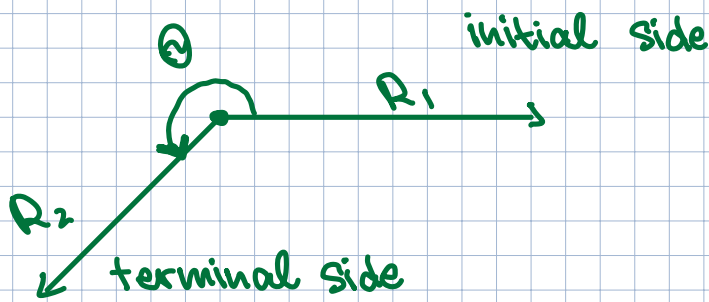
Def.

Let θ be an angle at the center of a circle of radius 1. The measure of θ in radians (rad) is the length of that portion of the circle subtended by θ , which is the portion of the circumference (—). Note that the unit of length measurement is immaterial. As long as

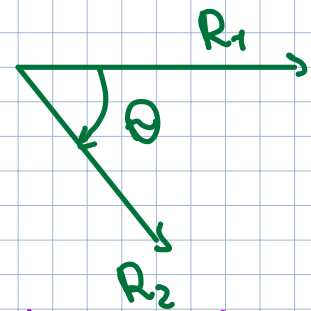
Radian measure of θ



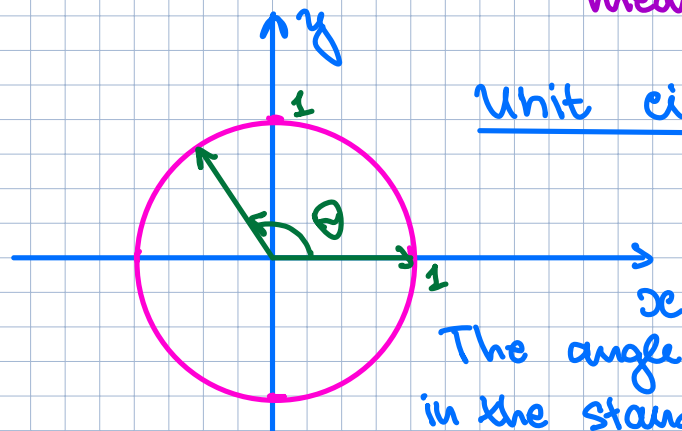
the circle has a radius 1, the length of the subtended portion of the circle is defined to be the radian measure of the angle.



Positive angle measure



Negative angle measure



Unit circle

The angle θ in the standard position

2.

Formula (Angle measurement conversion)

Since $180^\circ = \pi \text{ rad}$

\Downarrow

$$1^\circ = \frac{\pi}{180^\circ} \text{ rad} \quad \text{and} \quad \left(\frac{180^\circ}{\pi}\right) = 1 \text{ rad}$$

Therefore,

1. $x^\circ = x \left(\frac{\pi}{180} \right) \text{ rad}$

2. $x \text{ rad} = x \left(\frac{180}{\pi} \right)^\circ$

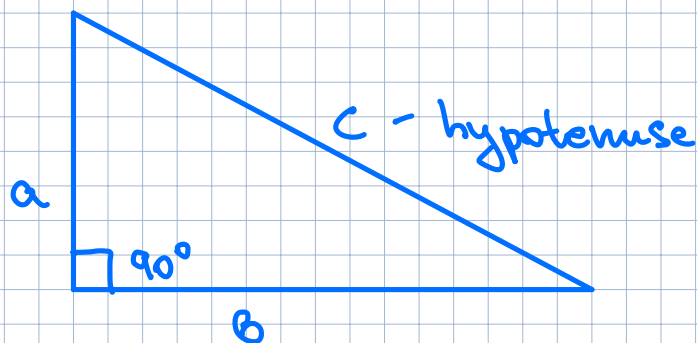
$$1 \text{ rad} \approx 57.296^\circ.$$

Example

• $\frac{\pi}{3} \text{ rad} = \frac{\pi}{3} \left(\frac{180}{\pi} \right)^\circ = 60^\circ$

• $270^\circ = 270 \frac{\pi}{180} = \frac{3\pi}{2} \text{ rad}$

3.



Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

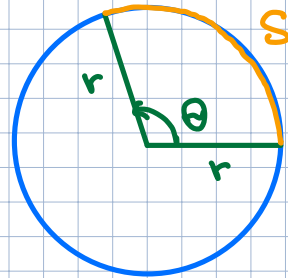
$$30^\circ = \frac{\pi}{6}$$

$$45^\circ = \frac{\pi}{4}$$

$$60^\circ = \frac{\pi}{3}$$

$$90^\circ = \frac{\pi}{2}$$

4.

Formula (Arc Length)

Given a circle of radius r , the length S of the arc subtended by a central angle θ (in radians) is given by the following formula

$$S = \left(\frac{\theta}{2\pi} \right) (2\pi r) = r\theta$$

Def. (Angular Speed and Linear Speed)

If an object moves along an arc of a circle defined by a central angle θ in time t , the object is said to have an angular speed ω given by

$$\omega = \frac{\theta}{t}.$$

If the circle has a radius of r , the distance traveled in time t is the arc length S , and the linear speed v is given by

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Caution!

The arc length and angular speed formulas, as well as the area formula that follows, are only true for angles measured in radians. Equivalent but less convenient formulas can be derived for angles measured in terms of degrees.

5.

Def. A sector of a circle is the portion of a circle between two radii.

Formula:

The area A of a sector with a central angle of θ in a circle of radius r is

$$A = \left(\frac{\theta}{2\pi} \right) (\pi r^2) = \frac{r^2 \theta}{2}.$$