SECTION 1.5

1.5.1 (a) $4992 = \frac{X}{(1.08)^{1/2}} \rightarrow X = 5187.84$

(b) $4992 = \frac{X}{\left[1+(.08)\left(\frac{1}{2}\right)\right]} \rightarrow X = 5191.68$

(c) $4992 = X(1-.08)^{1/2} \rightarrow X = 5204.52$

(d) $4992 = X\left[1-(.08)\left(\frac{1}{2}\right)\right] \rightarrow X = 5200$

1.5.2 With a quoted discount rate of .940, the price of a 91-day T-Bill should be $100(1-\frac{28}{360}\times.00050) = 99.996111$ as quoted.

The investment rate is found as

$$\left(\frac{100}{99.996111}-1\right)\times\frac{365}{28} = .00051, \text{ as quoted.}$$

1.5.3 $d_j = \frac{A(r+T)-A(r)}{A(r+T)} = 1-\frac{A(r)}{A(r+T)}$, and

$j = \frac{A(r+T)-A(r)}{A(r)} = \frac{A(r+T)}{A(r)}-1$

$\rightarrow 1-d_j = \frac{A(r)}{A(r+T)} = \frac{1}{1+j} \rightarrow$

(a) $d_j = \frac{j}{1+j}$ and (b) $j = \frac{d_j}{1-d_j}$.

1.5.4 $1.15 = (1-d)(1.3) \rightarrow d = .1154$

1.5.5 Bruce's interest in year 11: $100(1-d)^{-10}\cdot\left[(1-d)^{-1}-1\right] = X$.
Robbie's interest in year 17:

$$50(1-d)^{-16}\cdot\left[(1-d)^{-1}-1\right] = X = 100(1-d)^{-10}\cdot\left[(1-d)^{-1}-1\right]$$

$\rightarrow 50(1-d)^{-16} = 100(1-d)^{-10} \rightarrow (1-d)^6 = .5 \rightarrow d = .1091$

$\rightarrow X = 38.9$

1.5.6 The present value of 1 due in n years is $(1-d)^n$, so the accumulated value after n years of an initial investment of 1 is

$$\frac{1}{(1-d)^n} = (1-d)^{-n}.$$

1.5.7 The initial deposit of 10 grows to $10\left(1-\frac{d}{4}\right)^{-40}$ at the end of 10 years (40 quarters), and then continues to grow at 3% per half year after that. The accumulated value of the initial deposit of 10 at the end of 30 years is $10\left(1-\frac{d}{4}\right)^{-40}\times(1.03)^{40}$ (20 more years, 40 more half-years at 3% per half-year).

The second deposit is 20 made at time 15. The accumulated value of the second deposit at time 30 (15 years after the second deposit) is $20(1.03)^{30}$ (15 years is 30 half-years).

The total accumulated value at the end of 30 years is

$$10\left(1-\tfrac{d}{4}\right)^{-40}\times(1.03)^{40}+20(1.03)^{30} = 100.$$

Solving for d results in $d = .0453$.

This question is from the May 2003 Course 2 exam that was conducted jointly by the Society of Actuaries and the Casualty Actuarial Society. It should be noted that the nominal interest rate notation $i^{(m)}$ and nominal discount rate notation $d^{(m)}$ is not always specifically used on the professional actuarial exams. In this example, the notation d was a nominal annual rate of discount compounded quarterly.

1.5.8 (a) Bank pays

$1-d\cdot\frac{n}{365} = \frac{1}{1+i\cdot\frac{n}{365}} \rightarrow i = \frac{365}{n}\left[\frac{1}{1-d\cdot\frac{n}{365}}-1\right] = \frac{d}{1-d\cdot\frac{n}{365}}$.

As n increases, i increases.

(b) From (a) $1-dt = \frac{1}{1+it} \rightarrow d = \frac{i}{1+it}$. If $i=.11$ then

$t = 1 \rightarrow d = .099099$, $t = .50 \rightarrow d = .104265$,

$t = \frac{1}{12} \rightarrow d = .109001$.