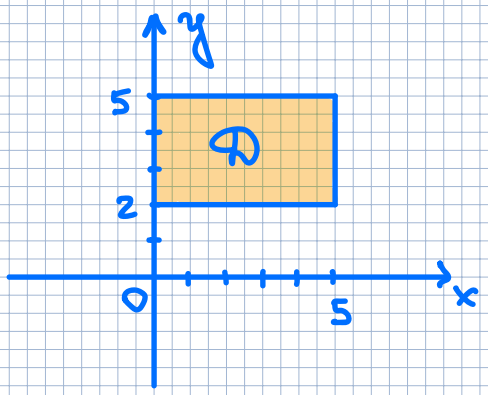
WRH-8-Solutions

15.4: 1,6
15.5: 2,11

15.4

① $0 \le x \le 5$
$2 \le y \le 5$
$\sigma(x,y) = 2x+4y$



The total charge is

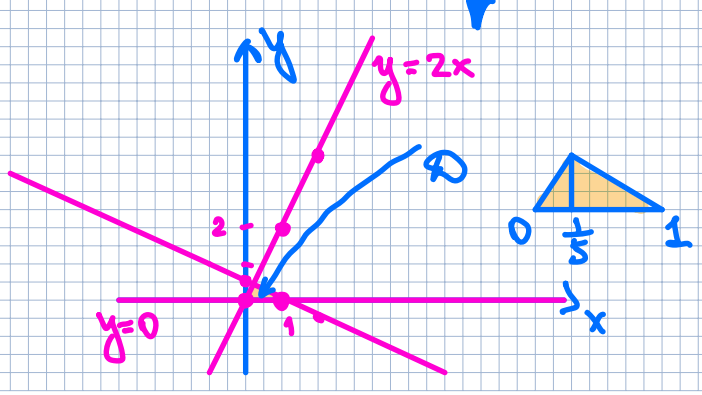
$$Q = \iint_D \sigma(x,y)\,dA =$$

$$Q = \int_0^5 \int_2^5 (2x+4y)\,dy\,dx = \int_0^5 (2xy+2y^2)\Big|_2^5\,dx =$$

$$= \int_0^5 (10x+50-4x-8)\,dx = \int_0^5 (6x+42)\,dx =$$

$$= \left(\frac{6x^2}{2}+42x\right)\Big|_0^5 = 3\cdot 25 + 210 = \boxed{285\ C}$$

⑥ D: $y=0$
$y=2x$
$x+2y=1$
$P(x,y)=x$



$$m = \iint_D \rho(x,y)\, dA$$

$$\begin{cases} y = 2x \\ x + 2y = 1 \end{cases} \qquad x = 1 - 2y$$

$$y = 2(1-2y) = 2 - 4y$$

$$5y = 2$$

$$y = \frac{2}{5} \Rightarrow x = 1 - \frac{4}{5} = \boxed{\frac{1}{5}}$$

Thus,

$$m = \int_0^{\frac{1}{5}} \int_0^{2x} x \, dA + \int_{\frac{1}{5}}^{1} \int_0^{\frac{1-x}{2}} x \, dA =$$

$$= \int_0^{\frac{1}{5}} xy \Big|_0^{2x} dx + \int_{\frac{1}{5}}^{1} xy \Big|_0^{\frac{1-x}{2}} dx =$$

$$= \int_0^{\frac{1}{5}} 2x^2 \, dx + \int_{\frac{1}{5}}^{1} \left(\frac{x}{2} - \frac{x^2}{2} \right) dx =$$

$$= \frac{2x^3}{3} \Big|_0^{\frac{1}{5}} + \left(\frac{x^2}{4} - \frac{x^3}{6} \right) \Big|_{\frac{1}{5}}^{1} =$$

$$= \frac{2}{3} \cdot \frac{1}{125} + \frac{1}{4} - \frac{1}{6} - \frac{1}{100} + \frac{1}{6} \frac{1}{125}$$

$$= \frac{2\cdot 2\cdot 100 + 6\cdot 125\cdot 25 - 125\cdot 100 - 6\cdot 125 + 100}{6\cdot 125\cdot 100} =$$

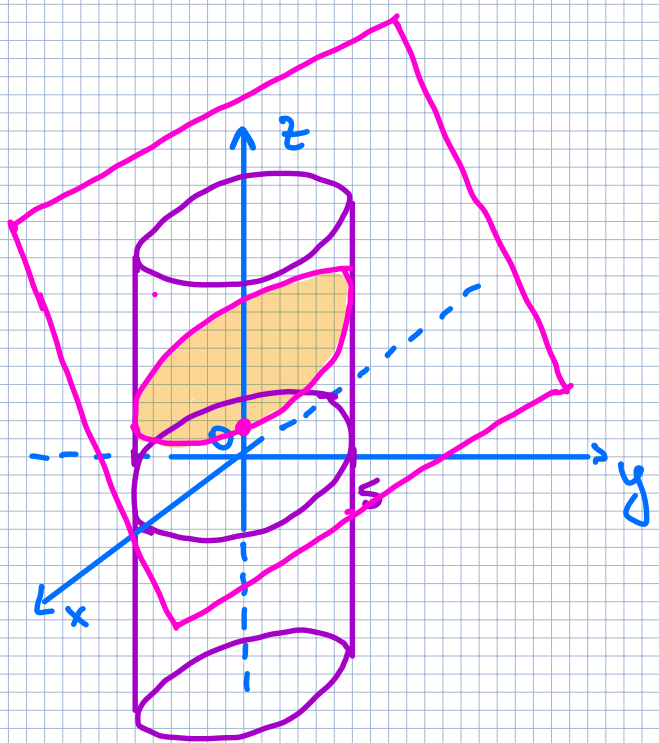
$$= \frac{400 + 18750 - 12500 - 750 + 100}{6\cdot 125\cdot 100} =$$

$$= \frac{6000}{6\cdot 125\cdot 100} = \frac{10}{125} = \boxed{\frac{2}{3}}$$

15.5

② $6x + 4y + 2z = 1$

$x^2 + y^2 = 25$



$x = 5$
$y = 0$

$30 + 2z = 1$

$2z = -29$

$z = -\frac{29}{2}$

$y = 5$
$x = 0$

$z = -\frac{19}{2}$

$$A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$2z = 1 - 6x - 4y$

$z = \frac{1}{2} - 3x - 2y$

$D: \quad x^2 + y^2 = 25$

$f(x,y) = z$

$f_x = -3$

$f_y = -2$

$$A(S) = \iint_D \sqrt{9 + 4 + 1}\, dA =$$

$$= \iint_D \sqrt{14}\, dA = \sqrt{14} \int_0^5 \int_0^{2\pi} r\, d\theta\, dr =$$

$$= \sqrt{14} \cdot 2\pi \left. \frac{r^2}{2} \right|_0^5 = \boxed{25\sqrt{14}\,\pi}$$

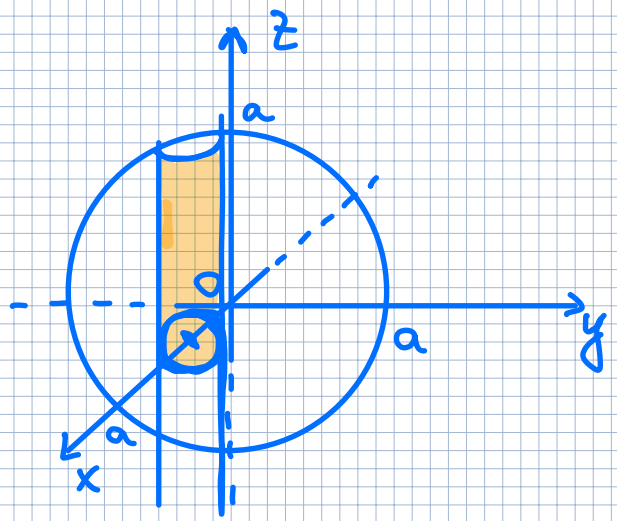
(11)

$$x^2 + y^2 + z^2 = a^2$$

$$x^2 + y^2 = ax$$

$$x^2 - ax + y^2 = 0$$

$$\left(x - \frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4}$$

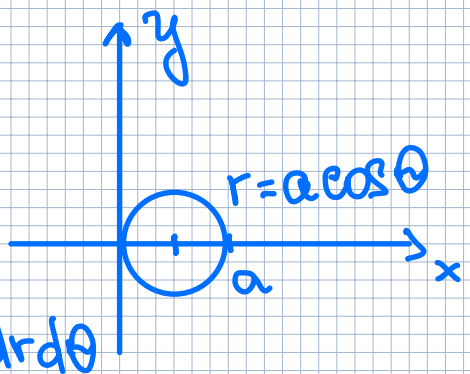


$$A(S) = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\, dA$$

$$f(x,y) = \sqrt{a^2 - x^2 - y^2}$$

$$f_x = \frac{-x}{\sqrt{a^2 - x^2 - y^2}}$$

$$f_y = \frac{-y}{\sqrt{a^2 - x^2 - y^2}}$$



$$A(S) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{a\cos\theta} \sqrt{\frac{r^2}{a^2 - r^2} + 1}\, r\, dr\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{a\cos\theta} \frac{ar}{\sqrt{a^2 - r^2}}\, dr\, d\theta =$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(-a\sqrt{a^2 - r^2} \right) \Big|_0^{a\cos\theta} d\theta =$$

$$= 2a^2 \int_0^{\frac{\pi}{2}} \left(1 - \sqrt{1 - \cos^2\theta} \right) d\theta$$

$$= a^2\pi - 2a^2\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta = \boxed{a^2(\pi - 2)}$$