1.4.5 Quarterly effective rate is $\frac{.0325}{4} = .008125$. Initial amount invested after commission is .99. At the end of 3 months, the accumulated value is .99(1.008125). This is then subject to the 1% commission for the rollover and then the 3-month interest rate of .008125. At the end of the year, the accumulated value is $[.99(1.008125)]^4 = .992198 = 1-.0078$. The effective after-commission return is −.78%.

1.4.6

$$i^{(.5)} = .5\left[(1.10)^{1/.5}-1\right] = .105$$
$$i^{(.25)} = .25\left[(1.1)^{1/.25}-1\right] = .116025$$
$$i^{(.1)} = .10\left[(1.10)^{1/.1}-1\right] = .159374$$
$$i^{(.01)} = .01\left[(1.10)^{1/.01}-1\right] = 137.796$$

1.4.7 From November 9 to January 1 (53 days) Smith earns (two full months) interest of $\frac{2}{12}(.1125)(1000) = 18.75$. Thus, Smith earns a 53-day effective rate of interest of .01875. The equivalent effective annual rate of interest is $i = (1.01875)^{365/53} - 1 = .1365$.

1.4.8 Left on deposit for a year at $i^{(12)} = .09$, X accumulates to $X(1.0075)^{12}$. If the monthly interest is reinvested at monthly rate .75%, the accumulated value at the end of the year is

$$X + X(.0075)\left[(1.0075)^{11} + (1.0075)^{10} + \cdots + (1.0075) + 1\right].$$

Since $1 + r + r^2 + \cdots + r^k = \frac{r^{k+1}-1}{r-1}$, it follows that the total at the end of the year with reinvestment of interest is

$$X\left[1 + (.0075)\cdot\frac{(1.0075)^{12}-1}{1.0075-1}\right] = X(1.0075)^{12}.$$

1.4.9 (a) We wish to show that

$$f'(m) = f(m)\cdot\left[\ln\left(1+\frac{j}{m}\right) - \frac{\frac{j}{m}}{1+\frac{j}{m}}\right] > 0.$$ First, $f(m) > 0$, since $j > 0$. Also, if $x > 0$ and $h(x) = \ln(1+x) - \frac{x}{1+x}$, then $h'(x) = \frac{x}{(1+x)^2} > 0$, and since $h(0) = 0$, it follows that $h(x) > 0$ for all $x > 0$. Letting $x = \frac{j}{m} > 0$, we see that $\ln\left(1+\frac{j}{m}\right) - \frac{\frac{j}{m}}{1+\frac{j}{m}} > 0$, which implies that $f'(m) > 0$.

(b) $g'(m) = (1+j)^{1/m} - 1 - \frac{(1+j)^{1/m}\cdot\ln(1+j)}{m}$

$$= (1+j)^{1/m}\cdot\left[(1-\ln(1+j)^{1/m}\right] - 1.$$

But $x[1-\ln x]$ has a maximum of 1 at $x = 1$, so that with $(1+j)^{1/m} = x$, we see that $g'(m) < 0$ for $m > 1$.

(c) Consider $\ln[f(m)] = m\cdot\ln\left(1+\frac{j}{m}\right) = \frac{\ln\left(1+\frac{j}{m}\right)}{\frac{1}{m}}$. Then

$$\lim_{m\to\infty}\ln[f(m)] = \lim_{m\to\infty}\frac{\ln\left(1+\frac{j}{m}\right)}{\frac{1}{m}} = \lim_{m\to\infty}\frac{\frac{1}{1+(j/m)}\cdot\left(-\frac{j}{m^2}\right)}{-\frac{1}{m^2}} = j.$$

Thus $\lim_{m\to\infty} f(m) = e^j$.

(d) $g(m) = \frac{(1+j)^{1/m}-1}{\frac{1}{m}}$, $\lim_{m\to\infty} g(m) = \lim_{m\to\infty}\frac{(1+j)^{1/m}\cdot\left(-\frac{\ln(1+j)}{m^2}\right)}{-\frac{1}{m^2}}$

$= \ln(1+j)$, since $\lim_{m\to\infty}(1+j)^{1/m} = 1$.

1.4.10 We want to find the smallest integer m so that

$f(m) = \left[1+\frac{.17}{m}\right]^m \geq 1.18$, $f(2) = 1.1772$, $f(3) = 1.1798$, $f(4) = 1.1811 \rightarrow m = 4$.

With 16%, we see that $\lim_{m\to\infty}\left[1+\frac{.16}{m}\right]^m = e^{.16} = 1.1735$, so that no matter how many times per year compounding takes place, a nominal rate of interest of 16% cannot accumulate to an effective rate of more than 17.35%.