Formulas & Definitions: Section 16-4

Definition: A simple closed curve C has a positive orientation if it refers to a single *counter-clockwise* traversal of C.

Green's Theorem: Let C be positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

Particular cases:

$$P(x,y) = 0, \quad P(x,y) = -y, \quad P(x,y) = -\frac{1}{2}x$$

$$Q(x,y) = x, \quad Q(x,y) = 0, \quad Q(x,y) = \frac{1}{2}x$$

Then Green's Theorem gives the following formulas for the area of D:

$$A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C x\,dy - y\,dx$$