Formulas & Definitions: Section 12-4

Definition: If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, then the **cross product** of a and b is the vector

$$a \times b = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle.$$

Theorem: The vector $a \times b$ is orthogonal to both a and b.

Theorem: If θ is the angle between a and b ($0 \le \theta \le \pi$), then

$$|a \times b| = |a|\,|b|\,\sin\theta.$$

Corollary:

- Two nonzero vectors a and b are parallel if and only if

$$a \times b = 0.$$

- The length of the cross product $a \times b$ is equal to the area of the parallelogram determined by a and b.

$$\boxed{\mathrm{i} \times j = k, \quad j \times k = i, \quad k \times i = j, \quad j \times i = -k, \quad k \times j = -i, \quad i \times k = -j}$$

Properties of the Cross Product: If a, b, and c are vectors and β is a scalar, then

1. $a \times b = -b \times a$
2. $(\beta a) \times b = \beta(a \times b) = a \times (\beta b)$
3. $a \times (b + c) = a \times b + b \times c$
4. $(a + b) \times c = a \times c + b \times c$
5. $a \cdot (b \times c) = (a \times b) \cdot c$
6. $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$

Proposition: The volume of the parallelepiped determined by the vectors a, b, and c is the magnitude of their scalar triple product:

$$V = |a \cdot (b \times c)|.$$