10 > MATHEMATICS OF INVESTMENT AND CREDIT

1.2.16 (a)
$$B_1 = B_0(1+i) + \sum_{k=1}^{n} a_k \left[1 + i(1-t_k) \right]$$

$$= B_0 + \sum_{k=1}^{n} a_k + \left[B_0 + \sum_{k=1}^{n} a_k (1-t_k) \right] \cdot i$$

(b) The balance is B_0 for t_1 years, $B_0 + a_1$ for $t_2 - t_1$ years, $B_0 + a_1 + a_2$ for $t_3 - t_2$ years, $B_0 + a_1 + a_2 + \cdots + a_n$ for $1 - t_n$ years. The average balance is

$$\overline{B} = \frac{B_0t_1 + (B_0 + a_1)(t_2 - t_1) + (B_0 + a_1 + a_2)(t_3 - t_2) + \dots + (B_0 + a_1 + a_2 + \dots + a_n)(1 - t_n)}{t_1 + (t_2 - t_1) - (t_3 - t_2) + \dots + (1 - t_n)}$$

$$= B_0 + a_1(1-t_1) + a_2(1-t_2) + \dots + a_n(1-t_n)$$

$$= B_0 + \sum_{k=1}^n a_k(1-t_k).$$

(c) Follows directly from (a) and (b).

1.2.17 The difference between the two payment plans is that the first 2 payments are deferred for 2 months, so the saving is

$$30\Big[(1+\nu) - (\nu^{24} + \nu^{25})\Big] = 12.68.$$

Alternatively, the present value under the current payment plan is

$$30[1+\nu+\nu^2+\cdots+\nu^{23}] = 643.67.$$

The present value under Smith's proposed payment plan is

$$30[v^2 + \dots + v^{25}] = 643.67v^2 = 630.99,$$

Saving is 12.68.

SOLUTIONS TO L'EXTBOOK EXERCISES 4 11

1.2.18 (a) Minimum monthly balance for January 2015 is 2500, for February 2015 is 6000, and for March 2015 is 9500. Interest earned is $(.10)\left(\frac{1}{12}\right)[2500+6000+9500] = 150$.

Balance on March 31 is 2500(4) + 1000(3) + 150 = 13,150.

(b) Minimum daily balance is 2500 for January 1-15, 3500 for January 16-31, 6000 for February 1-15, 7000 for February 16-28, 9500 for March 1-15, and 10,500 for March 16-31. Interest earned is

$$(.10) \left(\frac{1}{365}\right) \left((2500(15) + 3500(16) + 6000(15) + 7000(13) + 9500(15) + 10,500(16) \right) = 160.27.$$

Balance on March 31 is

$$2500(4) + 1000(3) + 160.27 = 13,160.27.$$

(c) Minimum monthly balance for January 2015 is 2500, so interest on January 31 is $\frac{10}{12}(2500) = 20.83$, so balance on January 31 (after deposit and interest) is 6020.83.

Minimum monthly balance for February 2015 is 6020.83, so interest on February 28 is $\frac{10}{12}$ (6020.83) = 50.17, so balance on February 28 is 9571.00.

Minimum monthly balance for March 2015 is 9571, so interest on March 31 is $\frac{10}{12}$ (9571) = 79.76, so balance on March 31 is 13,150.76.

(d) Minimum daily balance is 2500 for January 1-15 and 3500 for January 16-31, so interest on January 31 is 25.62.

Minimum daily balance is 6025.62 for February 1-15 and 7025.62 for February 16-28, so interest on February 28 is 49.79.

Minimum daily balance is 9575.41 for March 1-15 and 10,575.41 for March 16-31, so interest on March 31 is 85.71.

Balance on March 31 is 13,161.12.