

SECTION 1.4

1.4.1 $m=1$ implies interest convertible annually ($m=1$ time per year), which implies the effective annual interest rate $i^{(1)}=i=.12$. We use Equation (1.5) to solve for i for the other values of m , as shown below.

m (Effective Period)	$\frac{1}{m}$ -year effective interest rate $\frac{i^{(m)}}{m}$	$i = \left[1 + \frac{i^{(m)}}{m}\right]^m - 1$
1 (1 year)	$\frac{i^{(1)}}{1} = \frac{.12}{1} = .12$	$(1.12)^1 - 1 = .12$
2 (6 months)	$\frac{i^{(2)}}{2} = \frac{.12}{2} = .06$	$(1.06)^2 - 1 = .1236$
3 (4 months)	$\frac{i^{(3)}}{3} = \frac{.12}{3} = .04$	$(1.04)^3 - 1 = .124864$
4 (3 months)	$\frac{i^{(4)}}{4} = \frac{.12}{4} = .03$	$(1.03)^4 - 1 = .125509$
6 (2 months)	$\frac{i^{(6)}}{6} = \frac{.12}{6} = .02$	$(1.02)^6 - 1 = .126162$
8 (1.5 months)	$\frac{i^{(8)}}{8} = \frac{.12}{8} = .015$	$(1.015)^8 - 1 = .126593$
12 (1 months)	$\frac{i^{(12)}}{12} = \frac{.12}{12} = .01$	$(1.01)^{12} - 1 = .126825$
52 (1 week)	$\frac{i^{(52)}}{52} = \frac{.12}{52} = .0023$	$\left(1 + \frac{.12}{52}\right)^{52} - 1 = .12689$
365 (1 day)	$\frac{i^{(365)}}{365} = \frac{.12}{365} = .00033$	$\left(1 + \frac{.12}{365}\right)^{365} - 1 = .127475$
∞	$\lim_{y \rightarrow \infty} \left(1 + \frac{.12}{y}\right)^y - 1 = e^{.12} - 1 = .127497$	

1.4.2 (a) $1000v_{.045}^{20} = 414.64$

(b) $1000v_{.015}^{60} = 409.30$

(c) $1000v_{.0075}^{120} = 407.94$

1.4.3 Equivalent effective annual rates are

Mountain Bank: $(1.075)^2 - 1 = .155625$

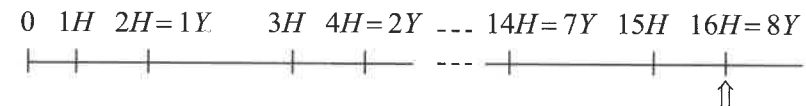
River Bank:

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} - 1 \geq .155625$$

$$\rightarrow \left(1 + \frac{i^{(365)}}{365}\right) \geq (.155625)^{1/365} = 1.000396356$$

$$\rightarrow i^{(365)} \geq .144670$$

1.4.4 The last 6 months of the 8th year is the time from the end of the 15th to the end of the 16th half-year.



The amount of interest earned in Eric's account in the 16th half-year is the change in balance from time 15H to time 16H.

The balances at those points are $X\left(1 + \frac{i}{2}\right)^{15}$ and $X\left(1 + \frac{i}{2}\right)^{16}$.

The amount of interest earned by Eric in the period is

$$X\left(1 + \frac{i}{2}\right)^{16} - X\left(1 + \frac{i}{2}\right)^{15} = X\left(1 + \frac{i}{2}\right)^{15} \left(\frac{i}{2}\right).$$

The balance in Mike's account at the end of the 15th half-year (7.5 years) is $2X(1+7.5i)$, and the balance at the end of the 16th half-year (8 years) is $2X(1+8i)$.

The interest earned by Mike in that period is

$$2X(1+8i) - 2X(1+7.5i) = 2X(.5i) = 2X\left(\frac{i}{2}\right).$$

We are told that Eric and Mike earn the same amount of interest.

Therefore, $X\left(1 + \frac{i}{2}\right)^{15} \left(\frac{i}{2}\right) = 2X\left(\frac{i}{2}\right)$, so that $\left(1 + \frac{i}{2}\right)^{15} = 2$.

Then $i = 2[2^{1/15} - 1] = .0946$.