Assessment 3 Practice Solutions

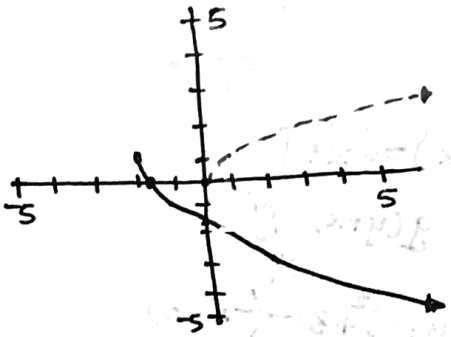
Graph using transformations

a) $f(x) = -2\sqrt{x+2} + 1$

basic: $f(x) = \sqrt{x}$

transformations:
- left 2 units
- stretched vertically by 2
- reflected about x-axis
- up 1 unit

Intercepts:
- $(0, -2\sqrt{2}+1)$
- $(-7/4, 0)$

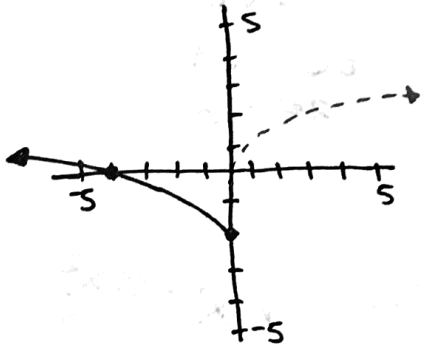


b) $f(x) = \sqrt{-x} - 2$

basic: $f(x) = \sqrt{x}$

transformations:
- reflect about y axis
- down 2 units

Intercepts:
- $(0, -2)$
- $(-4, 0)$

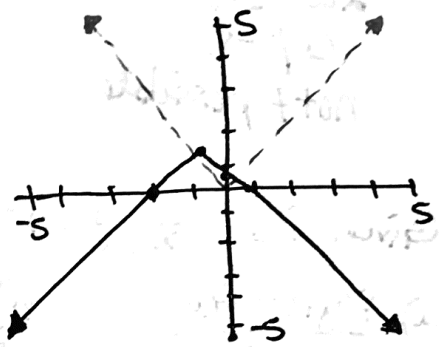


c) $g(x) = -|x + 2/3| + 4/3$

basic: $g(x) = |x|$

transformations:
- left 2/3 units
- reflect about x-axis
- up 4/3 units

Intercepts:
- $(0, 2/3)$
- $(-2, 0)$
- $(2/3, 0)$

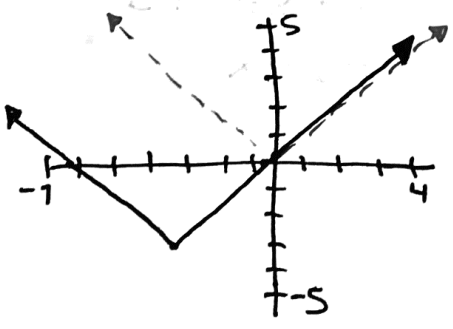


d) $g(x) = |-(x+\pi)| - 3$

basic: $g(x) = |x|$

transformations:
- right π units
- reflect about y axis
- down 3 units

Intercepts:
- $(0, \pi - 3)$
- $(3 - \pi, 0)$
- $(-3 - \pi, 0)$

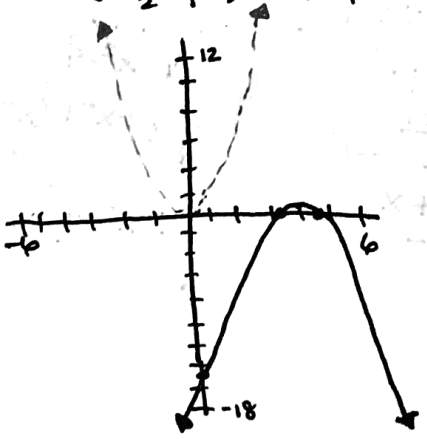


e) $h(x) = -(x-4)^2 + \frac{1}{2}$

basic: $h(x) = x^2$

transformations:
- right 4 units
- reflect about x axis
- up 1/2 unit

Intercepts
- $(0, -31/2)$
- $\left(\frac{8+\sqrt{2}}{2}, 0\right)$ $\left(\frac{8-\sqrt{2}}{2}, 0\right)$

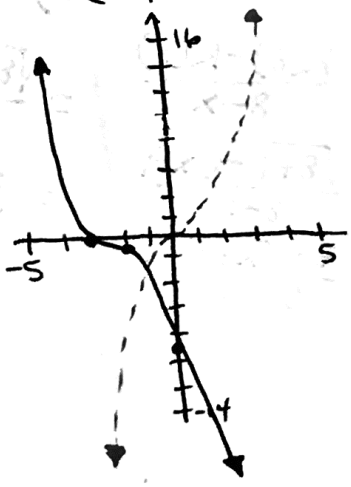


f) $h(x) = -(x+2)^3 - 1$

basic: $h(x) = x^3$

transformations:
- left 2 units
- reflect about x axis
- down 1 unit

Intercepts:
- $(0, -9)$
- $(-3, 0)$



Given $K=\{(-1,-3),(2,0),(\frac{3}{2},-1)\}$ $f(x)=\frac{2x}{x-4}$, $g(x)=\sqrt{x+1}$, $h(x)=4$

a) $f(0)$

$f(0)=\frac{2(0)}{0-4}$

$=0$

b) $g(-3)$

$g(-3)=\sqrt{-3+1}$

$=$ undefined

c) $K(-1)$

$K(-1)=-3$

d) $h(2)$

$h(2)=4$

e) domain of K

$D_K=\{-1,\frac{3}{2},2\}$

f) domain of f

$D_f=(-\infty,4)\cup(4,\infty)$

g) domain of g

$D_g=[-1,\infty)$

h) a if $f(a)=2$

$\frac{2a}{a-4}=2$

$2a=2a-8$

$0\neq -8$

not possible

i) b if $g(b)=6$

$6=\sqrt{b+1}$

$36=b+1$

$35=b$

Given $f(x)=\frac{2}{x}$, $g(x)=-x^2+3$, $h(x)=\frac{3+x^2}{x^2-4}$, $k(x)=\sqrt{x+1}$

a) $\frac{f(y+1)-f(y)}{y}$

$=\frac{\frac{2}{y+1}-\frac{2}{y}}{y}$

$=\frac{2y-2(y+1)}{y^2(y+1)}$

$=\frac{-2}{y^2(y+1)}$

b) Domain of h

$D_h=(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

c) $\frac{g(y+x)-g(x)}{y}$

$=\frac{(-(y+x)^2+3)-(-x^2+3)}{y}$

$=\frac{-y^2-2xy-x^2+3+x^2-3}{y}$

$=\frac{-y^2-2xy}{y}$

$=-y-2x$

d) $\frac{K(8)-K(x)}{8-x}$

$=\frac{\sqrt{8+1}-\sqrt{x+1}}{8-x}$

$=\frac{3-\sqrt{x+1}}{8-x}$

e) $(h\circ k)(x)$

$=\frac{3+(\sqrt{x+1})^2}{(\sqrt{x+1})^2-4}$

$=\frac{3+x+1}{x+1-4}$

$=\frac{4+x}{x-3}$

f) $(k\circ h)(x)$

$=\sqrt{\frac{3+x^2}{x^2-4}+1}$

$=\sqrt{\frac{3+x^2+(x^2-4)}{x^2-4}}$

$=\sqrt{\frac{2x^2-1}{x^2-4}}$

g) $(g\circ k)(x)$

$=-(\sqrt{x+1})^2+3$

$=-(x+1)+3$

$=2-x$

Find f^{-1}, $D_{f^{-1}}$, and $R_{f^{-1}}$.

a) $f(x) = \frac{2x+1}{x-1}$

$x = \frac{2y+1}{y-1}$

$x(y-1) = 2y+1$

$xy - x = 2y+1$

$xy - 2y = x+1$

$y(x-2) = x+1$

$y = \frac{x+1}{x-2}$

$f^{-1}(x) = \frac{x+1}{x-2}$

$D_{f^{-1}}: (-\infty, 2) \cup (2, \infty)$

$R_{f^{-1}}: (-\infty, 1) \cup (1, \infty)$

b) $f(x) = 3\sqrt{2x+1}$

$x = 3\sqrt{2y+1} \quad (y \geq -\frac{1}{2})$

$\frac{x}{3} = \sqrt{2y+1}$

$\frac{x^2}{9} = 2y+1$

$\frac{x^2}{9} - 1 = 2y$

$\frac{x^2}{18} - \frac{1}{2} = y$

$f^{-1}(x) = \frac{1}{18}x^2 - \frac{1}{2}$

$D_{f^{-1}}: [0, \infty)$

$R_{f^{-1}}: [-\frac{1}{2}, \infty)$

c) $f(x) = (x-1)^2 + 6 \; ; x \geq 1$

$x = (y-1)^2 + 6 \; ; y \geq 1$

$x - 6 = (y-1)^2$

$\pm\sqrt{x-6} = y-1$

$1 \pm \sqrt{x-6} = y$

$f^{-1}(x) = 1 + \sqrt{x-6}$

$D_{f^{-1}}: [6, \infty)$

$R_{f^{-1}}: [1, \infty)$

Find the vertex, intercepts, range, axis of symmetry, & sketch.

a) $f(x) = 6x^2 - 19x - 7$

$f(x) = 6(x - \frac{19}{12})^2 - \frac{529}{24}$

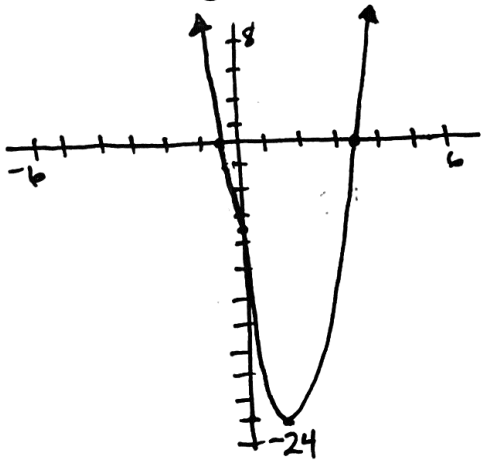
Vertex: $(\frac{19}{12}, -\frac{529}{24})$

Intercepts:

$(-\frac{1}{3}, 0), (\frac{7}{2}, 0), (0, -7)$

Range: $[-\frac{529}{24}, \infty)$

axis of sym.: $x = \frac{19}{12}$



b) $y = (x + \frac{1}{2})^2 - \frac{5}{4}$

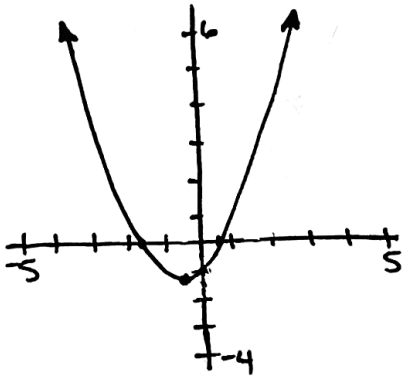
Vertex: $(-\frac{1}{2}, -\frac{5}{4})$

Intercepts:

$(0, -1), (\frac{-1+\sqrt{5}}{2}, 0), (\frac{-1-\sqrt{5}}{2}, 0)$

Range: $[-\frac{5}{4}, \infty)$

axis of sym: $x = -\frac{1}{2}$



c) $y = 2x^2 + 10x + 11$

$y = 2(x + \frac{5}{2})^2 - \frac{3}{2}$

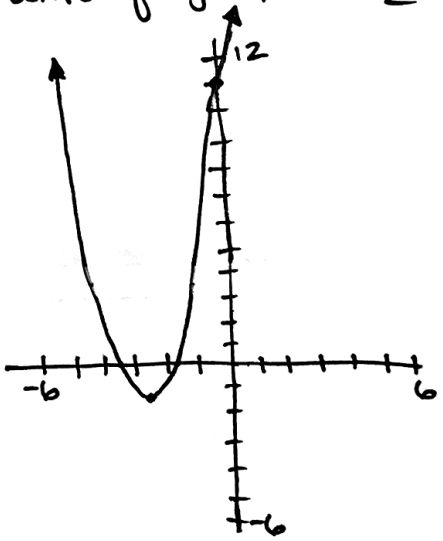
Vertex: $(-\frac{5}{2}, -\frac{3}{2})$

Intercepts:

$(0, 11), (\frac{-5+\sqrt{3}}{2}, 0), (\frac{-5-\sqrt{3}}{2}, 0)$

Range: $[-\frac{3}{2}, \infty)$

axis of sym: $x = -\frac{5}{2}$



d) $f(x) = 3x^2 + 4x + 3$

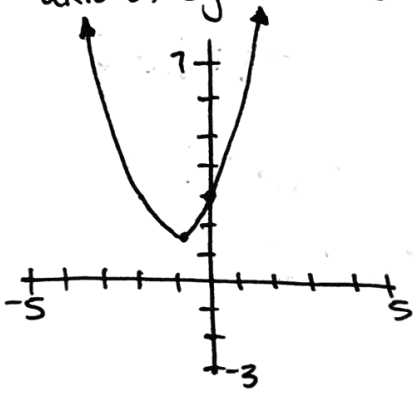
$f(x) = 3(x + \frac{2}{3})^2 + \frac{5}{3}$

Vertex: $(-\frac{2}{3}, \frac{5}{3})$

Intercepts: $(0,3)$

Range: $[\frac{5}{3}, \infty)$

axis of sym: $x = -\frac{2}{3}$



e) $g(x) = x^2 + 4x + 3$

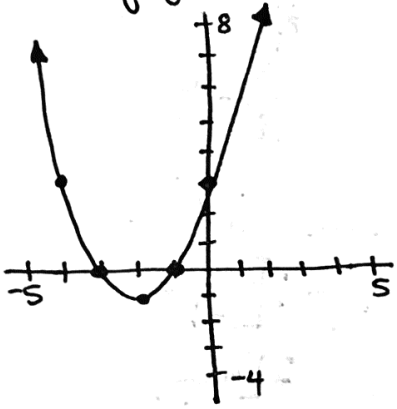
$g(x) = (x+2)^2 - 1$

vertex: $(-2, -1)$

Intercepts:
$(0,3), (-3,0), (-1,0)$

Range: $[-1, \infty)$

axis of sym: $x = -2$



f) $h(x) = 2x^2 - 4x + 3$

$h(x) = 2(x-1)^2 + 1$

vertex: $(1,1)$

Intercepts: $(0,3)$

Range: $[1, \infty)$

axis of sym: $x = 1$