Formulas & Definitions: Section 14-2

Definition: Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then we say that the **limit of** $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $(x, y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then

$$|f(x, y) - L| < \varepsilon.$$

Proposition: If $f(x, y) \longrightarrow L_1$ as $(x, y) \longrightarrow (a, b)$ along a path C_1 and $f(x, y) \longrightarrow L_2$ as $(x, y) \longrightarrow (a, b)$ along a path C_2, where $L_1 \neq L_2$, then

$$\lim_{(x,y)\to(a,b)} f(x, y)$$

does not exist.

Definition: A function f of two variables is called **continuous at** (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

We say f is **continuous on** D if f is continuous at every point (a, b) in D.

Definition: The function f is **continuous** at (a, b, c) if

$$\lim_{(x,y,z)\to(a,b,c)} f(x, y, z) = f(a, b, c).$$

Definition: If f is defined on a subset D of $\mathbb{R}^n$, then $\lim_{x\to a} f(x) = L$ means that for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that if $x \in D$ and $0 < |x - a| < \delta$ then

$$|f(x) - L| < \varepsilon.$$