Name:

Logistics

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz will be available starting from **5:00 PM until midnight** on scheduled week day (Thursday).
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. (a) Reduce the equation $x^2 - y^2 + z^2 - 4x - 2z = 0$ to the standard form.
   (b) Classify the surface from the part (a).
   (c) Sketch the surface from part (a).

(a) $x^2 - y^2 + z^2 - 4x - 2z = 0$

$(x^2 - 4x + 4) - y^2 + (z^2 - 2z + 1) - 4 - 1 = 0$

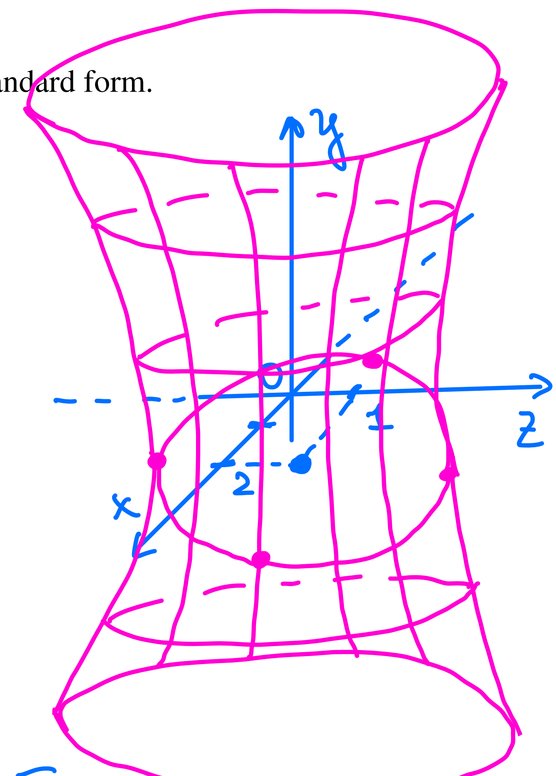
$(x-2)^2 - y^2 + (z-1)^2 = 5$

$\frac{(x-2)^2}{5} + \frac{(z-1)^2}{5} = 1 + \frac{y^2}{5}$

(b) This is the hyperboloid of one sheet along y-axis.

$$\boxed{\frac{(x-2)^2}{(\sqrt{5})^2} + \frac{(z-1)^2}{(\sqrt{5})^2} = 1 + \frac{y^2}{(\sqrt{5})^2}}$$

If $y=0$: we have a circle with $r=\sqrt{5}$.



2. Find the limit of the given vector function

$$\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\mathbf{i} + \sqrt{t+8}\,\mathbf{j} + \frac{\sin(\pi t)}{\ln t}\mathbf{k}\right) =$$

$= i \lim_{t\to 1} \frac{t^2-t}{t-1} + j \lim_{t\to 1} \sqrt{t+8} + k \lim_{t\to 1} \frac{\sin(\pi t)}{\ln t} = i \lim_{t\to 1} \frac{t\cancel{(t-1)}}{\cancel{t-1}} + j\cdot\sqrt{9} +$

L'H rule

$+ k \lim_{t\to 1} \frac{\pi\cos(\pi t)}{\frac{1}{t}} = i\cdot 1 + j\cdot 3 + k\cdot(-\pi) = \boxed{\langle 1, 3, -\pi\rangle.}$

3. **[5 points]** For the given vector function $r(t) = e^{2t}\,\mathbf{i} + e^{t}\,\mathbf{j}$ and $t = 0$ find:

   (a) Tangent vector $r'(t)$.

   (b) Sketch the position vector $r(t)$ and the tangent vector $r'(t)$ for the given value of t.

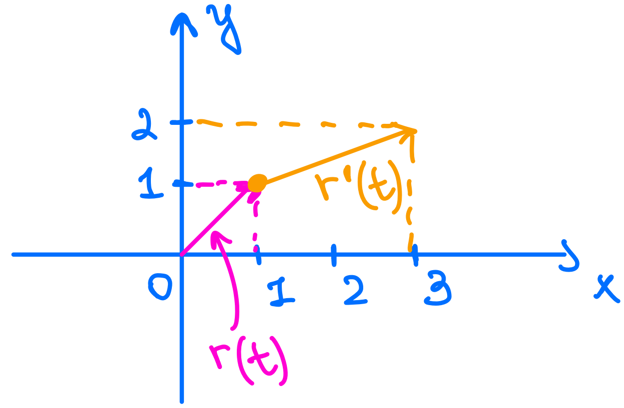
(a) $r'(t) = 2e^{2t}\,i + e^{t}\,j = \langle 2e^{2t}, e^{t} \rangle$

(b) $\boxed{\mathbb{R}^2}$

For $t=0$:

$r(0) = \langle 1, 1 \rangle$

$r'(0) = \langle 2, 1 \rangle$



4. **[5 points]** Evaluate the following integral

$$\int \left(te^{2t}\,\mathbf{i} + \frac{t}{1-t}\,\mathbf{j} + \frac{1}{\sqrt{1-t^2}}\,\mathbf{k} \right) dt =$$

$$= i\int te^{2t}\,dt + j\int \frac{t}{1-t}\,dt + k\int \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$= \frac{1}{2} i \int t\, d(e^{2t}) - j \int \frac{1-t-1}{1-t}\,dt + k \cdot \arcsin t =$$

$$= \frac{1}{2} i \left(t \cdot e^{2t} - \int e^{2t}\,dt \right) - j \int \left(1 - \frac{1}{1-t} \right) dt + k \arcsin t =$$

$$= \frac{1}{2} i \left(t \cdot e^{2t} - \frac{1}{2} e^{2t} \right) - j \left(t + \ln|1-t| \right) + k \arcsin t +$$

$$+ \langle c_1, c_2, c_3 \rangle = \boxed{\left\langle \frac{te^{2t}}{2} - \frac{e^{2t}}{4} + c_1, -t - \ln|1-t| + c_2, \arcsin t + c_3 \right\rangle}$$