# Formulas & Definitions: Section 14-7

**Definition 1:** A function of two variables has a **local maximum** at $(a, b)$ if $f(x, y) \leq f(a, b)$ when $(x, y)$ is near $(a, b)$. The number $f(a, b)$ is called a **local maximum value**. If $f(x, y) \geq f(a, b)$ when $(x, y)$ is near $(a, b)$, then $f$ has a **local minimum** at $(a, b)$ and $f(a, b)$ is a **local minimum value**.

**Remark:** If the inequalities in Def.1 hold for all points $(x, y)$ in the domain of $f$, then $f$ has an **absolute maximum** (or **absolute minimum**) at $(a, b)$.

**Theorem:** If $f$ has a local maximum or minimum at $(a, b)$ and the first-order partial derivatives of $f$ exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

**Second Derivatives Test:** Suppose the second partial derivatives of $f$ are continuous on a disk with center $(a, b)$, and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

(a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

(b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

(c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum. In this case the point $(a, b)$ is called a **saddle point** of $f$ and the graph of $f$ crosses its tangent plane at $(a, b)$.

**Remark:** If $D = 0$, the test gives no information.

**Extreme Value Theorem for Functions of Two Variables:** If $f$ is continuous on a closed, bounded set $D$ in $\mathbb{R}^2$, then $f$ attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points $(x_1, y_1)$ and $(x_2 . y_2)$ in $D$.

**Algorithm of finding an absolute max and min values of $f$:** To find the absolute maximum and minimum values of a continuous function $f$ on a closed, bounded set $D$:

1. Find the values of $f$ at the critical points of $f$ in $D$.
2. Find the extreme values of $f$ on the boundary of $D$.
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.