Formulas & Definitions: Section 14-6

Definition: The **directional derivative** of f at (x_0, y_0) in the direction of a unit vector $u = \langle a, b \rangle$ is

$$D_u f(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb) - f(x_0, y_0)}{h}$$

if this limit exists.

Theorem: If f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $u = \langle a, b \rangle$ and

$$D_u f(x, y) = f_x(x, y)a + f_y(x, y)b.$$

Definition: If f is a function of two variables x and y, then the **gradient** of f is the vector function ∇f defined by

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j.$$

We also have that

$$\boxed{\mathrm{D}_u f(x, y) = \nabla f(x, y) \cdot u}$$

Definition: The **directional derivative** of f at (x_0, y_0, z_0) in the direction of a unit vector $u = \langle a, b, c \rangle$ is

$$D_u f(x_0, y_0, z_0) = \lim_{h \to 0} \frac{f(x_0 + ha, y_0 + hb, z_0 + hc) - f(x_0, y_0, z_0)}{h}$$

if this limit exists.

We also have

$$\boxed{\nabla f(x, y, z) = \langle f_x, f_y, f_z \rangle = \tfrac{\partial f}{\partial x} i + \tfrac{\partial f}{\partial y} j + \tfrac{\partial f}{\partial z} k}$$

and

$$\boxed{\mathrm{D}_u f(x, y, z) = \nabla f(x, y, z) \cdot u}$$

Theorem: Suppose f is a differentiable function of two or three variables. The maximum value of the directional derivative $D_u f(x)$ is $|\nabla f(x)|$ and it occurs when u has the same direction as the gradient vector $\nabla f(x)$.

Tangent plane equation to the level surface:

$$\boxed{\mathrm{F}_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0 z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0}$$