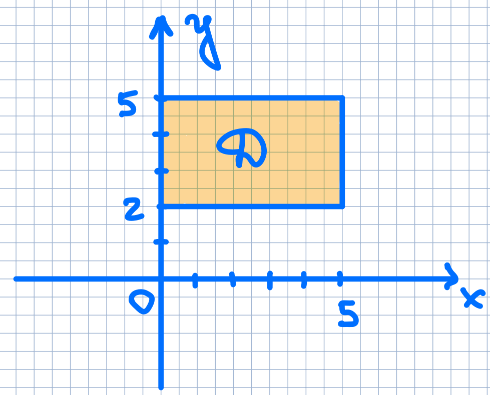
WRH-8-Solutions

15.4: 1, 6
15.5: 2, 11

15.4

① $0 \leq x \leq 5$
$2 \leq y \leq 5$
$\sigma(x,y) = 2x+4y$



The total charge is

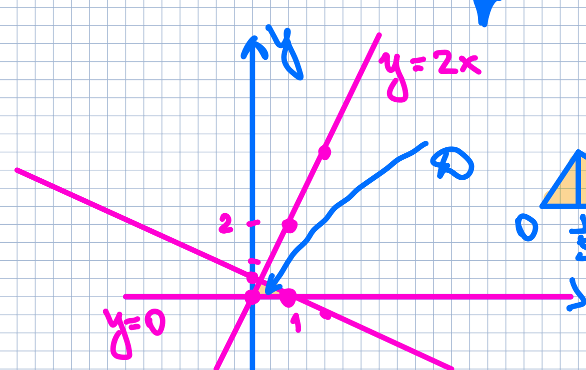
$$Q = \iint_D \sigma(x,y)\,dA =$$

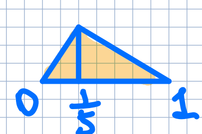
$$Q = \int_0^5 \int_2^5 (2x+4y)\,dy\,dx = \int_0^5 \left(2xy+2y^2\right)\Big|_2^5 dx =$$

$$= \int_0^5 (10x+50-4x-8)\,dx = \int_0^5 (6x+42)\,dx =$$

$$= \left(\frac{6x^2}{2}+42x\right)\Big|_0^5 = 3\cdot 25+210 = \boxed{285\ C}$$

⑥ D: $y=0$
$y=2x$
$x+2y=1$
$\rho(x,y)=x$





$$\boxed{m = \iint_D \rho(x,y)\, dA}$$

$$\begin{cases} y = 2x \\ x + 2y = 1 \end{cases} \qquad x = 1 - 2y$$

$$y = 2(1-2y) = 2 - 4y$$

$$5y = 2$$

$$y = \frac{2}{5} \Rightarrow x = 1 - \frac{4}{5} = \boxed{\frac{1}{5}}$$

Thus,

$$m = \int_0^{\frac{1}{5}} \int_0^{2x} x\, dA + \int_{\frac{1}{5}}^{1} \int_0^{\frac{1-x}{2}} x\, dA =$$

$$= \int_0^{\frac{1}{5}} xy \Big|_0^{2x} dx + \int_{\frac{1}{5}}^{1} xy \Big|_0^{\frac{1-x}{2}} dx =$$

$$= \int_0^{\frac{1}{5}} 2x^2\, dx + \int_{\frac{1}{5}}^{1} \left(\frac{x}{2} - \frac{x^2}{2}\right) dx =$$

$$= \frac{2x^3}{3} \Big|_0^{\frac{1}{5}} + \left(\frac{x^2}{4} - \frac{x^3}{6}\right) \Big|_{\frac{1}{5}}^{1} =$$

$$= \frac{2}{3} \cdot \frac{1}{125} + \frac{1}{4} - \frac{1}{6} - \frac{1}{100} + \frac{1}{6} \frac{1}{125} = \frac{2}{25}$$

$$\bar{x} = \frac{1}{m}\int_0^{\frac{1}{5}}\int_0^{2x} x^2\,dA + \frac{1}{m}\int_{\frac{1}{5}}^{1}\int_0^{\frac{1-x}{2}} x^2\,dA =$$

$$= \frac{25}{2}\int_0^{1/5} x^2 y \Big|_0^{2x} dx + \frac{25}{2}\int_{\frac{1}{5}}^{1} x^2 y \Big|_0^{\frac{1-x}{2}} dx =$$

$$= \frac{25}{2}\int_0^{1/5} 2x^3\,dx + \frac{25}{4}\int_{\frac{1}{5}}^{1} (x^2 - x^3)\,dx =$$

$$= 25\,\frac{x^4}{4}\Big|_0^{1/5} + \frac{25}{4}\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_{1/5}^{1} =$$

$$= \frac{25}{4}\cdot\frac{1}{5^4} + \frac{25}{4}\left(\frac{1}{3} - \frac{1}{4} - \frac{1}{5^3\cdot 3} + \frac{1}{4\cdot 5^4}\right) =$$

$$= \frac{1}{100} + \frac{\cancel{25}}{4}\,\frac{4\cdot 625 - 3\cdot 625 - 20 + 3}{3\cdot 4\cdot 5^{\cancel{4}\,2}} =$$

$$= \frac{1}{100} + \frac{608}{1200} = \frac{12+608}{1200} = \frac{620}{1200} = \frac{62}{120} = \boxed{\frac{31}{60}}$$

$$\bar{y} = \frac{1}{m}\int_0^{1/5}\int_0^{2x} xy\,dy\,dx + \frac{1}{m}\int_{1/5}^{1}\int_0^{\frac{1-x}{2}} xy\,dy\,dx =$$

$$= \boxed{\frac{7}{60}}$$

$$\boxed{m = \frac{2}{25} \quad , \quad (\bar{x}, \bar{y}) = \left(\frac{31}{60}, \frac{7}{60}\right)}$$

$$= \frac{2\cdot 2\cdot 100 + 6\cdot 125\cdot 25 - 125\cdot 100 - 6\cdot 125 + 100}{6\cdot 125\cdot 100} =$$

$$= \frac{400 + 18750 - 12500 - 750 + 100}{6\cdot 125\cdot 100} =$$

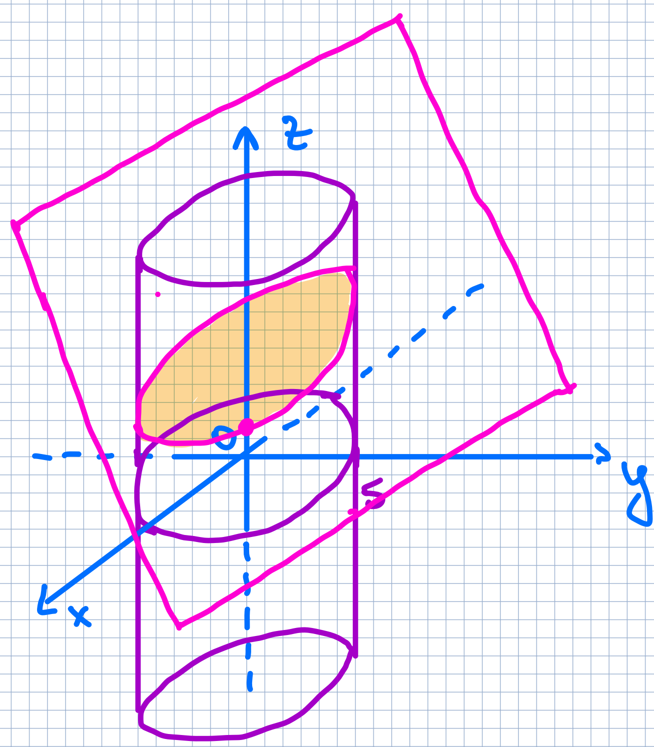
$$= \frac{6000}{6\cdot 125\cdot 100} = \frac{10}{125} = \boxed{\frac{2}{25}}$$

$\boxed{15.5}$

2

$6x + 4y + 2z = 1$

$x^2 + y^2 = 25$



$x = 5$
$y = 0$

$30 + 2z = 1$

$2z = -29$

$z = -\frac{29}{2}$

$y = 5$
$x = 0$

$z = -\frac{19}{2}$

$$A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\; dA$$

$$2z = 1 - 6x - 4y$$

$$z = \frac{1}{2} - 3x - 2y$$

$D: \quad x^2+y^2=25$

$f(x,y)=z$

$f_x=-3$

$f_y=-2$

$$A(S)=\iint_D \sqrt{9+4+1}\,dA=$$

$$=\iint_D \sqrt{14}\,dA=\sqrt{14}\int_0^5\int_0^{2\pi} r\,d\theta\,dr=$$

$$=\sqrt{14}\cdot\cancel{2}\pi\,\frac{r^2}{\cancel{2}}\Big|_0^5=\boxed{25\sqrt{14}\,\pi}$$

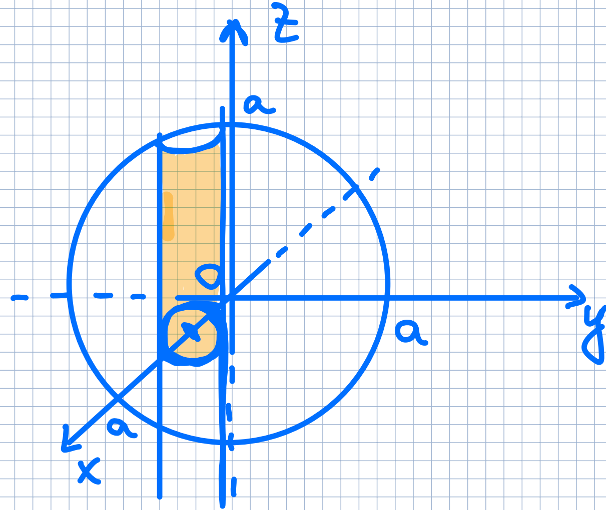
11

$$x^2+y^2+z^2=a^2$$

$$x^2+y^2=ax$$

$$x^2-ax+y^2=0$$

$$\left(x-\frac{a}{2}\right)^2+y^2=\frac{a^2}{4}$$

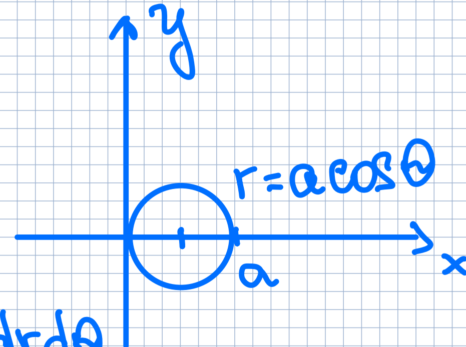


$$A(S) = \iint_{D} \sqrt{f_x^2 + f_y^2 + 1}\; dA$$

$$f(x,y) = \sqrt{a^2 - x^2 - y^2}$$

$$f_x = \frac{-x}{\sqrt{a^2 - x^2 - y^2}}$$

$$f_y = \frac{-y}{\sqrt{a^2 - x^2 - y^2}}$$



$$A(S) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{a\cos\theta} \sqrt{\frac{r^2}{a^2 - r^2} + 1}\; r\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{a\cos\theta} \frac{ar}{\sqrt{a^2 - r^2}}\, dr\, d\theta =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(-a\sqrt{a^2 - r^2}\right)\Big|_{0}^{a\cos\theta} d\theta =$$

$$= 2a^2 \int_{0}^{\frac{\pi}{2}} \left(1 - \sqrt{1 - \cos^2\theta}\right) d\theta$$

$$= a^2\pi - 2a^2\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = \boxed{a^2(\pi - 2)}$$