Formulas & Definitions: Section 15-8

Spherical coordinate system (conversion formulas): To convert from spherical to rectangular coordinates, we use the equations

$$x = \rho \sin \varphi \cos \theta, \quad y = \rho \sin \varphi \sin \theta, \quad z = \rho \cos \varphi.$$

The distance formula shows that

$$\rho^2 = x^2 + y^2 + z^2.$$

Evaluating triple integrals with spherical coordinates:

$$\iiint_E f(x, y, z)\, dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi) \rho^2 \sin \varphi \, d\rho \, d\theta \, d\varphi,$$

where E is a spherical wedge given by

$$E = \{(\rho, \theta, \varphi) \mid a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \varphi \leq d\}.$$