

Formulas & Definitions: Section 14-5

The chain rule (case 1): Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t . Then z is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}.$$

The chain rule (case 2): Suppose that $z = f(x, y)$ is a differentiable function of x and y , where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t . Then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s}, \quad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}.$$

The chain rule (general version): Suppose that u is a differentiable function of the n variables x_1, x_2, \dots, x_n and each x_j is a differentiable function of the m variables t_1, t_2, \dots, t_m . Then u is a function of t_1, t_2, \dots, t_m and

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial t_i} + \dots + \frac{\partial u}{\partial x_n} \frac{\partial x_n}{\partial t_i}.$$

Implicit Differentiation:

- Suppose that an equation of the form $F(x, y) = 0$ defines y implicitly as a differentiable function of x , that is, $y = f(x)$, where $F(x, f(x)) = 0$ for all x in the domain of f . Then

$$\frac{dy}{dx} = -\frac{\partial F / \partial x}{\partial F / \partial y} = -\frac{F_x}{F_y}.$$

- Suppose that z is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$. This means that $F(x, y, f(x, y)) = 0$ for all (x, y) in the domain of f . Then

$$\frac{\partial z}{\partial x} = -\frac{\partial F / \partial x}{\partial F / \partial z}, \quad \frac{\partial z}{\partial y} = -\frac{\partial F / \partial y}{\partial F / \partial z}.$$