Universität Zürich
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Foundations of Computing II
Assignment 1

Formal Languages, Automata, Regular Expressions

Distributed: 21.09.2020 – Due Date: 04.10.2020

Upload your solutions to the OLAT system.

1.1 Alphabets, Words, Languages

a) Let $X = \{1321, 2222, 31\}$, $Y = \{\varepsilon, 11, 21\}$, and $Z = \{\varepsilon, u, bddd\}$ be languages over the alphabet $\{1, 2, 3, u, b, d\}$; let $\circ$ denote the concatenation operator.

(i) Give the set of strings in X^* that are of length 4.

(ii) Give the set of strings in $X \circ Y$ that are of length 6.

(iii) Give the set of strings in $(Y \cup Z) \circ X$ that are of length 5 or less.

b) We consider two languages $\{1\}$ and $\{2\}$ that contain only one word each. You are only asked to explain your arguments in words; no formal arguments are required.

(i) Explain why $(\{1\}^*\{2\}^*)^* = (\{1, 2\}^*)^2$.

(ii) Explain why $(\{1\}^*\{2\}^*)^* \neq (\{1, 2\}^2)^*$.

1.2 Finite Automata

If you want to create graphs, `TIKZ` is a nice tool to generate diagrams from code.

a) Draw (either by hand or by using a drawing tool) a finite automaton (DFA) for each of the following languages.

(i) The language $L_1 = \{aab, aaab, b\}$ over the alphabet $\{a, b\}$.

(ii) The language L_2 over the alphabet $\{a, b, c\}$ consisting of all words that start with aba and contain at least one c.

(iii) The language L_3 over the alphabet $\{0, 1\}$ that consists of the words that are the binary representation of even numbers. All representations (except 0) should start with a 1; for instance, the word 100 is in L_3, but 11, 0100, and 101 are not in L_3.

b) It is important to get the quantifiers straight in this context. All the following languages are over the alphabet $\{0, 1\}$.

(i) Draw a DFA for the language $L_1 = \{01\}$.

(ii) Draw a DFA for the language $L_2 = \{0011\}$.

(iii) This can be generalized for arbitrary natural numbers. For a given $k \in \mathbb{N}$, sketch how an automaton for the language $L_k = \{0^k 1^k\}$ would look like.

(iv) However, explain on an intuitive level, in two or three sentences, where the problem lies, if one would want to create a DFA for the language $L = \{0^k 1^k \mid k \in \mathbb{N}\}$. Note that L contains all words $\varepsilon, 01, 0011, 000111, \ldots$. Later, we will even *prove* that there cannot be a DFA for this language.

1.3 Cycles in Finite Automata

Prove *by contradiction* that every DFA contains a cycle.