MATH F253X: Quiz 4: Sections 14.1-14.6 **Solutions** June 23, Summer 2022

Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[5 points]** Find and sketch the domain of the following function

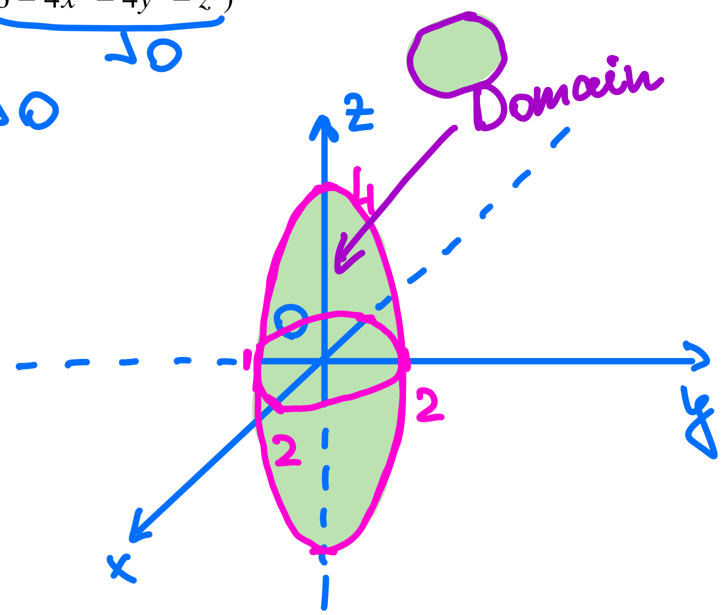
$$f(x, y, z) = \ln(\underbrace{16 - 4x^2 - 4y^2 - z^2}_{>0})$$

Domain: $16 - 4x^2 - 4y^2 - z^2 > 0$

$-4x^2 - 4y^2 - z^2 > -16$

$x^2 + y^2 + \frac{z^2}{4} < 4$

$\underbrace{\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} < 1}_{\text{ellipsoid (inside)}}$



2. **[5 points]** Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + 3y^2}{3x^2 + y^2}.$$

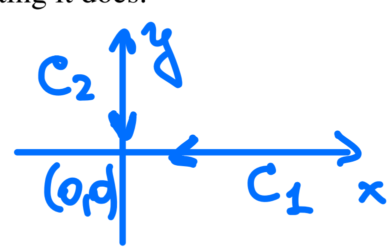
Either show it does not exist, or give strong evidence for suspecting it does.

C_1 path: $y = 0$

$\lim_{x \to 0} \frac{x^2}{3x^2} = \frac{1}{3}$

C_2 path: $x = 0$

$\lim_{y \to 0} \frac{3y^2}{y^2} = 3$



Thus, the limit DNE. We have two paths to reach (0,0) and two different limit values.

3. **[5 points]** Give an equation for the **linear (tangent plane) approximation** to $f(x, y) = e^{x-y}$ at the point $(2, 2)$, and use it to estimate $f(2.1, 2.2)$.

$L(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$f_x = e^{x-y}$ $\quad f(2,2) = 1$

$f_y = -e^{x-y}$

$f_x(2,2) = 1$

$f_y(2,2) = -1$

Hence, $L(x,y) \approx 1 + 1\cdot(x-2) - 1(y-2) = 1 + x - \cancel{2} + \cancel{2} - y =$

$= 1 + x - y$

$f(2.1, 2.2) = L(2.1, 2.2) \approx 1 + 2.1 - 2.2 = 0.9$

4. **[5 points]** Use the **Chain Rule** to compute $\frac{dh}{dt}(0)$, where

$$h(t) = f(\underbrace{t^2 + t - 3}_{x}, \underbrace{-2e^{5t} + 1}_{y}) \quad \text{and} \quad f(x, y) = x^2y + 3xy^4.$$

$\frac{dh}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$

$\frac{df}{dx} = 2xy + 3y^4$ $\quad \frac{dx}{dt} = 2t + 1$

$\frac{df}{dy} = x^2 + 12xy^3$ $\quad \frac{dy}{dt} = -10e^{5t}$

$\frac{dh}{dt} = (2xy + 3y^4)(2t+1) + (x^2 + 12xy^3)(-10e^{5t}) =$

$= \left(2(t^2+t-3)(-2e^{5t}+1) + 3(-2e^{5t}+1)^4\right)(2t+1) +$

$+ \left((t^2+t-3)^2 + 12(t^2+t-3)(-2e^{5t}+1)^3\right)(-10e^{5t})$

$\frac{dh}{dt}(0) = (-6\cdot(-1) + 3\cdot 1)\cdot 1 + (9 + 12\cdot(-3)(-1))(-10) = 9 - 450 = -441$