Formulas & Definitions: Section 16-3

Theorem 1: Let C be a smooth curve given by the vector function $r(t)$, $a \leq t \leq b$. Let f be a differentiable function of two or three variables whose gradient vector ∇f is continuous on C. Then

$$\int_C \nabla f \cdot dr = f(r(b)) - f(r(a))$$

Definition: If F is a continuous vector field with domain D, we say that the line integral

$$\int_C F \cdot dr$$

is independent of path if

$$\int_{C_1} F \cdot dr = \int_{C_2} F \cdot dr$$

for any two paths C_1 and C_2 in D that have the same initial points and the same terminal points.

Definition: A curve is called **closed** if its terminal point coincides with its initial point, that is, $r(b) = r(a)$.

Theorem 2:

$$\int_C F \cdot dr$$

is independent of path in D if and only if

$$\int_C F \cdot dr = 0$$

for every closed path C in D.

Definition:

- The region D is open if for every point P in D there is a disk with center P that lies entirely in D.
- The region D is connected if any two points in D can be joined by a path that lies in D.

Theorem 3: Suppose F is a vector field that is continuous on an open connected region D. If $\int_C F \cdot dr$ is independent of path in D, then F is a conservative vector field on D; that is, there exists a function f such that $\nabla f = F$.

Theorem 4: If $F(x, y) = P(x, y)i + Q(x, y)j$ is a conservative vector field, where P and Q have continuous first-order partial derivatives on a domain D, then throughout D we have

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Definition:

- A curve C is a simple curve if it does not intersect itself anywhere between its endpoints.
- A simply-connected region in the plane is a connected region D such that every simple closed curve in D encloses only points that are in D.

Theorem 5: Let $F = Pi + Qj$ be a vector field on an open simply-connected region D. Suppose that P and Q have continuous first-order partial derivatives and

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

throughout D. Then F is conservative.

Conservation of Energy: If an object moves from one point A to another point B under the influence of a conservative force field, then the sum of its potential energy and its kinetic energy remains constant, that is

$$P(A) + K(A) = P(B) + K(B).$$