1. Vector/scalar projection of vector u along vector v.

Problem:

vector projection: $u = \langle 1, 2, 3 \rangle$

$v = \langle 0, 2, 1 \rangle$

$\text{proj}_v u = \frac{u \cdot v}{|v|^2} v$

$u \cdot v = 1 \cdot 0 + 4 + 3 = 7$

$|v| = \sqrt{4+1} = \sqrt{5}$

$\text{proj}_v u = \frac{7\langle 0, 2, 1 \rangle}{\sqrt{5}} =$

$= \langle 0, \frac{14}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$

2. The condition of two vectors being parallel. $\parallel$

Problem: $u = \langle 1, 2, 4 \rangle$ Find c for which $u \parallel v$

$v = \langle 2, 4, c \rangle$

$u \parallel v \iff u = kv$

$$\begin{cases} 1 = k \cdot 2 \Rightarrow k = \frac{1}{2} \\ 2 = k \cdot 4 \Rightarrow k = \frac{1}{2} \\ 4 = k \cdot c \Rightarrow 4 = \frac{1}{2} \cdot c \Rightarrow \boxed{c = 8} \end{cases}$$

3. The condition of two vectors being orthogonal.

Problem: $u = \langle 1, 3, -1 \rangle$ Find c for which $u \perp v$

$v = \langle 0, c, 2 \rangle$

$u \perp v \iff u \cdot v = 0$

$1 \cdot 0 + 3 \cdot c - 1 \cdot 2 = 0$

$3c - 2 = 0$

$\boxed{c = \frac{2}{3}}$

4. Dot product/cross product of two vectors.

Problem: $u = \langle 2, 0, 1 \rangle$, $v = \langle 3, 1, 2 \rangle$. Find $u \times v$.

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix} = (0-1)i - (4-3)j + (2-0)k =$$

$$= -i - j + 2k = \langle -1, -1, 2 \rangle$$

5. Draw the trajectory of the vector function $r(t)$ for a given value t.

Problem: $r(t) = \langle t, \cos t, \sin t \rangle$, $0 \le t \le 2\pi$

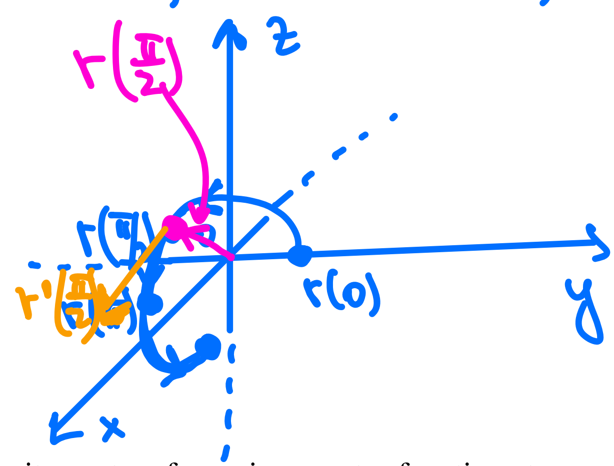
$r(0) = \langle 0, 1, 0 \rangle$

$r\left(\frac{\pi}{2}\right) = \langle \frac{\pi}{2}, 0, 1 \rangle$

$r(\pi) = \langle \pi, -1, 0 \rangle$

$r(2\pi) = \langle 2\pi, 1, 0 \rangle$

$r\left(\frac{3\pi}{2}\right) = \langle \frac{3\pi}{2}, 0, -1 \rangle$



6. Draw the position and velocity vectors for a given vector function at $t = a$.

Problem: $r(t) = \langle t, \cos t, \sin t \rangle$

position: $r\left(\frac{\pi}{2}\right) = \langle \frac{\pi}{2}, 0, 1 \rangle$ $\quad r\left(\frac{\pi}{2}\right)$

velocity: $r' = \langle 1, -\sin t, \cos t \rangle$

$r'\left(\frac{\pi}{2}\right) = \langle 1, -1, 0 \rangle$ $\quad r'\left(\frac{\pi}{2}\right)$

7. Calculating a speed for a given vector function $r(t)$.

Problem: $r(t) = \langle t^2, t^3, 2t^4 \rangle$

speed $= v = |v(t)| = |r'(t)|$

$v = |\langle 2t, 3t^2, 8t^3 \rangle|$

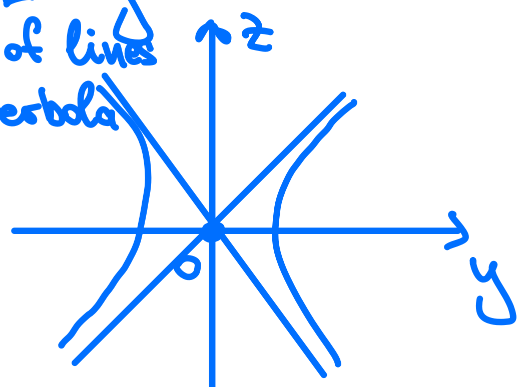
$v = \sqrt{4t^2 + 9t^4 + 64t^6}$

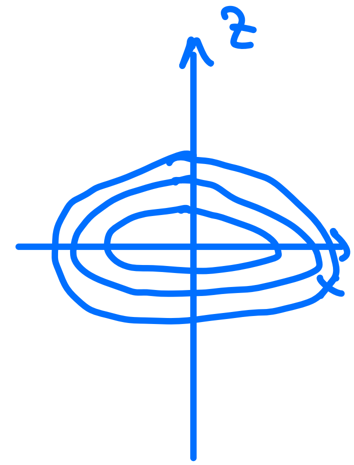
8. Sketching traces for a given surface.

Problem: $x^2 + 4z^2 = y^2$

traces: $x = 0$ – pair of lines

$x = \pm 2$ – hyperbola

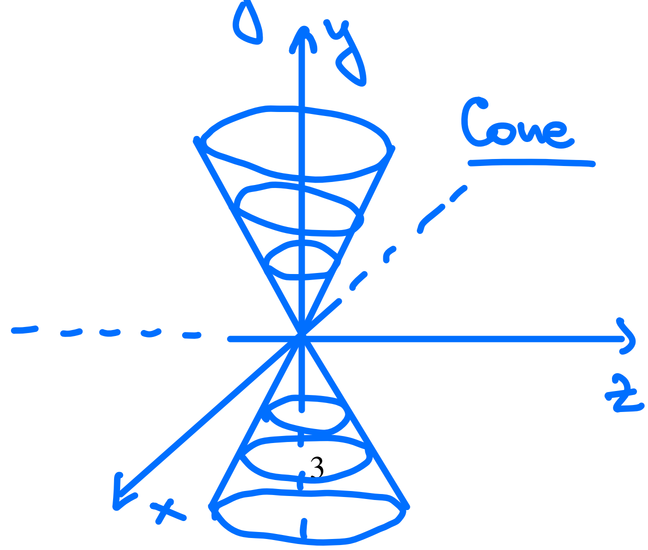




traces: $y = \pm 2$ - ellipses

9. Sketching surfaces.

Problem: $x^2 + 4z^2 = y^2$



10. Finding an equation of the plane that goes through the point A and is parallel to the given vector.

Problem: $A(1,2,-1)$
$n = \langle 3,2,1 \rangle$

Plane eq.: $3(x-1) + 2(y-2) + 1(z+1) = 0$
$3x - 3 + 2y - 4 + z + 1 = 0$
$\boxed{3x + 2y + z - 6 = 0}$

11. Finding the point of intersection of the line and the plane.

Problem: line $\ell(t) = \langle \overset{x}{1+t}, \overset{y}{2-t}, \overset{z}{2t} \rangle$
Plane: $2x + y - z = 9$

Intersection: $2(1+t) + 2 - t - 2t = 9$
$2 + \cancel{2t} + 2 - t - \cancel{2t} = 9$
$t = 4 - 9 = -5$
$(1-5, 2+5, -10) = \boxed{(-4,7,-10)}$

12. Finding the angle between the line and the normal vector to the plane.

Problem: $\ell(t) = \langle 1+t, 2-t, 2t \rangle$
$\vec{n} = \langle 2,1,-1 \rangle$
$\vec{u} = \langle 1,-1,2 \rangle$

$$\cos\theta = \frac{u \cdot n}{|u| \cdot |n|} = \frac{2-1-2}{\sqrt{6}\,\sqrt{6}} = -\frac{1}{6}$$

$\theta = \arccos\left(-\frac{1}{6}\right)$

13. Finding an equation of the plane containing the point A and the line $\vec{l}(t)$.

Problem: We have $\vec{u} = \langle 1,-1,2 \rangle$
We have $B = (1,2,0)$ } from $l(t)$
We are also given $\boxed{A = (1,2,-1)}$

We find $\vec{AB} = (0,0,1)$

We calculate

$$\vec{AB} \times \vec{u} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \langle 1,1,0 \rangle$$

Then plane eq. is
$1\cdot(x-1) + 1\cdot(y-2) + 0\cdot(z+1) = 0$

14. Recovering velocity vector function from the acceleration vector function.

Problem: $a(t) = \langle t, 1, t^2 \rangle$
$v(0) = \langle 0,-1,1 \rangle$

$$v(t) = \int a(t)\,dt = \boxed{\langle \frac{t^2}{2}, t, \frac{t^3}{3} \rangle + C}$$

$$v(0) = \langle 0,0,0 \rangle + C = \langle 0,-1,1 \rangle$$

$$C = \langle 0,-1,1 \rangle$$

15. Recovering the distance from the velocity by knowing that it is the integral of the speed over time.

Problem: Given velocity: $v(t) = \langle t, t^2, -t \rangle$
Find $d(t)$ from $t=0$ to $t=2$

$$d(2)-d(0) = \int_0^2 |v(t)|\,dt = \int_0^2 \sqrt{t^2+t^4+t^2}\,dt =$$

$$= \int_0^2 t\sqrt{1+t^2}\,dt = \dots \quad \text{(use substitution)}$$

16. Calculation the curvature of the trajectory.

Problem: $r(t) = \langle 1, t, 2t^2 \rangle$

$$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r' = \langle 0, 1, 4t \rangle$

$r'' = \langle 0, 0, 4 \rangle$

plug and calculate

17. Finding the unit tangent vector. $T(t)$

Problem:

$r(t) = \langle e^t, 1, t \rangle$

$r'(t) = \langle e^t, 0, 1 \rangle$

$$T(t) = \frac{r'(t)}{|r'(t)|}$$

$$T(t) = \frac{\langle e^t, 0, 1 \rangle}{\sqrt{e^{2t} + 1}}$$

18. Finding the tangential component of acceleration.

Problem:

$$a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|}$$

So, if $r(t) = \langle \sin t, t, \cos t \rangle$

Then

$r' = \langle \cos t, 1, -\sin t \rangle$

$r'' = \langle -\sin t, 0, -\cos t \rangle$

$$a_T = \frac{0}{\sqrt{2}} = 0$$

19. Evaluating a definite integral.

Problem:

$$\int_1^2 \left(\frac{1}{1+t} i + te^t j + t^{3/2} k \right) dt =$$

$$= i \int_1^2 \frac{1}{1+t}\, dt + j \int_1^2 te^t\, dt + k \int_1^2 t^{3/2}\, dt =$$

$$= i \left(\ln|1+t| \right)\Big|_1^2 + j \left(te^t + e^t \right)\Big|_1^2 + k \frac{t^{5/2}}{5/2}\Big|_1^2 =$$

$= \ldots\ldots$ (just plug boundaries)