Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[5 points]** For the given function $f(x, y) = x^2 \ln(y)$, the point $P(3, 1)$, and the unit vector $u = (-5/13)\mathbf{i} + (12/13)\mathbf{j}$ find:

   (a) the gradient of f

   $f_x = 2x \ln y$

   $f_y = \frac{x^2}{y}$

   $\nabla f = \langle f_x, f_y \rangle$

   $\nabla f = \langle 2x \ln y, \frac{x^2}{y} \rangle$

   (b) evaluate the gradient at the point P

   $\nabla f(3,1) = \boxed{\langle 0, 9 \rangle}$

   (c) find the rate of change of f at P in the direction of the vector u

   $\nabla f \cdot u = \langle 0, 9 \rangle \cdot \langle -\frac{5}{13}, \frac{12}{13} \rangle =$

   $= 0 \cdot \left(-\frac{5}{13}\right) + 9 \cdot \frac{12}{13} = \boxed{\frac{108}{13}}$

2. **[5 points]** Find and classify (**using the Second Derivatives Test**) all critical points of

$$f(x, y) = y(e^x - 1)$$

$$\begin{cases} f_x = ye^x = 0 \\ f_y = e^x - 1 = 0 \end{cases} \Rightarrow \begin{matrix} ye^x = 0 \Rightarrow \boxed{y=0} \text{ or } e^x \cancel{=} 0 \\ e^x = 1 \Rightarrow \boxed{x=0} \end{matrix}$$

Critical Points: $(0,0)$

$f_{xx} = ye^x \qquad f_{xy} = f_{yx} = e^x$

$f_{yy} = 0$

$$D = \begin{vmatrix} ye^x & e^x \\ e^x & 0 \end{vmatrix} = -e^{2x}$$

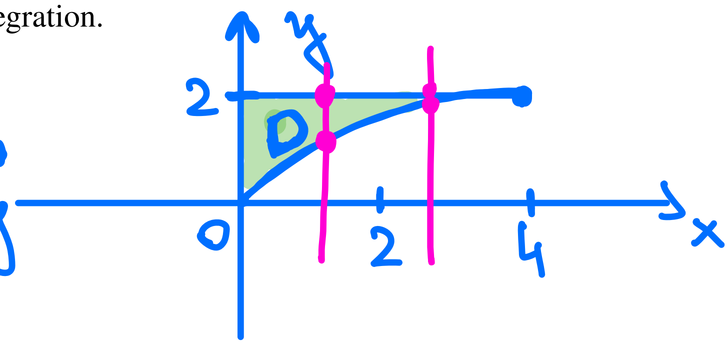
$D < 0$ for all $x \in \mathbb{R}$. Hence, $(0,0)$ is a saddle point. and f crosses tan. plane at $(0,0)$. $f(0,0) = 0$.

3. **[10 points]** For the given iterated integral

$$\int_0^2 \int_0^{y^2} x^2 y \, dx \, dy$$

(a) Draw the region of integration.

$D = \{(x,y) : 0 \le x \le y^2, \; 0 \le y \le 2\}$



(b) Evaluate the integral over the region drawn in part (a).

$$\int_0^2 \int_0^{y^2} x^2 y \, dx \, dy = \int_0^2 \frac{x^3}{3} y \Big|_0^{y^2} dy = \int_0^2 \frac{y^7}{3} \, dy =$$

$$= \frac{y^8}{24} \Big|_0^2 = \frac{2^8}{24} = \boxed{\frac{32}{3}}$$

(c) Reverse the order of integration. Evaluate the integral over a new region.

$$\int_0^4 \int_{\sqrt{x}}^2 x^2 y \, dy \, dx = \int_0^4 \frac{x^2 y^2}{2} \Big|_{\sqrt{x}}^2 dx = \int_0^4 \left(\frac{x^2 \cdot 4}{2} - \frac{x^2 \cdot x}{2} \right) dx =$$

$$= \left(\frac{2x^3}{3} - \frac{x^4}{8} \right) \Big|_0^4 = \frac{2}{3} \cdot 64 - \frac{256}{8} = \frac{1024 - 768}{24} = \boxed{\frac{32}{3}}$$