Formulas & Definitions: Section 15-2

Definition. Let the function F is given by

$$F(x,y) = \begin{cases} f(x,y) & \text{if} \quad (x,y) \in D, \\ 0 \quad \text{if} \quad (x,y) \in R \backslash D. \end{cases}$$

If F is integrable over R, then we define the double integral of f over D by

$$\iint\limits_D f(x,y)\,dA = \iint\limits_R F(x,y)\,dA.$$

Statement 1. If f is continuous on a type I region D such that

$$D = \{(x,y) \,|\, a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

then

$$\iint\limits_D f(x,y)\,dA = \int\limits_a^b \int\limits_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx.$$

Statement 2. If f is continuous on a type II region D such that

$$D = \{(x,y) \,|\, c \le y \le d, h_1(y) \le x \le h_2(y)\}$$

then

$$\iint\limits_D f(x,y)\,dA = \int\limits_c^d \int\limits_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy.$$

Properties of Double Integrals.

1.
$$\iint\limits_D [f(x,y) + g(x,y)]\,dA = \iint\limits_D f(x,y)\,dA + \iint\limits_D g(x,y)\,dA$$

2.
$$\iint\limits_D cf(x,y)\,dA = c\iint\limits_D f(x,y)\,dA,$$

   where c is a constant

3. If $f(x,y) \ge g(x,y)$ for all (x,y) in D, then
$$\iint\limits_D f(x,y)\,dA \ge \iint\limits_D g(x,y)\,dA$$

4. If $D = D_1 \cup D_2$, where D_1 and D_2 do not overlap except perhaps on their boundaries, then
$$\iint\limits_D f(x,y)\,dA = \iint\limits_{D_1} f(x,y)\,dA + \iint\limits_{D_2} f(x,y)\,dA$$

5.

$$\iint_D 1\, dA = A(D)$$

6. If $m \leq f(x,y) \leq M$ for all (x,y) in D, then

$$mA(D) \leq \iint_D f(x,y)\, dA \leq MA(D)$$