Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

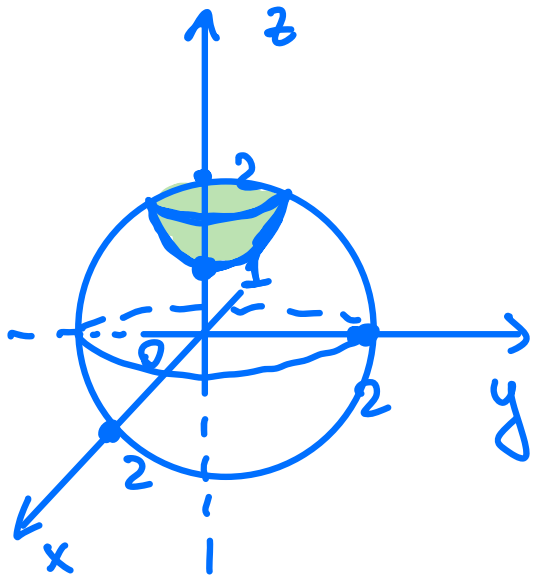
1. **[10 points]** Use **cylindrical coordinates** to find the mass of the solid enclosed below by the paraboloid $z = x^2 + y^2 + 1$ and above by the sphere $x^2 + y^2 + z^2 = 4$ if the density function is given by $\rho(x, y, z) = \dfrac{2}{z^2}$.

   Hint: use the following formula

$$m = \iiint_E \rho(x, y, z)\, dV.$$

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$ $\quad dV = r\,dr\,d\theta$

$$m = \int_1^2 \int_0^{2\pi} \int_0^1 \frac{2}{z^2}\, r\,dr\,d\theta\,dz = (=)$$



$1 \le x^2 + y^2 + 1 \le 2$

$1 \le r^2 + 1 \le 2$

$0 \le r^2 \le 1$

$0 \le r \le 1$

$$(=) \int_1^2 \frac{2}{z^2}\,dz \cdot \int_0^{2\pi} d\theta \cdot \int_0^1 r\,dr = -\frac{2}{z}\Big|_1^2 \cdot 2\pi \cdot \frac{1}{2} = \pi\left(-1 + 2\right) = \pi$$

3. **[5 points]** Find the Jacobian of the transformation:

$$x = ue^v, \quad y = ve^u.$$

$$J = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} e^v & ue^v \\ ve^u & e^u \end{vmatrix} = e^{u+v} - uve^{u+v}$$

$$\left|\begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{matrix}\right|$$

$$= \boxed{e^{u+v}(1-uv)}$$

4. **[5 points]** Sketch the vector field **F** on the xy-plane:

$$F(x,y) = y\,\mathbf{i} + (x+y)\,\mathbf{j}.$$

(x,y)	$\langle y, x+y \rangle$	
$(0,0)$	$\langle 0,0 \rangle$	
$(1,0)$	$\langle 0,1 \rangle$	1
$(0,1)$	$\langle 1,1 \rangle$	$\sqrt{2}$
$(1,1)$	$\langle 1,2 \rangle$	$\sqrt{5}$