Name:

Instructions. You have 30 minutes to work on the problem below. This problem will give you 5 extra credit points to your MT-2 exam result. You have to show full work to receive 5 points.

Books, notes, and other resources are not allowed.

1. **[5 points]** Use Lagrange multipliers to find the maximum product of two positive numbers satisfying $x + y^2 = 5$.

$x \geq 0$
$y \geq 0$

$f(x,y) = x \cdot y \to \max$

$g(x,y) = x + y^2 - 5$

$\nabla f = \lambda \nabla g$

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \end{cases}$$

$f_x = y \quad f_y = x \quad g_x = 1 \quad g_y = 2y$

$$\begin{cases} y = 1 \cdot \lambda \Rightarrow y = \lambda \\ x = \lambda \cdot 2y \Rightarrow x = 2\lambda^2 \end{cases}$$

$2\lambda^2 + \lambda^2 = 5$

$3\lambda^2 = 5 \Rightarrow \lambda = \pm \frac{\sqrt{5}}{\sqrt{3}}$

1) $\lambda = \frac{\sqrt{5}}{\sqrt{3}} \Rightarrow x = \frac{10}{3}, \; y = \frac{\sqrt{5}}{\sqrt{3}}$

2) $\lambda = -\frac{\sqrt{5}}{\sqrt{3}} \Rightarrow x = \frac{10}{3}, \; y = -\frac{\sqrt{5}}{\sqrt{3}}$

$f_{max} = \frac{10}{3} \cdot \frac{\sqrt{5}}{\sqrt{3}} = \frac{10\sqrt{5}}{3\sqrt{3}}$