Formulas & Definitions: Section 13-2

Definition: The derivative r' of a vector function r is

$$\frac{dr}{dt} = r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}.$$

Definition: The unit tangent vector is

$$T(t) = \frac{r'(t)}{|r'(t)|}.$$

Theorem: If $r(t) = \langle f(t), g(t), h(t) \rangle = f(t)i + g(t)j + h(t)k$, where f, g, and h are differentiable functions, then

$$r'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)i + g'(t)j + h'(t)k.$$

Theorem: Suppose u and v are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

- $\frac{d}{dt}[u(t) + v(t)] = u'(t) + v'(t)$
- $\frac{d}{dt}[cu(t)] = cu'(t)$
- $\frac{d}{dt}[f(t)u(t)] = f'(t)u(t) + f(t)u'(t)$
- $\frac{d}{dt}[u(t) \cdot v(t)] = u'(t) \cdot v(t) + u(t) \cdot v'(t)$
- $\frac{d}{dt}[u(t) \times v(t)] = u'(t) \times v(t) + u(t) \times v'(t)$
- $\frac{d}{dt}[u(f(t))] = f'(t)u'(f(t))$

Definition: The definite integral of a continuous vector function $r(t)$ is

$$\int_a^b r(t)\,dt = \Big(\int_a^b f(t)\,dt\Big)i + \Big(\int_a^b g(t)\,dt\Big)j + \Big(\int_a^b h(t)\,dt\Big)k,$$

and

$$\int_a^b r(t)\,dt = R(t)\Big|_a^b = R(b) - R(a).$$