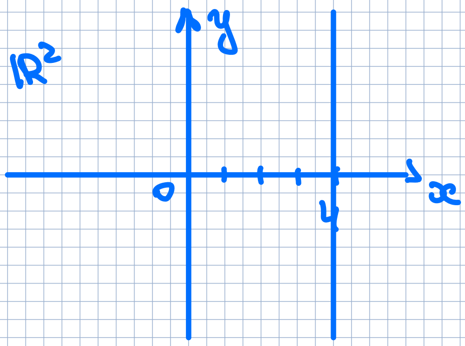
WRH - 1 - Solutions

12.1: 5, 35
12.2: 19, 27
12.3: 4, 39
12.4: 2, 23

$\boxed{12.1}$

5

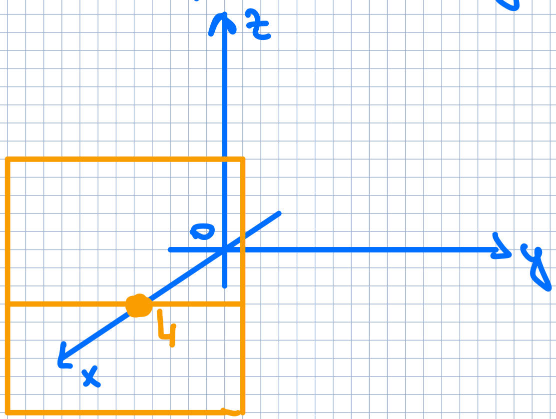
$\mathbb{R}^2$: $x = 4$ is a vertical line.



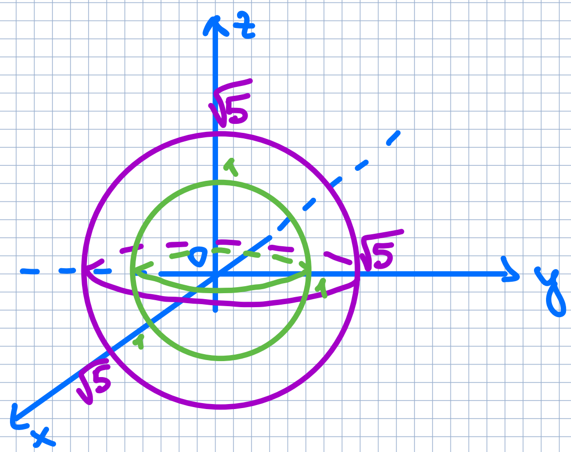
$\mathbb{R}^3$: $x=4$ is a plane. It is a set of points

$\{(x,y,z) \mid x=4, y\in\mathbb{R}, z\in\mathbb{R}\}$

This plane is parallel to yz-plane.



(35) $1 \le x^2 + y^2 + z^2 \le 5$



This is the set of all points on or between spheres with radii 1 and $\sqrt{5}$ and centers (0,0,0).

12.2

(19) $a = \langle -3, 4 \rangle$
$b = \langle 9, -1 \rangle$

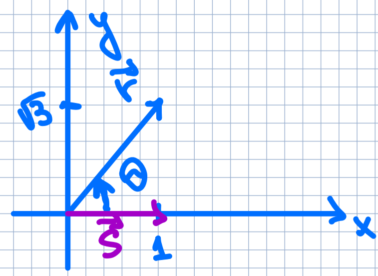
$a + b = \langle -3+9, 4-1 \rangle = \boxed{\langle 6, 3 \rangle}$

$4a + 2b = 4\langle -3, 4 \rangle + 2\langle 9, -1 \rangle = \langle -12, 16 \rangle + \langle 18, -2 \rangle =$
$= \boxed{\langle 6, 14 \rangle}$

$|a| = \sqrt{9+16} = \boxed{5}$

$|a - b| = |\langle -3-9, 4+1 \rangle| = |\langle -12, 5 \rangle| =$
$= \sqrt{144+25} = \boxed{13}$

(27) $\vec{r} = i + \sqrt{3}\, j$



$\vec{r} = i + \sqrt{3}\, j = \langle 1, \sqrt{3} \rangle$

$\vec{s} = i = \langle 1, 0 \rangle$

$$\cos\theta = \frac{(r,s)}{|r| \cdot |s|} = \frac{1+0}{1 \cdot 2} = \frac{1}{2}$$

$$\boxed{\theta = \frac{\pi}{3}}$$

12.3

4

$a = \langle 6, -2, 3 \rangle$

$b = \langle 2, 5, -1 \rangle$

$a \cdot b = 6 \cdot 2 + (-2) \cdot 5 + 3 \cdot (-1) = 12 - 10 - 3 = \boxed{-1}$

39

$a = \langle -5, 12 \rangle$

$b = \langle 4, 6 \rangle$

$$\text{comp}_a b = \frac{a \cdot b}{|a|}$$

$a \cdot b = -20 + 72 = 52$

$|a| = \sqrt{25 + 144} = 13$

$$\boxed{\text{comp}_a b = 4}$$

$$\text{proj}_a b = \frac{a \cdot b}{|a|^2} a$$

$$\text{proj}_a b = \frac{52}{13^2} \langle -5, 12 \rangle = \langle -\frac{20}{13}, \frac{48}{13} \rangle$$

$$\boxed{\text{proj}_a b = \langle -\frac{20}{13}, \frac{48}{13} \rangle}$$

12.4

2

$a = \langle 4, 3, -2 \rangle$

$b = \langle 2, -1, 1 \rangle$

$$a \times b = \begin{vmatrix} i & j & k \\ 4 & 3 & -2 \\ 2 & -1 & 1 \end{vmatrix} = i \begin{vmatrix} 3 & -2 \\ -1 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -2 \\ 2 & 1 \end{vmatrix} +$$

$$+ k \begin{vmatrix} 4 & 3 \\ 2 & -1 \end{vmatrix} = i \cdot 1 - j \cdot 8 + k(-10) =$$

$$= \langle 1, -8, -10 \rangle$$

$(a \times b) \cdot a = 4 - 24 + 20 = 0$ ✓

$(a \times b) \cdot b = 2 + 8 - 10 = 0$ ✓

23 $\quad a \times b = -b \times a$

$a = \langle a_1, a_2, a_3 \rangle$
$b = \langle b_1, b_2, b_3 \rangle$

Proof

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$-b \times a = \begin{vmatrix} i & j & k \\ -b_1 & -b_2 & -b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = -\begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} =$$

$$= \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = a \times b$$