## Logistics

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.

- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.

- The quiz will be available starting from **5:00 PM until midnight** on scheduled week day (Thursday).

- The quiz have **20 points** in total.

- There is **no extension or quiz retake**.

- Show your full work to receive a full credit on each problem.

1. **[5 points]** Sketch the region in $\mathbb{R}^3$ represented by the inequality

$$x^2 + y^2 \le 9 = 3^2$$

$x^2 + y^2 = 9$ is a cylinder with radius 3.

$z = k \in \mathbb{R}$ is a scalar

$x^2 + y^2 \le 9$ is the inside part of a cylinder + the boundary
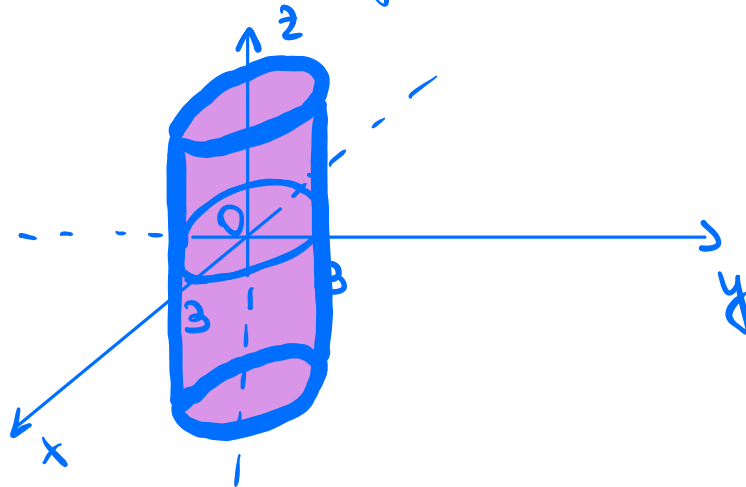
2. **[5 points]** Find a unit vector that has the same direction as the given vector $-3i + 6j + 2k$.

$$v = -3i + 6j + 2k = \langle -3, 6, 2 \rangle$$

Then $|v| = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

$$u = \frac{v}{|v|} = \frac{-3i + 6j + 2k}{7} = -\frac{3}{7}i + \frac{6}{7}j + \frac{2}{7}k =$$

$$= \boxed{\langle -\frac{3}{7}, \frac{6}{7}, \frac{2}{7} \rangle.}$$

3. **[5 points]**

   (a) Determine whether the given vectors $\langle -5, 4, -2 \rangle$ and $\langle 3, 4, -1 \rangle$ are orthogonal. *(Fully justify your answer)*

   (b) For vectors $a = 2j - 4k$ and $b = -i + 3j + k$ find the cross product $a \times b$.

(a) $u = \langle -5, 4, -2 \rangle$

$v = \langle 3, 4, -1 \rangle$

$u \perp v$ if and only if $u \cdot v = 0$

$u \cdot v = -5 \cdot 3 + 4 \cdot 4 + (-2) \cdot (-1) = -15 + 16 + 2 = 18 - 15 = 3$

Hence, u and v are not orthogonal.

(B) $a \times b = \begin{vmatrix} i & j & k \\ 0 & 2 & -4 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 0 & -4 \\ -1 & 1 \end{vmatrix} j + \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} k =$

$= 14i + 4j + 2k = \langle 14, 4, 2 \rangle$

4. **[5 points]**

   (a) Find an equation of the plane that goes through the point $(5, 3, 5)$ and has a normal vector $i + 4j + k$.

   (b) Use intercepts to sketch the plane from part (a).

(a) In general form for the plane that goes through $P(x_0, y_0, z_0)$ and is $\parallel$ to $v = \langle a, b, c \rangle$ has an equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Thus, $v = \langle 1, 4, 1 \rangle$

$P(5, 3, 5)$

$1(x - 5) + 4(y - 3) + 1 \cdot (z - 5) = 0$

$x - 5 + 4y - 12 + z - 5 = 0$

$x + 4y + z = 22$

(B)

$$x = 0$$
$$y = 0 \implies z = 22$$

$$y = 0$$
$$z = 0 \implies x = 22$$

$$x = 0$$
$$z = 0 \implies y = \frac{22}{4} = 5.5$$