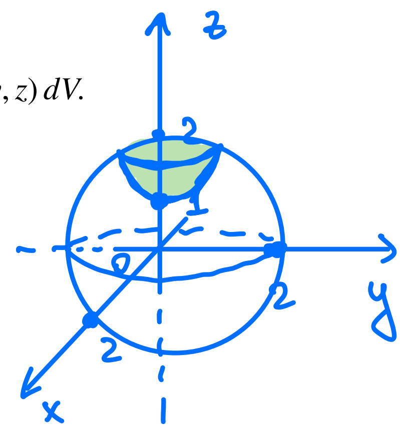
Student Name:

- The quiz is closed book, closed notes, and calculator free. No form of collaboration or help is allowed.
- The quiz is **45 minutes** long. This time includes downloading, working on, and submitting a quiz **in a PDF format via Gradescope**.
- The quiz have **20 points** in total.
- There is **no extension or quiz retake**.
- Show your full work to receive a full credit on each problem.

1. **[10 points]** Use **cylindrical coordinates** to find the mass of the solid enclosed below by the paraboloid $z = x^2 + y^2 + 1$ and above by the sphere $x^2 + y^2 + z^2 = 4$ if the density function is given by $\rho(x, y, z) = \frac{2}{z^2}$.

   Hint: use the following formula

$$m = \iiint_E \rho(x, y, z)\, dV.$$



$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad dV = r\,dr\,d\theta$$

$$m = \int_0^{2\pi}\int_0^1\int_{r^2+1}^{\sqrt{4-r^2}} \frac{2}{z^2}\, r\, dz\, dr\, d\theta =$$

$$1 \le x^2+y^2+1 \le 2$$
$$1 \le r^2+1 \le 2$$
$$0 \le r^2 \le 1$$
$$2 \quad 0 \le r \le 1$$

$$z = r^2+1$$
$$z^2 = 4-r^2$$
$$z = \sqrt{4-r^2}$$
$$r^2+1 \le z \le \sqrt{4-r^2}$$

$$= \int_0^{2\pi} d\theta \int_0^1 r\left(-\frac{2}{z}\right)\Bigg|_{r^2+1}^{\sqrt{4-r^2}} dr\, d\theta = 2\pi(-2)\int_0^1 \left(\frac{r}{\sqrt{4-r^2}} - \frac{r}{r^2+1}\right) dr =$$

$$= -4\pi\left(-\sqrt{4-r^2} - \frac{1}{2}\ln|r^2+1|\right)\Bigg|_0^1 = -4\pi\left(-\sqrt{3} - \frac{1}{2}\ln 2 + 2 + 0\right) = \boxed{4\pi\left(\sqrt{3}+2+\frac{1}{2}\ln 2\right)}.$$

3. **[5 points]** Find the Jacobian of the transformation:

$$x = ue^v, \quad y = ve^u.$$

$$J = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} e^v & ue^v \\ ve^u & e^u \end{vmatrix} = e^{u+v} - uve^{u+v}$$

$$\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \boxed{e^{u+v}(1-uv)}$$

4. **[5 points]** Sketch the vector field **F** on the xy-plane:

$$F(x, y) = y\,\mathbf{i} + (x + y)\,\mathbf{j}.$$

(x,y)	$\langle y, x+y \rangle$
$(0,0)$	$\langle 0,0 \rangle$
$(1,0)$	$\langle 0,1 \rangle$ 1
$(0,1)$	$\langle 1,1 \rangle$ 2
$(1,1)$	$\langle 1,2 \rangle$ 3