Formulas & Definitions: Section 16-6

Definition: The set of all points (x, y, z) in $\mathbb{R}^3$ such that

$$x = x(u, v), \quad y = y(u, v), \quad z = z(u, v)$$

and (u, v) varies throughout D, is called a **parametric surface** S and above equations are called **parametric equations** of S.

Definition: Let

$$r(u, v) = x(u, v)i + y(u, v)j + z(u, v)k$$

be a vector function. Then

$$r_v = \frac{\partial x}{\partial v}(u_0, v_0)i + \frac{\partial y}{\partial v}(u_0, v_0)j + \frac{\partial z}{\partial v}(u_0, v_0)k$$

and

$$r_u = \frac{\partial x}{\partial u}(u_0, v_0)i + \frac{\partial y}{\partial u}(u_0, v_0)j + \frac{\partial z}{\partial u}(u_0, v_0)k$$

If $r_u \times r_v$ is not 0, then the surface S is called **smooth**. For a smooth surface, the **tangent plane** is the plane that contains the tangent vectors r_u and r_v, and the vector $r_u \times r_v$ is a normal vector to the tangent plane.

Definition: If a smooth parametric surface S is given be the equation

$$r(u, v) = x(u, v)i + y(u, v)jk + z(u, v)k, \quad (u, v) \in D$$

and S is covered just once as (u, v) ranges throughout the parameter domain D, then the **surface area** of S is

$$A(S) = \iint_D |r_u \times r_v| \, dA$$

where

$$r_u = \frac{\partial x}{]\partial u}i + \frac{\partial y}{\partial u}j + \frac{\partial z}{\partial u}k, \quad r_v = \frac{\partial x}{]\partial v}i + \frac{\partial y}{\partial v}j + \frac{\partial z}{\partial v}k$$

Surface Area of the Graph of a Function: For a special case of a surface S with equation $z = f(x, y)$, where (x, y) lies in D and f has continuous partial derivatives, we take x and y as parameters. Then

$$|r_x \times r_y| = \sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}$$

and the surface area formula becomes

$$A(S) = \iint_D \sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2} \, dA$$