Formulas & Definitions: Section 16-2

Definition: If f is defined on a smooth curve C given by

$$x = x(t), \quad y = y(t), \quad a \le t \le b,$$

then the line integral of f along C is

$$\int_C f(x, y)\, ds = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*, y_i^*)\, \Delta s_i$$

if this limit exists.

Formula: The following formula can be used to evaluate the line integral:

$$\int_C f(x, y)\, ds = \int_a^b f(x(t), y(t)) \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$$

Formula: The following formulas say that line integrals with respect to x and y can also be evaluated by expressing everything in terms of t: $x = x(t)$, $y = y(t)$, $dx = x'(t)dt$, $dy = y'(t)dt$.

$$\int_C f(x, y)\, dx = \int_a^b f(x(t), y(t))\, x'(t)\, dt,$$

$$\int_C f(x, y)\, dx = \int_a^b f(x(t), y(t))\, y'(t)\, dt$$

Definition: A vector representation of the line segment that starts at r_0 and ends at r_1 is given by

$$r(t) = (1 - t)r_0 + tr_1, \quad 0 \le t \le 1.$$

Line Integrals in Space: We evaluate the line integral in $\mathbb{R}^3$ by using the following formula:

$$\int_C f(x, y, z)\, ds = \int_a^b f(x(t), y(t), z(t)) \sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2}\, dt$$

Definition: Let F be a continuous vector field defined on a smooth curve C given by a vector function $r(t)$, $a \le t \le b$. Then the **line integral of F along C** is

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt = \int_C F \cdot T\, ds,$$

where $T(x, y, z)$ is a unit tangent vector at the point (x, y, z) on C.

Also,

$$\int_C F \cdot dr = \int_C P\, dx + Q\, dy + R\, dz,$$

where $F = Pi + Qj + Rk$.