Name: Solutions ______ / 40

Assessment 5 Instructions:

- The AS-5 is 10 problems and is worth 40 points.
- You will have 1 hour to complete AS-5.
- The AS-5 is closed book and closed notes.
- **Calculators are not allowed** on the AS-5.
- Show all your work for full credit and box your final answer.

1. [4 points] Use the properties of logarithms to **expand** the following expressions as much as possible.

$$\log\left(\frac{10}{\sqrt{x+y}}\right)$$

$$\log\left(\frac{10}{\sqrt{x+y}}\right) = \log 10 - \log\sqrt{x+y} = \boxed{1 - \frac{1}{2}\log(x+y)}$$

2. [4 points] Solve the following exponential and logarithmic equations.

a. $e^{4x} = e^{3x+14}$

$$4x = 3x + 14$$

$$\boxed{x = 14}$$

b. $\log_5 x^2 = 3$

$$5^3 = x^2$$

$$\boxed{x = \sqrt{5^3} = 5\sqrt{5}}$$

3. [4 points] Convert each of the following angle measures as directed.

a. Express $\frac{3\pi}{2}$ in degrees.

$$\frac{3\pi}{2}\cdot\left(\frac{180}{\pi}\right)^\circ = \frac{3\cdot \cancel{180}\,90}{\cancel{2}} = 270^\circ$$

b. Express -144° in radians.

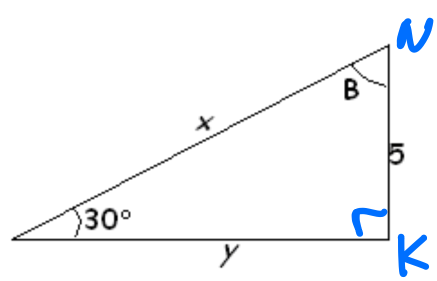
$$-144\cdot\frac{\pi}{180} = -\frac{72}{90}\pi = -\frac{36}{45}\pi = -\frac{12}{15}\pi = \boxed{-\frac{4}{5}\pi}$$

4. [4 points] Find the area of the sector of a circle of radius 20 ft with a central angle of 138°. *(Hint: $A = \frac{r^2\theta}{2}$)*

$$A = \frac{r^2\theta}{2} \qquad \theta = 138^\circ = 138\cdot\frac{\pi}{180} = \frac{69}{90}\pi = \frac{23}{30}\pi$$

$$A = \frac{20^2}{2}\cdot\frac{23}{30}\pi = \frac{20\cancel{0}}{3\cancel{0}}\cdot 23\pi = \frac{460}{3}\pi\ (ft^2)$$

5. [4 points] Use the information contained in the figure to determine the values of the six trigonometric functions of an angle B.



Since $\triangle MNK$ is a right triangle, we have

$x = 10$

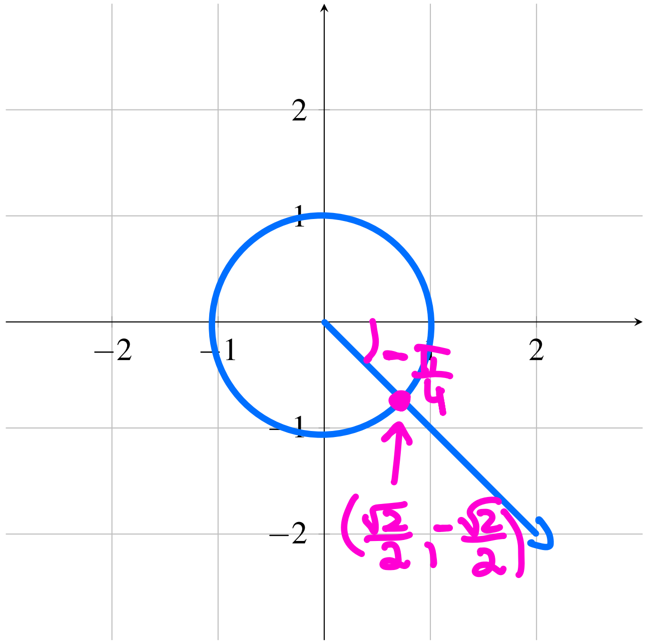
Then $y^2 = 10^2 - 5^2 = 100 - 25 = 75$

$y = 5\sqrt{3}$

$\sin B = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ $\cos B = \frac{5}{10} = \frac{1}{2}$ $\sec B = 2$

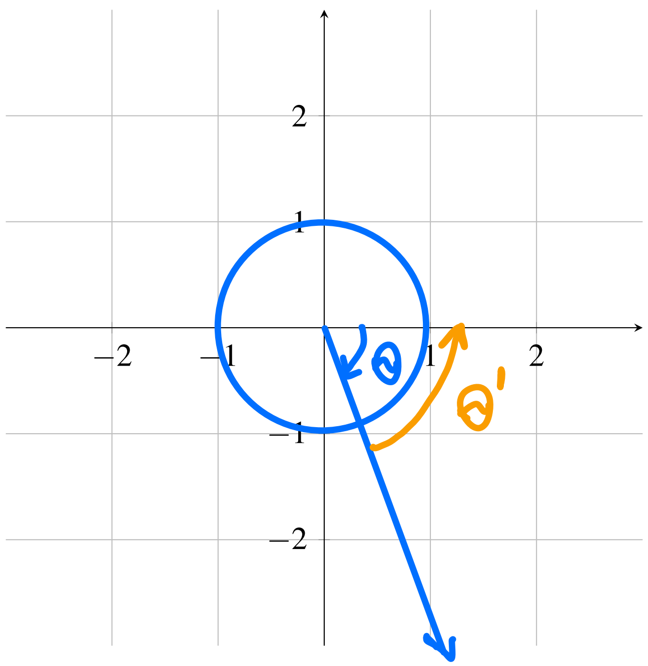
$\tan B = \frac{5\sqrt{3}}{5} = \sqrt{3}$ $\cot B = \frac{1}{\sqrt{3}}$ $\csc B = \frac{2}{\sqrt{3}}$

6. [4 points] Determine the point (x, y) on the unit circle associated with the real number $s = -\dfrac{\pi}{4}$. Sketch the the unit circle and the point (x, y) on it on the plane below.



$(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

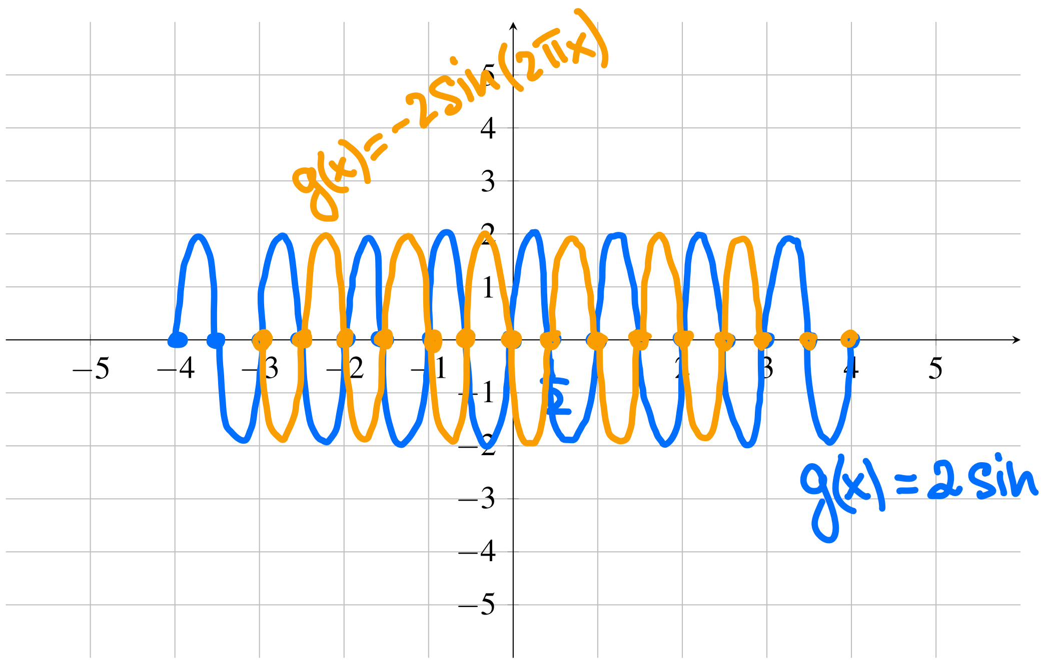
7. [4 points] Determine the reference angle associated with the given angle $\theta = -60^\circ$. Sketch both angles θ and θ' on the plane below.



$\boxed{\theta' = 60^\circ}$

$\theta = 60^\circ$

8. **[4 points]** Sketch the graph of the function $g(x) = -2\sin(2\pi x)$. State **precisely** the amplitude, frequency and phase shift for the given function.



$|a| = 2$

$\nu = \frac{B}{2\pi} = \frac{2\pi}{2\pi} = 1$

No phase shift

$2\pi x = 0$ $\quad$ $2\pi x = 2\pi$

$x = 0$ $\quad$ $x = 1$

9. **[4 points]** Evaluate the following expressions

a. $\arccos(-1) = \pi$

$\cos\varphi = -1 \Rightarrow \varphi = \boxed{\pi}$

b. $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$\tan\varphi = -\sqrt{3} \Rightarrow \varphi = \boxed{-\frac{\pi}{3}}$

10. [4 points] Use trigonometric identities to simplify the expression

$$\frac{1}{\sec^2 x} + \sin x \cdot \cos\left(\frac{\pi}{2} - x\right)$$

$$\frac{1}{\sec^2 x} + \sin x \cdot \sin x = \frac{1}{\sec^2 x} + \sin^2 x = \cos^2 x + \sin^2 x =$$

$$= \boxed{1}$$