THEORETICAL PART:

Solutions

Definition (Power Functions):

A **power function** is a function of the form $f(x) = ax^r$, where a and r are real numbers.

Types of Power functions:

1. Power functions of the form $f(x) = ax^n$.
2. Power functions of the form $f(x) = ax^{-n}$, where $a \in \mathbb{R}$, $n \in \mathbb{N}$; we need to consider cases when n is even and when n is odd.
3. Power functions of the form $f(x) = ax^{\frac{1}{n}}$, where $a \in \mathbb{R}$, $n \in \mathbb{N}$; we need to consider cases when n is even and when n is odd.

Absolute Value Function:

The basic absolute value function is $f(x) = |x|$.

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0 \end{cases}$$

The Greatest Integer Function:

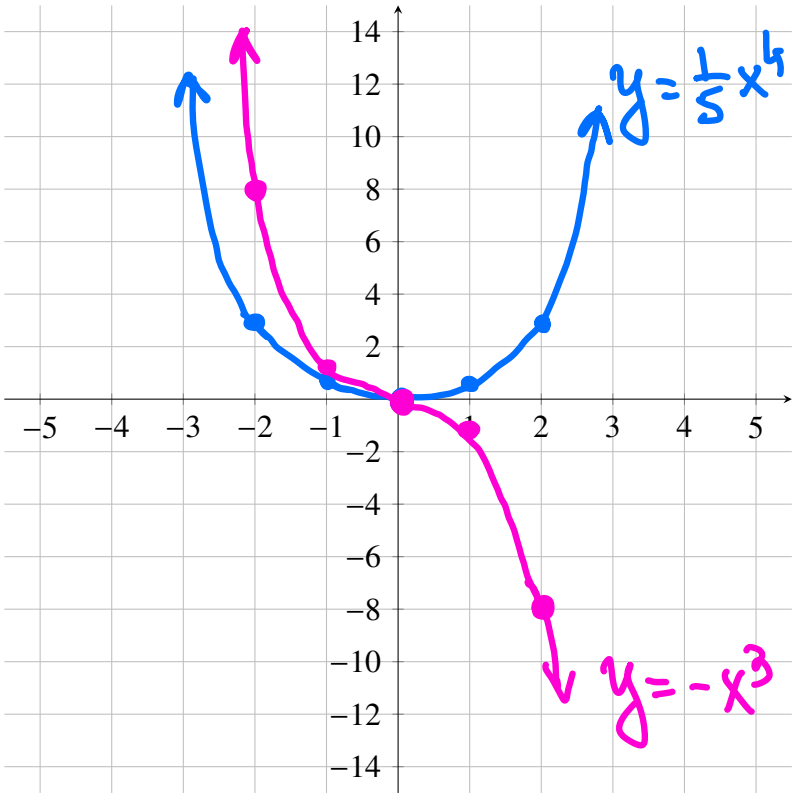
The greatest integer function, $f(x) = [\![x]\!]$, is a function commonly encountered in computer science applications. It is defined as follows: the **greatest integer of** x is the largest integer less than or equal to x. For instance, $[\![4.3]\!] = 4$ and $[\![-2.9]\!] = -3$.

Piecewise-Defined Function:

A **piecewise-defined function** is a function defined in terms of two or more formulas, each valid for its own unique portion of the real number line.

PRACTICAL PART:

1. Sketch the graphs of the following functions:
   (a) $f(x) = \frac{1}{5}x^4$
   (b) $f(x) = -x^3$



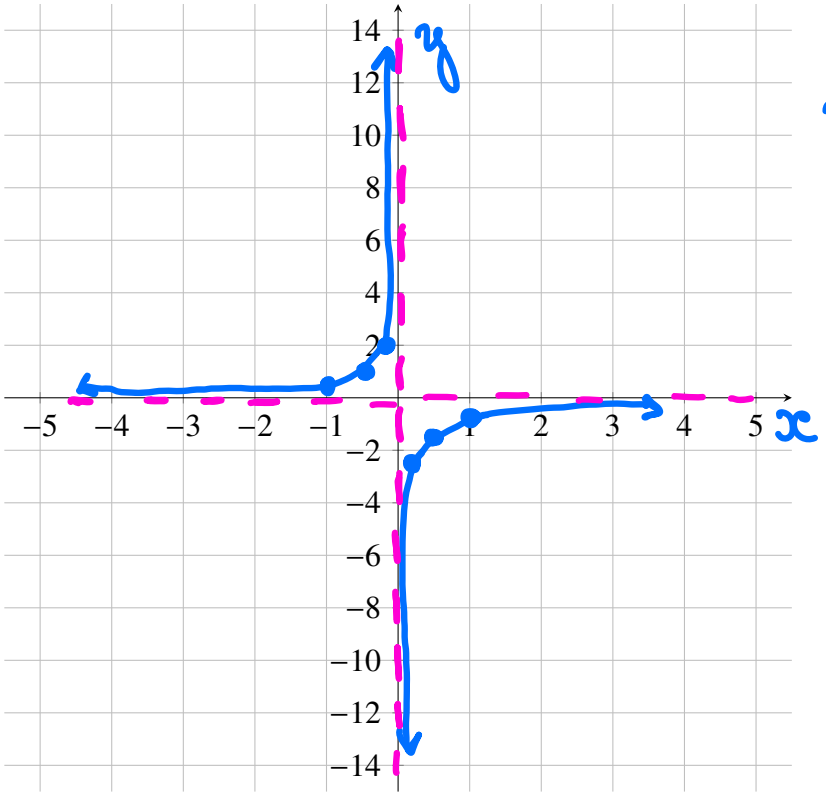
$y = \frac{1}{5}x^4$

x	y
0	0
1	$\frac{1}{5}$
−1	$\frac{1}{5}$
2	3.2
−2	3.2

$y = -x^3$

x	y
0	0
−1	+1
2	−8
−2	8

2. Sketch the graph of the function $f(x) = -\frac{1}{4x}$.

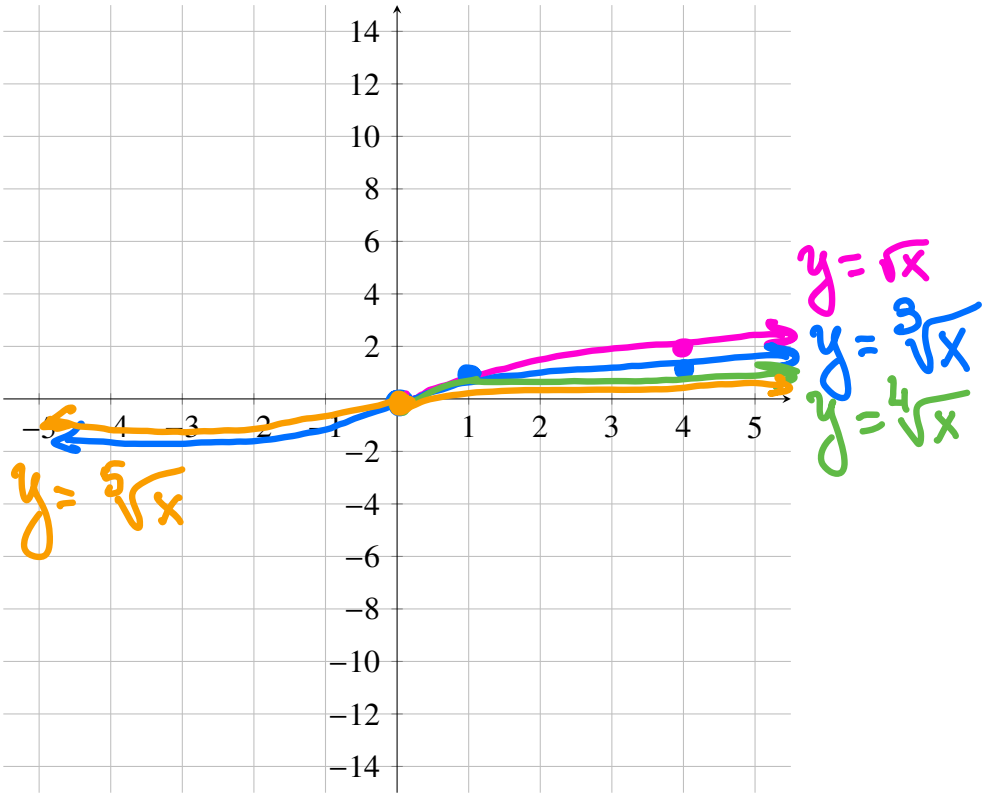


$y = -\frac{1}{4} \cdot \frac{1}{x}$

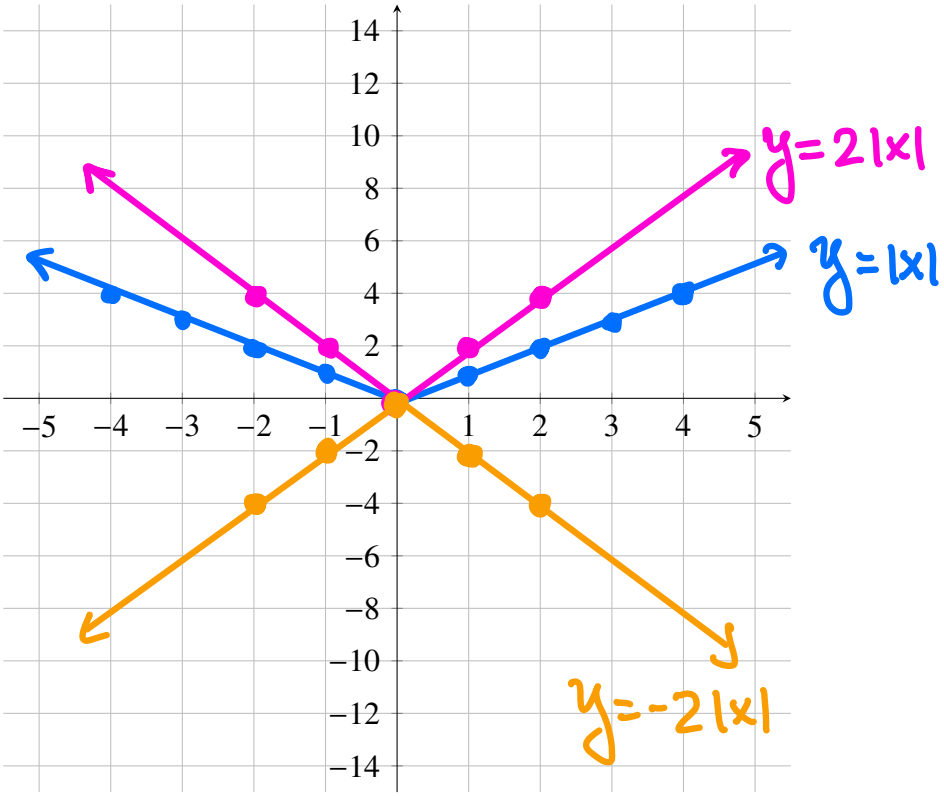
$x \neq 0$

x	y
1	$-\frac{1}{4}$
−1	$\frac{1}{4}$
$\frac{1}{2}$	$-\frac{1}{2}$
$-\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{4}$	−1
$-\frac{1}{4}$	1

3. Sketch the graph of the function $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{x}$, $h(x) = \sqrt[4]{x}$, $k(x) = \sqrt[5]{x}$.



4. Sketch the graph of the function $f(x) = -2|x|$.



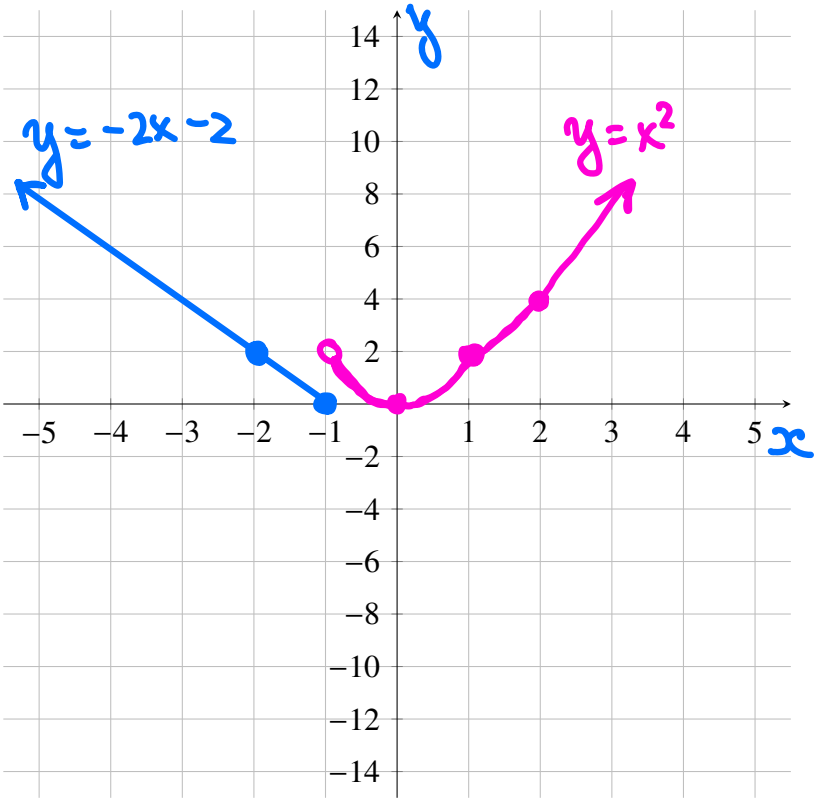
y = 2|x|

x	y
0	0
1	2
-1	2
2	4
-2	4

y = -2|x|

x	y
0	0
1	-2
-1	-2

5. Sketch the graph of the function $f(x) = \begin{cases} -2x - 2, & x \leq -1, \\ x^2, & x > -1 \end{cases}$.



For $x \leq -1$

$y_1 = -2x - 2$ - line

x	y
0	-2
-1	0
-2	2

For $x > -1$

$y_2 = x^2$ - parabola