Trabajo Práctico N°1

Simplificación de las funciones gracias a los operadores aplicando las propiedades del Álgebra de Boole postuladas en el PDF "Álgebra de Boole" de Acerbi y "Fundamentos de sistemas digitales", de Floyd, específicamente del capítulo 4 de este último libro.

1. $Z = (\bar{A} \cdot D \cdot (\bar{A}\bar{D}))(D\bar{B}AC B\bar{D}\bar{A}\bar{D}\bar{C}B)$

Propiedad conmutativa $(A \cdot B = B \cdot A)$

$Z = (\bar{A} \cdot D\bar{D}(\bar{A}))(D\bar{B}AC B\bar{D}\bar{A}\bar{D}\bar{C}B)$

$D \cdot \bar{D} = 0$, Regla 8 del Álgebra de Boole $(A \cdot \bar{A} = 0)$

$Z = (\bar{A} \cdot 0 \cdot (\bar{A}))(D\bar{B}ACB\bar{D}\bar{A}\bar{D}\bar{C}B)$

Al tener en cuenta la regla 3 del Álgebra de Boole $(A \cdot 0 = 0)$ y siendo toda la función múltiples operaciones AND, podemos concluir:

$Z = 0$

2. $Z = (BA + C\bar{A})\cdot(D\bar{A}\bar{B}\bar{C}A\bar{D})\cdot(\bar{C}A + C\bar{D}\bar{B}A)$

$\downarrow$ Propiedad conmutativa

$$Z = (BA + C\bar{A})\cdot(D\underbrace{A\bar{A}}_{A\cdot\bar{A}=0,\ \text{regla 8 de Floyd}}\bar{B}\bar{C}\bar{D})\cdot(\bar{C}A + C\bar{D}\bar{B}A)$$

$$Z = (BA + C\bar{A})\cdot(\underbrace{D\cdot 0\cdot\bar{B}\bar{C}\bar{D}}_{A\cdot 0=0,\ \text{regla 3 de Floyd}})\cdot(\bar{C}A + C\bar{D}\bar{B}A)$$

$$Z = \underbrace{(BA + C\bar{A})\cdot 0\cdot(\bar{C}\bar{A} + C\bar{D}\bar{B}A)}_{A\cdot 0=0,\ \text{regla 3 de Floyd}}$$

$Z = 0$

3. $Z = (\bar{A}CB\bar{C}\bar{B}\bar{A})\cdot(\bar{D}\bar{\bar{B}}\bar{C} + \bar{B}DA\bar{C} + AD\bar{C})$

$\downarrow$ Propiedad conmutativa

$$Z = (\underbrace{C\bar{C}}_{C\cdot\bar{C}=0,\ \text{regla 8 de Floyd}}\bar{A}B\bar{B}\bar{A})\cdot(\bar{D}B\bar{C} + \bar{B}DA\bar{C} + AD\bar{C})$$

$$Z = \underbrace{(0\cdot\bar{A}B\bar{B}\bar{A})}_{A\cdot 0=0,\ \text{regla 3 de Floyd}}\cdot(\bar{D}B\bar{C} + \bar{B}DA\bar{C} + AD\bar{C})$$

$$Z = \underbrace{0\cdot(\bar{D}B\bar{C} + \bar{B}DA\bar{C} + AD\bar{C})}_{A\cdot 0=0,\ \text{regla 3 de Floyd}}$$

$Z = 0$

4. $Z = (\overline{B}\,\overline{A}\,C + D\,A\,C) + (\overline{D}\,B\,C + \overline{C}\,A\,\overline{D}\,\overline{B}) + \underbrace{(B\,\overline{C}\,D\,A\,\overline{A} \cdot \overline{B}\,D\,C\,\overline{A})}$

$A \cdot \overline{A} = 0$, y $A \cdot 0 = 0$, el término se anula (3era y 8va regla de Floyd)

$Z = (\overline{B}\,\overline{A}\,C + D\,A\,C) + (\overline{D}\,B\,C + \overline{C}\,A\,\overline{D}\,\overline{B})$

(Propiedad asociativa de la suma $(A+B)+C = A+(B+C)$)

$Z = \overline{B}\,\overline{A}\,C + D\,A\,C + \overline{D}\,B\,C + \overline{C}\,A\,\overline{D}\,\overline{B}$

~~5. Z = C Ā B~~

5.

$Z = (\overline{A}\,B\,\underbrace{\overline{C}\,C}_{\overline{C}\cdot C = 0,\ 8^{va}\ \text{regla}}\,\overline{A})(B\,\overline{C}\,B\,C\,\overline{D}\,\overline{A})(\overline{D}\,B + D\,\overline{C}\,B + \overline{C}\,A) \cdot (\overline{A}\,C\,\overline{D}\,B\,\overline{D}\,C)$

$Z = \underbrace{(\overline{A}\,B \cdot 0 \cdot \overline{A})(B\,\overline{C}\,B\,C\,\overline{D}\,\overline{A})(\overline{D}\,B + D\,\overline{C}\,B + \overline{C}\,A) \cdot (\overline{A}\,C\,\overline{D}\,B\,\overline{D}\,C)}$

$A \cdot 0 = 0$, 3era regla, observando que son 4 operaciones AND.

$Z = 0$

6. $Z = (\overline{C}\,\overline{D}\,\overline{A}\,\overline{B}\,C\,\overline{A}\,\overline{D}) + (\overline{B}\,\overline{A} + \overline{D}\,\overline{C}\,\overline{A}\,B) + (C\,\overline{D}\,\overline{C}\,\overline{B}\,D\,C\,D\,\overline{A}\,B)$

Teniendo en cuenta la propiedad conmutativa, se agrupan $\overline{B} \cdot B = 0$ (8va regla), sabiendo que $A \cdot 0 = 0$, lo consiguiente es la anulación del término:

$Z = (\underbrace{\overline{C}\,\overline{D}\,\overline{A}\,\overline{B}\,C}_{\text{Idem.}}\,\overline{A}\,\overline{D}) + (\overline{B}\,\overline{A} + \overline{D}\,\overline{C}\,\overline{A}\,B)$

$Z = \bar{B}\bar{A} + \bar{D}\bar{C}\bar{A}B$

7. $Z = (\bar{A}\bar{D}BCDA\bar{D}A\bar{B})(\bar{C}\bar{A} + D\bar{B}CA + A\bar{D}\bar{C})$

Por el análisis lógico ya desarrollado, el primer término será 0 siempre. Al ser el término un multiplicando, toda la función resultará en 0:

$Z = 0$

8. $Z = (\bar{A}\bar{C}\bar{D}B\bar{B}C\bar{A}\bar{D}DAB\bar{C}) \cdot (\bar{B}A + C\bar{A}D\bar{B} + \bar{D}\bar{C}\bar{B}A)$

Análogamente, en esta función se procede con la misma lógica al anterior; concluyendo en lo mismo.

$Z = 0$

9. $(\bar{B}\bar{D}CA\bar{B}C\bar{A}) + (C\bar{D}A\bar{B}AD)$

Ambos términos de esta suma lógica resultan en 0 (debido a la propiedad conmutativa la 3era y 8va regla del Álgebra de Boole).

$Z = 0 + 0$

Utilizando la propiedad de Idempotencia $(A+A=A)$:

$Z = 0$

10. $Z=(DB\bar{C}A\bar{B}\bar{\bar{B}}C)+(\bar{A}\bar{B}\bar{D}CAC)+(BA\bar{C}D+\bar{B}\bar{C}\bar{A})+$
$+(C\bar{D}ABA\bar{B})$

Asimismo, se aplicarán las mismas leyes en los primer, segundo y cuarto términos

$Z=0+0+BA\bar{C}D+\bar{B}\bar{C}\bar{A}+0$

Por propiedad de Idempotencia: $(A+A=A$

$Z=BA\bar{C}D+\bar{B}\bar{C}\bar{A}+0$

Por "elemento neutro". $(A+0=A)$

$Z=BA\bar{C}D+\bar{B}\bar{C}\bar{A}$

11 $Z=(\bar{B}CDB)+(D\bar{A}C\bar{D}BC\bar{A})+(ADC\bar{B}+\bar{B}D)$

En el mismo sentido que en el caso anterior, los primeros dos términos se transforman en 0, y al ser el elemento neutro de la suma se ven a su vez anulados.

$Z=(ADC\bar{B}+\bar{B}D)$

$\downarrow$ ley distributiva, de Floyd (Factor Común)

$Z=\bar{B}D\underbrace{(AC+1)}$

$AC+1=1$, ya que la 2da regla (Floyd) postula que $A+1=1$

$Z=\bar{B}D(1)$

Tal como enuncia la 4ta regla (Floyd), $A \cdot 1 = A$.
Por consiguiente

$Z = \bar{B}D$

12. $Z = (\bar{D}\bar{A}C + \bar{C}\bar{A}) \cdot (\bar{D}A\bar{C}B + DB\bar{C}A)$

ley distributiva (Floyd)

$$Z = [\bar{A}(\bar{D}C + \bar{C})] \cdot [A \cdot B \cdot \bar{C} \cdot \underbrace{(\bar{D} + D)}]$$

$A + \bar{A} = 1$ 6ta regla (Floyd)
$A \cdot 1 = A$ 4ta regla (Floyd)

$$Z = \underbrace{[\bar{A}(\bar{D}C + \bar{C})] \cdot [AB\bar{C}]}$$

ley distributiva (Floyd) $A(B+C) = AB + AC$

$$Z = A \cdot \bar{A} \cdot (\bar{D}C + \bar{C}) \cdot B\bar{C}$$

Recurriendo a las 3era y 8va reglas del Álgebra de Boole, ya establecidas, podemos colegir que

$Z = 0$

13. $(B\bar{D}DC) \cdot (\bar{A}CAC\bar{B}D\bar{D}A\bar{B}) \cdot (C\bar{A}DCAB) \cdot$
$\cdot (DAC + C\bar{B})$

Aplicando el mismo método que en anteriores funciones, se utilizan las reglas 3era y 8va (Floyd) y se reduce la función a:

$Z = 0$

14. $Z = (A\bar{B}\bar{C}\bar{D}\bar{B}CA\bar{D}) + (C\bar{B}\bar{A}\bar{D}\bar{C}\bar{B}\bar{A}C\bar{D}B) + (\bar{B}\bar{C} + BD)$

Los primeros dos términos se anulan aplicando la ley conmutativa (Floyd), la 8va regla y la 3era, al igual que anteriormente:

$Z = \bar{B}\bar{C} + BD$

15. $Z = (\bar{D}CA + C\bar{A}\bar{D}) \cdot (\bar{A}\bar{D}C\bar{B}\bar{D}\bar{C}\bar{A}) \cdot (ADC + \bar{B}AC) \cdot (C\bar{D}AB + \bar{A}\bar{D}C)$

Ídem que en el caso anterior, en este caso al ser operaciones AND las que juntan a los paréntesis, toda la función resultará en 0 ya que $A \cdot 0 = 0$ (3era regla):

$Z = 0$

16. $(\bar{A}DBD\bar{A}C\bar{A}D) + (B\bar{D}AD) + (\bar{A}\bar{B} + D\bar{B}\bar{C}) = Z$

Idempotencia 8va y 3era reglas (Floyd)

$Z = 0\,\bar{A}DBC + 0 + \bar{A}\bar{B} + D\bar{B}\bar{C}$

$A + 0 = A$, 1era regla (Floyd)

$Z = \bar{A}DBC + \bar{A}\bar{B} + D\bar{B}\bar{C}$

17. $Z = (\bar{A}\bar{D}\bar{C} + \bar{A}\bar{D} + \bar{A}\bar{B}) + (\bar{A}CBD\bar{B}C) + (\bar{D}\bar{B}C + CD) + (\bar{A}\bar{D}\bar{B}CB\bar{C}D\bar{A}C\bar{A}\bar{B})$

Se anulan los 2do y 4to términos ya que B y $\bar{B}$ no pueden ser ambos 1 (variables [illegible])

$$Z = (\underbrace{\bar{A}\bar{D}\bar{C} + \bar{A}\bar{D} + \bar{A}\bar{B}}_{\text{ley distributiva, factor común } \bar{A}\bar{D} \text{ (Floyd)}}) + (\underbrace{\bar{D}\bar{B}C + CD}_{\text{ley distributiva, factor común } C \text{ (Floyd)}})$$

$$Z = [\bar{A}\bar{D}(\underbrace{\bar{C}+1}_{\text{2da ley (Floyd)}}) + \bar{A}\bar{B}] + C(\underbrace{\bar{D}\bar{B} + D}_{\text{11era regla (Floyd)}})$$

$A + 1 = 1$ $\qquad\qquad$ $A + \bar{A}B = A + B$

$$Z = \underbrace{\bar{A}\bar{D}\cdot 1}_{\text{4ta regla (Floyd)}} + \bar{A}\bar{B} + \underbrace{C(D + \bar{B})}_{\text{ley distributiva (Floyd)}}$$

$A \cdot 1 = A$ $\qquad\qquad$ $A(B+C) = AB + AC$

$$Z = \bar{A}\bar{D} + \bar{A}\bar{B} + CD + C\bar{B}$$

18. $(\bar{B}ADDB\bar{C}\bar{C}BA)\cdot(DABC\bar{A}CDCA)\cdot(\bar{B}A\bar{C}\bar{D}\bar{C}B)\cdot$
$\cdot(\bar{D}B\bar{A}\bar{D}\bar{B}B\bar{A}\bar{D}) = Z$

Mediante el uso de la conmutatividad y las 3era y 8va reglas, se infiere que:

$Z = 0$

19. $Z = (ACDB + BA) + (CA\bar{D}\bar{A}\bar{B}\bar{C}DA\bar{B}) + (\bar{A}DB + \bar{B}CD + D\bar{B}) +$
$+ (\bar{B}\bar{A}D + DBC\bar{A})$

El segundo término se anula y se aplica factor común (ley distributiva) a los restantes como se ve:

$$Z = AB\underbrace{(CD+1)}_{\substack{A+1=1 \\ 2^{da}\text{ regla}}} + D(\bar{A}B + \underbrace{\bar{B}C + \bar{B}}_{\substack{A+AB=A \\ 10^{ma}\text{ regla}}}) + \bar{A}D\underbrace{(\bar{B}+BC)}_{\substack{A+\bar{A}B=A+B \\ 11^{ma}\text{ regla}}}$$

$$Z = \underbrace{AB\cdot 1}_{\substack{A\cdot 1=A \\ 4^{ta}\text{ regla}}} + D\underbrace{(\bar{A}B+\bar{B})}_{\substack{A+\bar{A}B=A+B \\ 11^{ma}\text{ regla}}} + \bar{A}D(\bar{B}+C)$$

$$Z = AB + \underbrace{D(\bar{B}+\bar{A})}_{\text{ley distributiva}} + \underbrace{\bar{A}D(\bar{B}+C)}$$

$$Z = \underbrace{AB + \overbrace{D\bar{B}+D\bar{A}} + \overbrace{\bar{A}D\bar{B}+\bar{A}DC}}_{\text{ley Conmutativa}}$$

$$Z = \underbrace{D\bar{B}+D\bar{B}\bar{A}}_{\substack{A+AB=A \\ 10^{ma}\text{ regla}}} + \underbrace{\bar{A}D+\bar{A}D\bar{B}}_{\substack{A+AB=A \\ 10^{ma}\text{ regla}}} + AB$$

$$Z = \underbrace{D\bar{B}+D\bar{A}}_{\substack{\text{Factor común } D \\ \text{(ley distributiva)}}} + AB$$

$$Z = D(\bar{A}+\bar{B}) + AB$$

Se puede analizar la operación $\bar{A}+\bar{B}$ como $\overline{AB}$, ya que la operación AND negada es equivalente a una OR:

$Z = \underbrace{D(\overline{AB}) + AB}$

$A + \bar{A}B = A + B$

11ma regla

$Z = AB + D$

20. $Z = (AD + CDB + DB\bar{A}C) \cdot (\bar{D}\bar{B}CA + C\bar{B}) \cdot (D\bar{A}BC\bar{C}B)$
$\cdot (C\bar{B}D\bar{A} + \bar{D}A\bar{B} + B\bar{A}\bar{D}\bar{C})$

Dando por entendido que el tercer paréntesis resulta únicamente en 0, y siendo que el resto de paréntesis están en conjunción con este, se desprende que:

$Z = 0$,
por 3era y 8va reglas enunciadas en la obra de Floyd.

21. $Z = (B\bar{A}C\bar{D} + CBA) \cdot (\bar{B}\bar{A}\bar{C}\bar{C}\bar{B}AD) \cdot (\bar{C}DAB\bar{C}D\bar{B}A)$

Remitiéndonos a las propiedades y el análisis anteriores, demostramos que:

$Z = 0$

22. $(\bar{C}DB\bar{A}B\bar{A}) \cdot (B\bar{C}AD\bar{A}D\bar{C}BB\bar{C}\bar{D}) \cdot (AC\bar{D}AD\bar{C}\bar{B})$

Invocando lo ya demostrado, llegamos a:

$Z = 0$

$23\ (C\bar{A} + D\bar{A}\bar{B}\bar{C}\bar{A}C)\cdot(CAD\bar{A}D\bar{C}\bar{B}) = Z$

Reactivando las nociones previamente construidas, de ellas se sigue que:

$Z = 0$

24.

$Z = (\bar{D}\bar{C}\bar{C}ABD\bar{A}\bar{B}\bar{D})\cdot(D\bar{C}\bar{B}A + B\bar{A}CD + \bar{B}C\bar{D}\bar{A})$

Trayendo nuevamente los análisis ya deducidos, se deriva que:

$Z = 0$

$25\ (\overbrace{ACBD\bar{D}C\bar{A}}^{3^{era}\text{ y }8^{va}\text{ reglas}}) + (AB\bar{C} + \bar{D}\bar{B}) + (\underbrace{\bar{D}\bar{A}\bar{B}C + C\bar{D}\bar{B}}_{10^{ma}\text{ regla}}) +$

$+ (AC\bar{B} + \bar{B}\bar{A}) = Z$

$A + AB = A$

$\bar{D}\bar{A}\bar{B}C + C\bar{D}\bar{B} = C\bar{D}\bar{B}$

Anulando el primer término y reduciendo el tercero como se muestra, luego de aplicar leyes conmutativas y asociativas podemos expresar la función como

$Z = AB\bar{C} + \underbrace{\bar{D}\bar{B} + \bar{B}\bar{D}C}_{A + AB = A\ \ 10^{ma}\text{ regla}} + AC\bar{B} + \bar{B}\bar{A}$

$Z = AB\bar{C} + \bar{D}\bar{B} + AC\bar{B} + \bar{B}\bar{A}$

$$Z = AB\bar{C} + \bar{D}\bar{B} + \underbrace{AC\bar{B} + \bar{B}\bar{A}}_{\text{Factor común } \bar{B}}$$

$$Z = AB\bar{C} + \bar{D}\bar{B} + \bar{B}(\underbrace{AC + \bar{A}}_{A+\bar{A}B = A+B \quad 11^{va} \text{ regla}})$$

$$Z = AB\bar{C} + \bar{D}\bar{B} + \underbrace{\bar{B}(\bar{A} + C)}_{\text{ley distributiva}}$$

$$Z = AB\bar{C} + \bar{D}\bar{B} + \bar{B}\bar{A} + \bar{B}C$$

26 $Z = (\bar{D}CB + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B}) + (\bar{A}\bar{C}B + \bar{A}\bar{D}\bar{C}B) + (B\bar{D}\bar{C} + BD\bar{C}A + DA)$

ley asociativa

$$Z = \bar{D}CB + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B} + \underbrace{\bar{A}\bar{C}B + \bar{A}\bar{D}\bar{C}B}_{A+AB=A \quad 10^{ma} \text{ regla}} + \underbrace{B\bar{D}\bar{C} + BD\bar{C}A}_{\text{Factor común } B\bar{C}} + DA$$

$$Z = \bar{D}CB + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B} + \bar{A}\bar{C}B + B\bar{C}(\underbrace{\bar{D} + DA}_{A+\bar{A}B=A+B \quad 11^{ma} \text{ regla}}) + DA$$

$$Z = \bar{D}CB + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B} + \bar{A}\bar{C}B + \underbrace{B\bar{C}(\bar{D} + A)}_{\text{ley distributiva}} + DA$$

$$Z = \underbrace{\bar{D}CB}_{} + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B} + \bar{A}\bar{C}B + \underbrace{B\bar{C}\bar{D}}_{} + B\bar{C}A + DA$$

Factor común $\bar{D}B$

$$Z = \bar{D}B(\underbrace{C + \bar{C}}_{A+\bar{A}=1 \quad 6^{ta} \text{ regla}}) + \bar{D}\bar{B}A + D\bar{A}\bar{C}\bar{B} + \bar{A}\bar{C}B + B\bar{C}A + DA$$

Véase que 1 es el elemento neutro de la multi-plicación, por lo tanto:

$$Z = \underbrace{\overline{D}B + \overline{D}\,\overline{B}A}_{\text{Factor común } \overline{D}} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + B\overline{C}A + DA$$

$$Z = \overline{D}\underbrace{(B + \overline{B}A)}_{\substack{A + \overline{A}B = A + B \\ 11^{era} \text{ regla}}} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + B\overline{C}A + DA$$

$$Z = \underbrace{\overline{D}(B + A)}_{\text{ley distributiva}} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + B\overline{C}A + DA$$

$$Z = \overline{D}B + \underbrace{\overline{D}A}_{} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + B\overline{C}A + \underbrace{DA}_{}$$

Factor común A

$$Z = \overline{D}B + A\underbrace{(\overline{D} + D)}_{\substack{A + \overline{A} = 1,\ 6^{ta} \text{ regla} \\ (\text{elemento neutro})}} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + B\overline{C}A$$

$$Z = \overline{D}B + \underbrace{A}_{} + D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B + \underbrace{B\overline{C}A}_{}$$

$A + AB = A$
10^{ma} regla (y leyes asociativa y conmutativa)

$$Z = \overline{D}B + A + \underbrace{D\overline{A}\,\overline{C}\,\overline{B} + \overline{A}\,\overline{C}B}_{\text{Factor común } \overline{A}\,\overline{C}}$$

$$Z = \overline{D}B + A + \overline{A}\,\overline{C}\underbrace{(D\overline{B} + B)}_{\substack{A + \overline{A}B = A + B \\ 10^{ma} \text{ regla}}}$$

$$Z = \overline{D}B + A + \underbrace{\overline{A}\,\overline{C}(B + D)}_{\text{ley distributiva}}$$

$$Z = \underbrace{\bar{D}B + A + \bar{A}\bar{C}B + \bar{A}\bar{C}D}_{\text{Ley conmutativa}}$$

$$Z = \underbrace{A + \bar{A}\bar{C}B}_{} + \bar{A}\bar{C}D + \bar{D}B$$

$A + \bar{A}B = A + B$

11era regla

$$Z = \underbrace{A}_{} + \bar{C}B + \underbrace{\bar{A}\bar{C}D}_{} + \bar{D}B$$

$A + \bar{A}B = A + B$

11era regla

$$Z = A + \underbrace{\bar{C}B}_{} + \bar{C}D + \bar{D}B$$

elemento neutro de la multiplicación (1)

$$Z = A + \bar{C}B \cdot \underbrace{1}_{} + \bar{C}D + \bar{D}B$$

$A + \bar{A} = 1$

6ta regla

$$Z = A + \underbrace{\bar{C}B(D + \bar{D})}_{\text{ley distributiva}} + \bar{C}D + \bar{D}B$$

$$Z = \underbrace{A + \bar{C}BD + \bar{C}B\bar{D} + \bar{C}D + \bar{D}B}_{\text{leyes conmutativa y asociativa}}$$

$$Z = A + (\underbrace{\bar{C}BD + \bar{C}D}_{}) + (\underbrace{\bar{C}B\bar{D} + \bar{D}B}_{})$$

$A + AB = A$

10ma regla

$$Z = A + D\bar{C} + B\bar{D}$$

27. ~~C~~

$Z = (A\bar{C}\bar{B}BDCA) \cdot (BD + BC\bar{A}D + \bar{D}C\bar{B}) \cdot (\bar{D}B + \bar{C}B\bar{D} + B$

Refiriendonos a las reglas que establecen la presencia de dos elementos opuestos en esta función la transforman inmediatamente en

$Z = 0$

28. $Z = (B\bar{A}C\bar{B}\bar{C}) + (D\bar{B}\bar{D}B\bar{A}\bar{B}A) + (DCB\bar{C}\bar{D}B\bar{D}\bar{B}) +$
$+ (A\bar{D} + D\bar{C}B + DC\bar{A}B)$

Los primeros tres términos son anulados, el cuarto permanece debido a que es una suma lógica:

$Z = A\bar{D} + \underbrace{D\bar{C}B + DC\bar{A}B}_{\text{Factor común } DB}$

$Z = A\bar{D} + DB(\underbrace{\bar{C} + C\bar{A}}_{\substack{A + \bar{A}B = A + B \\ 11^{va} \text{ regla}}})$

$Z = A\bar{D} + \underbrace{DB(\bar{C} + \bar{A})}_{\text{ley distributiva}}$

$Z = A\bar{D} + DB\bar{C} + DB\bar{A}$

29. $Z = (\bar{C}DBD) + (B\bar{A}D\bar{C}\bar{D}BC\bar{A}) + (\bar{D}B + \bar{D}CB + \bar{D}\bar{B}\bar{A}\bar{C})$

El segundo término se anula (3era y 8va reglas) y al primero se le eliminan redundancias (1ra regla $A \cdot A = A$): Se aplica ley asociativa:

$$Z = \bar{C}D\bar{B} + \underbrace{\bar{D}B + \bar{D}\bar{C}B}_{\substack{A + AB = A \\ \text{10ma regla}}} + \bar{D}\bar{B}\bar{A}\bar{C}$$

$$Z = \bar{C}D\bar{B} + \underbrace{\bar{D}B + \bar{D}\bar{B}\bar{A}\bar{C}}_{\text{Factor común } \bar{D}}$$

$$Z = \bar{C}D\bar{B} + \bar{D}(\underbrace{B + \bar{B}\bar{A}\bar{C}}_{\substack{A + \bar{A}B = A + B \\ \text{11era regla}}})$$

$$Z = \bar{C}D\bar{B} + \underbrace{\bar{D}(B + \bar{A}\bar{C})}_{\text{ley distributiva}}$$

$$Z = \bar{C}D\bar{B} + \bar{D}B + \bar{D}\bar{A}\bar{C}$$

30. $Z = (C\bar{D}A\bar{B}ADC) + (B\bar{C} + \bar{B}DCA)$

El primer termino es anulado resultando en lo siguiente:

$$Z = B\bar{C} + \bar{B}DCA$$

31.

$$Z=(A\bar{C}B\bar{A}\bar{C}\bar{B}C\bar{A}D\bar{B})\cdot(\bar{A}CBBA\bar{C}\bar{A}B)\cdot(\bar{B}\bar{C}\bar{A}\bar{D}\bar{C}\bar{B}\bar{D})\cdot$$
$$\cdot(CD\bar{B}A\bar{D}\bar{C}AB\bar{A}DC)$$

A sabiendas de que esto pueda sonar reiterativo, basándome en las 3era y 8va reglas, indefectiblemente el resultado de esta función es:

$$Z=0$$

32.

$$Z=(\bar{A}D\bar{B}+\bar{B}A\bar{D}+AD\bar{C})\cdot(\underbrace{C\bar{B}DA+\bar{B}A\bar{D}}_{\text{Factor común } A\bar{B}})$$

$$Z=(\bar{A}D\bar{B}+\bar{B}A\bar{D}+AD\bar{C})\cdot[A\bar{B}(\underbrace{CD+\bar{D}}_{\substack{A+\bar{A}B=A+B \\ 11^{era}\text{ regla}}})]$$

$$Z=(\bar{A}D\bar{B}+\bar{B}A\bar{D}+AD\bar{C})\cdot[A\bar{B}(\bar{D}+C)]$$

$$Z=\underbrace{(\bar{A}D\bar{B}+\bar{B}A\bar{D}+AD\bar{C})\cdot[A\bar{B}\bar{D}+A\bar{B}C]}_{\substack{12^{da}\text{ regla} \\ (A+B)(A+C)=A+BC}}$$

$$Z=A\bar{B}\bar{D}+[\underbrace{(\bar{A}D\bar{B}+AD\bar{C})(A\bar{B}C)}_{\text{ley distributiva}}]$$

$$Z=A\bar{B}\bar{D}+\bar{A}\bar{B}DA\bar{B}C+AD\bar{C}A\bar{B}C$$

Los segundo y tercer términos se anulan, sabiendo que 0 es el elemento neutro de la suma:

$Z = A\overline{B}\overline{D}$

33.

$Z = (\overline{D}A\overline{A}CD\overline{B}A) + (\overline{A}D\overline{D}C\overline{D}BC\overline{A}) + (\overline{A}C\overline{D}AB\overline{C}) +$
$+ (\overline{A}\overline{D}B + BD)$

Permanece únicamente el último término, al aplicar las reglas 3era y 8va:

$Z = \underbrace{\overline{A}\overline{D}B + BD}$

Factor común B

$Z = B(\underbrace{\overline{A}\overline{D} + D})$

$A + \overline{A}B = A + B$
11era regla

$Z = \underbrace{B(D + \overline{A})}$

ley distributiva

$Z = BD + B\overline{A}$

34.

$Z = (\overline{C}\overline{D} + A\overline{D}B\overline{C} + \overline{B}AC\overline{D}) \cdot (D\overline{B} + \overline{D}BA) \cdot (B\overline{A}C + AD\overline{C} + \overline{C}\overline{B}D)$

Tal como sucede con otros ejercicios, se puede prever que la función resultará en 0 al multiplicar los sumandos de los paréntesis con el uso de la ley distributiva:

$Z = 0$

35.

$Z = (AC\bar{B}\bar{D} + DC\bar{A}B)(\bar{D}AC\bar{B} + A\bar{B} + B\bar{D}\bar{C}A)\cdot(\bar{C}\bar{A}B\bar{C}\bar{D}A)\cdot$
$\cdot(A\bar{D}\bar{B}C\bar{D}\bar{B}A\bar{C}\bar{C}\bar{B}\bar{A})$

Subrayando nuevamente que se presenta una función constituida por conjunciones, la presencia de $A\cdot\bar{A} = 0$ anula completamente la función:

$Z = 0$

36.

$Z = (\bar{D}BBA\bar{C}\bar{B}\bar{D})\cdot(\bar{D}\bar{B}\bar{A}C + C\bar{D}\bar{A})\cdot(A\bar{B}DB)\cdot(D\bar{B}\bar{C}\bar{A}\bar{B}\bar{A})$

Con el riesgo de parecer redundante, remarco que en este otro caso la función vuelve a anularse por completo:

$Z = 0$

37.

$Z = (ABD + \bar{B}\bar{D}\bar{C} + CBD)\cdot(BAD + C\bar{D})\cdot(\bar{A}CD\bar{C}BDA)\cdot$
$\cdot(B\bar{A}CD + C\bar{D}\bar{A} + D\bar{C})$

Aunque repita lo anterior, aplicando las 3^{era} y 8^{va} reglas (Floyd) resulta que:

$Z = 0$

38.

$$Z=(B\bar{C}\bar{B}D\bar{A}\bar{C}\bar{D}\bar{A}B\bar{C})+(C\bar{D}\bar{B}A\bar{A}\bar{D}CB)+(\bar{C}\bar{D}D\bar{A}\bar{A}\bar{B}D\bar{C})+$$
$$+(DCAC\bar{D}\bar{A}B\bar{C}D\bar{B})$$

No sin cierta reiteración, insisto en que la función se anula por completo. En este caso, aún siendo una suma, todos los términos se anulan y, debido a que $0+0+0+0=0$, se deduce que:

$$Z=0$$

39.

$$Z=(C\bar{B}\bar{D}+A\bar{D}\bar{B})+(\bar{A}\overbrace{BB\bar{A}\bar{A}}^{7^{ma}\text{ regla } A\cdot A=A}C\bar{D})+(\underbrace{DA\bar{C}\bar{B}+\bar{B}\bar{A}}_{\substack{A+\bar{A}B=A\\ 11^{era}\text{ regla}}})+$$
$$+(\underbrace{A\bar{B}+\bar{B}\bar{C}AD+CAB}_{\text{Factor común } A})$$

$$Z=(C\bar{B}\bar{D}+A\bar{D}\bar{B})+(\bar{A}BC\bar{D})+\bar{B}\bar{A}+D\bar{C}\bar{B}+[A\cdot(\underbrace{\bar{B}+\bar{B}\bar{C}D}_{\substack{A+AB=A\\ 10^{ma}\text{ regla}}}+CB)]$$

$$Z=(C\bar{B}\bar{D}+A\bar{D}\bar{B})+(\bar{A}BC\bar{D})+\bar{B}\bar{A}+D\bar{C}\bar{B}+[A(\underbrace{\bar{B}+CB}_{\substack{A+\bar{A}B=A+B\\ 11^{era}\text{ regla}}})]$$

$$\underbrace{Z=(C\bar{B}\bar{D}+A\bar{D}\bar{B})+(\bar{A}BC\bar{D})+\bar{B}\bar{A}+D\bar{C}\bar{B}+[\underbrace{A\cdot(\bar{B}+C)}_{\text{ley distributiva}}]}_{\text{ley asociativa}}$$

$$Z=C\bar{B}\bar{D}+A\bar{D}\bar{B}+\bar{A}BC\bar{D}+\bar{B}\bar{A}+D\bar{C}\bar{B}+A\bar{B}+AC$$

$$Z = C\bar{B}\bar{D} + \underbrace{A\bar{D}\bar{B}} + \bar{A}BC\bar{D} + \bar{B}\bar{A} + D\bar{C}\bar{B} + \underbrace{A\bar{B}} + AC$$

$A + AB = A$
10ma regla

$$Z = \underbrace{C\bar{B}\bar{D} + \bar{A}BC\bar{D}}_{\bar{B}\qquad C} + \underbrace{\bar{B}\bar{A} + D\bar{C}\bar{B} + A\bar{B}}_{\text{Factor común } \bar{B}} + \underbrace{AC}_{\text{Factor común}}$$

$$Z = \bar{B}(\underbrace{C\bar{D} + \bar{A} + \bar{C}D + A}) + C(\underbrace{\bar{A}B\bar{D} + A})$$

$A + \bar{A} = 1$, 6ta regla
$A + 1 = 1$, 2da regla

$A + \bar{A}B = A + B$
11era regla

$$Z = \underbrace{\bar{B}(1)}_{\substack{A.1 = A \\ \text{4ta regla}}} + \underbrace{C\cdot(A + B\bar{D})}_{\text{ley distributiva}}$$

$$Z = \underbrace{\bar{B}} + CA + \underbrace{CB\bar{D}}$$

$A + \bar{A}B = A + B$
11era regla

$$Z = \bar{B} + CA + C\bar{D}$$

40.

$$Z = (\bar{B}AA\bar{B}\bar{D}\bar{A}\bar{B}\bar{C}) + (\bar{A}DC + \bar{C}AB) + (B\bar{C}\bar{D}\bar{C}\bar{B}\bar{B}\bar{D}\bar{C}A)$$

Volviendo, quizás innecesariamente, sobre lo ya desarrollado, los primer y último términos de esta función se anulan, resultando en:

$$Z = \bar{A}DC + \bar{C}AB$$

Confección de mapas de Karnaugh y simplificación de las ecuaciones por minterms y máxterms, habiendo tenido en cuenta el análisis previo:

Z_m = minterm

Z_M = máxterm

1. $Z = 0$

D	C	B	A	Z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$ (no hay ninguna agrupación de unos)

$Z_M = 0$ (se agrupan todos los bits, así que DCBA cambian todos al solaparse)

Esta tabla de verdad se omitirá en las siguientes ecuaciones y solo se mostrará Z y el decimal formado por DCBA, a efectos de eficiencia expositiva.

2. Z = 0

Dec	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

3. Z = 0

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

4

DEC	Z
0	0
1	1
2	0
3	0
4	1
5	0
6	1
7	1
8	0
9	0
10	0
11	0
12	1
13	1
14	0
15	1

DC \ BA	00	01	11	10
00	0	1	0	0
01	1	0	1	1
11	1	1	1	0
10	0	0	0	0

$Z_m = \overline{D}\,\overline{C}\,\overline{B}A + \overline{D}C\overline{A} + DC\overline{B} + CBA$

$\overline{Z_M} = D\overline{C} + \overline{D}C\overline{B}A + DB\overline{A} + \overline{C}B + \overline{A}\,\overline{C}$) [illegible]

$\overline{\overline{Z_M}} = \overline{D\overline{C} + \overline{D}C\overline{B}A + DB\overline{A} + \overline{C}B + \overline{A}\,\overline{C}}$

$Z_M = \overline{D} \div \overline{\overline{C}} \div \overline{\overline{D}} \div \overline{C} \div \overline{\overline{B}} \div \overline{A} \div \overline{D} \div \overline{B} \div \overline{\overline{A}} \div \overline{\overline{C}} \div \overline{B} \div \overline{\overline{A}} \div \overline{\overline{C}}$

$Z_M = (\overline{D} + C)(D + \overline{C} + B + \overline{A})(\overline{D} + \overline{B} + A)(C + \overline{B})(A + C)$

5. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

6. $Z = \bar{B}\bar{A} + \bar{D}\bar{C}\bar{A}B$

DEC	Z
0	1
1	0
2	1
3	0
4	1
5	0
6	0
7	0
8	1
9	0
10	0
11	0
12	1
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	1	0	0	1
01	1	0	0	0
11	1	0	0	0
10	1	0	0	0

$Z_m = \bar{B}\bar{A} + \bar{D}\bar{C}\bar{A}$

$\bar{Z}_M = A + BC + DB$

$\bar{\bar{Z}}_M = \overline{A + BC + DB}$

Teorema de de Morgan

$Z_M = \bar{A} \cdot \bar{B} + \bar{C} \cdot \bar{D} + \bar{B}$

$Z_M = \bar{A}(\bar{B} + \bar{C})(\bar{D} + \bar{B})$

7. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

8. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

9. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

10. $Z = BA\overline{C}D + \overline{B}\overline{C}\overline{A}$

DEC	Z
0	1
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	1
9	0
10	0
11	1
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	0	0
10	1	0	1	0

$Z_m = D\overline{C}BA + \overline{C}\overline{B}\overline{A}$

$\overline{Z_M} = C + \overline{B}A + B\overline{A} + B\overline{D}$ de Morgan

$\overline{\overline{Z_M}} = \overline{C + \overline{B}A + B\overline{A} + B\overline{D}}$

$Z_M = \overline{C} \mp \overline{\overline{B}} \mp \overline{A} \mp \overline{B} \mp \overline{\overline{A}} \mp \overline{B} \mp \overline{\overline{D}}$

$Z_M = \overline{C}(B + \overline{A})(\overline{B} + A)(\overline{B} + D)$

11. $Z = \overline{B}D$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	1
9	1
10	0
11	0
12	1
13	1
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	0	0
10	1	1	0	0

$Z_m = D\overline{B}$

$\overline{Z_M} = \overline{D} + B$ de Morgan

$\overline{\overline{Z_M}} = \overline{\overline{D} + B}$

$Z_M = \overline{\overline{D}} \mp \overline{B}$

$Z_M = D\overline{B}$

12. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

13. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

14. $Z = \bar{B}\bar{C} + BD$

DEC	Z
0	1
1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	1
9	1
10	1
11	1
12	0
13	0
14	1
15	1

DC \ BA	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	0	1	1
10	1	1	1	1

$Z_m = \bar{B}\bar{C} + DB$

$\bar{Z}_M = C\bar{B} + B\bar{D}$

$\bar{\bar{Z}}_M = \overline{C\bar{B} + B\bar{D}}$ de Morgan

$Z_M = \bar{C} \div \bar{\bar{B}} \mp \bar{B} \div \bar{\bar{D}}$

$Z_M = (\bar{C} + B)(\bar{B} + D)$

15. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

16. $Z = \bar{A}DCB + \bar{A}\bar{B} + D\bar{B}\bar{C}$

DEC	Z
0	1
1	0
2	0
3	0
4	1
5	0
6	0
7	0
8	1
9	1
10	0
11	0
12	1
13	0
14	1
15	0

DC \ BA	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	0	1
10	1	1	0	0

$Z_m = D\bar{C}\bar{B} + DC\bar{A} + \bar{B}\bar{A}$

$\bar{Z}_M = \bar{C}B + B\bar{D} + A\bar{D} + AC$

de Morgan

$\bar{\bar{Z}}_M = \overline{\bar{C}B + B\bar{D} + A\bar{D} + AC}$

$Z_M = \bar{\bar{C}} \div \bar{B} \mp \bar{B} \div \bar{\bar{D}} \mp \bar{A} \div \bar{\bar{D}} \mp \bar{A} \div \bar{C}$

$Z_M = (C + \bar{B})(\bar{B} + D)(\bar{A} + D)(\bar{A} + \bar{C})$

17 $Z = \bar A \bar D + \bar A \bar B + CD + C\bar B$

DEC	Z
0	1
1	0
2	1
3	0
4	1
5	1
6	1
7	0
8	1
9	0
10	0
11	0
12	1
13	1
14	1
15	1

DC \ BA	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	1	1	1	1
10	1	0	0	0

$Z_m = \bar D \bar A + \bar B \bar A + C\bar B + DC$

$\bar Z_M = \bar C A + BA\bar D + D\bar C B$ de Morgan

$\bar{\bar Z}_M = \overline{\bar C A + BA\bar D + D\bar C B}$

$Z_M = \bar{\bar C}\,\bar{\cdot}\,\bar A\,\bar{+}\,\bar B\,\bar{\cdot}\,\bar A\,\bar{\cdot}\,\bar{\bar D}\,\bar{+}\,\bar D\,\bar{\cdot}\,\bar{\bar C}\,\bar{\cdot}\,\bar B$

$Z_M = (C+\bar A)(\bar B+\bar A+D)(\bar D+C+\bar B)$

18. $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

19. $Z = AB + D$

DEC	Z
0	0
1	0
2	0
3	1
4	0
5	0
6	0
7	1
8	1
9	1
10	1
11	1
12	1
13	1
14	1
15	1

DC \ BA	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	1	1
10	1	1	1	1

$Z_m = BA + D$

$\bar Z_M = \bar B \bar D + \bar A \bar D$ de Morgan

$\bar{\bar Z}_M = \overline{\bar B \bar D + \bar A \bar D}$

$Z_M = \bar{\bar B}\,\bar{\cdot}\,\bar{\bar D}\,\bar{+}\,\bar{\bar A}\,\bar{\cdot}\,\bar{\bar D}$

$Z_M = (B+D)(A+D)$

20. $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

21. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

22. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

23. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

24. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

25. $Z = AB\bar{C} + \bar{D}\bar{B} + \bar{B}\bar{A} + \bar{B}C$

DEC	Z
0	1
1	1
2	0
3	1
4	1
5	1
6	0
7	0
8	1
9	0
10	0
11	1
12	1
13	1
14	0
15	0

DC \ BA	00	01	11	10
00	1	1	1	0
01	1	1	0	0
11	1	1	0	0
10	1	0	1	0

$Z_m = C\bar{B}\bar{A} + \bar{D}\bar{B} + \bar{B}A + C\bar{B}$

$\bar{Z}_M = \bar{B}AD\bar{C} + B\bar{A} + CB$

$\bar{\bar{Z}}_M = \overline{\bar{B}AD\bar{C} + B\bar{A} + CB}$ de Morgan

$Z_M = \bar{\bar{B}} \cdot \bar{A} \cdot \bar{D} \cdot \bar{\bar{C}} + \bar{B} \cdot \bar{\bar{A}} + \overline{C \cdot B}$

$Z_M = (B + \bar{A} + \bar{D} + C)(\bar{B} + A)(\bar{C} + \bar{B})$

26. $Z = A + D\bar{C} + B\bar{D}$

DEC	Z
0	0
1	1
2	1
3	1
4	0
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	0
13	1
14	0
15	1

DC \ BA	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	1	0
10	1	1	1	1

$Z_m = B\bar{D} + \bar{C}D + A$

$\overline{Z_M} = \bar{B}\bar{A}\bar{D} + DC\bar{A}$

de Morgan

$\overline{\overline{Z_M}} = \overline{\bar{B}\bar{A}\bar{D} + DC\bar{A}}$

$Z_M = \bar{\bar{B}} \cdot \bar{\bar{A}} \cdot \bar{\bar{D}} + \bar{D} \cdot \bar{C} \cdot \bar{\bar{A}}$

$Z_M = (B + A + D)(\bar{D} + \bar{C} + A)$

27. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

28. $Z = A\bar{D} + DB\bar{C} + DB\bar{A}$

DEC	Z
0	0
1	1
2	0
3	1
4	0
5	1
6	0
7	1
8	0
9	0
10	1
11	1
12	0
13	0
14	1
15	0

DC \ BA	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	0	0	1
10	0	0	1	1

$Z_m = \bar{C}AB + DB\bar{A} + A\bar{D}$

$\overline{Z_M} = ADC + \bar{B}D + \bar{A}\bar{D}$

de Morgan

$\overline{\overline{Z_M}} = \overline{ADC + \bar{B}D + \bar{A}\bar{D}}$

$Z_M = \bar{A} \cdot \bar{D} \cdot \bar{C} + \bar{\bar{B}} \cdot \bar{D} + \bar{\bar{A}} \cdot \bar{\bar{D}}$

$Z_M = (\bar{A} + \bar{D} + \bar{C})(B + \bar{D})(A + D)$

29. $Z = \bar{C}D\bar{B} + \bar{D}B + \bar{D}\bar{A}\bar{C}$

DEC	Z
0	1
1	0
2	1
3	1
4	0
5	0
6	1
7	1
8	1
9	1
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	1	0	1	1
01	0	0	1	1
11	0	0	0	0
10	1	1	0	0

$Z_m = B\bar{D} + \bar{C}\bar{D}\bar{A} + D\bar{C}\bar{B}$

$\overline{Z_M} = \bar{B}A\bar{D} + C\bar{B} + DB$

de Morgan

$\overline{\overline{Z_M}} = \overline{\bar{B}A\bar{D} + C\bar{B} + DB}$

$Z_M = \bar{\bar{B}} \cdot \bar{A} \cdot \bar{\bar{D}} + \bar{C} \cdot \bar{\bar{B}} + \bar{D} \cdot \bar{B}$

$Z_M = (B + \bar{A} + D)(\bar{C} + B)(\bar{D} + \bar{B})$

30. $Z = B\bar{C} + \bar{B}DCA$

DEC	Z
0	0
1	0
2	1
3	1
4	0
5	0
6	0
7	0
8	0
9	0
10	1
11	1
12	0
13	1
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	0	0	0	0
10	0	0	1	1

$Z_m = \bar{C}B + DAC\bar{B}$

$\bar{Z}_M = BC + \bar{A}C + \bar{C}\bar{B} + \bar{D}C$

de Morgan

$\bar{\bar{Z}}_M = \overline{BC + \bar{A}C + \bar{C}\bar{B} + \bar{D}C}$

$Z_M = \bar{B} \div \bar{C} \mp \bar{\bar{A}} \div \bar{C} \mp \bar{\bar{C}} \div \bar{\bar{B}} \mp \bar{\bar{D}} \div \bar{C}$

$Z_M = (\bar{B}+\bar{C})(A+\bar{C})(C+B)(D+\bar{C})$

31. $Z = 0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

32. $Z = A\bar{B}\bar{D}$

DEC	Z
0	0
1	1
2	0
3	0
4	0
5	1
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	1	0	0
01	0	1	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = A\bar{B}\bar{D}$

$\bar{Z}_M = \bar{A} + D + B$

de Morgan

$\bar{\bar{Z}}_M = \overline{\bar{A} + D + B}$

$Z_M = \bar{\bar{A}} \mp \bar{D} \mp \bar{B}$

$Z_M = A\bar{D}\bar{B}$

33. $Z = BD + B\bar{A}$

DEC	Z
0	0
1	0
2	1
3	0
4	0
5	0
6	1
7	0
8	0
9	0
10	1
11	1
12	0
13	0
14	1
15	1

DC \ BA	00	01	11	10
00	0	0	0	1
01	0	0	0	1
11	0	0	1	1
10	0	0	1	1

$Z_m = DB + B\bar{A}$

$\bar{Z}_M = A\bar{D} + \bar{B}$

de Morgan

$\bar{\bar{Z}}_M = \overline{A\bar{D} + \bar{B}}$

$Z_M = \bar{A} \div \bar{\bar{D}} \mp \bar{\bar{B}}$

$Z_M = (\bar{A}+D)B$

34. $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

35. $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_n = 0$

36

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_n = 0$

37 $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

38. $Z=0$

DEC	Z
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0

DC \ BA	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$Z_m = 0$

$Z_M = 0$

39. $Z = \overline{B} + CA + C\overline{D}$

DEC	Z
0	1
1	1
2	0
3	0
4	1
5	1
6	1
7	1
8	1
9	1
10	0
11	0
12	1
13	1
14	0
15	1

DC \ BA	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	1	1	1	0
10	1	1	0	0

$Z_m = CA + C\overline{D} + \overline{B}$

$\overline{Z}_M = B\overline{C} + DB\overline{A}$

de Morgan

$\overline{\overline{Z}}_M = \overline{B\overline{C} + DB\overline{A}}$

$Z_M = \overline{B} \cdot \overline{\overline{C}} \cdot \overline{D} \cdot \overline{B} \cdot \overline{\overline{A}}$

$Z_M = (\overline{B} + C)(\overline{D} + \overline{B} + A)$

40. $Z = \overline{A}DC + \overline{C}AB$

DEC	Z
0	0
1	0
2	0
3	1
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	1
12	1
13	0
14	1
15	0

DC \ BA	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	1	0	0	1
10	0	0	1	1

$Z_m = D\overline{A}C + \overline{C}BA$

$\overline{Z}_M = \overline{A}\,\overline{C} + \overline{D}C + \overline{B}A + AC$

de Morgan

$\overline{\overline{Z}}_M = \overline{\overline{A}\,\overline{C} + \overline{D}C + \overline{B}A + AC}$

$Z_M = \overline{\overline{A}} \cdot \overline{\overline{C}} \cdot \overline{\overline{D}} \cdot \overline{C} \cdot \overline{\overline{B}} \cdot \overline{A} \cdot \overline{A} \cdot \overline{C}$

$Z_M = (A + C)(D + \overline{C})(B + \overline{A})(\overline{A} + \overline{C})$

Las simulaciones seran entregadas en Proteus de manera digital