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Formale Grundlagen der Informatik I - Assignment 2

Hand out: 05.03.2020 - Due to: 19.03.2020

Please upload your solutions to the Olat system.

2.1 Circuits

- a) **(6 Min)** Given the following truth table, derive and draw the corresponding circuit for the output s .

a	b	c	s
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

2.2 The Logic of Quantified Statements

a) (3 Min) Consider the following grid and colored figures:

●	■			●
●		●		
■	●	▲		
	■		●	▲
	■	●		▲

The domain of variables (V) is the set of all figures on the grid. Function names can be figure forms (eg. $\text{Triangle}(x)$ means x is a triangle), figure positions, colors or other attributes.

For example, $\text{IsLeftOf}(x, y)$ returns true if x is on the left field of y .

$\text{IsNextTo}(x, y)$ returns true if x is a neighbour of y (horizontally or vertically, but not diagonally).

Decide if the following statements are true or false for the grid given above. Please give a short reason for your decision.

i. $\exists x \in V : \text{Hexagon}(x)$

ii. $\exists x \in V : \text{Red}(x) \wedge \text{Square}(x)$

iii. $\forall x \in V : \neg \text{Yellow}(x) \wedge \text{Square}(x)$

iv. $\forall x \in V : \neg \text{Green}(x) \vee \text{Circle}(x)$

v. $\exists x, y \in V : \text{Square}(x) \wedge \text{Triangle}(y) \wedge \text{IsLeftOf}(x, y)$

vi. $\forall x, y \in V : \neg \text{Square}(x) \wedge \neg \text{Circle}(y) \wedge \text{IsNextTo}(x, y)$

b) (3 Min) Which of the following is contradicting “For all cats there exists a dog, who hates this cat.”? More than one answer may be correct.

i. Every cat hates every dog.

ii. Every dog likes the same cat.

- iii. Every dog hates the same cat.
- iv. Every dog likes every cat.
- v. One cat is liked by all dogs.
- vi. Some cats are hated by some dogs.
- vii. All cats are liked by some dogs.
- viii. Every cat is liked by every dog.
- ix. No dog likes every cat.

2.3 Number Theory and Proofs

a) **(2 Min)** Prove the following statement: There are distinct integers m and n such that $\frac{1}{m} + \frac{1}{n}$ is an integer.

b) **(3 Min)** Prove that the product of any two consecutive positive integers is even.

c) (3 Min) Use proof by contradiction to show that for all positive integers m , $7m + 1$ is not divisible by 7.

d) (7 Min) Given are $a, b, c \in \mathbb{Z}$. Prove or disprove the following statements:

- i. If $b \bmod a = 0$ and $c \bmod a = 0$, then $(b + c) \bmod a = 0$.
- ii. If $c \bmod a = 0$ and $c \bmod b = 0$, then $c \bmod (a \cdot b) = 0$.
- iii. If $n \in \mathbb{N}$ is even, then n^3 is even too.
- iv. If $n \in \mathbb{N}$ is odd, then are $a, b \in \mathbb{N}$ so that $n = a^2 - b^2$.
- v. If p is a prime number and $n \in \mathbb{N}$ is smaller than p , then $\binom{p}{n} \bmod p = 0$.
- vi. If $n \in \mathbb{N}$, then $\binom{2n}{n}$ is even.