Formulas & Definitions: Section 14-3

Definition:

- The partial derivative of f with respect to x at (a,b) and denote it by $f_x(a,b)$ is
$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}.$$
- The partial derivative of f with respect to y at (a,b) and denote it by $f_y(a,b)$ is
$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}.$$

Definition: If f is a function of two variables, its **partial derivatives** are the functions f_x and f_y defined by

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}, \quad f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}.$$

Notations for partial derivatives: If $z = f(x,y)$, we write

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f,$$
$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f.$$

Rule for finding partial derivatives of $z = f(x,y)$:

1. To find f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.
2. To find f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.

Second Partial Derivatives: If $z = f(x,y)$, then

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}, \quad f_{yy} = \frac{\partial^2 f}{\partial y^2},$$
$$f_{xy} = \frac{\partial^2 f}{\partial x \partial y}, \quad f_{yx} = \frac{\partial^2 f}{\partial y \partial x}.$$

Clairaut's Theorem: Suppose f is defined on a disk D that contains the point (a,b). If functions f_{xy} and f_{yx} are both continuous on D, then

$$\boxed{\mathrm{f}_{xy}(a,b) = f_{yx}(a,b)}$$

Partial Differential Equations:

- Laplace's equation
$$u_{xx} + u_{yy} = 0.$$
- Wave equation
$$u_{tt} = a^2 u_{xx}.$$