Formulas & Definitions: Section 15-3

Change to Polar Coordinates in a Double Integral: If f is continuous on a polar rectangle R given by $0 \le a \le r \le b$, $\alpha \le \theta \le \beta$, where $0 \le \beta - \alpha \le 2\pi$, then

$$\iint\limits_R f(x,y) dA = \int\limits_{\alpha}^{\beta} \int\limits_a^b f(r\cos\theta, r\sin\beta) r dr d\theta.$$

Statement: If f is continuous on a polar region of the form

$$D = \{(r, \theta) \mid \alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta)\}$$

then

$$\iint\limits_{D} f(x,y) dA = \int\limits_{\alpha}^{\beta} \int\limits_{h_{1}(\theta)}^{h_{2}(\theta)} f(r\cos\theta, r\sin\theta) r dr d\theta.$$