## THEORETICAL PART:

### Definitions:

- A polynomial in the variable $x$ of degree $n$ can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

  where $a_n, \cdots, a_0 \in \mathbb{R}$, $a_n \neq 0$, and $n$ is nonnegative integer.

- **Basic operations with polynomials:** addition, subtraction, multiplication, division (will be considered later)
- **Special Product Formulas:** Let $A$ and $B$ be algebraic expressions. Then

$$(A - B)(A + B) = A^2 - B^2$$

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(A - B)^2 = A^2 - 2AB + B^2$$

- The polynomial is **factorable** if it can be written as a product of two or more polynomials with integer coefficients. If it cannot be done, the polynomial is **irreducible** or **prime**.
- The **greatest common factor (GCF)** among all the terms is the product of all the factors common to each.

### Factoring Special Binomials:

In the following equations, $A$ and $B$ are algebraic expressions.

- **Difference of two squares:**

$$A^2 - B^2 = (A - B)(A + B)$$

- **Difference of two cubes:**

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

- **Sum of two cubes:**

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

### Factoring a Trinomial by Grouping:

To factor the trinomial $ax^2 + bx + c$, perform the following steps:

- Multiply $a$ and $c$.
- Factor $ac$ into two integers whose sum is $b$. If no such factors exist, the trinomial is irreducible over the integers.
- Rewrite $b$ in the trinomial with the sum found in step 2, and distribute. The resulting polynomial of four terms may now be factored by grouping.

**Perfect Square Trinomials:**

Let $A$ and $B$ be algebraic expressions. Then

$$A^2 + 2AB + B^2 = (A + B)^2$$

$$A^2 - 2AB + B^2 = (A - B)^2$$

**PRACTICAL PART:**

1. Classify each of the following expressions as either polynomial or not a polynomial. For those that are polynomials, identify the degree of the polynomial and the number of terms.

   (a) $17x^2y^5 + 2z^3 - 4$

   (b) $3n^4m^{-3} + n^2m$

   (c) $3x^{\frac{3}{2}} - 2x$

2. Add and subtract the following polynomials:

   (a) $(x^2y - xy - 6y) + (xy^2 + xy + 6x) =$

   (b) $(-8x^4 + 13 - 9x^2) - (8 - 2x^4) =$

3. Multiply the polynomials:

   (a) $(x^2 - 2y)(x^2 + y) =$

(b) $(2xy^2 + 4y - 6x)(x^2y - 5xy) =$

4. Use a special product formula to perform the indicated operations:

(a) $(x - 3y)^2 =$

(b) $\left(\frac{1}{x} - y\right)\left(\frac{1}{x} + y\right) =$

5. Factor each polynomial by factoring out the greatest common factor:

(a) $12x^5 - 4x^2 + 8x^3z^3 =$

(b) $(x^2 + y)^3 + 3(x^2 + y)^2 =$

6. Factor the following polynomials by grouping:

(a) $ax - ay - bx + by =$

(b) $4x - 2x^2 - 2x^3 + x^4 =$

7. Use the special factoring patterns to factor the following binomials:

   (a) $49a^2 - 144b^2 =$

   (b) $27a^9 + 8b^{12} =$

   (c) $343y^9 - 27x^3z^6 =$

8. Factor the following trinomial by grouping:

$$6x^2 - x - 12 =$$

9. Factor the algebraic expressions:

   (a) $x^2 - 4x + 4 =$

   (b) $25y^2 + 10y + 1 =$

   (c) $x^2 + 6x + 9 =$

10. Factor the following expressions with noninteger rational exponents:

    (a) $2x^{-2} + 3x^{-1} =$

    (b) $(5x + 7)^{\frac{7}{3}} - (5x + 7)^{\frac{4}{3}} =$

    (c) $5x^{-4} - 4x^{-5}y =$