SECTION 1.4

1.4.1 $m=1$ implies interest convertible annually (m=1 time per year), which implies the effective annual interest rate $i^{(1)}=i=.12$. We use Equation (1.5) to solve for i for the other values of m, as shown below.

m **(Effective Period)**	$\frac{1}{m}$-year effective interest rate $\frac{i^{(m)}}{m}$	$i = \left[1+\frac{i^{(m)}}{m}\right]^m - 1$
1 (1 year)	$\frac{i^{(1)}}{1}=\frac{.12}{1}=.12$	$(1.12)^1-1 = .12$
2 (6 months)	$\frac{i^{(2)}}{2}=\frac{.12}{2}=.06$	$(1.06)^2-1 = .1236$
3 (4 months)	$\frac{i^{(3)}}{3}=\frac{.12}{3}=.04$	$(1.04)^3-1 = .124864$
4 (3 months)	$\frac{i^{(4)}}{4}=\frac{.12}{4}=.03$	$(1.03)^4-1 = .125509$
6 (2 months)	$\frac{i^{(6)}}{6}=\frac{.12}{6}=.02$	$(1.02)^6-1 = .126162$
8 (1.5 months)	$\frac{i^{(8)}}{8}=\frac{.12}{8}=.015$	$(1.015)^8-1 = .126593$
12 (1 months)	$\frac{i^{(12)}}{12}=\frac{.12}{12}=.01$	$(1.01)^{12}-1 = .126825$
52 (1 week)	$\frac{i^{(52)}}{52}=\frac{.12}{52}=.0023$	$\left(1+\frac{.12}{52}\right)^{52}-1 = .12689$
365 (1 day)	$\frac{i^{(365)}}{365}=\frac{.12}{365}=.00033$	$\left(1+\frac{.12}{365}\right)^{365}-1 = .127475$
∞	$\lim_{y\to\infty}\left(1+\frac{.12}{y}\right)^y-1 = e^{.12}-1 = .127497$	

1.4.2 (a) $1000v_{.045}^{20}=414.64$

(b) $1000v_{.015}^{60}=409.30$

(c) $1000v_{.0075}^{120}=407.94$

1.4.3 Equivalent effective annual rates are
Mountain Bank: $(1.075)^2-1 = .155625$
River Bank:

$$\left(1+\frac{i^{(365)}}{365}\right)^{365}-1 \geq .155625$$

$$\rightarrow \left(1+\frac{i^{(365)}}{365}\right) \geq (1.155625)^{1/365} = 1.000396356$$

$$\rightarrow i^{(365)} \geq .144670$$

1.4.4 The last 6 months of the 8^{th} year is the time from the end of the 15^{th} to the end of the 16^{th} half-year.

0 $1H$ $2H=1Y$ $3H$ $4H=2Y$ --- $14H=7Y$ $15H$ $16H=8Y$

⇑

The amount of interest earned in Eric's account in the 16^{th} half-year is the change in balance from time $15H$ to time $16H$.

The balances at those points are $X\left(1+\frac{i}{2}\right)^{15}$ and $X\left(1+\frac{i}{2}\right)^{16}$.
The amount of interest earned by Eric in the period is

$$X\left(1+\frac{i}{2}\right)^{16} - X\left(1+\frac{i}{2}\right)^{15} = X\left(1+\frac{i}{2}\right)^{15}\left(\frac{i}{2}\right).$$

The balance in Mike's account at the end of the 15^{th} half-year (7.5 years) is $2X(1+7.5i)$, and the balance at the end of the 16^{th} half-year (8 years) is $2X(1+8i)$.

The interest earned by Mike in that period is

$$2X(1+8i)-2X(1+7.5i) = 2X(.5i) = 2X\left(\frac{i}{2}\right).$$

We are told that Eric and Mike earn the same amount of interest.
Therefore, $X\left(1+\frac{i}{2}\right)^{15}\left(\frac{i}{2}\right) = 2X\left(\frac{i}{2}\right)$, so that $\left(1+\frac{i}{2}\right)^{15}=2$.

Then $i = 2[2^{1/15}-1] = .0946$.