# Formulas & Definitions: Section 12-3

**Definition:** If $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, then the **dot product** of $a$ and $b$ is the number $a \cdot b$ given by

$$\boxed{\text{a} \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3}$$

**Properties of the Dot Product:** If $a, b$, and $c$ are vectors in $V_3$ and $\alpha$ is a scalar, then

1. $a \cdot a = |a|^2$
2. $a \cdot (b + c) = a \cdot b + a \cdot c$
3. $0 \cdot a = 0$
4. $a \cdot b = b \cdot a$
5. $(\alpha a) \cdot b = \alpha(a \cdot b) = a \cdot (\alpha b)$

**Theorem:** If $\theta$ is the angle between the vectors $a$ and $b$, then

$$\boxed{\text{a} \cdot b = |a|\,|b| \cos\theta}$$

**Corollary:** If $\theta$ is the angle between the nonzero vectors $a$ and $b$, then

$$\cos\theta = \frac{a \cdot b}{|a|\,|b|}.$$

**Proposition:** Two vectors $a$ and $b$ are orthogonal if and only if $a \cdot b = 0$.

**Definition:**

- Scalar projection of $b$ onto $a$ is
$$comp_a b = \frac{a \cdot b}{|a|}.$$
- Vector projection of $b$ onto $a$ is
$$proj_a b = \frac{a \cdot b}{|a|^2}\, a.$$