4.1 , find $n_+ - n_-$ in terms of N & physical params

(eq 4.3) $\frac{n_-}{n_+} = e^{\Delta E/kT}$

$n^- = n^+(e^{\Delta E/kT}$

$N = n_+ + n_-$

$N = n_+(1 + e^{-\Delta E/kT})$

$n_+ = \left(\frac{1 + e^{\Delta E/kT}}{N}\right)$

$n_- = n^+(e^{\Delta E/kT}) = \left(e^{-\Delta E/kT}\left(\frac{1 + e^{\Delta E/kT}}{N}\right)\right)$

$n_+ - n_- = \frac{1 + e^{-\Delta E/kT}}{N} - \left(e^{-\Delta E kT}\left(\frac{1 + e^{-\Delta/kT}}{N}\right)\right)$

$n_+ - n_- = \frac{1 + e^{-\Delta E/kT}}{N}\left(1 - e^{-\Delta E/kT}\right)$

$$\boxed{n_+ - n_- = \frac{1 - e^{-2\Delta E/kT}}{N}}$$

b) $I_z = 1/2$ & kT large compared to ΔE

$n_+ - n_- = \frac{1 - e^{0}}{N}$ $\quad -2\Delta E/kT \to 0$ when $kT >>> 2\Delta E$

$n_+ - n_- = \frac{1 - 1}{N}$

$$\boxed{n_+ - n_- = 0}$$

7.2

material with M_0 + T_1 + T_2

$90°$ excitation find $|M(t)|$

Show that if $T_2 < T_1$ $|M(t)| < M_0$

$M_z(t) = M_0(1 - e^{-t/T_1})$

$M_{xy}(t) = M_0(e^{-t/T_2})$

$|M(t)| = \sqrt{M_z^2 + M_{xy}^2}$

$|M(t)| = \sqrt{M_0^2(1-e^{-t/T_1})^2 + M_0^2(e^{-2t/T_2})}$

$$\boxed{|M(t)| = M_0\sqrt{1 - 2e^{-t/T_1} + e^{-2t/T_1} + e^{-2t/T_2}}}$$

$T_2 < T_1$

$\frac{|M(t)|}{M_0} < 1$

$\sqrt{1 - 2e^{-t/T_1} + e^{-2t/T_1} + e^{-2t/T_2}}^{\,2} < 1^2$

$1 - 2e^{-t/T_1} + e^{-2t/T_1} + e^{-2t/T_2} < 1$
$-1 \qquad\qquad -1$

$-2e^{-t/T_1} + e^{-2t/T_1} + e^{-2t/T_2} < 0$

$e^{-2t/T_2} < 2e^{-t/T_1} - e^{-2t/T_1}$

$e^{-2t/T_2} + e^{-2t/T_1} < 2e^{-t/T_1}$ if $T_1 < T_2$

if $T_1 = T_2$

$\ln(\cancel{2}e^{-2t/T_1} < \ln(\cancel{2}e^{-t/T_1})$

$-2\cancel{t/T_1} < -\cancel{t/T_1}$

$-2 < -1$ ✓

therefore if $T_1 < T_2$

than $e^{-2t/T_2} < e^{-2t/T_1}$

$e^{-2t/T_1} < e^{-t/T_1}$

So

$e^{-2t/T_2} + e^{-2t/T_1} < e^{-t/T_1}$ ✓

4.4 A & B have same M_0 with relaxation (T_{1A}, T_{2A}) & (T_{1B}, T_{2B})

$$\Delta S_{xy}(t) = M_{xyA}(t) - M_{xyB}(t)$$

$$\Delta S_z(t) = M_{zA}(t) - M_{zB}(t)$$

Find an expression for the time that maximizes ΔS_{xy}

$$|\Delta S_{xy}(t)| = |M_0 (e^{-t/T_{2A}} - e^{-t/T_{2B}})|$$

$$\left|\frac{\Delta S_{xy}(t)}{dt}\right| = \frac{M_0 (e^{-t/T_{2A}} - e^{-t/T_{2B}})}{dt}$$

$$\frac{\Delta S_{xy}(t)}{dt} = M_0\left(-\frac{1}{T_{2A}} e^{-t} + \frac{1}{T_{2B}} e^{-t/T_{2B}}\right.$$

$$0 = -\frac{1}{T_2} e^{-t/T_{2A}} + \frac{1}{T_{2B}} e^{-t/T_{2B}}$$

$$\frac{1}{T_2} e^{-t/T_{2A}} = \frac{1}{T_{2B}} e^{-t/T_{2B}}$$

$$\frac{T_{2B}}{T_{2A}} = \frac{e^{-t/T_{2B}}}{e^{-t/T_{2A}}}$$

$$\ln \frac{T_{2B}}{T_{2A}} = \ln e^{-t(T_{2B} - T_{2A})}$$

$$\ln\left(\frac{T_{2B}}{T_{2A}}\right) = -t(T_{2B} - T_{2A}$$

$$\boxed{\frac{-\ln\left(\frac{T_2B}{T_2A}\right)}{T_2B - T_2A} = t_{max_{XY}}}$$

6)

$$\Delta S_2(t) = M_0\left(\cancel{1} - e^{-t/T_1A} - \cancel{(1} - e^{-t/T_1B}\right)$$

$$\frac{\Delta S_2(t)}{dt} = \frac{M_0\left(-e^{-t/T_1A} + e^{-t/T_1B}\right)}{dt}$$

$$\cancel{0} = \cancel{M_0}\left(\frac{1}{T_1A}e^{-t/T_1A} - \frac{1}{T_1B}e^{-t/T_1B}\right)$$

$$\frac{1}{T_1B}e^{-t/T_1B} = \frac{1}{T_1A}e^{-t/T_1A}$$ Same as before

$$\boxed{T_{max_2} = \left|\left(\frac{-\ln\left(\frac{T_2B}{T_2A}\right)}{T_2B - T_2A}\right)\right|}$$

$B_0 = 1T$

4.4c

$B = 1\,T$

$$T_{max_{T_2}} = \left| \frac{-\ln\left(\frac{T_2 B}{T_2 A}\right)}{T_2 B - T_2 A} \right|$$

A → gray matter

B → white matter

$T_2 A = 0.1$

$T_2 B = 0.092$

$$T_{max_2} = \frac{-\ln\left(\frac{0.092}{0.1}\right)}{0.092 - 0.1} = 0.08338810$$

$$\boxed{t_{max} = 10.4227\,s}$$

$$T_{max_2} = \left| \frac{-\ln\left(\frac{T_1 B}{T_1 A}\right)}{T_1 B - T_1 A} \right|$$

$T_1 A = 0.82$

$T_1 B = 0.68$

$$T_{max_2} = \frac{-\ln\left(\frac{0.68}{0.82}\right)}{0.68 - 0.82}$$ $$\boxed{T_{max_2} = 1.337\,s}$$