Student Name: Student Number:

## Foundations of Computing II Assignment 4

Non-Context-Freeness, Pushdown Automata

Distributed: 02.11.2020 - Due Date: 22.11.2020

Upload your solutions to the OLAT system.

## 4.1 The Pumping Lemma for Context-Free Languages

We have proven the pumping lemma for context-free languages by assuming that the given CFG G is in Chomsky normal form.

Explain how an alternative proof can work without assuming that G is in Chomsky normal form. However, you are allowed to assume that G is normalized besides from that; in particular, its does not contain  $\varepsilon$ -productions, unit productions, or any useless symbols.

## 4.2 Non-Context-Freeness

Use the pumping lemma for context-free languages to prove that the following languages are not context-free.

a) 
$$L_1 = \{ w \in \{0,1\}^* \mid w = 1^k 0^{2k} 1^k \text{ for some } k \in \mathbb{N} \}$$

**b)** 
$$L_2 = \{ww^{\mathsf{R}}w \mid w \in \{a, b\}^*\}$$

c) 
$$L_3 = \{0^k 1^{k^2} \mid k \in \mathbb{N}\}$$

## 4.3 Pushdown Automata

Give pushdown automata that recognize the following languages.

a) 
$$L_4 = \{ w \in \{0,1\}^* \mid w = w^{\mathsf{R}} \text{ and the length of } w \text{ is odd} \}$$

**b)** 
$$L_5 = \{w \in \{0,1\}^* \mid |w|_0 = |w|_1\}$$