WRH-7-Solutions

15.1 : 9, 16, 27
15.2: 7, 24
15.3: 10, 15

15.1

9) $\iint_R \sqrt{2}\, dA = ?$

$R = \{(x,y) \mid 2 \le x \le 6,\ -1 \le y \le 5\}$

$$\iint_R \sqrt{2}\, dA = \int_2^6 \int_{-1}^5 \sqrt{2}\, dy\, dx = \sqrt{2} \int_2^6 6\, dx = \sqrt{2} \cdot 6 \cdot 4 =$$

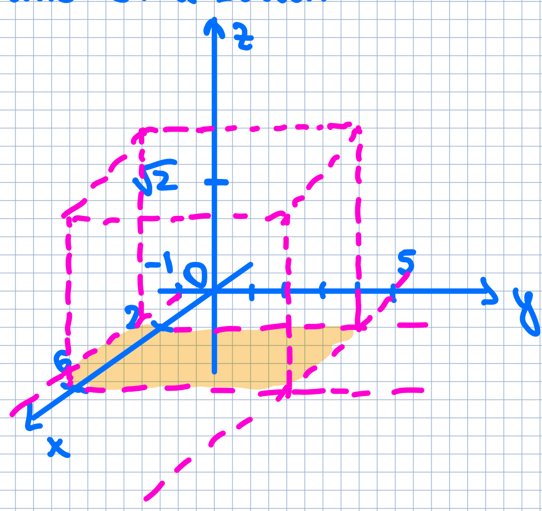
$$= \boxed{24\sqrt{2}}$$

As a volume of a solid:

$z = \sqrt{2}$

$V = \sqrt{2} \cdot 6 \cdot 4 =$

$= 24\sqrt{2}.$



16 $$\int_0^1\int_0^1 (x+y)^2\,dx\,dy = \int_0^1\int_0^1 (x^2+2xy+y^2)\,dx\,dy =$$

$$= \int_0^1 \left(\frac{x^3}{3}+x^2y+xy^2\right)\Big|_0^1 dy = \int_0^1 \left(\frac{1}{3}+y+y^2\right)dy =$$

$$= \left(\frac{1}{3}y+\frac{y^2}{2}+\frac{y^3}{3}\right)\Big|_0^1 = \frac{1}{3}+\frac{1}{2}+\frac{1}{3} = \frac{2}{3}+\frac{1}{2} =$$

$$= \frac{4+3}{6} = \boxed{\frac{7}{6}}$$

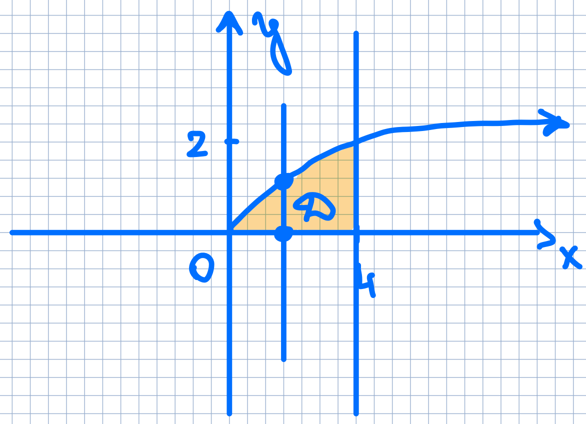
27 $$\iint_R x\sec^2 y\,dA\ ,\quad R=\{(x,y)\mid 0\le x\le 2,\ 0\le y\le \tfrac{\pi}{4}\}$$

$$\int_0^2\int_0^{\frac{\pi}{4}} x\sec^2 y\,dy\,dx = \int_0^2 x\,dx\int_0^{\frac{\pi}{4}}\sec^2 y\,dy =$$

$$= \left(\frac{x^2}{2}\right)\Big|_0^2 \tan y\Big|_0^{\frac{\pi}{4}} = 2\cdot(1-0) = \boxed{2}$$

15.2

7 $$\iint_D \frac{y}{x^2+1}\,dA\ ,\quad D=\{(x,y)\mid 0\le x\le 4,\ 0\le y\le \sqrt{x}\}$$



$$\iint_D \frac{y}{x^2+1}\,dA = \int_0^4 \int_0^{\sqrt{x}} \frac{y}{x^2+1}\,dy\,dx =$$

$$= \int_0^4 \frac{1}{x^2+1} \left.\frac{y^2}{2}\right|_0^{\sqrt{x}} dx = \int_0^4 \frac{1}{x^2+1} \frac{x}{2}\,dx =$$

$$= \frac{1}{2}\int_0^4 \frac{x}{x^2+1}\,dx = \frac{1}{4}\int_0^4 \frac{d(x^2+1)}{x^2+1} =$$

$$= \frac{1}{4} \ln|x^2+1| \Big|_0^4 = \frac{1}{4}(\ln 17 - 0) = \boxed{\frac{1}{4}\ln 17}$$

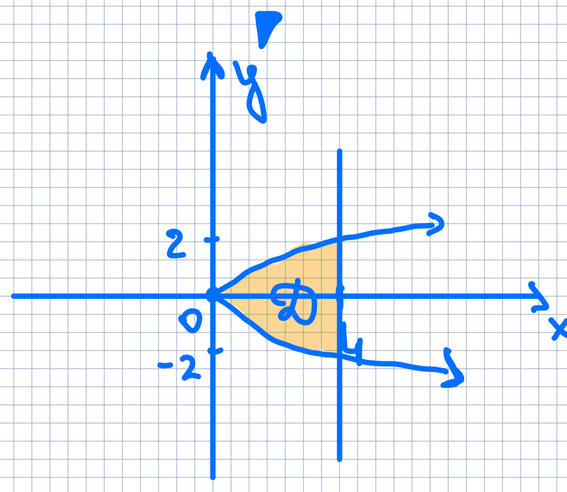
24

$z = 1 + x^2y^2$

$D:\ x = y^2,\ x = 4$

$V - ?$

$$V = \iint_D z(x,y)\,dA$$



$$V = \int_{\cancel{-2}\,0}^{2}\int_{y^2}^{4}(1+x^2y^2)\,dx\,dy = \int_{0\,\cancel{-2}}^{2}\left(x+\frac{x^3y^2}{3}\right)\Bigg|_{y^2}^{4} dy =$$

$$= \int_{\cancel{-2}\,0}^{2}\left(4+\frac{61}{3}y^2-\frac{1}{3}y^8\right)dy = \left(4y+\frac{61}{9}y^3 - \right.$$

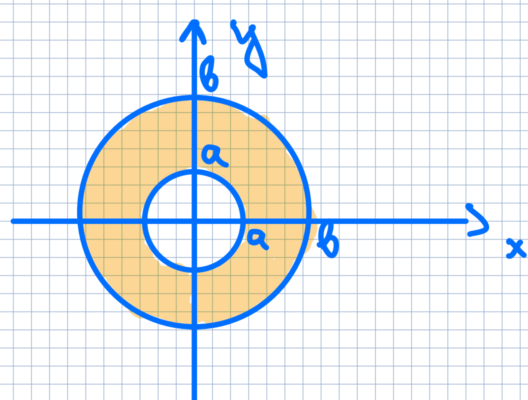
$$\left.-\frac{1}{27}y^9\right)\Bigg|_{\cancel{-2}\,0}^{2} = 8+\frac{488}{9}-\frac{512}{27}\cancel{+8+\frac{488}{9}-\frac{512}{7}}$$

$$= \boxed{\cancel{\frac{2336}{27}}} \qquad \frac{216+1464-512}{27} = \frac{1168}{27}$$

$\boxed{15.3}$

(10) $\iint_R \frac{y^2}{x^2+y^2}\,dA$,

R: $x^2+y^2=a^2$
$x^2+y^2=b^2$, $0<a<b$



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$x^2+y^2=r^2=a^2$

$x^2+y^2=r^2=b^2$

$a \le r \le b$

$0 \le \theta \le 2\pi$

$dA = r\,dr\,d\theta$

$$\iint_R \frac{y^2}{x^2+y^2}\,dA = \int_a^b \int_0^{2\pi} \frac{r^2 \sin^2\theta}{r^2}\, r\,dr\,d\theta =$$

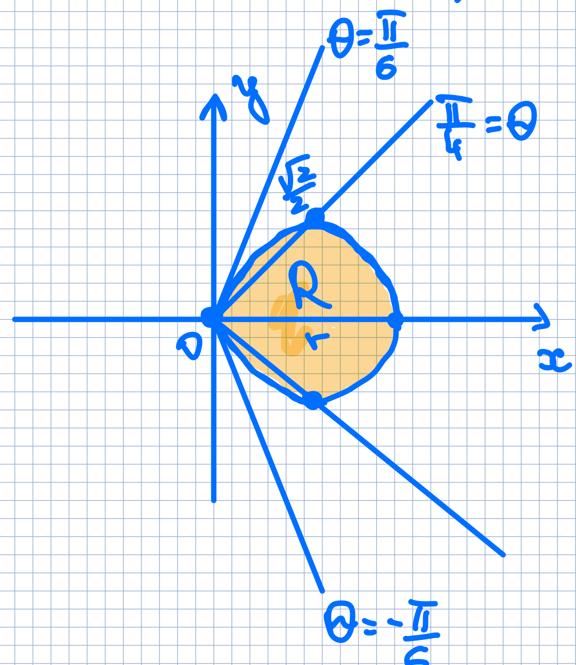
$$= \int_a^b r\,dr \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\theta)\,d\theta = \frac{r^2}{2}\Big|_a^b \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}(b^2 - a^2)\cdot \frac{1}{2}\, 2\pi = \boxed{\frac{\pi}{2}(b^2 - a^2)}$$

15

$r = \cos 3\theta$

$\theta = 0$
$r = 1$
$\theta = \frac{\pi}{4}$
$r = \cos \frac{3\pi}{4}$
$\theta = \frac{\pi}{2}$
$r = 0$
$\theta = \pi$
$r = 1$



$$\iint_R dA = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \int_0^{\cos 3\theta} r\,dr\,d\theta = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{6}} r^2\Big|_0^{\cos 3\theta} d\theta =$$

$$= \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2 3\theta \, d\theta = \frac{1}{4}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1+\cos 6\theta)\, d\theta =$$

$$= \frac{1}{2}\int_{0}^{\frac{\pi}{6}} (1+\cos 6\theta)\, d\theta = \frac{1}{2}\left(\theta + \frac{1}{6}\sin 6\theta\right)\Bigg|_{0}^{\frac{\pi}{6}} =$$

$$= \boxed{\frac{\pi}{12}}$$