Formulas & Definitions: Section 16-1

Definition: Let D be a set in $\mathbb{R}^2$ (a plane region). A **vector field** on $\mathbb{R}^2$ is a function F that assigns to each point (x, y) in D a two-dimensional vector $F(x, y)$.

$$F(x, y) = P(x, y)i + Q(x, y)j = \langle P(x, y), Q(x, y)\rangle$$

or

$$F = Pi + Qj$$

Definition: Let E be a subset of $\mathbb{R}^3$. A vector field on $\mathbb{R}^3$ is a function F that assigns to each point (x, y, z) in E a three-dimensional vector $F(x, y, z)$.

$$F(x, y, z) = P(x, y, z)i + Q(x, y, z)j + R(x, y, z)k.$$

Statement: If f is a scalar function of three variables, its gradient is a vector field on $\mathbb{R}^3$ given by

$$\nabla f(x, y, z) = f_x(x, y, z)i + f_y(x, y, z)j + f_z(x, y, z)k.$$

Definition: A vector field F is called **conservative vector field** if it is the gradient of some scalar function, that is, if there exists a function f such that $F = \nabla f$. In this situation f is called a potential function for F.