Exercisesummary

27.02.20

Definitions

nonempty The solution has to have at least one element in it. I.e. the empty set is not accepted as a solution.

pairwise different In a group of elements, each element has to be different to each other one.

unique In the first Assignment (task 1.2b) "Is your solution unique?" can be rephrased to "Is your solution the only one possible?".

domain For a given Relation $R \subseteq A \times B$, A is the domain.

codomain For the same Relation, B is the codomain.

image The image of a Relation is the subset of the codomain, which gets treated by the relation ($\{y \in B \mid \exists x \in A: (x, y) \in R\}$).

left-total A relation $R \subseteq A \times B$ is left-total if $\forall x \in A: \exists y \in B: (x, y) \in R$. In other words: Each element of A gets treated by R.

right-total Same as left-total, but for B ($\forall y \in B: \exists x \in A: (x, y) \in R$).

left-unique Each element in the codomain is allowed to be treated at maximum once in the relation.

right-unique Same as left-unique but for the domain.

mutally excluding Two statements A and B are mutually excluding, if A excludes B and B excludes A. E.g. “The class is silent.” and “The class is talking.” are mutually excluding, since silence isn't compatible with talking and talking isn't compatible with silence.

Reading sets

$\{x \in \mathbb{R} \mid \frac{\lvert x \rvert}{\pi} \in \mathbb{N}\}$	Every multiple (also negatives) of π.
$\{x \in \mathbb{R} \mid x \notin \mathbb{Q}\}$	The irrationals

Implications

Given two statements $A =$ “It rains.” and $B =$ “The street is wet.”, $A \to B$ is obvious. But $B \to A$ is not always true. The street could also be wet, because a pipe is broken, or someone emptied a bucket of water on the street.

As an additional example (which did not get mentioned in the exercise!), consider the following statement:

“If a given number is smaller than 10, it is also smaller than 100.”

Obviously, this statement is true. Here, $A =$ “A given number is smaller than 10” and $B =$ “A given number is smaller than 100.”. The implication is $A \to B$.

Some examples for each possible entry in the truthtable.
True $\to$ True

“If 5 is smaller than 10, it is also smaller than 100.”

A is True, B is True and the implication is also True.

False $\to$ False

“If 500 is smaller than 10, it is also smaller than 100.”

A is False, B is False, but the implication itself is True.

False $\to$ True

“If 50 is smaller than 10, it is also smaller than 100.”

A is False, B is True, but the implication itself is True again.

In this implication, it's not possible to construct an example, where A would be True and B would be False, which would make the whole implication False. Nevertheless, this example should give a bit of an insight, why False $\rightarrow$ True is still a True/valid implication.