Name:

Instructions. (100 points) You have two hours. The exam is closed book, closed notes, and only simple calculators are allowed. Show all your work in order to receive full credit.

1. **[8 points]** Consider the points $A = (1, 0, -1)$, $B = (-2, 1, 3)$ and $C = (-1, 1, 0)$.

   (a) (3 pts) Give a parameterization of the straight line segment from B to C. **Be sure to state what the parameter may range over**.

   (b) (5pts) Find an equation (not a parameterization) for the plane containing points A, B, C.

2. **[8 points]** Find the region of integration

$$\int_0^4 \int_{\sqrt{y}}^2 e^{(x^3+1)} dx\, dy.$$

   Then use your sketch to reverse the order of integration and evaluate the integral.

3. **[11 points]** Assume a particle has velocity $v(t) = (t^2 + 1)\,\mathbf{i} + 2e^t\,\mathbf{j} + (1 - t)\,\mathbf{k}$, $t \geq 1$ with speed measured in m/s.

   (a) (3pts) Find acceleration of the particle at $t = 2$.

   (b) (4pts) Set the formula for the distance traveled from $t = 1$ s to $t = 3$ s. (DO NOT EVALUATE)

   (c) (4pts) Find the position vector $r(t)$ at all times if $r(1) = \mathbf{i} - 2\mathbf{k}$.

4. **[8 points]** Use Lagrange multipliers to find the maximum product of two positive numbers satisfying $x^2 + y = 4$.

5. **[12 points]** Compute the surface integral

$$\iint_S x^2 \, dS,$$

where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

6. **[8 points]** Sketch the two surfaces

$$x^2 + y^2 = 9, \quad y + z = 4.$$

Highlight their curve of intersection. Give a parameterization of that curve.

7. **[10 points]** Find all critical points of the function

$$f(x, y) = x^2 - 4xy + 6y^2$$

and, to the extent possible, determine whether they are local maxima, local minima, or saddle points.

8. **[9 points]** Use cylindrical coordinates to set the formula for the volume of the solid E that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$. (DO NOT EVALUATE)

9. **[8 points]** Find a parametric representation for the cylinder

$$x^2 + y^2 = 16, \quad 0 \leq z \leq 1.$$

10. **[8 points]** Consider the force field

$$F(x, y) = \langle x^2, xy \rangle$$

(a) (5pts) Find a potential function for $F(x, y)$.

(b) (3pts) Find the work done by the force field $F(x, y)$ on a particle that moves once around the circle $x^2 + y^2 = 4$ oriented in the counterclockwise direction.

11. **[10 points]** Use Green's Theorem to evaluate the line integral

$$\int_C ye^x \, dx + 2e^x \, dy$$

along the positively oriented curve C, where C is the triangle with vertices $(0, 0)$, $(3, 0)$, and $(0, 3)$.

12. **[Extra Credit, 8 points]** Let $f(x, y) = \dfrac{y}{x^2} + y^2 x$.

    (a) (4pts) Find the directional derivative of f at $(1, 2)$ when moving towards $(1, 0)$? What does this mean for function values?

    (b) (4pts) Let $x(s, t) = ts^2$ and $y(s, t) = 4t - s$. Use the appropriate chain rule to find $\dfrac{\partial f}{\partial t}$. Your final answer should only contain s and t, but DO NOT simplify.

Formulas:

- Surface integral formula

$$\iint_S f(x, y, z)\, dS = \iint_D f(r(u, v))\, |r_u \times r_v|\, dA$$

- The work done by a force field on a particle formula

$$W = \int_C F \cdot dr$$