

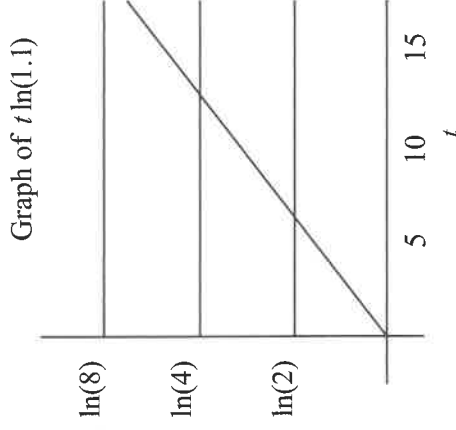
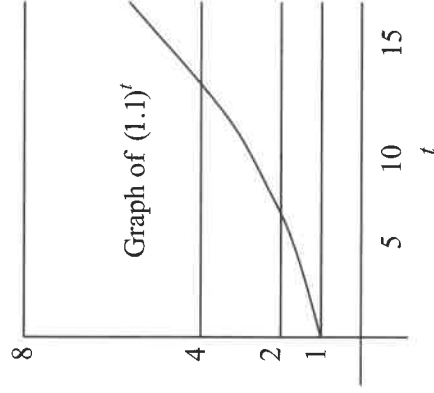
1.1.13 We use the following result from calculus: if f and g are differentiable functions such that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$, then $f(b) < g(b)$.

- (i) Suppose $0 < t < 1$. Let $f(i) = (1+i)^t$ and $g(i) = 1 + i \cdot t$. Then $f'(0) = g'(0) = 1$. If we can show that $f'(i) < g'(i)$ for any $i > 0$, then we can use the calculus result above to conclude that $f(i) < g(i)$ for any $i > 0$. First note that $f'(i) = t \cdot (1+i)^{t-1}$ and $g'(i) = t$. Since $i > 0$, it follows that $1 + i > 1$, and since $t < 1$, it follows that $t - 1 < 0$. Then $(1+i)^{t-1} < 1$, so $f'(i) < g'(i)$.

This completes the proof of part (i).

- (ii) Suppose that $t > 1$. Let $f(i) = 1 + i \cdot t$ and $g(i) = (1+i)^t$. Again $f(0) = g(0)$. If we can show that $f'(i) < g'(i)$ for any $i > 0$, then we can use the calculus result above to conclude that $f(i) < g(i)$ for any $i > 0$. Since $t > 1$ and $i > 0$ it follows that $t - 1 > 0$ and $1 + i > 1$. Thus $(1+i)^{t-1} > 1$, and it follows that $f'(i) = t < t \cdot (1+i)^{t-1} = g'(i)$. This completes the proof of part (ii).

1.1.14 Original graph is $y = (1+i)^t$. New graph is $10^y = (1+i)^t$, or, equivalently, $y = t \cdot \frac{\ln(1+i)}{\ln(10)}$, so that y is now a linear function of t .



SECTIONS 1.2 AND 1.3

1.2.1 Present value is

$$5000 \left[\frac{1}{1.06} + \frac{1}{(1.06)^2} + \frac{1}{(1.06)^3} + \frac{1}{(1.06)^4} \right] = 17,325.53.$$

1.2.2 Amount now required is

$$25,000[v^{17} + v^{15} + v^{12}] + 100,000[v^{20} + v^{18} + v^{15}] = 75,686$$

1.2.3 $28 = 15 + 16.50v \rightarrow v = .78779 \rightarrow i = .2692$

1.2.4 $1000 \cdot v_{.06}^3 \cdot v_{.07}^4 \cdot v_{.09}^3 = 494.62$

1.2.5 Equation of value on July 1, 2013 is

$$200(1.04) + 300v = 100(1.04)^4 + X \rightarrow X = 379.48.$$