# WRH-10-Solutions

15.9: 1, 15
16.1: 3, 25
16.2: 4, 19
16.3: 5, 13

## 15.9

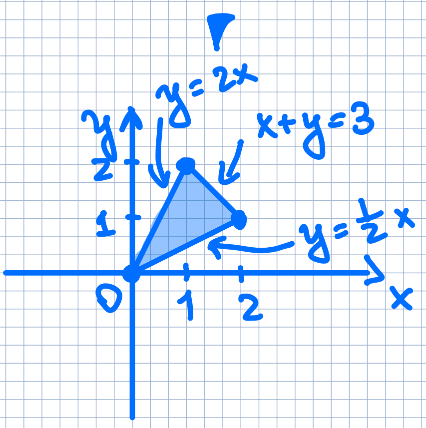
**1**

$x = 2u + v$
$y = 4u - v$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -2-4 = \boxed{-6}$$

**15**

$$\iint_R (x-3y)\, dA$$




R: $(0,0), (2,1), (1,2)$

$$\begin{cases} x = 2u + v \\ y = u + 2v \end{cases}$$

$$x - 3y = 2u + v - 3(u + 2v) = \boxed{-u - 5v}$$

$$J = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$y = \frac{1}{2}x$: $\quad u + 2v = u + \frac{1}{2}v \Rightarrow v = 0$

$x + y = 3$: $\quad 2u + v + u + 2v = 3 \Rightarrow u + v = 1$

$y = 2x$: $\quad u + 2v = 4u + 2v \Rightarrow u = 0$

Thus $\quad 0 \le u \le 1$

$\quad\quad 0 \le v \le 1-u$

$$\iint_R (x - 3y)\, dA = \int_0^1 \int_0^{1-u} (-u - 5v)|3|\, dv\, du =$$

$$= -3 \int_0^1 \left(uv + \frac{5}{2}v^2\right)\Big|_0^{1-u} du =$$

$$= -3 \int_0^1 \left(u - u^2 + \frac{5}{2}(1-u)^2\right) du = \boxed{-3}$$

**16.1**

(3) $\quad F(x,y) = -\frac{1}{2}i + (y-x)j$

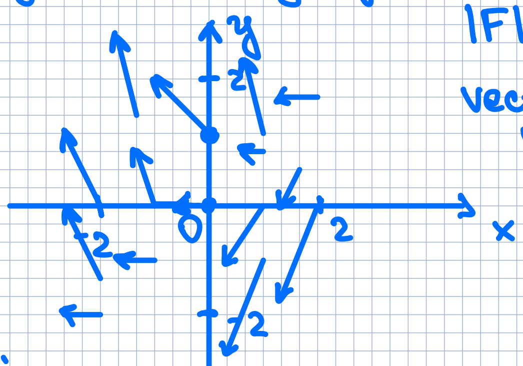
$F(0,0) = -\frac{1}{2}i$

$F(0,1) = -\frac{1}{2}i + j$

$F(1,0) = -\frac{1}{2}i - j$

$F(2,2) = -\frac{1}{2}i$

$F(-1,0) = -\frac{1}{2}i + j$




$|F| = \sqrt{1/4 + (y-x)^2}$

vectors along $y = x$: horizontal with length $\frac{1}{2}$

(25) $\quad f(x,y) = \frac{1}{2}(x-y)^2$

$\nabla f = \langle f_x, f_y \rangle$

$f_x = x - y$

$f_y = -(x-y) = -x+y = y-x$

$\nabla f = \langle x-y, y-x \rangle$

$(1,1): \nabla f = \langle 0,0 \rangle$

$(1,0): \nabla f = \langle 1,-1 \rangle$

$(0,1): \nabla f = \langle -1,1 \rangle$

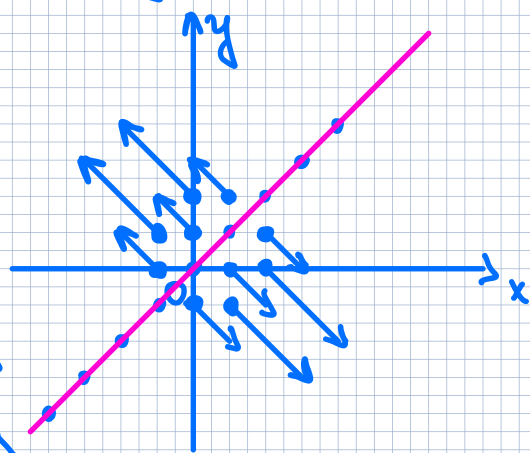
$(-1,0): \nabla f = \langle -1,1 \rangle$

$(0,-1): \nabla f = \langle 1,-1 \rangle$

$(2,1): \nabla f = \langle 1,-1 \rangle$

$(1,2): \nabla f = \langle -1,1 \rangle$

$(2,0): \nabla f = \langle 2,-2 \rangle$



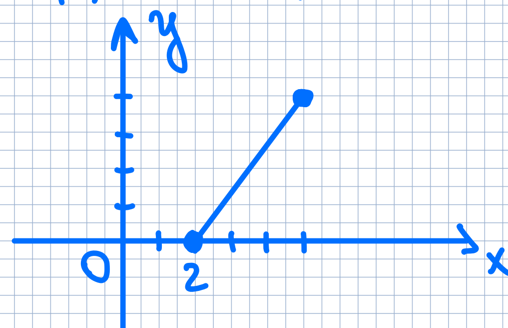

$|\nabla f| = \sqrt{2}\,|x-y|$

$\nabla f = \langle 0,0 \rangle$ along $y=x$

$\boxed{16.2}$

4. $\int_C x e^y \, ds$ , $C : (2,0)$ to $(5,4)$

$\frac{x-2}{3} = \frac{y-0}{4} = t$



$$\begin{cases} x = 3t+2 \\ y = 4t \end{cases} \quad 0 \leq t \leq 1$$

$$\int_C x e^y \, ds = \int_0^1 (3t+2)e^{4t} \cdot \sqrt{3^2+4^2}\, dt =$$

$$= 5\int_0^1 (3t+2)e^{4t}\, dt = 5\left(\frac{1}{4}(2+3t)e^{4t} - \frac{3}{16}e^{4t}\right)\Bigg|_0^1 =$$

$$= \boxed{\frac{85}{16}e^4 - \frac{25}{16}}$$

19

$$F(x,y) = xy^2 i - x^2 j$$

$$r(t) = t^3 i + t^2 j, \quad 0 \leq t \leq 1$$

$$F(r(t)) = t^3(t^2)^2 i - (t^3)^2 j = t^7 i - t^6 j$$

$$r'(t) = 3t^2 i + 2t j$$

$$\int_C F \cdot dr = \int_0^1 F(r(t)) \cdot r'(t)\, dt = \int_0^1 (t^7 \cdot 3t^2 - t^6 \cdot 2t)\, dt =$$

$$= \int_0^1 (3t^9 - 2t^7)\, dt = \left(\frac{3}{10}t^{10} - \frac{1}{4}t^8\right)\Bigg|_0^1 = \boxed{\frac{1}{20}}$$

$\boxed{16.3}$

5 $F(x,y) = y^2 e^{xy} i + (1+xy) e^{xy} j$

$\frac{\partial}{\partial y}\left(y^2 e^{xy}\right) = 2y e^{xy} + y^2 \cdot x e^{xy} = (2y + xy^2) e^{xy}$

$\frac{\partial}{\partial x}\left((1+xy) e^{xy}\right) = y e^{xy} + y e^{xy} + xy \cdot y e^{xy} =$

$= (2y + xy^2) e^{xy}$

$\frac{\partial x}{\partial y} = \frac{\partial y}{\partial x}$ and the $\mathrm{Dom}(F) = \mathbb{R}^2$ is open and simply-connected.

Hence, $F$ is conservative.

Thus, there $\exists$ $f$: $\nabla f = F$.

$f_x(x,y) = y^2 e^{xy}$ (1)

$f_y(x,y) = (1+xy) e^{xy}$ (2)

From (1): $f(x,y) = y e^{xy} + g(y)$

$f_y(x,y) = (1+xy) e^{xy} + g'(y)$

Thus

$(1+xy) e^{xy} = (1+xy) e^{xy} + g'(y)$

$g'(y) = 0 \Rightarrow g(y) = \text{const}$

Hence, $\boxed{f(x,y) = y e^{xy} + \text{const}}$ is a potential function for F

(13) $F(x,y) = x^2y^3\, i + x^3y^2\, j$

C: $r(t) = \langle t^3 - 2t,\ t^3 + 2t \rangle, \quad 0 \le t \le 1$

(a) If $F = \nabla f$ : $f_x(x,y) = x^2y^3 \quad (1)$

$f_y(x,y) = x^3y^2 \quad (2)$

$(1) \Rightarrow f(x,y) = \frac{1}{3}x^3y^3 + g(y)$

$f_y(x,y) = x^3y^2 + g'(y)$

But $f_y(x,y) = x^3y^2$. So $g'(y) = 0 \Rightarrow g(y) = \text{const}$

We can take const = 0.

Hence, $\boxed{f(x,y) = \frac{1}{3}x^3y^3.}$

(b) C is a smooth curve

$r(0) = (0,0)$

$r(1) = (-1,3)$

$$\int_C F \cdot dr = \int_C \nabla f \cdot dr = f(-1,3) - f(0,0) = -9 - 0 = \boxed{-9}$$