WRH-6-Solutions

14.5: 2, 13

14.6: 4, 12

14.7: 5, 19

14.8: 4

14.5

(2)

$$z = \frac{x-y}{x+2y}, \quad x = e^{\pi t}, \quad y = e^{-\pi t}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} =$$

$$= \frac{(x+2y)\cdot 1 - (x-y)\cdot 1}{(x+2y)^2}\pi e^{\pi t} + \frac{(x+2y)(-1) - (x-y)\cdot 2}{(x+2y)^2}(-\pi)e^{-\pi t}$$

$$= \frac{3y}{(x+2y)^2}\pi e^{\pi t} + \frac{-3x}{(x+2y)^2}(-\pi e^{\pi t}) =$$

$$= \boxed{\frac{3\pi}{(x+2y)^2}\left(ye^{\pi t} + xe^{-\pi t}\right)}$$

(13)

$p(t) = f(g(t), h(t))$

f is differ.

$g(2) = 4, \quad g'(2) = -3$

$h(2) = 5, \quad h'(2) = 6$

$f_x(4,5) = 2, \quad f_y(4,5) = 8$

$$p'(t) = f_x \cdot \frac{dx}{dt} + f_y \frac{dy}{dt}$$

When $t=2$: $x=g(2)=4$
$y=h(2)=5$

So, $p'(2)=f_x(4,5)\,g'(2)+f_y(4,5)\,h'(2)=$
$=2\cdot(-3)+8\cdot 6=\boxed{42}$

$\boxed{14.6}$

(4) $f(x,y)=xy^3-x^2$, $(1,2)$, $\theta=\frac{\pi}{3}$

$D_u f(x_0,y_0)=f_x(x_0,y_0)\cos\theta+f_y(x_0,y_0)\sin\theta$

$f_x(x,y)=y^3-2x$
$f_y(x,y)=3xy^2$
$f_x(1,2)=8-2=6$
$f_y(1,2)=3\cdot 4=12$

$D_u f(1,2)=6\cdot\cos\frac{\pi}{3}+12\sin\frac{\pi}{3}=6\cdot\frac{1}{2}+12\cdot\frac{\sqrt{3}}{2}=$
$=\boxed{3+6\sqrt{3}}$

(12) $f(x,y)=\dfrac{x}{x^2+y^2}$, $(1,2)$, $v=\langle 3,5\rangle$

$D_u f(x_0,y_0)=f_x(x_0,y_0)u_1+f_y(x_0,y_0)u_2$

$$f_x(x,y) = \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{-2xy}{(x^2+y^2)^2}$$

$$f_x(1,2) = \frac{-1+4}{(1+4)^2} = \frac{3}{25}$$

$$f_y(1,2) = \frac{-2\cdot 2}{25} = -\frac{4}{25}$$

$$u = \frac{V}{|V|} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$$

$$D_u f(1,2) = \frac{3}{25}\cdot\frac{3}{\sqrt{34}} - \frac{20}{25\cdot\sqrt{34}} = \frac{-11}{25\sqrt{34}}$$

14.7

5 $f(x,y) = x^2 + xy + y^2 + y$

SOT:

$$f_x(x,y) = 2x+y = 0$$

$$f_y(x,y) = x+2y+1 = 0$$

$$\begin{cases} 2x+y=0 \\ x+2y+1=0 \quad |\cdot 2 \end{cases}$$

$$y-4y-2=0$$

$$-3y=2$$

$$y=-\frac{2}{3} \Rightarrow x=\frac{1}{3}$$

Hence, $\left(\frac{1}{3}, -\frac{2}{3}\right)$ is a critical point.

$$D = f_{xx}(a,b)\, f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$$

$$f_{xx} = 2$$
$$f_{yy} = 2$$
$$f_{xy} = 1$$

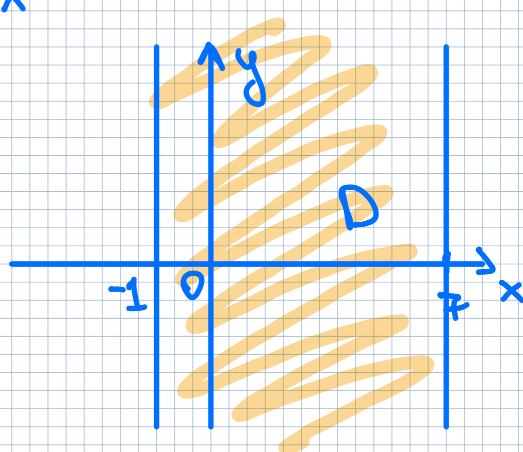
$D = 2 \cdot 2 - 1 = 3 > 0$ for all $(x,y) \in Dom(f)$

Since $D > 0$ and $f_{xx} > 0$, then $f\left(\frac{1}{3}, -\frac{2}{3}\right)$ is a loc. min and

$$f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}.$$

(19) $f(x,y) = y^2 - 2y\cos x$

$-1 \le x \le 7$



$$\begin{cases} f_x = 2y\sin x = 0 \\ f_y = 2y - 2\cos x = 0 \end{cases}$$

$$y = \cos x$$

$$2\cos x \sin x = 0$$

$f_x = 0 \Rightarrow y=0$ or $x = 0, \pi, 2\pi$

If $y=0$, then

f_y: $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

If $x = 0, 2\pi$, then

f_y: $2y - 2 = 0 \Rightarrow y = 1$

If $x = \pi$, then

f_y: $2y + 2 = 0 \Rightarrow y = -1$

Critical points: $\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), (0,1), (2\pi, 1),$
$(\pi, -1)$

$f_{xx} = 2y\cos x$
$f_{yy} = 2 + 2\sin x$
$f_{xy} = 2\sin x$
$D = 2y\cos x \cdot (2 + 2\sin x) - 4\sin^2 x$

$D\left(\frac{\pi}{2}, 0\right) = -4 < 0$
$D\left(\frac{3\pi}{2}, 0\right) = -4 < 0$
$D(0,1) = 4 > 0$
$D(2\pi, 1) = 4 > 0$
$D(\pi, -1) = 4 > 0$

$f_{xx}(0,1) = 2 > 0$
$f_{xx}(2\pi, 1) = 2 > 0$
$f_{xx}(\pi, -1) = 2 > 0$

Hence, $f(0,1), f(2\pi,1), f(\pi,-1)$ are loc. min
and $f(0,1) = f(2\pi,1) = f(\pi,-1) = -1$.

Since $D(\frac{\pi}{2},0) < 0$ and $D(\frac{3\pi}{2},0) < 0$, then
$(\frac{\pi}{2},0)$ and $(\frac{3\pi}{2},0)$ are saddle points.

$\boxed{14.8}$

4 $f(x,y) = 3x + y$
$x^2 + y^2 = 10$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = c \end{cases}$$

$$\nabla f = \langle f_x, f_y \rangle = \langle 3, 1 \rangle$$

$$\nabla g = \langle g_x, g_y \rangle = \langle 2x, 2y \rangle$$

$$\begin{cases} 3 = \lambda\, 2x \\ 1 = \lambda\, 2y \\ x^2 + y^2 = 10 \end{cases}$$

$$x = \frac{3}{2\lambda} \Rightarrow x \neq 0$$

$$y = \frac{1}{2\lambda} \Rightarrow y \neq 0$$

$$\frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 10$$

$$\frac{10}{4\lambda^2} = 10 \implies 4\lambda^2 = 1$$

$$\lambda = \pm \frac{1}{2}$$

If $\lambda = \frac{1}{2}$: $x = 3$, $y = 1$ $(3,1)$

If $\lambda = -\frac{1}{2}$: $x = -3$, $y = -1$ $(-3,-1)$

Possible extreme values at points: $(3,1)$ and $(-3,-1)$.

$f(3,1) = 10$

$f(-3,-1) = -10$

Thus $f_{max} = 10$ on $x^2 + y^2 = 10$

$f_{min} = -10$ on $x^2 + y^2 = 10$.