Formulas & Definitions: Section 15-1

Definition 1. The **double integral** of f over the rectangle R is

$$\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

if this limit exists.

Statement 1. If $f(x,y) \geq 0$, then the volume V of the solid that lies above the rectangle R and below the surface $z = f(x,y)$ is

$$V = \iint_R f(x,y)\,dA.$$

Midpoint Rule for Double Integrals.

$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(\bar{x}_i, \bar{y}_j)\Delta A$$

where $\bar{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and $\bar{y}_j$ is the midpoint of $[y_{j-1}, y_j]$.

Fubini's Theorem. If f is continuous on the rectangle $R = \{(x,y)\,|\,a \leq x \leq b, c \leq y \leq d\}$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

More generally, this is true if we assume that f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

Statement 2. The double integral of f can be written as the product of two single integrals:

$$\iint_R g(x)h(y)\,dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$

where $R = [a,b] \times [c,d]$.

Formula. The **Average value** of a function f of two variables defined on a rectangle R is

$$f_{ave} = \frac{1}{A(R)} \iint_R f(x,y)\,dA$$

where $A(R)$ is the area of R.