

WRH-7-Solutions

15.1: 9, 16, 27

15.2: 7, 24

15.3: 10, 15

15.1

$$\textcircled{9} \iint_R \sqrt{z} \, dA = ?$$

$$R = \{(x, y) \mid 2 \leq x \leq 6, -1 \leq y \leq 5\}$$

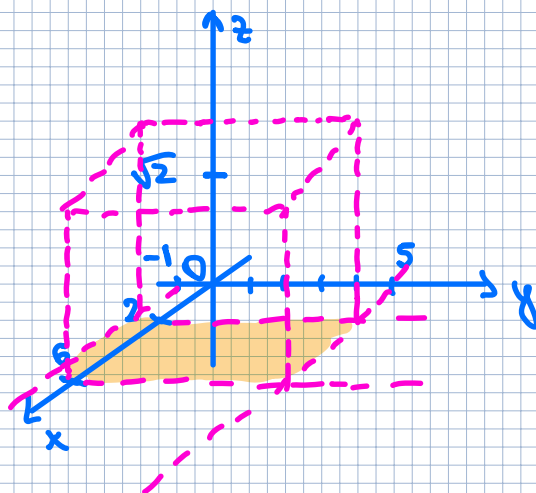
$$\iint_R \sqrt{z} \, dA = \int_2^6 \int_{-1}^5 \sqrt{z} \, dy \, dx = \sqrt{z} \int_2^6 6 \, dx = \sqrt{z} \cdot 6 \cdot 4 =$$

$$= \boxed{24\sqrt{z}}$$

As a volume of a solid:

$$z = \sqrt{z}$$

$$V = \sqrt{z} \cdot 6 \cdot 4 =$$
$$= 24\sqrt{z}.$$



16

$$\begin{aligned}
 \int_0^1 \int_0^1 (x+y)^2 dx dy &= \int_0^1 \int_0^1 (x^2 + 2xy + y^2) dx dy = \\
 &= \int_0^1 \left(\frac{x^3}{3} + x^2 y + x y^2 \right) \Big|_0^1 dy = \int_0^1 \left(\frac{1}{3} + y + y^2 \right) dy = \\
 &= \left(\frac{1}{3} y + \frac{y^2}{2} + \frac{y^3}{3} \right) \Big|_0^1 = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{2}{3} + \frac{1}{2} = \\
 &= \frac{4+3}{6} = \boxed{\frac{7}{6}}
 \end{aligned}$$

27

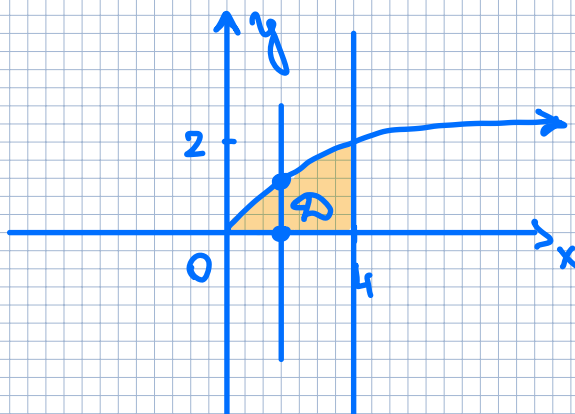
$$\iint_R x \sec^2 y dA, \quad R = \{(x,y) \mid 0 \leq x \leq 2, 0 \leq y \leq \frac{\pi}{4}\}$$

$$\begin{aligned}
 \int_0^2 \int_0^{\frac{\pi}{4}} x \sec^2 y dy dx &= \int_0^2 x dx \int_0^{\frac{\pi}{4}} \sec^2 y dy = \\
 &= \left(\frac{x^2}{2} \right) \Big|_0^2 + \tan y \Big|_0^{\frac{\pi}{4}} = 2 \cdot (1 - 0) = \boxed{2}
 \end{aligned}$$

15.2

7

$$\iint_D \frac{y}{x^2 + 1} dA, \quad D = \{(x,y) \mid 0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}\}$$



$$\begin{aligned}
 \iint_D \frac{y}{x^2+1} dA &= \int_0^4 \int_0^{\sqrt{x}} \frac{y}{x^2+1} dy dx = \\
 &= \int_0^4 \frac{1}{x^2+1} \left. \frac{y^2}{2} \right|_0^{\sqrt{x}} dx = \int_0^4 \frac{1}{x^2+1} \frac{x}{2} dx = \\
 &= \frac{1}{2} \int_0^4 \frac{x}{x^2+1} dx = \frac{1}{4} \int_0^4 \frac{d(x^2+1)}{x^2+1} = \\
 &= \frac{1}{4} \ln|x^2+1| \Big|_0^4 = \frac{1}{4} (\ln 17 - 0) = \boxed{\frac{1}{4} \ln 17}
 \end{aligned}$$

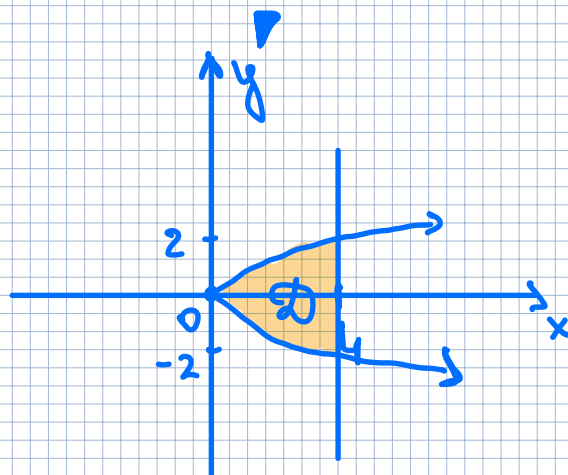
(24)

$$z = 1 + x^2 y^2$$

$$D: x = y^2, x = 4$$

V = ?

$$V = \iint_D z(x,y) dA$$



$$\begin{aligned}
 V &= \int_{-2}^2 \int_{y^2}^4 (1+x^2y^2) dx dy = \int_{-2}^2 \left(x + \frac{x^3 y^2}{3} \right) \Big|_{y^2}^4 dy = \\
 &= \int_{-2}^2 \left(4 + \frac{61}{3} y^2 - \frac{1}{3} y^8 \right) dy = \left(4y + \frac{61}{9} y^3 - \right. \\
 &\quad \left. - \frac{1}{27} y^9 \right) \Big|_{-2}^2 = 8 + \frac{488}{9} - \frac{512}{27} + 8 + \frac{488}{9} - \frac{512}{27} \\
 &= \boxed{\frac{2336}{27}}
 \end{aligned}$$

15.3

(10) $\iint_R \frac{y^2}{x^2+y^2} dA,$

$R: \begin{cases} x^2+y^2 = a^2 \\ x^2+y^2 = b^2 \end{cases}, \quad 0 < a < b$

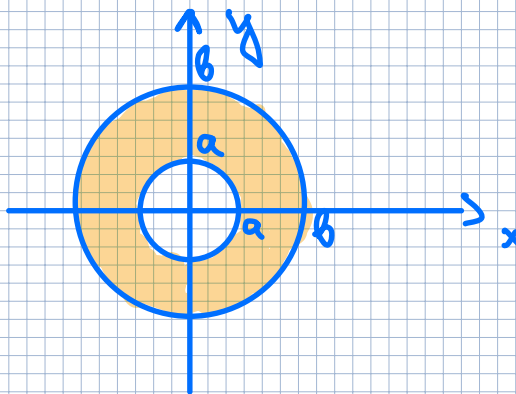
$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$x^2 + y^2 = r^2 = a^2$$

$$x^2 + y^2 = r^2 = b^2$$

$$a \leq r \leq b$$

$$0 \leq \theta \leq 2\pi$$



$$dA = r dr d\theta$$

$$\iint_R \frac{y^2}{x^2 + y^2} dA = \int_a^b \int_0^{2\pi} \frac{r^2 \sin^2 \theta}{r^2} r dr d\theta =$$

$$= \int_a^b r dr \int_0^{2\pi} \frac{1}{2} (1 - \cos 2\theta) d\theta = \frac{r^2}{2} \Big|_a^b \left(\theta - \frac{\sin 2\theta}{2} \right) \Big|_0^{2\pi}$$

$$= \frac{1}{2} (b^2 - a^2) \cdot \frac{1}{2} 2\pi = \boxed{\frac{\pi}{2} (b^2 - a^2)}$$

(15)

$$r = \cos 3\theta$$

$$\theta = 0$$

$$r = 1$$

$$\theta = \frac{\pi}{4}$$

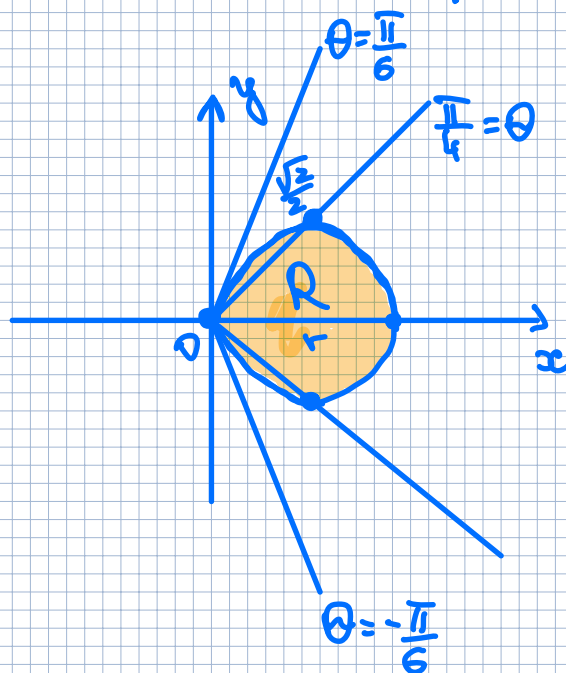
$$r = \cos \frac{3\pi}{4}$$

$$\theta = \frac{\pi}{2}$$

$$r = 0$$

$$\theta = \pi$$

$$r = 1$$



$$\iint_R dA = \int_{-\pi/6}^{\pi/6} \int_0^{\cos 3\theta} r dr d\theta = \frac{1}{2} \int_{-\pi/6}^{\pi/6} r^2 \Big|_0^{\cos 3\theta} d\theta =$$

$$= \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2 3\theta \, d\theta = \frac{1}{4} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1 + \cos 6\theta) \, d\theta =$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} (1 + \cos 6\theta) \, d\theta = \frac{1}{2} \left(\theta + \frac{1}{6} \sin 6\theta \right) \Big|_0^{\frac{\pi}{6}} =$$

$$= \boxed{\frac{\pi}{12}}$$

