Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[10 points]** Evaluate the integral

$$\int_C e^x \, dx,$$

where C is the arc of the curve $x = y^3$ from $(-1, -1)$ to $(1, 1)$.

$y = y$, $x = y^3$, $dx = 3y^2 dy$

$-1 \le y \le 1$

$$\int_C e^x dx = \int_{-1}^{1} e^{y^3} \cdot 3y^2 dy = \int_{-1}^{1} e^u du =$$

$u = y^3$

$du = 3y^2 dy$

$-1 \le u \le 1$

$$= e^u \Big|_{-1}^{1} = e^1 - e^{-1} = \boxed{e - \frac{1}{e}}$$

3. **[5 points]** Determine whether or not

$$F(x, y) = \underbrace{(y^2 - 2x)}_{P}\,\mathbf{i} + \underbrace{2xy}_{Q}\,\mathbf{j}$$

is a conservative vector field.

$$\frac{\partial P}{\partial y} = 2y \qquad \frac{\partial Q}{\partial x} = 2y$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Also, $Dom(F) = \mathbb{R}^2$ which is open and simply-connected. Hence,

$\boxed{\text{F is a conservative vector field.}}$

4. **[5 points]** Evaluate the following integral using Green's Theorem

$$\oint_C y\,dx - x\,dy,$$

where C is the circle with center the origin and radius 4.

$$\oint_C y\,dx - x\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_D (-1-1)\,dA =$$

$$= \iint_D -2\,dA = \int_0^{2\pi}\int_0^4 -2\cdot r\,dr\,d\theta =$$

$$= -2\cdot 2\pi \left.\frac{r^2}{2}\right|_0^4 = -4\pi\cdot\frac{16}{2} = \boxed{-32\pi}$$