Section 6.5. Exponential and Logarithmic Equations

1. Converting between exponential and logarithmic forms.
2. Applications of exponential and logarithmic equations.
3. Analysis of a stock market investment.

$\boxed{1.}$

Properties:

1. The equations $x = a^y$ and $y = \log_a x$ are equivalent, and are, respectively, the exponential form and the logarithmic form of the same statement.
2. The inverse of the function $f(x) = a^x$ is $f^{-1}(x) = \log_a x$, and vice versa.
3. A consequence of the last point is that

$$\log_a(a^x) = x$$

and

$$a^{\log_a x} = x.$$

$$\log_a 1 = 0 \quad \text{and} \quad \log_a a = 1.$$

4. $$\log_a (xy) = \log_a x + \log_a y$$

5. $$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

6. $$\log_a (x^r) = r \log_a x$$

<u>Example</u>

1) exponential equation:

$$3^{2-5x} = 11$$

$$2 - 5x = \log_3 11$$

$$-5x = \log_3 11 - 2$$

$$x = -\frac{1}{5} \log_3 11 + \frac{2}{5}$$

$$\boxed{x = -\frac{1}{5} \frac{\ln 11}{\ln 3} + \frac{2}{5}}$$

2) Logarithmic equation:

$$\log_7 (3x - 2) = 2$$

$$3x - 2 = 7^2$$
$$3x = 51$$
$$\boxed{x = 17}$$

$\boxed{2.}$

Example (Compounding Interest)

Rita is saving up money for a down payment on a new car. She currently has \$5500 but she knows she can get a loan at a lower interest rate if she can put down \$6000. If she invests her \$5500 in a money market account that earns an annual interest rate of 4.8 % compounded monthly, how long will it take her to accumulate the \$6000?

Solution

$P = 5500$

$A(t) = 6000$

$r = 0.048$

$n = 12$

$$6000 = 5500\left(1+\frac{0.048}{12}\right)^{12t}$$

$$\frac{60\cancel{00}}{55\cancel{00}} = \left(1+\frac{0.048}{12}\right)^{12t}$$

$$\frac{12}{11} = \left(1+\frac{0.048}{12}\right)^{12t}$$

$$\ln\left(\frac{12}{11}\right) = 12t\,\ln\left(1+\frac{0.048}{12}\right)$$

$$\ln\left(\frac{12}{11}\right) = 12t\,\ln(1.004)$$

$$t \approx 1.82 \text{ (years)}$$

$\boxed{3.}$ Analysis of a Stock Market Investment (Read at home).