

# 46755 – Assignment 1

Deadline: March 20, 2023 (11:59pm)

February 5, 2023

## Case study

The case study is based on the IEEE 24-node reliability test system ([link](#)), including 12 conventional generating units, 6 wind farms, 17 demands, and 34 transmission lines. In case there is a lack of input data for some items, **please select arbitrary (reasonable) values**, and mention it in your project report. For the technical details of conventional generators and transmission lines, this [link](#) might be helpful. Please assume the production cost of wind farms is zero. For the bid price of price-elastic demands, please consider comparatively high prices (compared to the generation cost of conventional units) such that the majority of demands will be supplied. For wind data, one potential data source is this [link](#) (you can normalize such data for your case study). For transmission lines, you can simply consider an identical reactance for all lines (e.g., 0.002 p.u.  $\rightarrow$  susceptance = 500 p.u.).

## Step 1: Copper-plate single hour

In lecture 2 you learn how to develop a market-clearing optimization model for a copper-plate power system (i.e., without modeling transmission network) and with a single hour.

- 1.1 Please write the corresponding optimization problem in a compact way (similar to the last slide of lecture 2) and then derive the corresponding KKT conditions.
- 1.2 Please compute the values for the market-clearing price (uniform pricing scheme), the social welfare, the profit of every supplier (both types of conventional units or and wind farms) and the utility of every demand. By utility for a demand, we mean her power consumption  $\times$  [bid price - market price]. For the market price, please justify the value obtained using the KKT conditions.

## Step 2: Copper-plate multiple hours

This step is the extension of what you learn in lecture 2. Please extend the optimization model in Step 1 by including multiple time periods (here, 24 hours). This means you may need to define a new index, e.g.,  $t$ , running from 1 to 24. Some input data are varying across hours, e.g., wind power and demand level. We are interested in enforcing new constraints that link

hours, the so-called *inter-temporal* constraints<sup>1</sup>. The specific type of inter-temporal constraints we are interested in this step is the one imposed by hydrogen demand. For that, please consider at least two out of 6 wind farms are equipped by their local electrolyzer. Each electrolyzer consumes wind power to electrolyze water and therefore produce hydrogen. Imagine the size of each electrolyzer is the half of the installed capacity of the corresponding wind farm. Assume each electrolyzer is always on, and produces 18 kg hydrogen by consuming 1 MW electricity<sup>2</sup>. The only operational cost of the electrolyzer corresponds the power consumption, otherwise we ignore all other potential costs, e.g., the water cost. The inter-temporal constraint related to the hydrogen demand is that each electrolyzer should produce at least 30 tons over the day (not hour), so this provides a degree of freedom how to meet the minimum daily hydrogen demand over 24 hours. Accordingly,

- 2.1 Please update the formulation for the market-clearing optimization problem. Reporting KKTs are not needed in Step 2 and next steps.
- 2.2 Please obtain hourly market-clearing prices as well as the total social welfare and total profits of suppliers over 24 hours (not hourly but daily values). Please explore how these market-clearing outcomes are changed by altering the limit of 30 tonnes.
- 2.3 Imagine we co-optimize for power and hydrogen systems, such that there is a fixed revenue of \$3/kg by selling hydrogen. While keeping the daily hydrogen demand, add this revenue term to the objective function of the market-clearing optimization problem. Please discuss the change in the market-clearing outcomes. Please conduct a sensitivity analysis with respect to the hydrogen price of \$3/kg, and explore and discuss whether it matters we allow electrolyzers to consume not only the local wind power but also consume power produced by other suppliers<sup>3</sup>.

## Step 3: Network constraints

In lecture 3 you learn how to model power flow across network and enforce power transmission network constraints. Please extend your market-clearing optimization model in Step 2.1 (with no hydrogen revenue term) by enforcing network constraints.

- 3.1 *Nodal model*: Please explain how the market-clearing optimization model should be updated. Then, please derive hourly nodal market-clearing prices. Are nodal prices in every given hour necessarily identical? Please conduct a sensitivity analysis (by gradually changing the capacity of one or more transmission lines) and discuss your results.
- 3.2 *Zonal model*: Please explain how the market-clearing optimization model should be updated. Please split your power network to at least 3 zones. For different values of available

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<sup>1</sup>A common inter-temporal constraint is the ramping up (down) limit of conventional generating units, enforcing how much their power generation can be increased (decreased) in the current hour compared to that in the previous hour. Another example of inter-temporal constraints is those imposed by energy storage systems.

<sup>2</sup>For simplicity, we assume the efficiency of the electrolyzer is fixed (i.e., 18 kg hydrogen production by consuming 1 MW power). However, in general it depends on the loading level of the electrolyzer. Please feel free to read more about electrolyzers [here](#).

<sup>3</sup>Note by allowing this the produced hydrogen might be no longer green.

transfer capacity (ATC), derive zonal market prices. How different are they compared to nodal prices? What are the implications of nodal vs zonal frameworks to market participants (e.g., in terms of profit for generators)?

## Step 4: Balancing market

In lecture 4 you learn about the balancing market. The day-ahead market outcomes are fixed. For simplicity, please discard intra-day market. The realized wind power production of each farm is equal to her day-ahead forecast plus a forecast error, which could be a positive value (wind excess) or a negative value (wind deficit). Please arbitrarily sample the forecast error (in MW) of each farm in every hour from a normal probability distribution function of  $\mathcal{N}(0, x)$ . Please tune  $x$  yourself in a reasonable way depending on the installed capacity and the day-ahead forecast of the farm in question. The load curtailment cost is \$500/MWh.

- 4.1 Considering the network and 24-hour time period, let us assume electrolyzers are the only balancing service providers (i.e., conventional generators and demands are not balancing service providers). In the real-time operation when the true wind power is realized, electrolyzers can quickly change their consumption level in every hour compared to their day-ahead plan. However, their minimum daily hydrogen demand constraint should still be fulfilled. At every hour, each electrolyzer offers to reduce her consumption at the cost of 10% higher than the day-ahead price, while she pays for extra consumption at the price of 15% lower than the day-ahead price. Please develop an optimization model and explore how electrolyzers could offset the total wind power deviation in the system over the day.
- 4.2 Now, please pick one out of 24 hours, and discard network constraints for simplicity. Let us assume demand side (including electrolyzers) do not provide balancing services, but all conventional generators do. Each conventional generator offers the upward balancing service (if she can, depending on her day-ahead schedule) at a price equal to the day-ahead price plus 12% of her production cost. Similarly, she offers the downward balancing service (if she can, depending on her day-ahead schedule) at a price equal to the day-ahead price minus 15% of her production cost. On top of wind deviations for that hour, please assume there is an unexpected failure (outage) in one of conventional generators. Please develop an optimization model to clear the balancing market for the given hour, and derive the balancing market-clearing price. Finally, please calculate the total profit of conventional generating units in the given hour (in day-ahead and balancing).

## Step 5: Reserve market

In lectures 5 and 6 you learn about different ancillary services and how TSO clears a reserve market for the corresponding service in advance. Here, we assume a single generic service, for which we develop a reserve market. Imagine, according to the TSO's quantification, the hourly upward reserve requirement in the reserve market is 20% and the hourly downward reserve requirement is 15% of the total demand in the corresponding hour. The service providers are electrolyzers and conventional generators. While discarding network constraints,

- 5.1 Please develop optimization models to clear reserve and day-ahead markets *sequentially* (following the current practice in the European electricity markets) and report hourly market-clearing outcomes for 24 hours.
- 5.2 Please develop an optimization model to clear the US-style market (the *joint* reserve and energy market in the day-ahead stage). Compare the market-clearing outcomes (schedules and prices) achieved with those of the European style.