# Formulas & Definitions: Section 16-5

**Definition:** If $F = Pi + Qj + Rk$ is a vector field on $\mathbb{R}^3$ and the partial derivatives of $P, Q$, and $R$ all exist, then the curl of $F$ is the vector field on $\mathbb{R}^3$ defined by

$$\operatorname{curl} F = \Big(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\Big)i + \Big(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\Big)j + \Big(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\Big)k$$

or

$$\operatorname{curl} F = \nabla \times F$$

**Theorem:** If $f$ is a function of three variables that has continuous second-order partial derivatives, then

$$\operatorname{curl}(\nabla f) = 0.$$

**Theorem:** If $F$ is a vector field defined on all of $\mathbb{R}^3$ whose component functions have continuous partial derivatives and $\operatorname{curl} F = 0$, then $F$ is a conservative vector field.

**Definition:** If $F = Pi + Qj + Rk$ is a vector field on $\mathbb{R}^3$ and $\partial P/\partial x$, $\partial Q/\partial y$, and $\partial R/\partial z$ exist, then the **divergence of** $F$ is the function of three variables defined by

$$\operatorname{div} F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

or

$$\operatorname{div} F = \nabla \cdot F.$$

**Theorem:** If $F = Pi+Qj+Rk$ is a vector field on $\mathbb{R}^3$ and $P, Q, R$ have continuous second-order partial derivatives, then

$$\operatorname{div} \operatorname{curl} F = 0.$$

**Definition:** If $\operatorname{div} F = 0$, then $F$ is said to be **incompressible**.

The operator

$$\nabla^2 = \nabla \cdot \nabla$$

is called the **Laplace operator**.

**Vector forms of Green's Theorem:**

First vector form:

$$\oint_C F \cdot dr = \iint_D (\operatorname{curl} \mathrm{F}) \cdot k \, dA.$$

Second vector form:

$$\oint_C F \cdot n \, ds = \iint_D \operatorname{div} F(x, y) \, dA.$$