Name:

Instructions. (100 points) You have two hours. The exam is closed book, closed notes, and only simple calculators are allowed. Show all your work in order to receive full credit.

1. **[6 points]** Consider the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}.$$
Either show it does not exist, or give strong evidence for suspecting it does.

2. **[10 points]** For the given function
$$f(x, y) = x^2y - y^2x$$
(a) (5 pts) Use the chain rule to compute $\frac{dg}{dt}(0)$, where:
$$g(t) = f(t^2 + e^{2t}, 2t + 1).$$
(b) (5 pts) Give an equation for the linear (tangent plane) approximation to f at te point $(1, -1)$, and use it to estimate $f(1.1, -0.9)$.

3. **[12 points]** Evaluate the integral

$$\int_0^{\pi/2}\int_0^{x} \sin(y)\, dy\, dx$$

fully, by first drawing the region of integration, and then reversing the order of integration.

4. **[12 points]** Find the classify (using the Second Derivative Test) all critical points of

$$f(x, y) = x^2 + xy + y^2 + y.$$

5. **[8 points]** Give an equation for the tangent plane to the surface

$$\frac{xy}{z} + e^x \ln(z + 2y) = 2$$

at the point $(2, 1, 1)$.

6. **[10 points]** Use polar coordinates to find the volume of the solid bounded above by the paraboloid $z = x^2 + y^2$ and below by the disk $x^2 + y^2 \leq 25$.

7. **[16 points]** Find the mass and the center of mass of a triangular lamina with vertices $(0,0)$, $(1,0)$, $(0,2)$ if the density function is $\rho(x,y) = x + 2$.

   (a) **[4 points]** Draw the triangular lamina.

   (b) **[6 points]** Use the formula

   $$m = \iint_D \rho(x,y)\,dA$$

   to find the mass of the lamina.

   (c) **[6 points]** Use formulas

   $$\bar{x} = \frac{1}{m}\iint_D x\,\rho(x,y)\,dA, \quad \bar{y} = \frac{1}{m}\iint_D y\,\rho(x,y)\,dA$$

   to set up the expressions for coordinates of the center of mass of the lamina. DO NOT EVALUATE THE INTEGRALS.

8. **[10 points]** Use Lagrange multipliers to find the maximum surface area of the rectangular box whose total edge length is $200\,cm$.

9. **[16 points]** For the given function $f(x, y) = y^2 e^{xy}$, the point $P(0, 1)$, and the directional vector $u = \langle 3/5, 4/5 \rangle$

   (a) **[5 points]** Find the gradient of f at the point P.

   (b) **[5 points]** Find the rate of change of f at P in the direction of the vector u.

   (c) **[6 points]** Fully set up bounds and integrand for computing the **surface area** of f over the region $[-1, 1] \times [-1, 2]$. DO NOT EVALUATE.