Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz will be available starting from **5:00 PM until midnight** on scheduled week day (Thursday).
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[5 points]** Find the **length** of the following curve

$$r(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \quad 0 \le t \le 1$$

$$L = \int_0^1 \sqrt{0 + (2t)^2 + (3t^2)^2}\, dt = \int_0^1 \sqrt{4t^2 + 9t^4}\, dt =$$

$$= \int_0^1 t\sqrt{4+9t^2}\, dt = \frac{1}{18}\int_4^{13} \sqrt{u}\, du = \frac{1}{18}\frac{u^{3/2}}{3/2}\Big|_4^{13} = \frac{1}{27}\left(\sqrt{13^3} - \sqrt{4^3}\right).$$

$4 + 9t^2 = u$

$18t\, dt = du$

$4 \le u \le 13$

2. **[5 points]** Find the unit tangent **T(t)** and the unit normal **N(t)** vectors for the given vector function at $t = 0$:

$$r(t) = \langle t, 3\sin t, 3\cos t\rangle$$

$T(t) = \dfrac{r'(t)}{|r'(t)|}$

$r'(t) = \langle 1, 3\cos t, -3\sin t\rangle$

$r'(0) = \langle 1, 3, 0\rangle$

$|r'(0)| = \sqrt{1+9} = \sqrt{10}$

$T(0) = \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0\right\rangle$

$N(t) = \dfrac{T'(t)}{|T'(t)|}$

$T(t) = \left\langle \frac{1}{\sqrt{10}}, \frac{3\cos t}{\sqrt{10}}, \frac{-3\sin t}{\sqrt{10}}\right\rangle$

$T'(t) = \frac{1}{\sqrt{10}}\langle 0, -3\sin t, -3\cos t\rangle$

$|T'(t)| = \frac{3}{\sqrt{10}}$

$N(t) = \frac{1}{3}\langle 0, -3\sin t, -3\cos t\rangle$

$N(0) = \langle 0, 0, -1\rangle$

3. **[5 points]** Find the velocity and speed of a particle with the given position function

$$r(t) = \langle t^2, 2t, \ln t\rangle$$

$v = r'(t) = \langle 2t, 2, \frac{1}{t} \rangle$

$v = |v| = \sqrt{4t^2 + 4 + \frac{1}{t^2}} = \frac{1}{t}\sqrt{4t^4 + 4t^2 + 1} =$

$= \frac{1}{t}\sqrt{(2t^2+1)^2} = \frac{2t^2+1}{t} = 2t + \frac{1}{t}.$

4. **[5 points]** Find the **tangential** and **normal** components of the **acceleration** vector for the following vector function

$$r(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$$

$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|}$

$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|}$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$|r'(t)| = \sqrt{1 + 4t^2 + 9t^4}$

$a_T = \frac{4t + 18t^3}{\sqrt{1+4t^2+9t^4}}$

$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} =$

$= (12t^2 - 6t^2)i - (6t)j + (2)k =$

$= \langle 6t^2, -6t, 2 \rangle$

$|r' \times r''| = \sqrt{36t^4 + 36t^2 + 4}$

$a_N = \frac{\sqrt{36t^4 + 36t^2 + 4}}{\sqrt{1 + 4t^2 + 9t^4}}$