WRH-6-Solutions

14.5: 2, 13

14.6: 4, 12

14.7: 5, 19

14.8: 4

14.5

(2)

$$z = \frac{x-y}{x+2y}, \quad x = e^{\pi t}, \quad y = e^{-\pi t}$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} =$$

$$= \frac{(x+2y)\cdot 1 - (x-y)\cdot 1}{(x+2y)^2}\pi e^{\pi t} + \frac{(x+2y)(-1) - (x-y)\cdot 2}{(x+2y)^2}(-\pi)e^{-\pi t}$$

$$= \frac{3y}{(x+2y)^2}\pi e^{\pi t} + \frac{-3x}{(x+2y)^2}(-\pi e^{\pi t}) =$$

$$= \boxed{\frac{3\pi}{(x+2y)^2}\left(ye^{\pi t} + xe^{-\pi t}\right)}$$

(13)

$p(t) = f(g(t), h(t))$

f is differ.

$g(2) = 4, \quad g'(2) = -3$

$h(2) = 5, \quad h'(2) = 6$

$f_x(4,5) = 2, \quad f_y(4,5) = 8$

$$p'(t) = f_x \cdot \frac{dx}{dt} + f_y \frac{dy}{dt}$$

When $t=2$: $x = g(2) = 4$
$y = h(2) = 5$

So, $p'(2) = f_x(4,5)\, g'(2) + f_y(4,5)\, h'(2) =$
$= 2\cdot(-3) + 8\cdot 6 = \boxed{42}$

14.6

(4) $f(x,y) = xy^3 - x^2$, $(1,2)$, $\theta = \frac{\pi}{3}$

$D_u f(x_0,y_0) = f_x(x_0,y_0)\cos\theta + f_y(x_0,y_0)\sin\theta$

$f_x(x,y) = y^3 - 2x$
$f_y(x,y) = 3xy^2$
$f_x(1,2) = 8 - 2 = 6$
$f_y(1,2) = 3\cdot 4 = 12$

$D_u f(1,2) = 6\cdot\cos\frac{\pi}{3} + 12\sin\frac{\pi}{3} = 6\cdot\frac{1}{2} + 12\cdot\frac{\sqrt{3}}{2} =$
$= \boxed{3 + 6\sqrt{3}}$

(12) $f(x,y) = \dfrac{x}{x^2+y^2}$, $(1,2)$, $v = \langle 3,5\rangle$

$D_u f(x_0,y_0) = f_x(x_0,y_0)\, u_1 + f_y(x_0,y_0)\, u_2$

$$f_x(x,y) = \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{-2xy}{(x^2+y^2)^2}$$

$$f_x(1,2) = \frac{-1+4}{(1+4)^2} = \frac{3}{25}$$

$$f_y(1,2) = \frac{-2\cdot 2}{25} = -\frac{4}{25}$$

$$u = \frac{v}{|v|} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$$

$$D_u f(1,2) = \frac{3}{25}\cdot\frac{3}{\sqrt{34}} - \frac{20}{25\cdot\sqrt{34}} = \frac{-11}{25\sqrt{34}}$$

$\boxed{14.7}$

5 $\quad f(x,y) = x^2 + xy + y^2 + y$

SDT:

$$f_x(x,y) = 2x+y = 0$$

$$f_y(x,y) = x+2y+1 = 0$$

$$\begin{cases} 2x+y=0 \\ x+2y+1=0 \quad |\cdot 2 \end{cases}$$

$$y-4y-2=0$$

$$-3y=2$$

$$y=-\frac{2}{3} \Rightarrow x=\frac{1}{3}$$

Hence, $\left(\frac{1}{3}, -\frac{2}{3}\right)$ is a critical point.

$$D = f_{xx}(a,b)\, f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$$

$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 1$

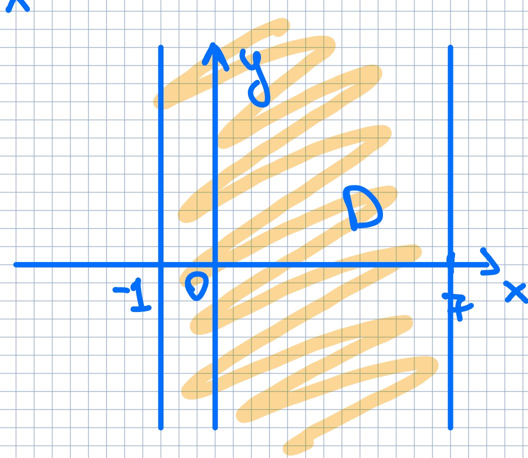
$D = 2 \cdot 2 - 1 = 3 > 0$ for all $(x,y) \in Dom(f)$

Since $D > 0$ and $f_{xx} > 0$, then $f\left(\frac{1}{3}, -\frac{2}{3}\right)$ is a loc. min and

$f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}.$

(19) $f(x,y) = y^2 - 2y\cos x$

$-1 \le x \le 7$



$$\begin{cases} f_x = 2y\sin x = 0 \\ f_y = 2y - 2\cos x = 0 \end{cases}$$

$y = \cos x$

$2\cos x \sin x = 0$

$f_x = 0 \Rightarrow y=0$ or $x = 0, \pi, 2\pi$

If $y=0$, then

$f_y: \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

If $x = 0, 2\pi$, then

$f_y: \quad 2y-2=0 \Rightarrow y=1$

If $x=\pi$, then

$f_y: \quad 2y+2=0 \Rightarrow y=-1$

Critical points: $\left(\frac{\pi}{2},0\right), \left(\frac{3\pi}{2},0\right), (0,1), (2\pi,1), (\pi,-1)$

$f_{xx} = 2y\cos x$

$f_{yy} = 2 \cancel{+2\sin x}$

$f_{xy} = 2\sin x$

$D = 2y\cos x \cdot (2 \cancel{+2\sin x}) - 4\sin^2 x = 4(y\cos x - \sin^2 x)$

$D\left(\frac{\pi}{2},0\right) = -4 < 0$ $\qquad$ $f_{xx}(0,1) = 2 > 0$

$D\left(\frac{3\pi}{2},0\right) = -4 < 0$ $\qquad$ $f_{xx}(2\pi,1) = 2 > 0$

$D(0,1) = 4 > 0$ $\qquad$ $f_{xx}(\pi,-1) = 2 > 0$

$D(2\pi,1) = 4 > 0$

$D(\pi,-1) = 4 > 0$

Hence, $f(0,1)$, $f(2\pi,1)$, $f(\pi,-1)$ are loc. min and $f(0,1) = f(2\pi,1) = f(\pi,-1) = -1$.

Since $D(\frac{\pi}{2},0) < 0$ and $D(\frac{3\pi}{2},0) < 0$, then $(\frac{\pi}{2},0)$ and $(\frac{3\pi}{2},0)$ are saddle points.

▼

$\boxed{14.8}$

4. $f(x,y) = 3x + y$

$x^2 + y^2 = 10$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = c \end{cases}$$

$$\nabla f = \langle f_x, f_y \rangle = \langle 3, 1 \rangle$$

$$\nabla g = \langle g_x, g_y \rangle = \langle 2x, 2y \rangle$$

$$\begin{cases} 3 = \lambda \, 2x \\ 1 = \lambda \, 2y \\ x^2 + y^2 = 10 \end{cases}$$

$$x = \frac{3}{2\lambda} \Rightarrow x \neq 0$$

$$y = \frac{1}{2\lambda} \Rightarrow y \neq 0$$

$$\frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 10$$

$$\frac{10}{4\lambda^2} = 10 \implies 4\lambda^2 = 1$$

$$\lambda = \pm \frac{1}{2}$$

If $\lambda = \frac{1}{2}$: $x = 3$, $y = 1$ $(3,1)$

If $\lambda = -\frac{1}{2}$: $x = -3$, $y = -1$ $(-3,-1)$

Possible extreme values at points: $(3,1)$ and $(-3,-1)$.

$f(3,1) = 10$

$f(-3,-1) = -10$

Thus $f_{max} = 10$ on $x^2+y^2=10$

$f_{min} = -10$ on $x^2+y^2=10$.