Universität Zürich
Institut für Informatik

FS 2020

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Formale Grundlagen der Informatik I - Assignment 1

Hand out: 20.02.2020 - Due to: 05.03.2020

Please upload your solutions to the Olat system.

1.1 Sets and Subsets

a) (1 Min) Write in your own words how to read the following sets:

i. $\{n \in \mathbb{Z} \mid n \notin \mathbb{N}\}$
The set of all integers n such that n is not a nonnegative integer.

ii. $\{x \in \mathbb{R}^- \mid x > -10\}$
The set of all negative real numbers x such that x is bigger than minus ten.

b) (3 Min) Answer the following questions with a short explanation:

i. Is $\emptyset \in \{\}$?
No. $\emptyset$ is equal to $\{\}$, but it is not an element (as indicated by $\in$) of the empty list $\{\}$.

ii. How many elements does the set $\{1, 1, 1, 2, 2\}$ contain?
The set contains the items 1 & 2. According to the axiom of extension the amount of times an element of a set appears when written is irrelevant for the determination of the set.

iii. How many elements does the set $\{1, 2, \{1, 2\}, \{\{1, 2\}, \{1, 2\}\}, \{2, 1\}\}$ contain?
It contains four elements. Sets inside another set are counted as one element, ignoring their length. The five elements are the two numbers 1 and 2, the set $\{1, 2\}$ and the set $\{\{1, 2\},\{1, 2\}\}$, which consists of another set (the two sets are equal and only count as one). The last written out set $\{2, 1\}$ is the same as $\{1, 2\}$ which is already named earlier and is not considered a new element.

iv. Is $\{2\} \in \{\{1\}, \{2\}\}$?
Yes. The bigger set consists of two elements $\{1\}$ and $\{2\}$. $\{2\}$ is therefor an element.

v. Is $0 \in \{\{0\}, \{1\}\}$?
No. The number 0 is not an element of the set that consists of two sets.

vi. Is $\{2\} \in \{\{1,2\}\}$?
No. The set $\{2\}$ is not equal to the set $\{1,2\}$ and is therefor not a part of the set which has one element $\{1,2\}$.

1.2 Relations and Functions

a) (1 Min) What is the difference between a function and a relation?
A relation is a subset of a cartesian product. For every ordered pair in this subset a property is true. The property does not have to be true for all ordered pairs of the cartesian product.
A function F is a relation with every element x of the domain having an element y in the co-domain, so that the relation is true. Every x only has exactly one y. If there are multiple elements $y, z, ...$ for which $(x,y),(x,z),(x,...) \in F$, then $y = z = ...$ or the relation is not a function. This means that functions are right-unique and left-total.

b) (5 Min) Let $A = \{2,4\}$ and $B = \{1,3,5\}$. Define the nonempty and pairwise different relations $U, V, W \subseteq A \times B$. Is your solution unique?

- $x \cdot y \geq 7 \rightarrow (x,y) \in U$.
  $(2,5),(4,3),(4,5)$ Unique.
- $(x,y) \in V \rightarrow x > y$.
  $(2,1),(4,3),(4,1)$ Unique.
- $x > y \rightarrow (x,y) \in W$.
  $(2,1),(4,3),(4,1)$ Unique.

i. For wich of the above tasks would the empty set be a valid solution if the additional constraints (pairwise difference and nonemptiness) weren't given?
None.

ii. Determine if the relations U, V and W are functions and reason in a few words.
None of them are functions. There isn't exactly one $y \in B$ for each $x \in A$.

c) (2 Min) Which attributes (left/right-total, left/right-unique) do the following relations $A, B, C, D \subseteq \mathbb{N} \times \mathbb{N}$ have?

- $A = \{(n,1) \mid n \in \mathbb{N}\}$
  Right-unique, left-total, right-total
- $B = \{(1,n) \mid n \in \mathbb{N}\}$
  Left-unique, left-total, right-total
- $C = \{(n,m) \mid n, m \in \mathbb{N}\}$
  Left-total, right-total
- $D = \{(n,n) \mid n \in \mathbb{N}\}$
  Left-total, right-total

1.3 Logical Equivalence

a) (4 Min) Write down the truth table for the following logical statement:

$$(a \wedge \neg b) \vee (\neg a \wedge b) \leftrightarrow \neg(a \wedge b) \vee (a \wedge b)$$

b) **(2 Min)** Please determine (in a plausible way) if the following statements are tautologies or contradictions.

i. $((p \land \neg q \land \neg r) \lor (p \land \neg q \land r)) \leftrightarrow \neg(p \lor q)$

ii. $(p \lor q) \lor \neg(p \land q)$

c) **(1 Min)** With a few words of explanation, determine if the following statements are mutually excluding.

- Susan speaks German and English. Oliver only speaks English.
- It is not the case that Oliver and Susan both speak German and English.

Statement 2 is only wrong if both speak both languages. As Oliver only speaks English, both statement 1 2 are both correct and are not mutually excluding.

1.4 Conditional Statements

a) **(4 Min)** Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Please define the variables you use at least once.
w = Walks like a duck
t = Talks like a duck
i = is a duck

i. If it walks like a duck and it talks like a duck, then it is a duck.
$w \land t \to i$

ii. Either it does not walk like a duck or it does not talk like a duck, or it is a duck.
$(\neg w \lor \neg t) \lor i$

iii. If it does not walk like a duck and it does not talk like a duck, then it is not a duck.
$(\neg w \land \neg t) \to \neg i$

iv. If it walks like a duck and doesn't talk like a duck, it is a duck, but if it doesn't walk like a duck and talks like a duck it's not a duck.
$(w \land \neg t \to i) \land (\neg w \land t \to \neg i)$