1.6.8 (a) $(1+i)^5 = \exp\left[\int_0^5 \left(.08+\frac{.025t}{t+1}\right) dt\right] = 1.616407 \rightarrow i = .1008$

(b) $1+i_1 = \exp\left[\int_0^1 \left(.08+\frac{.025t}{t+1}\right) dt\right] = 1.091629 \rightarrow i_1 = .091629$

$1+i_2 = \exp\left[\int_1^2 \left(.08+\frac{.025t}{t+1}\right) dt\right] = 1.099509 \rightarrow i_2 = .099509$

$i_3 = .102751, \quad i_4 = .104532, \quad i_5 = .105659$

(c) $1000 \cdot \exp\left[-\int_2^4 \left(.08+\frac{.025t}{t+1}\right) dt\right] = 821.00$

1.6.9 $Ke^{-2\delta} = 960,\ Ke^{-\delta} = 1200 \rightarrow e^{-\delta} = 1-d = .80 \rightarrow K = 1500$ and $d = .20$. If d changes to .10, then the present value becomes $1500(1-.10)^2 = 1215$.

1.6.10 $i = e^{\delta}-1,\ \delta' = 2\delta \rightarrow$

$i' = e^{\delta'}-1 = e^{2\delta}-1 = (1+i)^2-1 = 2i+i^2 > 2i,$

$d' = 1-e^{\delta'} = 1-e^{-2\delta} = 1-(1-d)^2 = 2d-d^2 < 2d$

1.6.11 (a) $1000(1.02)^2\left[1+(.08)\left(\frac{19}{365}\right)\right] = 1044.73$

(b) For $0 < t \le \frac{1}{4}$, $A(t) = 1000[1+(.08)t]$

for $\frac{1}{4} \le t \le \frac{1}{2}$, $A(t) = 1000(1.02)\left[1+(.08)\left(t-\frac{1}{4}\right)\right]$

for $\frac{1}{2} \le t \le \frac{3}{4}$, $A(t) = 1000(1.02)^2\left[1+(.08)\left(t-\frac{1}{2}\right)\right]$

for $\frac{3}{4} \le t \le 1$, $A(t) = 1000(1.02)^3\left[1+(.08)\left(t-\frac{3}{4}\right)\right]$

(c) For $0 < t = \frac{1}{4}$, $\delta_t = \frac{S'(t)}{S(t)} = \frac{.08}{1+(.08)t}$.

To find $\delta_{t+1/4}$, let $r = t+\frac{1}{4}$, or $t = r-\frac{1}{4}$.

Then

$$\delta_{t+1/4}, = \delta_r = \frac{S'\left(t+\frac{1}{4}\right)}{S\left(t+\frac{1}{4}\right)}$$

$$= \frac{S'(r)}{S(r)} = \frac{1000(1.02)(.08)}{1000(1.02)\left[1+(.08)\left(r-\frac{1}{4}\right)\right]} = \frac{.08}{1+(.08)t}.$$

The same occurs for $t+\frac{1}{2}$ and $t+\frac{3}{4}$.

1.6.12 (a) $\frac{A\left(t+\frac{1}{m}\right)-A(t)}{A\left(t+\frac{1}{m}\right)}$

(b) $d^{(m)} = m \cdot \frac{A\left(t+\frac{1}{m}\right)-A(t)}{A\left(t+\frac{1}{m}\right)} = \frac{A\left(t+\frac{1}{m}\right)-A(t)}{\frac{1}{m}\cdot A\left(t+\frac{1}{m}\right)}$

(c) Let $h = \frac{1}{m}$. Then $\lim_{m\to\infty} d^{(m)} = \lim_{h\to 0} \frac{A(t+h)-A(t)}{h \cdot A(t+h)} = \frac{A'(t)}{A(t)}$.

1.6.13 (a) $\delta_t = \frac{A'(t)}{A(t)} = \frac{a_1+2a_2t+\cdots+na_nt^{n-1}}{a_0+a_1t+\cdots+a_nt^n} \rightarrow \lim_{t\to\infty} \delta_t = 0$

(apply l'Hospital's rule)

(b) $A(t) = \exp\left[\int_0^t \delta_s\, ds\right] = \exp[k \cdot 2 \cdot t^{1/2}]$.

$$\lim_{t\to\infty} \frac{A(t)}{1+it} = \lim_{t\to\infty} \frac{e^{2kt^{1/2}} \cdot \frac{k}{t^{1/2}}}{i} = \infty$$

$$\lim_{t\to\infty} \frac{A(t)}{(1+i)^t} = \lim_{t\to\infty} \frac{e^{2kt^{1/2}} \cdot \frac{k}{t^{1/2}}}{(1+i)^t \cdot \ln(1+i)}$$

$$= \lim_{t\to\infty} \frac{1}{t^{1/2}\ln(1+i)} \cdot \frac{1}{\exp\left[t \cdot \ln(1+i) - 2kt^{1/2}\right]} = 0$$