

THEORETICAL PART:**Identities (Sum and Difference Identities):****Sine Identities**

$$\sin(u + v) = \sin u \cos v + \cos u \sin v, \quad \sin(u - v) = \sin u \cos v - \cos u \sin v$$

Cosine Identities

$$\cos(u + v) = \cos u \cos v - \sin u \sin v, \quad \cos(u - v) = \cos u \cos v + \sin u \sin v$$

Tangent Identities

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}, \quad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Theorem (Sum of Sines and Cosines)

$$A \sin(x) + B \cos(x) = \sqrt{A^2 + B^2} \left(\frac{A}{\sqrt{A^2 + B^2}} \sin(x) + \frac{B}{\sqrt{A^2 + B^2}} \cos(x) \right) = \sqrt{A^2 + B^2} \sin(x + \varphi),$$

where

$$\cos \varphi = \frac{A}{\sqrt{A^2 + B^2}}, \quad \sin \varphi = \frac{B}{\sqrt{A^2 + B^2}}$$

PRACTICAL PART:

1. Determine the exact value of $\sin 75^\circ$.

2. Determine the exact value of $\cos 75^\circ$.

3. Determine the exact value of $\tan(\pi/12)$.

4. Determine the exact value of $\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ$.

5. Use a difference identity to verify that $\sin\left(\frac{\pi}{2} - x\right) = \cos x$.

6. Evaluate the expression

$$\sin\left(\cos^{-1}(3/5) - \tan^{-1}(12/5)\right).$$

7. Express $\sin(\tan^{-1} x + \cos^{-1} x)$ as an algebraic function of x .

8. Express the function $f(x) = \sin x - \sqrt{3} \cos x$ in terms of a single sine function, and graph the result.