WRH-4 - Solutions

13.4: 6, 15, 37, 41

$\boxed{13.4}$

6

$r(t) = e^t i + e^{2t} j \,,\ t=0$

$v(t) = r'(t) = \langle e^t , 2e^{2t} \rangle$

$a(t) = r''(t) = \langle e^t , 4e^{2t} \rangle$

$v = |v(t)| = \sqrt{e^{2t}+4e^{4t}} = e^t\sqrt{1+4e^{2t}}$

$v(0) = \langle 1,2 \rangle$

$a(0) = \langle 1,4 \rangle$

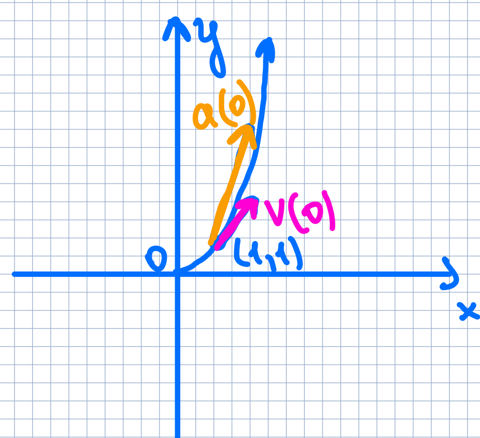
$x(t) = e^t$

$y(t) = e^{2t} = x^2$

$\boxed{y = x^2}$ parabola

At $t=0$: $x(0)=1$

$y(0)=1$



15

$a(t) = 2i + 2t\,k$

$v(0) = 3i - j = \langle 3,-1,0 \rangle$

$r(0) = j + k = \langle 0,1,1 \rangle$

$v(t) = \int a(t)\,dt = \langle 2t, 0, t^2 \rangle + \langle a,b,c \rangle$

$v(0) = \langle 0,0,0 \rangle + \langle a,b,c \rangle = \langle 3,-1,0 \rangle$

Hence,

$$\boxed{v(t) = \langle 2t+3, -1, t^2 \rangle}$$

$$r(t) = \int v(t)\,dt = \langle t^2+3t, -t, \frac{t^3}{3} \rangle + \langle \tilde{a}, \tilde{b}, \tilde{c} \rangle$$

$$r(0) = \langle 0,0,0 \rangle + \langle \tilde{a}, \tilde{b}, \tilde{c} \rangle = \langle 0,1,1 \rangle$$

Thus,

$$\boxed{r(t) = \langle t^2+3t, -t+1, \frac{t^3}{3}+1 \rangle}$$

▼

37) $r(t) = (t^2+1)\,i + t^3\,j, \; t \geq 0$

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|}$$

$$r'(t) = \langle 2t, 3t^2 \rangle$$

$$r''(t) = \langle 2, 6t \rangle$$

$$r'(t) \cdot r''(t) = 4t + 18t^3$$

$$|r'(t)| = \sqrt{4t^2 + 9t^4} = t\sqrt{4+9t^2}$$

Hence,

$$a_T = \frac{4t+18t^3}{t\sqrt{4+9t^2}} = \boxed{\frac{4+18t^2}{\sqrt{4+9t^2}}}$$

$$a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|}$$

$$r'(t)\times r''(t) = \begin{vmatrix} i & j & k \\ 2t & 3t^2 & 0 \\ 2 & 6t & 0 \end{vmatrix} =$$

$$= i\cdot 0 + j\cdot 0 + k\,(12t^2 - 6t^2) = \langle 0, 0, 6t^2 \rangle$$

$$|r'(t)\times r''(t)| = 6t^2$$

Hence,

$$a_N = \frac{6t^2}{t\sqrt{4+9t^2}} = \boxed{\frac{6t}{\sqrt{4+9t^2}}}$$

41

$$r(t) = \ln t\, i + (t^2+3t)\,j + 4\sqrt{t}\,k, \quad (0,4,4)$$

$$r'(t) = \langle \tfrac{1}{t}, 2t+3, \tfrac{2}{\sqrt{t}} \rangle$$

$$r''(t) = \langle -\tfrac{1}{t^2}, 2, -\tfrac{1}{t^{3/2}} \rangle$$

$$\ln t = 0 \Rightarrow t = 1$$

Hence,

$$r'(1) = \langle 1, 5, 2 \rangle$$

$$r''(1) = \langle -1, 2, -1 \rangle$$

$$a_T = \frac{-1+10-2}{\sqrt{1+25+4}} = \boxed{\frac{7}{\sqrt{30}}}$$

$$a_N = \frac{|r'(1) \times r''(1)|}{|r'(1)|} = \frac{\sqrt{81+1+49}}{\sqrt{30}} = \boxed{\sqrt{\frac{131}{30}}}$$