

Student Name:
Matrikel-Nr:

Formale Grundlagen der Informatik I - Assignment 1

Hand out: 20.02.2020 - Due to: 05.03.2020

Please upload your solutions to the Olat system.

1.1 Sets and Subsets

a) (1 Min) Write in your own words how to read the following sets:

i. $\{n \in \mathbb{Z} \mid n \notin \mathbb{N}\}$

ii. $\{x \in \mathbb{R}^- \mid x > -10\}$

b) (3 Min) Answer the following questions with a short explanation:

i. Is $\emptyset \in \{\}$?

ii. How many elements does the set $\{1, 1, 1, 2, 2\}$ contain?

iii. How many elements does the set $\{1, 2, \{1, 2\}, \{\{1, 2\}, \{1, 2\}\}, \{2, 1\}\}$ contain?

iv. Is $\{2\} \in \{\{1\}, \{2\}\}$?

v. Is $0 \in \{\{0\}, \{1\}\}$?

vi. Is $\{2\} \in \{\{1, 2\}\}$?

1.2 Relations and Functions

a) (1 Min) What is the difference between a function and a relation?

b) (5 Min) Let $A = \{2, 4\}$ and $B = \{1, 3, 5\}$. Define the nonempty and pairwise different relations $U, V, W \subseteq A \times B$. Is your solution unique?

- $x \cdot y \geq 7 \rightarrow (x, y) \in U$.

- $(x, y) \in V \rightarrow x > y$.

- $x > y \rightarrow (x, y) \in W$.

i. For which of the above tasks would the empty set be a valid solution if the additional constraints (pairwise difference and nonemptiness) weren't given?

ii. Determine if the relations U, V and W are functions and reason in a few words.

c) (2 Min) Which attributes (left/right-total, left/right-unique) do the following relations $A, B, C, D \subseteq \mathbb{N} \times \mathbb{N}$ have?

- $A = \{(n, 1) \mid n \in \mathbb{N}\}$

- $B = \{(1, n) \mid n \in \mathbb{N}\}$

- $C = \{(n, m) \mid n, m \in \mathbb{N}\}$

- $D = \{(n, n) \mid n \in \mathbb{N}\}$

1.3 Logical Equivalence

a) (4 Min) Write down the truth table for the following logical statement:

$$(a \wedge \neg b) \vee (\neg a \wedge b) \leftrightarrow \neg(a \wedge b) \vee (a \wedge b)$$

b) (2 Min) Please determine (in a plausible way) if the following statements are tautologies or contradictions.

i. $((p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r)) \leftrightarrow \neg(p \vee q)$

ii. $(p \vee q) \vee \neg(p \wedge q)$

c) (1 Min) With a few words of explanation, determine if the following statements are mutually excluding.

- Susan speaks German and English. Oliver only speaks English.
- It is not the case that Oliver and Susan both speak German and English.

1.4 Conditional Statements

a) (4 Min) Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Please define the variables you use at least once.

- i. If it walks like a duck and it talks like a duck, then it is a duck.
- ii. Either it does not walk like a duck or it does not talk like a duck, or it is a duck.
- iii. If it does not walk like a duck and it does not talk like a duck, then it is not a duck.
- iv. If it walks like a duck and doesn't talk like a duck, it is a duck, but if it doesn't walk like a duck and talks like a duck it's not a duck.