Universität Zürich
Institut für Informatik

FS 2020

Student Name:
Matrikel-Nr:

Formale Grundlagen der Informatik I - Assignment 3

Hand out: 19.03.2020 - Due to: 09.04.2020

Please upload your solutions to the Olat system.

3.1 Binomial Coefficients

a) **(1 Min)** Without proof: What is the relation between Pascal's triangle and the binomial coefficients?

b) **(2 Min)** Please give a recursive formula to calculate $\binom{n}{k}$ with $n, k \in \mathbb{N}$ that appears reasonable. Please give a short explanation.

3.2 Mathematical Induction and Proofs

a) **(4 Min)** Please describe, how induction works as a proof. For what kind of problem is it well suited and for what kind of problem is it badly applicable and why?

b) (2 Min) Name the four steps that you have to perform in every induction.

c) (9 Min) Prove the following statements using induction:

i. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}^{+}$.

ii. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}^{+}$.

d) (3 Min) Now prove $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}$ without induction.

e) (2 Min) Given $P(n) = (2^n < (n+1)!)\ \forall n \in \mathbb{N}^+$. ($P$ takes a positive integer and returns a boolean.)

i. Write $P(2)$, is $P(2)$ true?

ii. Write $P(k)$.

iii. Write $P(k+1)$.

iv. In a proof by mathematical induction that this inequality holds for all integers $n \geq 2$, what must be shown in the inductive step?