Solutions

THEORETICAL PART:

Definition:

A **rational expression** is an expression that can be written as a ratio of two polynomials $\frac{P}{Q}$. Such a fraction is undefined for any value(s) of the variable(s) for which $Q = 0$. A given rational expression is **simplified** or **reduced** when P and Q contain no common factors (other then 1 or -1).

Definition:

A **complex rational expression** is a fraction in which the numerator or denominator (or both) contains at least one rational expression.

Caution: Only common factors can be canceled!

$$\frac{\cancel{x}+4}{x^2} = \frac{4}{x} \quad \text{is incorrect}$$

PRACTICAL PART:

1. Simplify the following rational expressions, and indicate values of the variable that must be excluded:

(a)

$$\frac{x^3-8}{x^2-2x} = \frac{\cancel{(x-2)}(x^2+2x+4)}{x\cancel{(x-2)}} =$$

$$= \frac{x^2+2x+4}{x}$$

$x \neq 0, \ x \neq 2$

(b)

$$\frac{x^2-x-6}{3-x} = \frac{\cancel{(x-3)}(x+2)}{-\cancel{(x-3)}} =$$

$$= -(x+2)$$

$x \neq 3$

2. Add or subtract the rational expressions:

(a)

$$\frac{2x-1}{x^2+x-2} - \frac{2x}{x^2-4} =$$

$$= \frac{(2x-1)(x^2-4) - 2x(x^2+x-2)}{(x^2+x-2)(x^2-4)} =$$

$$= \frac{\cancel{2x^3} - 8x - x^2 + 4 - \cancel{2x^3} - 2x^2 + 4x}{(x^2+x-2)(x^2-4)} = \frac{-3x^2 - 4x + 4}{(x^2+x-2)(x^2-4)}$$

(b)

$$\frac{x+1}{x+3} + \frac{x^2+x-2}{x^2-x-6} - \frac{x^2-2x+9}{x^2-9} =$$

$$= \frac{(x+1)(x-3)(x+2) + (x^2+x-2)(x+3) - (x^2-2x+9)(x+2)}{(x+3)(x-3)(x+2)} =$$

$$= \frac{(x^2-2x-3)(x+2) + \cancel{x^3} + 3x^2 + x^2 + 3x - 2x - 6 - \cancel{x^3} - \cancel{2x^2} + \cancel{2x^2} + 4x - 9x - 18}{(x+3)(x-3)(x+2)}$$

$$= \frac{x^3 + \cancel{2x^2} - \cancel{2x^2} - 4x - 3x - 6 + 4x^2 - 4x - 24}{\cdots\cdot} = \frac{x^3 + 4x^2 - 11x - 30}{(x+3)(x-3)(x+2)}$$

3. Multiply or divide the rational expressions:

(a)

$$\frac{x^2+3x-10}{x+3} \cdot \frac{x-3}{x^2-x-2} =$$

$$= \frac{(x^2+3x-10)(x-3)}{(x+3)(x^2-x-2)} = \frac{x^3 - \cancel{3x^2} + \cancel{3x^2} - 9x + 30 - 10x}{(x+3)(x^2-x-2)} =$$

$$= \frac{x^3 - 19x + 30}{(x+3)(x^2-x-2)}$$

(b)

$$\frac{x^2+5x-14}{3x} \div \frac{x^2-4x+4}{9x^3} =$$

$$= \frac{x^2+5x-14}{\cancel{3x}} \cdot \frac{\overset{3}{\cancel{9}}x^{\cancel{3}\,2}}{x^2-4x+4} = \frac{3x^2(x^2+5x-14)}{x^2-4x+4}$$

4. Simplify the complex rational expressions:

(a)

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{\cancel{x}-\cancel{x}-h}{x(x+h)}}{h} =$$

$$= \frac{-\cancel{h}}{x(x+h)\cancel{h}} = \frac{-1}{x(x+h)}$$

(b)

$$\frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}} =$$

$$= \frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{\frac{y-x}{xy}}{\frac{y^2-x^2}{x^2y^2}} =$$

$$= \frac{(y-x)}{\cancel{xy}} \div \frac{x^{\cancel{2}}y^{\cancel{2}}}{(y^2-x^2)} = \frac{(\cancel{y-x})xy}{(\cancel{y-x})(y+x)} =$$

$$= \frac{xy}{y+x}$$