Name:

Instructions. (100 points) You have two hours. The exam is closed book, closed notes, and only simple calculators are allowed. Show all your work in order to receive full credit.

1. **[17 points]** Consider points $A(2, 3, 1)$ and $B(3, 4, c)$ and vectors $u = \langle 1, 2, 3\rangle$ and $v = \langle 1, 1, 2\rangle$.

   (a) (4pts) Find the vector projection of u along v.

   (b) (3 pts) Find all values of c such that the length of $\overrightarrow{AB}$ equals 5.

   (c) (3 pts) Find all values of c such that $\overrightarrow{AB}$ is parallel to v.

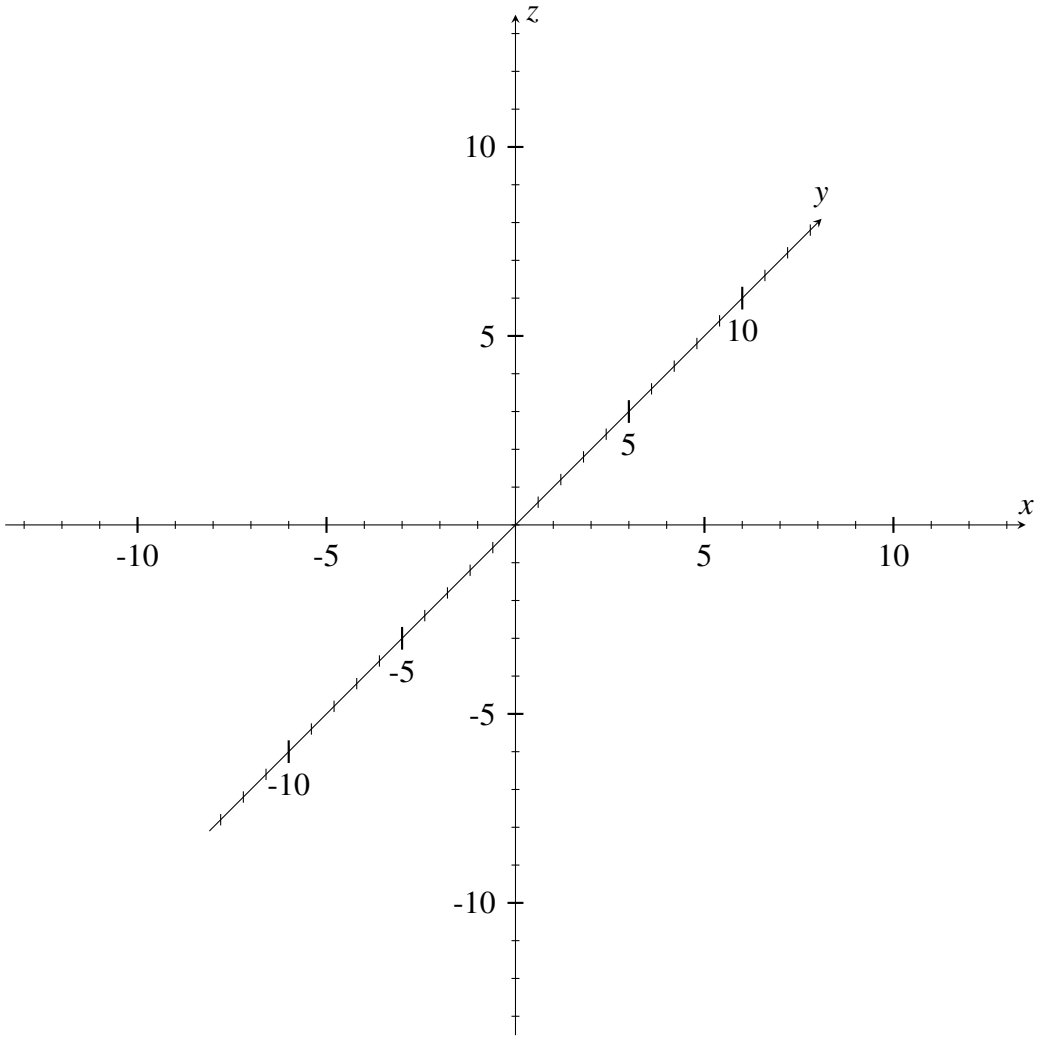
   (d) (3 pts) Find all values of c such that $\overrightarrow{AB}$ is orthogonal to u.

   (e) (4 pts) Find the cross product of vectors u and v.

2. **[15 points]** You are given the following space curve:

$$r(t) = \langle 2\cos(2t),\ 2\sin(2t),\ t\rangle, \quad 0 \le t \le \frac{5\pi}{2}$$

(a) (8 pts) Draw the trajectory of the vector function $r(t)$ for the given value t.



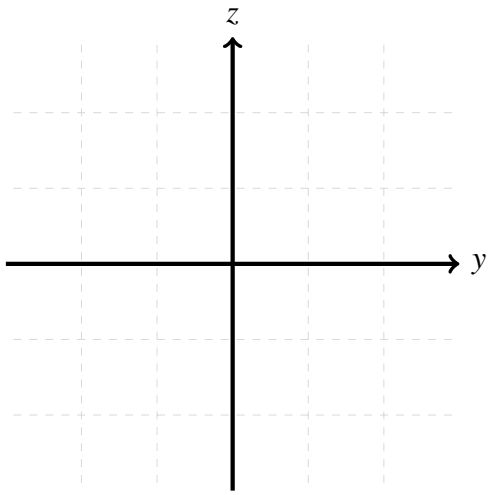
(b) (4 pts) Draw on the above trajectory the position and velocity vectors for $t = 2\pi$.

(c) (3 pts) Find the speed at time t and simplify your result.

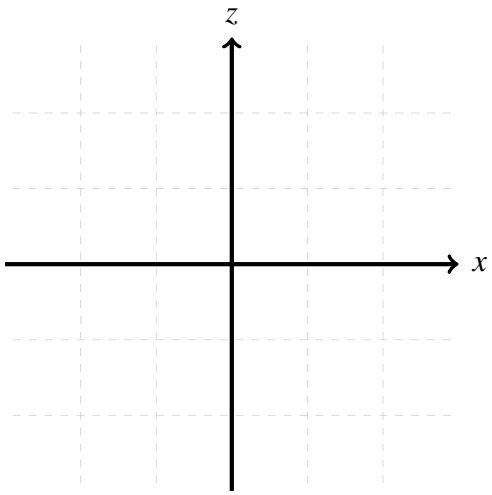
3. **[15 points]** Sketch the following surfaces.

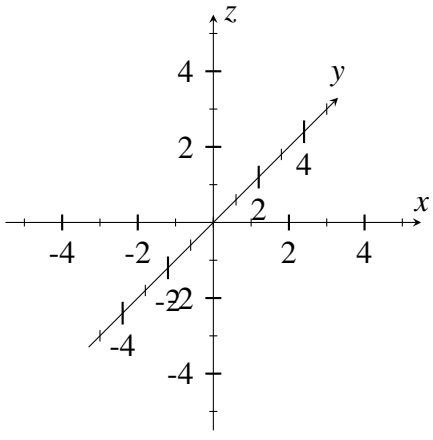
   (a) (10 pts) For $x = y^2 + 4z^2$, sketch the given traces, then the surface in 3-D.

   1) traces: $x = 0, 4$

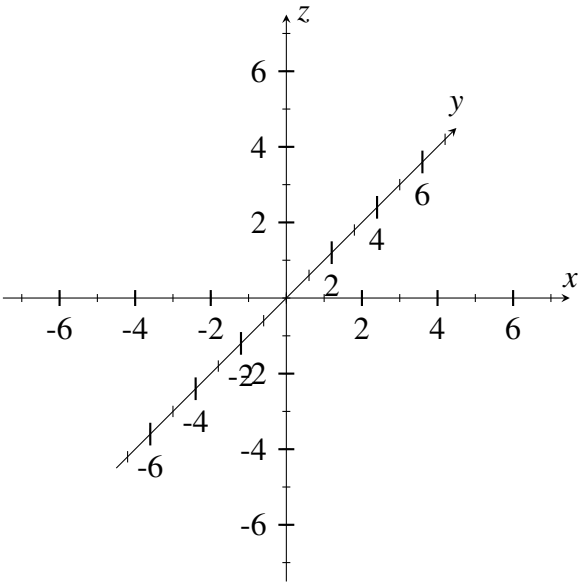


   2) traces: $y = 0, \pm 2$





(b) (5 pts) Sketch the surface $(x-1)^2 + y^2 + (z-3)^2 = 9$.



4. **[21 points]** Consider the following point, line, and plane:

$$B(3, -2, 1)$$
$$\vec{l}(t) = \langle t,\ 1-t,\ 2t+3 \rangle$$
$$Plane\ P : 2x + 5y - z = 10$$

(a) (5 pts) Give the equation of a plane parallel to the plane P that passes through B.

(b) (4 pts) Find the point of intersection of the line $\vec{l}(t)$ and the plane P.

(c) (5 pts) Find the angle the line $\vec{l}(t)$ makes with the normal to the plane P. (Your answer may involve an inverse trigonometric function.)

(d) (7 pts) Find an equation for the plane containing the point B and the line $\vec{l}(t)$.

5. **[8 points]** An object moves in 3-D with acceleration

$$a(t) = \langle \sin(t),\ 2\cos(t),\ 6t \rangle.$$

At time $t = 0$ it has velocity $\langle 0, 0, -1 \rangle$. Find a function $v(t) = r'(t)$ giving its velocity at all times $t > 0$.

6. **[20 points]** A particle moves with *velocity* $v(t) = \langle t^3, t^2, 2t \rangle$.

   (a) (7 pts) Find the distance the particle travels between times $t = 0$ and $t = 2$.

   (b) (8 pts) Calculate the curvature

$$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

   of the trajectory at time $t = 1$.

(c) (5 pts) Find the unit tangent vector **T(t)** and the tangential component of acceleration

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|}$$

at $t = 1$.

7. **[4 points]** Evaluate the following integral

$$\int_1^4 \left(2t^{3/2}\,\mathbf{i} + (t+1)\sqrt{t}\,\mathbf{k}\right) dt.$$