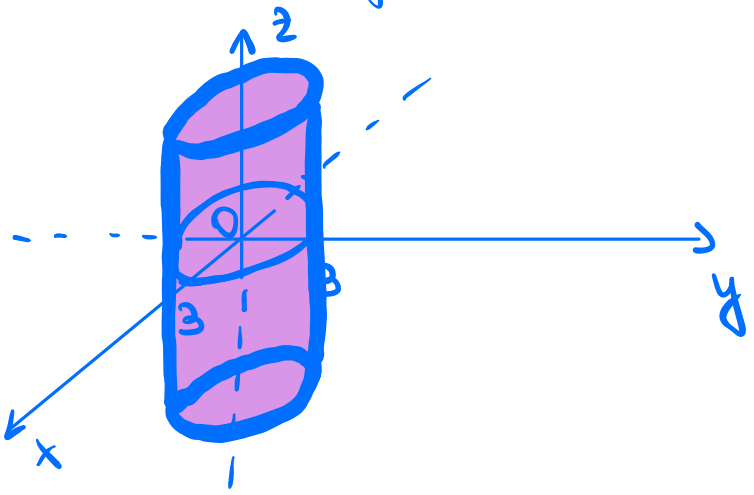
Logistics

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz will be available starting from **5:00 PM until midnight** on scheduled week day (Thursday).
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[5 points]** Sketch the region in $\mathbb{R}^3$ represented by the inequality

$$x^2 + y^2 \leq 9 = 3^2$$

$x^2+y^2=9$ is a cylinder with radius 3.
$z=k\in\mathbb{R}$ is a scalar
$x^2+y^2\leq 9$ is the inside part of a cylinder + the boundary



2. **[5 points]** Find a unit vector that has the same direction as the given vector $-3i + 6j + 2k$.

$$v = -3i+6j+2k = \langle -3, 6, 2\rangle$$

Then $|v| = \sqrt{9+36+4} = \sqrt{49} = 7$

$$u = \frac{v}{|v|} = \frac{-3i+6j+2k}{7} = -\frac{3}{7}i + \frac{6}{7}j + \frac{2}{7}k =$$

$$= \boxed{\left\langle -\frac{3}{7}, \frac{6}{7}, \frac{2}{7}\right\rangle}.$$

3. **[5 points]**

   (a) Determine whether the given vectors $\langle -5, 4, -2\rangle$ and $\langle 3, 4, -1\rangle$ are orthogonal. *(Fully justify your answer)*

   (b) For vectors $a = 2j - 4k$ and $b = -i + 3j + k$ find the cross product $a \times b$.

(a) $u = \langle -5, 4, -2\rangle$

$v = \langle 3, 4, -1\rangle$

$u \perp v$ if and only if $u \cdot v = 0$

$u \cdot v = -5 \cdot 3 + 4 \cdot 4 + (-2)\cdot(-1) = -15 + 16 + 2 = 18 - 15 = 3$

Hence, u and v are not orthogonal.

(b) $a \times b = \begin{vmatrix} i & j & k \\ 0 & 2 & -4 \\ -1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 0 & -4 \\ -1 & 1 \end{vmatrix} j + \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} k =$

$= 14i + 4j + 2k = \langle 14, 4, 2\rangle$

4. **[5 points]**

   (a) Find an equation of the plane that goes through the point $(5, 3, 5)$ and has a normal vector $i + 4j + k$.

   (b) Use intercepts to sketch the plane from part (a).

(a) In general form for the plane that goes through $P(x_0, y_0, z_0)$ and is $\perp$ to $v = \langle a, b, c\rangle$ has an equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Thus, $v = \langle 1, 4, 1\rangle$

$P(3, 5, 3)$

$1(x - 3) + 4(y - 5) + 1\cdot(z - 3) = 0$

$x - 3 + 4y - 20 + z - 3 = 0$

$$\boxed{x + 4y + z - 26 = 0}$$

(b)

$x = 0$
$y = 0$ $\Rightarrow$ $z = 26$

$y = 0$
$z = 0$ $\Rightarrow$ $x = 26$

$x = 0$
$z = 0$ $\Rightarrow$ $y = \frac{26}{4} = \frac{13}{2} = 6.5$