

Formulas & Definitions: Section 15-7

Cylindrical coordinate system (conversion formulas): To convert from cylindrical to rectangular coordinates, we use the equations

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z,$$

whereas to convert from rectangular to cylindrical coordinates, we use

$$r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}, \quad z = z.$$

Evaluating triple integrals with cylindrical coordinates: Suppose that f is continuous function and

$$E = \{(x, y, z) \mid (x, y) \in D, u_1(x, y) \leq z \leq u_2(x, y)\},$$

where D is given in polar coordinates by

$$D = \{(r, \theta) \mid \alpha \leq \theta \leq \beta, h_1(\theta) \leq h_2(\theta)\}.$$

Then

$$\iiint_E f(x, y, z) dV = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r \cos \theta, r \sin \theta)}^{u_2(r \cos \theta, r \sin \theta)} f(r \cos \theta, r \sin \theta, z) r dz dr d\theta.$$