WRH-3-Solutions

13.1: 2, 3, 12
13.2: 11, 23, 36
13.3: 1, 23

13.1

(2) $r(t) = \cos t\, i + \ln t\, j + \frac{1}{t-2} k$

$x(t) = \cos t$
$y(t) = \ln t$
$z(t) = \frac{1}{t-2}$

$\text{Dom}(x) = \mathbb{R}$
$\text{Dom}(y) = (0, \infty)$
$\text{Dom}(z) = \mathbb{R} \setminus \{2\}$

Thus, $\boxed{\text{Dom}(r(t)) = (0,2) \cup (2, \infty)}$

(3)

$$\lim_{t \to 0} \left(e^{-3t} i + \frac{t^2}{\sin^2 t} j + \cos 2t\, k \right) =$$

$$= \left(\lim_{t \to 0} e^{-3t} \right) i + \left(\lim_{t \to 0} \frac{t^2}{\sin^2 t} \right) j + \left(\lim_{t \to 0} \cos 2t \right) k$$

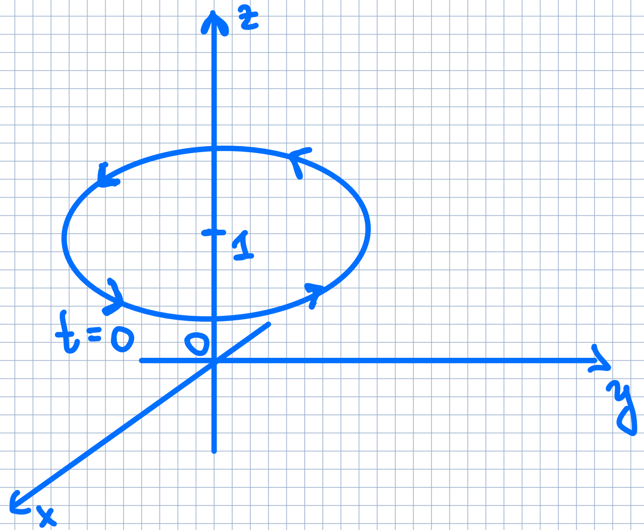
$$= 1 \cdot i + 1 \cdot j + 1 \cdot k = \boxed{\langle 1, 1, 1 \rangle}$$

(12) $r(t) = 2\cos t\, i + 2\sin t\, j + k$

$r(t) = \langle 2\cos t, 2\sin t, 1 \rangle$

$$\begin{cases} z = 1 \\ x = 2\cos t \\ y = 2\sin t \end{cases}$$

$x^2 + y^2 = 4$



13.2

(11) $r(t) = t^2 i + \cos(t^2) j + \sin^2 t\, k$

$r'(t) = (t^2)' i + (\cos(t^2))' j + (\sin^2 t)' k$

$r'(t) = \boxed{2t\, i - \sin(t^2) \cdot 2t\, j + 2\sin t \cos t\, k}$

(23) $x = t^2 + 1$

$y = 4\sqrt{t}$ $\quad P(2,4,1)$

$z = e^{t^2 - t}$

$r'(t) = \langle 2t, \frac{2}{\sqrt{t}}, e^{t^2-t}(2t-1) \rangle$

$2 = t^2 + 1 \Rightarrow \boxed{t=1}$

So $r'(1) = \langle \overset{a}{2}, \overset{b}{2}, \overset{c}{1} \rangle$

Thus, the tangent line parametric equations are

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

$$\boxed{\begin{cases} x = 2 + 2t \\ y = 4 + 2t \\ z = 1 + t \end{cases}}$$

36

$$\int_1^4 \left(2t^{3/2} i + (t+1)\sqrt{t}\, k\right) dt =$$

$$= 2i \int_1^4 t^{3/2}\, dt + k \int_1^4 (t+1)\sqrt{t}\, dt =$$

$$= 2i \left.\frac{2t^{5/2}}{5}\right|_1^4 + k \cdot \left.\left(\frac{2t^{5/2}}{5} + \frac{2t^{3/2}}{3}\right)\right|_1^4 =$$

$$= \frac{4}{5} i \left(32 - 1\right) + k\left(\frac{2}{5} \cdot 32 + \frac{2}{3} \cdot 8 - \frac{2}{5} - \frac{2}{3}\right) =$$

$$= \boxed{\frac{124}{5} i + \frac{256}{15} k}$$

$\boxed{13.3}$

1

$r(t) = \langle t, 3\cos t, 3\sin t \rangle$

$-5 \leq t \leq 5$

$r'(t) = \langle 1, -3\sin t, 3\cos t \rangle$

$|r'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t} = \sqrt{10}$

$$L = \int_{-5}^{5} \sqrt{10}\, dt = \sqrt{10} \cdot 10 = \boxed{10\sqrt{10}}$$

23

$r(t) = \sqrt{6}t^2 i + 2t j + 2t^3 k$

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r'(t) = \langle 2\sqrt{6}t, 2, 6t^2 \rangle$

$r''(t) = \langle 2\sqrt{6}, 0, 12t \rangle$

$|r'(t)| = \sqrt{24t^2 + 4 + 36t^4} = 2(3t^2 + 1)$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 2\sqrt{6}t & 2 & 6t^2 \\ 2\sqrt{6} & 0 & 12t \end{vmatrix} =$$

$$= i\left(24t\right) - j\left(24\sqrt{6}t^2 - 12\sqrt{6}\,t^2\right) +$$

$$+ k\left(-4\sqrt{6}\right) = 24t\,i - 12\sqrt{6}\,t^2 j - 4\sqrt{6}\,k$$

$$|r'(t) \times r''(t)| = \sqrt{(24)^2 t^2 + 144 \cdot 6 t^4 + 16 \cdot 6} =$$

$$= 4\sqrt{6}\left(3t^2 + 1\right).$$

Then,

$$\kappa(t) = \frac{4\sqrt{6}\left(3t^2+1\right)}{8\left(3t^2+1\right)^3} = \boxed{\frac{\sqrt{6}}{2\left(3t^2+1\right)^2}}$$