Investment Bundle Optimisation Model (Formal LP Formulation)

1 Problem statement

We consider a private credit lender with an existing portfolio of SME investments distributed across a finite set of economic sectors. The lender may deploy additional capital into a finite set of candidate SME investments, but may not divest existing positions. The aim is to construct an "investment bundle" (a selection and sizing of candidate investments) such that the resulting portfolio is as close as possible to a desired sectoral distribution, subject to a capital budget and a preference for lower-risk investments.

2 Sets and indices

$$
\begin{aligned}
&i \in \{1, \ldots, n\} && \text{sector index,} \\
&j \in \{1, \ldots, m\} && \text{candidate investment index.}
\end{aligned}
$$

3 Parameters

All monetary quantities are measured in GBP.

$$
\begin{aligned}
C_i &\geq 0 && \text{current capital invested in sector } i, \\
C &= \sum_{i=1}^{n} C_i && \text{total current portfolio size,} \\
d_i &\geq 0 && \text{desired portfolio weight for sector } i \text{ (decimal form),} \\
\sum_{i=1}^{n} d_i &= 1, && \\
a_j &> 0 && \text{deal size of candidate investment } j, \\
s(j) &\in \{1, \ldots, n\} && \text{sector associated with investment } j, \\
r_{1,j},\, r_{2,j},\, r_{3,j} &> 0 && \text{risk ratios for investment } j, \\
B &> 0 && \text{capital budget for new deployment.}
\end{aligned}
$$

Risk-score aggregation Each candidate investment is assessed using three standard credit risk metrics: the debt service coverage ratio ($r_{1,j}$), the liquidity ratio ($r_{2,j}$) and the solvency ratio ($r_{3,j}$).

We define the aggregate risk score as

$$
R_j = r_{1,j} + r_{2,j} + r_{3,j},
$$

where larger values correspond to lower overall credit risk and thus more desirable investments.

4 Decision variables

$y_j \in [0,1]$	fraction of candidate investment j undertaken,
$x_i \geq 0$	new capital allocated to sector i,
$X \geq 0$	total new capital deployed,
$t \geq 0$	total portfolio size after deployment,
$u_i \geq 0$	auxiliary variable for absolute deviation in sector i.

5 Capital accounting

The new capital allocated to each sector is the sum of deployed fractions of deals belonging to that sector:

$$x_i = \sum_{j:\, s(j)=i} a_j y_j \qquad \forall i \in \{1, \ldots, n\}. \tag{1}$$

Total new deployment is

$$X = \sum_{j=1}^{m} a_j y_j, \tag{2}$$

and the post-deployment portfolio size is

$$t = C + X. \tag{3}$$

The budget constraint is

$$X \leq B. \tag{4}$$

6 Sector alignment and deviations

After deployment, the *desired* GBP amount in sector i is $d_i t$. The *realised* GBP amount is $C_i + x_i$. Define the deviation in GBP:

$$g_i = (C_i + x_i) - d_i t \qquad \forall i. \tag{5}$$

We penalise the absolute deviation $|g_i|$ in the objective. To preserve linearity we introduce $u_i \geq 0$ satisfying

$$g_i \leq u_i \qquad \forall i, \tag{6}$$

$$-g_i \leq u_i \qquad \forall i. \tag{7}$$

At optimality, $u_i = |g_i|$.

7 Risk preference term

To prefer lower-risk investments, we add a separable penalty proportional to deployed capital in each deal and inversely proportional to its risk score:

$$\text{risk penalty for deal } j \;=\; \frac{a_j y_j}{R_j} \;=\; \frac{a_j y_j}{r_{1,j} + r_{2,j} + r_{3,j}}.$$

This term is linear in the decision variable y_j, since a_j and R_j are constant parameters.

8 Objective function

The objective trades off (i) sectoral misalignment in GBP and (ii) the risk penalty:

$$\min \ \sum_{i=1}^{n} u_i \ + \ \sum_{j=1}^{m} \frac{a_j y_j}{r_{1,j} + r_{2,j} + r_{3,j}}. \tag{8}$$

9 Complete linear program

$$\begin{aligned}
\min_{\{y_j, x_i, X, t, u_i\}} \quad & \sum_{i=1}^{n} u_i \ + \ \sum_{j=1}^{m} \frac{a_j y_j}{r_{1,j} + r_{2,j} + r_{3,j}} && && (9)\\
\text{s.t.} \quad & x_i = \sum_{j:\, s(j)=i} a_j y_j && \forall i && (10)\\
& X = \sum_{j=1}^{m} a_j y_j && && (11)\\
& t = C + X && && (12)\\
& X \le B && && (13)\\
& (C_i + x_i) - d_i t \le u_i && \forall i && (14)\\
& -\big((C_i + x_i) - d_i t\big) \le u_i && \forall i && (15)\\
& 0 \le y_j \le 1 && \forall j && (16)\\
& x_i \ge 0,\ u_i \ge 0 && \forall i, \qquad X \ge 0,\ t \ge 0. && (17)
\end{aligned}$$

10 Interpretation of the solution

An optimal solution yields $\{y_j^\star\}_{j=1}^m$, which defines the fraction of each candidate investment to include. The implied new deployment is $X^\star = \sum_j a_j y_j^\star \le B$ and the implied sector allocations are $x_i^\star = \sum_{j:s(j)=i} a_j y_j^\star$. The resulting post-deployment sector weights are

$$\widehat{w}_i \ = \ \frac{C_i + x_i^\star}{t^\star}, \qquad \text{where } t^\star = C + X^\star.$$

The model minimises the total absolute GBP deviation from the target sectoral allocation together with a capital-weighted penalty that discourages allocating to relatively riskier deals (as encoded by smaller $r_{1,j} + r_{2,j} + r_{3,j}$).