WRH-5-Solutions

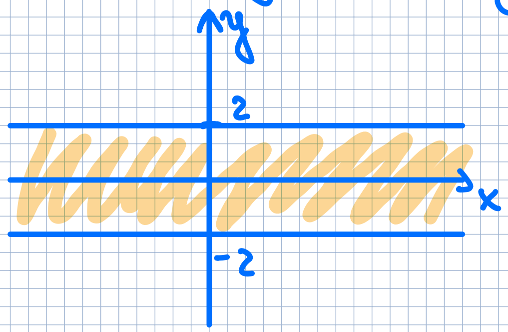
14.1 : 10, 45
14.2 : 6, 25, 39
14.3 : 15, 34, 47, 53, 67
14.4 : 1, 15, 25

$\boxed{14.1}$

(10) $F(x,y) = 1 + \sqrt{4-y^2}$

(a) $F(3,1) = 1 + \sqrt{4-1} = 1+\sqrt{3}$

(b) $\text{dom}(F) = \{(x,y) \mid x \in \mathbb{R},\ y \in [-2,2]\}$



(c) We know that

$$0 \leq \sqrt{4-y^2} \leq 2$$

So

$$1 \leq 1+\sqrt{4-y^2} \leq 3$$

Hence, $F \in [1,3]$.

(45) $f(x,y) = x^2 - y^2$

$$z = x^2 - y^2$$

$x^2 - y^2 = 0$

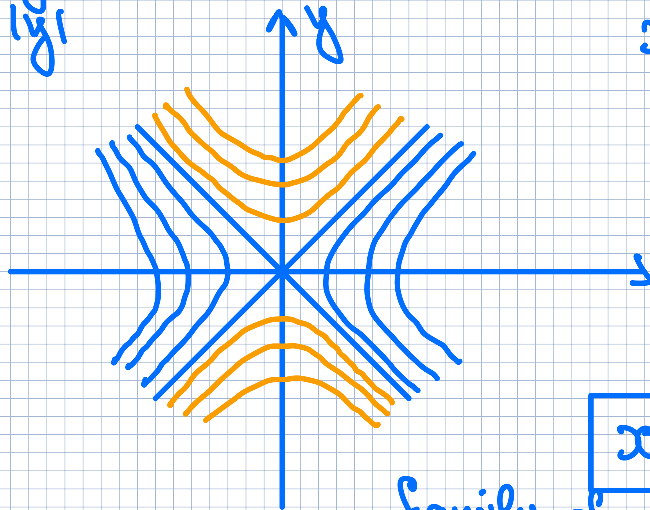
$x^2 = y^2$

$|x| = |y|$

$x^2 - y^2 = 1$

$y^2 = x^2 - 1$

$x^2 - y^2 = -1$



$\boxed{x^2 - y^2 = k}$

family of hyperbola's

14.2

(6) $\displaystyle\lim_{(x,y)\to(2,-1)} \frac{x^2y + xy^2}{x^2 - y^2} = \lim_{(x,y)\to(2,-1)} \frac{-4+2}{4-1} = \boxed{\frac{-2}{3}}$

We have that $(2,-1) \in \mathrm{Dom}(f)$.

(25) $g(t) = t^2 + \sqrt{t}$

$f(x,y) = 2x + 3y - 6$

$h(x,y) = g(f(x,y)) = g(2x+3y-6) =$

$= (2x+3y-6)^2 + \sqrt{2x+3y-6}$

f is continuous on $\mathbb{R}^2$

g is continuous on $\{t \mid t \geq 0\}$.

Hence, h is continuous on its domain

$$\{(x,y) \mid 2x+3y-6 \geq 0\} = \{(x,y) \mid y \geq 2 - \tfrac{2}{3}x\}$$

which consists of all points on or above the line $y = -\frac{2}{3}x + 2$.

(39) $$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = \lim_{r\to 0^+} \frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2} =$$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$= \lim_{r\to 0^+} r\left(\cos^3\theta + \sin^3\theta\right) = \boxed{0.}$$

14.3

(15) $f(x,y) = x^4 + 5xy^3$

$f_x = 4x^3 + 5y^3$

$f_y = 15xy^2$

(34) $w = y\tan(x+2z)$

$w_x = y \sec^2(x+2z)$

$w_y = \tan(x+2z)$

$w_z = 2y \sec^2(x+2z)$

(47)

$x^2 + 2y^2 + 3z^2 = 1$

$2x + 6z \frac{\partial z}{\partial x} = 0$

$\frac{\partial z}{\partial x} = -\frac{2x}{6z} = \boxed{-\frac{x}{3z}}$

$4y + 6z \frac{\partial z}{\partial y} = 0$

$\frac{\partial z}{\partial y} = -\frac{4y}{6z} = \boxed{-\frac{2y}{3z}}$

(53)

$f(x,y) = x^4 y - 2x^3 y^2$

$f_x = 4x^3 y - 6x^2 y^2$

$f_y = x^4 - 4x^3 y$

$f_{xx} = 12x^2 y - 12x y^2$

$f_{xy} = 4x^3 - 12x^2 y$

$f_{yx} = 4x^3 - 12x^2 y$

$f_{yy} = -4x^3$

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$W = \sqrt{u+v^2}$

$\frac{\partial W}{\partial u} = \frac{1}{2\sqrt{u+v^2}}$

$\frac{\partial^2 W}{\partial u^2} = \frac{1}{2}\left(-\frac{1}{2}\right)(u+v^2)^{-3/2} = -\frac{1}{4}(u+v^2)^{-3/2}$

$\frac{\partial^3 W}{\partial u^2 \partial v} = -\frac{1}{4}\cdot\left(-\frac{3}{\cancel{2}}\right)(u+v^2)^{-5/2}\cdot \cancel{2}v =$

$= \frac{3}{4} v (u+v^2)^{-5/2}$

14.4

1

$z = 2x^2 + y^2 - 5y$
$P(1, 2, -4)$

$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$z_0 = -4$

$f_x = 4x$ $\quad$ $f_x(x_0,y_0) = 4$

$f_y = 2y-5$ $\quad$ $f_y(x_0,y_0) = -1$

Hence, $z+4 = 4(x-1) + (-1)(y-2)$

$$z+4 = 4x - 4 - y + 2$$

$$\boxed{z = 4x - y - 6}$$

(15) $f(x,y) = 4\arctan(xy), \quad (1,1)$

$$f_x(x,y) = \frac{4y}{1+x^2y^2}$$

$$f_y(x,y) = \frac{4x}{1+x^2y^2}$$

$$f_x(1,1) = \frac{4}{2} = 2$$

$$f_y(1,1) = \frac{4}{2} = 2$$

f_x and f_y are continuous at $(1,1)$.
So f is differ. at $(1,1)$.

$$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) =$$

$$= 4 \cdot \frac{\pi}{4} + 2(x-1) + 2(y-1) =$$

$$= \boxed{\pi + 2x + 2y - 4}$$

(25) $z = e^{-2x} \cos 2\pi t$

$$\frac{dz}{dt} = e^{-2x} \cdot (-2) \frac{dx}{dt} \cos 2\pi t -$$

$$- e^{-2x} \sin 2\pi t \cdot (2\pi)$$

$$\boxed{dz = -2e^{-2x} \cos 2\pi t \; dx - 2\pi e^{-2x} \sin 2\pi t \; dt}$$