1.1.4 There are two (equivalent) ways to approach this problem. We can update the balance in the account at the time of each transaction until we reach the end of 10 years, and set the balance equal to 10,000 to solve for K:

Balance at $t=4$ (after interest and withdrawal) is

$$10{,}000(1.04)^4-(1.05)K;$$

balance at $t=5$ is

$$\left[10{,}000(1.04)^4-(1.05)K\right](1.04)-(1.05)K;$$

balance at $t=6$ is

$$\left[\left[10{,}000(1.04)^4-(1.05)K\right](1.04)-(1.05)K\right](1.04)-K;$$

at $t=7$ is

$$\left[\left[\left[10{,}000(1.04)^4-(1.05)K\right](1.04)-(1.05)K\right](1.04)-K\right](1.04)-K;$$

at $t=10$ there is 3 years of compounding from time 7, so that

$$\left[\left[\left[\left[10{,}000(1.04)^4-(1.05)K\right](1.04)-(1.05)K\right](1.04)-K\right](1.04)-K\right](1.04)^3 = 10{,}000.$$

Solving for K from this equation results in $K=979.93$.

Alternatively, we can accumulate to time 10 the initial deposit and the withdrawals separately. The balance at time 10 is

$$10{,}000(1.04)^{10}-K(1.05)(1.04)^6-K(1.05)(1.04)^5-K(1.04)^4-K(1.04)^3=10{,}000.$$

This is the same equation as in the first approach (and must result in the same value of K). In general, when using compound interest for a series of deposits and withdrawals that occur at various points in time, the balance in an account at any given time point is the accumulated values of all deposits minus the accumulated values of all withdrawals to that time point. This is also the idea behind the "dollar-weighted rate of return," which will be discussed later.

1.1.5 (a) Over 5 years the unit value has grown by a factor of $(1.10)(1.16)(1-.07)(1.04)(1.32)=1.629074$. The average annual (compound) growth is $(1.629074)^{1/5}=1.1025$, or average annual growth of 10.25% for 5 years.

(b) Five-year average annual return from January 1, 2006 to December 31, 2015 is j, where $(1+j)^5(1.17)^5=(1.13)^{10}$, so that $j=.0914$. Annual return for 2014 is k, where $(1+k)(1.22)=(1.15)^2$, so that $k=.084$.

(c) Over n years the growth is $(1+i_1)(1+i_2)\cdots(1+i_n)=(1+i)^n$, where the average annual (compound) interest rate is i, so that $i+1$ is the geometric mean of $1+i_1, 1+i_2, \ldots, 1+i_n$.

Thus, $1+i \le \frac{(1+i_1)+(1+i_2)+\cdots+(1+i_n)}{n} \to i \le \frac{i_1+i_2+\cdots+i_n}{n}$.

1.1.6 We equate the accumulated value of Joe's deposits with that of Tina's. Note that it is assumed that for simple interest, each new deposit is considered separately and begins earning simple interest from the time of the new deposit.

$$10[1+(.11)]+30[1+5(.11)] = 67.5$$
$$= 10(1.0915)^{10-n}+30(1.0915)^{10-2n}.$$

This can be solved by substituting in the possible values of n until the equation is satisfied. Alternatively, the equation can be rewritten as

$$67.5(1.0195)^{2n}-10(1.0915)^{10}(1.0915)^n-30(1.0915)^{10} = 0$$

which is a quadratic equation in $(1.0915)^n$. The solution is

$$(1.0915)^n=\frac{24\pm 141.5}{135}.$$

We ignore the negative root and get $(1.0915)^n=1.226 \to n=2.3$.