Formulas & Definitions: Section 16-7

Definition: Suppose that a surface S has a vector equation

$$r(u,v) = x(u,v)i + y(u,v)j + z(u,v)k, \quad (u,v) \in D.$$

Then the surface integral of f over the surface S is

$$\iint_S f(x,y,z)\,dS = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(P_{ij}^*)\,\Delta S_{ij}$$

or

$$\iint_S f(x,y,z)\,dS = \iint_D f(r(u,v))\,|r_u \times r_v|\,dA$$

Applications: If a thin sheet has the shape of a surface S and the density at the point (x,y,z) is $\rho(x,y,z)$, then the total **mass** of the sheet is

$$m = \iint_S \rho(x,y,z)\,dS$$

and the **center of mass** is $(\bar{x},\bar{y},\bar{z})$, where

$$\bar{x} = \frac{1}{m}\iint_S x\,\rho(x,y,z)\,dS, \quad \bar{y} = \frac{1}{m}\iint_S y\,\rho(x,y,z)\,dS, \quad \bar{z} = \frac{1}{m}\iint_S z\,\rho(x,y,z)\,dS$$

Graphs of functions: Any surface S with equation $z = g(x,y)$ can be regarded as a parametric surface with parametric equations

$$x = x, \quad y = y, \quad z = g(x,y).$$

Thus

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))\sqrt{(\partial z/\partial x)^2 + (\partial z/\partial y)^2 + 1}\,dA$$

Definition: If it is possible to choose a unit normal vector n at every such point (x,y,z) so that n varies continuously over S, then S is called an **oriented surface** and the given choice of n provides S with an **orientation**.

Definition: A surface S is **closed** if it is the boundary of a solid region E.

Definition: If F is a continuous vector field defined on an oriented surface S with unit normal vector n, then the **surface integral of F over S** is

$$\iint_S F \cdot dS = \iint_S F \cdot n\,dS = \iint_D F \cdot (r_u \times r_v)\,dA.$$

This integral is also called the **flux** of F across S.

Particular case: In the case of a surface S given by a graph $z = g(x, y)$, we obtain

$$\iint_S F \cdot dS = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA.$$

Applications:

- Gauss's Law (electric flux):

$$Q = \varepsilon_0 \iint_S E \cdot dS$$

- Conductivity of the substance:

$$\iint_S F \cdot dS = -K \iint_S \nabla u \cdot dS$$