

1.5.9 Suppose that the T-Bill's face amount is \$100. Then Smith purchases the bill for  $100\left[1 - \frac{182}{360}(.10)\right] = 94.94$  (nearest .01).

91 days later, the value of the T-Bill is

$$100\left[1 - \frac{91}{360}(.10)\right] = 97.47.$$

Smith's return for the 91 days is  $\frac{97.47}{94.94} - 1 = .0266$  (2.66%).

1.5.10 From Exercise 1.5.3, we have  $\frac{d^{(m)}}{m} = d_j$ , and  $\frac{i^{(m)}}{m} = j$ , so

$$(a) \quad \frac{d^{(m)}}{m} = d_j = \frac{j}{1+j} = \frac{\frac{i^{(m)}}{m}}{1 + \frac{i^{(m)}}{m}} \rightarrow d^{(m)} = \frac{i^{(m)}}{1 + \frac{i^{(m)}}{m}}, \text{ and}$$

$$(b) \quad i^{(m)} = \frac{d^{(m)}}{1 - \frac{d^{(m)}}{m}}.$$

$$1.5.11 \quad 1000 = 1200\left(\frac{1}{1+i}\right)(1-i) \rightarrow i = .0909$$

$$1.5.12 \quad 1000(1+j) = 1000 + 40(1+j)^2 \rightarrow (1+j)^2 - 25(1+j) + 25 = 0 \\ \rightarrow 1+j = 1.043561 \text{ or } 23.9564. \text{ Since } j < .10, \text{ it follows that } j = .0436.$$

## SECTION 1.6

1.6.1 Accumulated value at time 1 is

$$10,000 \times e^{\int_0^1 .05 dt} = 10,000 \times e^{.05} = 10,512.71.$$

Accumulated value at time 2 is

$$10,000 \times e^{\int_0^1 .05 dt + \int_1^2 [.05 + .02(t-1)] dt} = 10,000 \times e^{.05 + .06} = 11,162.78$$

$$1.6.2 \quad \left(1 + \frac{i^{(4)}}{4}\right)^{16} = e^{\int_0^3 .02t dt + \int_3^4 .045 dt} = e^{.09 + .045} \rightarrow i^{(4)} = .0339.$$

$$1.6.3 \quad \exp\left(\int_0^5 \frac{t}{k} dt\right) = (1-.04)^{-10} \rightarrow e^{125/3k} = 1.50414 \\ \rightarrow \frac{125}{3k} = \ln(1.50414) \rightarrow k = 102.$$

1.6.4 Effective annual rate for Tawny is  $i = (1.05)^2 - 1 = .1025$ .  
Tawny:  $\delta = \ln(1.1025) = .09758$ .

Fabio: Simple interest rate  $j \rightarrow \delta_i = \frac{j}{1+ij}$ .

At time 5,

$$.09758 = \frac{j}{1+5j} \rightarrow j = .1906 \rightarrow Z = 1000[1+5j] = 1953.$$

$$1.6.5 \quad 100\left[e^{\int_0^6 .01t^2 dt} - e^{\int_0^3 .01t^2 dt}\right] + X\left[e^{\int_3^6 .01t^2 dt} - 1\right] = X \\ \rightarrow 100(e^{.72} - e^{.09}) = X(2 - e^{.63}) \rightarrow X = 784.6.$$

1.6.6 Bruce's 6-month rate of interest is  $\frac{i}{2}$ , and 7.25 years is 14.5 6-month periods. Bruce's accumulated value after 7.25 years is  $100\left(1 + \frac{i}{2}\right)^{14.5} = 200$ . Solving for  $i$ , we get

$$\left(1 + \frac{i}{2}\right) = 2^{1/14.5} \rightarrow i = .0979.$$

Peter's account grows to  $100e^{7.25\delta} = 200$ , so that  $\delta = \frac{1}{7.25} \ln 2 = .0956$ . Then  $i - \delta = .0023 = .23\%$ .

$$1.6.7 \quad e^\delta \cdot (e^{1.5\delta})^4 = 1.36086 \rightarrow e^{7\delta} = 1.36086 \rightarrow 1+i = e^\delta = 1.045$$