Formulas & Definitions: Section 15-4

Total mass of the lamina. Let $\rho(x, y)$ be a continuous function on D. Then the total mass of the lamina is

$$m = \iint\limits_{D} \rho(x, y)\, dA.$$

The total charge. If an electric charge is distributed over the region D and the charge density is given by (x, y) at a point (x, y) in D, then the total charge Q is given by

$$Q = \iint\limits_{D} \sigma(x, y)\, dA.$$

Moments and Centers of Mass.

- The moment of the entire lamina about the x-axis is

$$M_x = \iint\limits_{D} y\, \rho(x, y)\, dA.$$

- The moment about the y-axis is

$$M_y = \iint\limits_{D} x\, \rho(x, y)\, dA.$$

Statement. The coordinates $(\bar{x}, \bar{y})$ of the center of mass of a lamina occupying the region D and having density function $\rho(x, y)$ are

$$\bar{x} = \frac{M_y}{M} = \frac{1}{m} \iint\limits_{D} x\, \rho(x, y)\, dA, \quad \bar{y} = \frac{M_x}{M} = \frac{1}{m} \iint\limits_{D} y\, \rho(x, y)\, dA$$

where the mass m is given by

$$m = \iint\limits_{D} \rho(x, y)\, dA.$$

Moment of Inertia. The moment of inertia of the lamina about the x-axis is given by:

$$I_x = \iint\limits_{D} y^2\, \rho(x, y)\, dA.$$

The moment of inertia about the y-axis is given by:

$$I_y = \iint\limits_{D} x^2\, \rho(x, y)\, dA.$$

The moment of inertia about the origin (polar moment of inertia) is given by:

$$I_0 = \iint\limits_D (x^2 + y^2)\, \rho(x, y)\, dA.$$

Probability. The joint density function of X and Y is a function f of two variables such that the probability that (X, Y) lies in a region D is

$$P((X, Y) \in D) = \iint\limits_D f(x, y)\, dA.$$

In particular, if the region is a rectangle, the probability that X lies between a and b and Y lies between c and d is

$$P(a \leq X \leq b, c \leq Y \leq d) = \int\limits_a^b \int\limits_c^d f(x, y)\, dy\, dx.$$

Expected values. If X and Y are random variables with joint density function f, we define the X-mean and Y-mean, also called the expected values of X and Y, to be

$$\mu_1 = \iint\limits_{\mathbb{R}^2} x\, f(x, y)\, dA, \quad \mu_2 = \iint\limits_{\mathbb{R}^2} y\, f(x, y)\, dA.$$