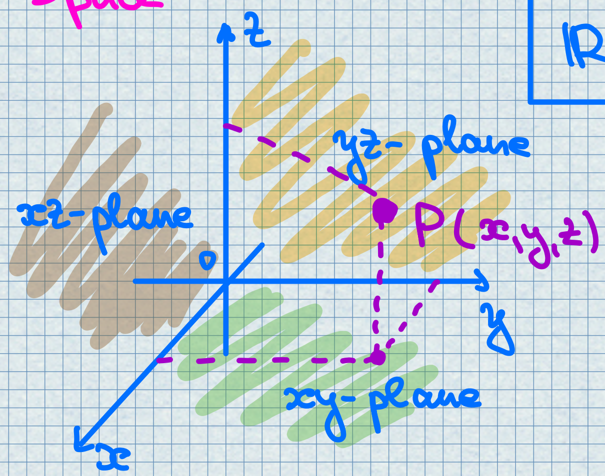
- **3D Space**



$$\boxed{\mathbb{R}^3} = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$$

This three planes divide space into 8 parts, called <u>octants</u>.

$P(a,b,0)$ is a projection of P onto xy- plane
$P(0,b,c)$ is a projection of P onto yz- plane
$P(a,0,c)$ is a projection of P onto xz-plane

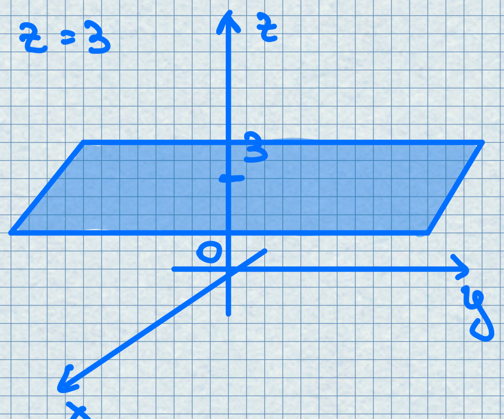
- **Surfaces**

In $\mathbb{R}^2$: $f(x,y) = c$ is a <u>curve</u>

In $\mathbb{R}^3$: $f(x,y,z) = c$ is a <u>surface</u>

<u>Example</u>

(a) $z = 3$

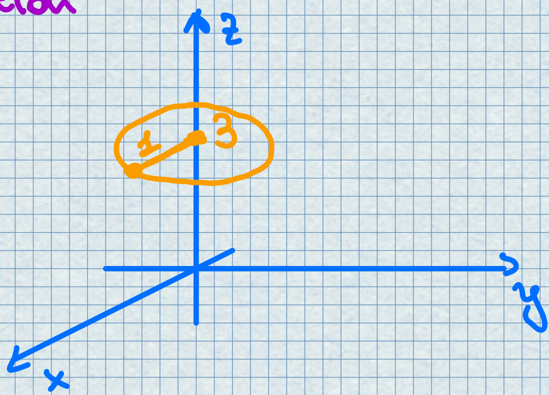


Example

Which points (x, y, z) satisfy equations

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 3$$

Solution

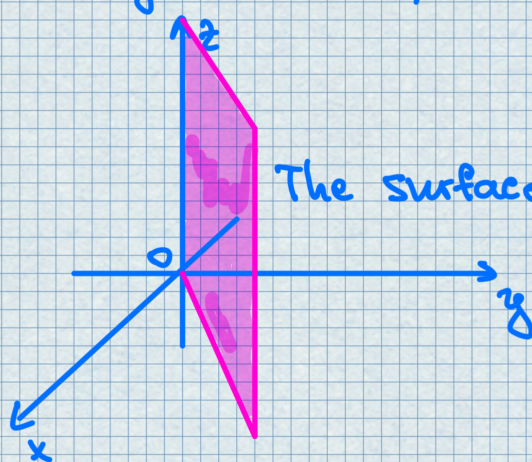


Example

Describe and sketch the surface in $\mathbb{R}^3$

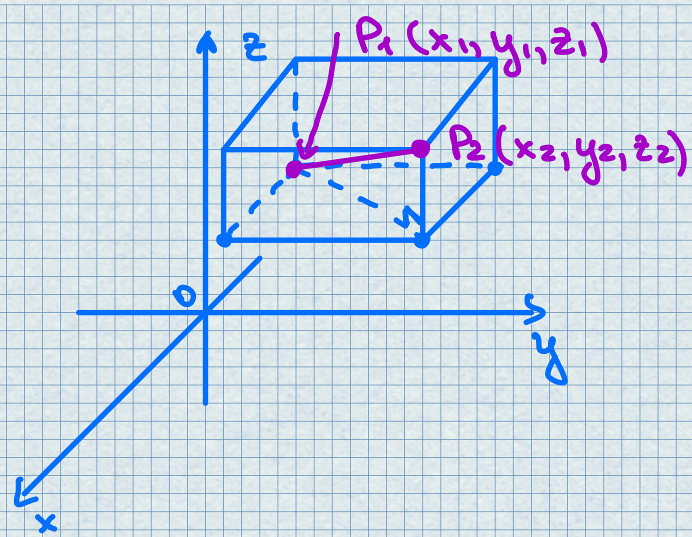
$$y = x$$

surface : $\{(x, y, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$

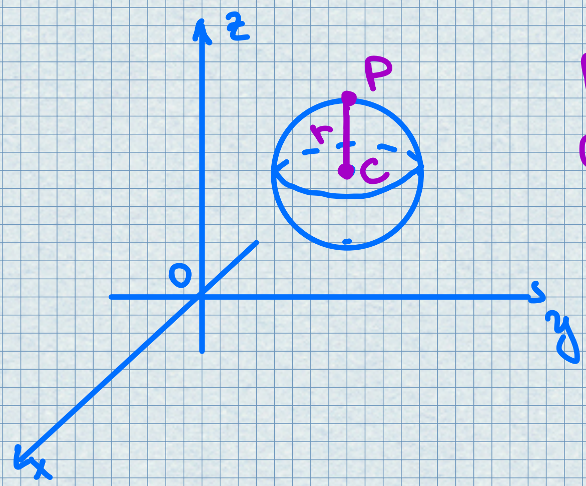


The surface in the I octant

- Distance and spheres



$$\boxed{|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



$P(x, y, z)$
$C(a, b, c)$

Equation of a Sphere

$$\boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2}$$

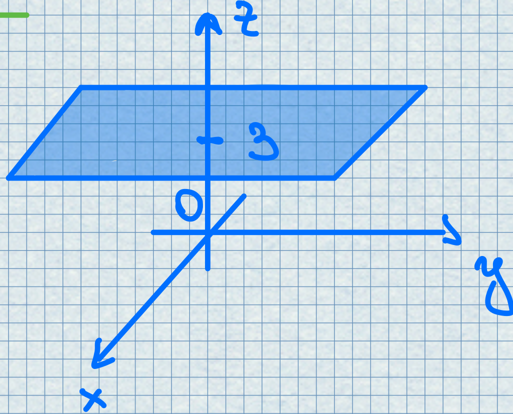
If $(a, b, c) = (0, 0, 0)$, then

$$\boxed{x^2 + y^2 + z^2 = r^2}$$

Examples

1. What surface in $\mathbb{R}^3$ is represented by the equation $z=3$?

Solution

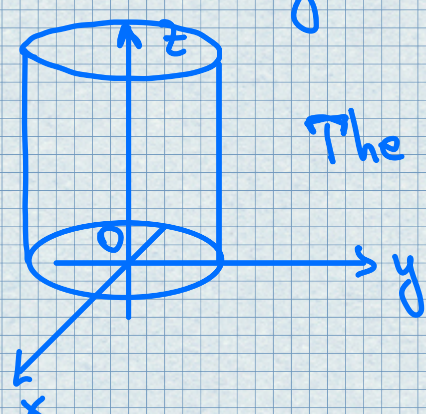


$\{(x,y,z) \mid z=3\}$
horizontal plane $\parallel$ to xy-plane

2. What does the equation $x^2+y^2=1$ represent as a surface in $\mathbb{R}^3$?

Solution

We have that $z=k$. So the surface $x^2+y^2=1$ in $\mathbb{R}^3$ consists of all possible horizontal circles $x^2+y^2=1,\ z=k$.



The cylinder

3. Find the distance from $P(2,-1,7)$ to $Q(1,-3,5)$.

Solution

$$|PQ| = \sqrt{(1-2)^2+(-3+1)^2+(5-7)^2} = 3$$

4. Show that $x^2+y^2+z^2+4x-6y+2z+6=0$ is the equation of a sphere.

Solution

$$(x^2+4x+4)+(y^2-6y+9)+(z^2+2z+1) = -6+4+9+1$$

$$\boxed{(x+2)^2+(y-3)^2+(z+1)^2 = 8}$$

Center: $(-2,3,-1)$

radius $= 2\sqrt{2}$.