Assignment 3 (ML for TS) - MVA

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in one report per pair of students.
- Rename your report and notebook as follows:
  `FirstnameLastname1_FirstnameLastname1.pdf` and
  `FirstnameLastname2_FirstnameLastname2.ipynb`.
  For instance, `LaurentOudre_CharlesTruong.pdf`.
- Upload your report (PDF file) and notebook (IPYNB file) using the link given in the email.

2 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \ldots, T-1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

To decompose an audio signal into a sequence of symbols, our procedure first computes the time-frequency representation of the signal using a spectrogram, focusing primarily on the DTMF frequency range (650–1800 Hz). Next, a change-point detection algorithm, in our case PELT, is applied to the spectrogram's intensity over time to identify the start and end points of symbols and silences. Each identified segment is analyzed to determine the two dominant frequencies with the highest energy, which are mapped to the corresponding symbol using a predefined frequency-symbol mapping. Furthermore, multiple hyperparameters are calibrated using a grid search approach applied to a training dataset to minimize classification errors. The method ensures robustness to noise and variations in symbol duration by filtering low-energy segments and validating frequency pairs. Finally, the accuracy of the predicted symbol sequence is measured using the Levenshtein distance, a metric determining the number of necessary changes to obtain one sequence from the other. By fine-tuning our hyperparameters (see notebook for more details), we were able to obtain high accuracy results for our training dataset (84%) and test dataset (100%).

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1: [7, 2, 1, C, 9, 9]
- Sequence 2: [1, #, 2, #]

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E. A specific node is denoted by v and a specific edge, by e. The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $u_1, u_2, \ldots, u_n$ respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \ldots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^{n} \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \tag{1}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_n]$ is the Fourier transform of f and $\hat{g}_m$ are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the $\hat{g}_m$; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel $\hat{g}_m$ is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \tag{2}$$

where $a := \lambda_n / (M + 1 - R)$,

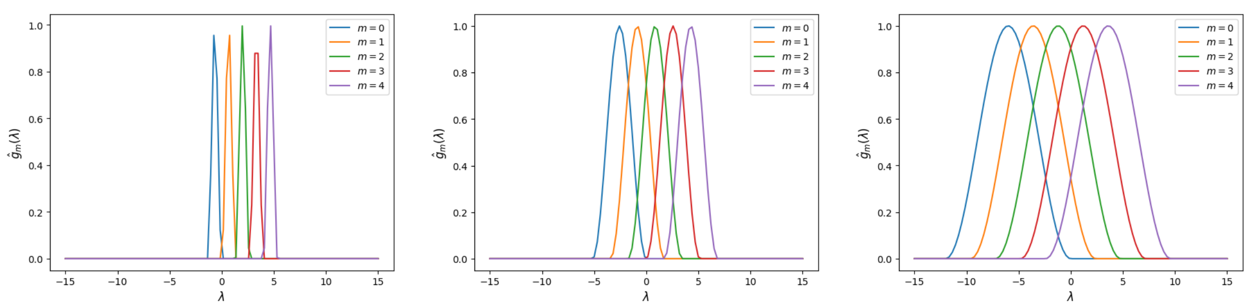
$$\hat{g}^U(\lambda) := \frac{1}{2}\left[1 + \cos\left(2\pi\left(\frac{\lambda}{aR} + \frac{1}{2}\right)\right)\right] \mathbb{1}(-Ra \leq \lambda < 0) \tag{3}$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions $\hat{g}_m$ for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R?

Answer 3



(a) $R = 1$ (b) $R = 3$ (c) $R = 5$

Figure 1: The SAGW kernels functions

By looking at the plots of $\hat{g}_m$ for different values of R, one can notice that R has a strong influence on the smoothness of the signal. While taking R smaller will focus on more localized patterns, larger values of R will take more vast and gradual phenomena into consideration.

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are ["AURAY", "BELLE ILELE TALUT", "BIGNAN", "DINARD", "GUERANDE", "GUISCRIFF SA", "ILE DE GROIX", "ILEDEBREHAT", "KERPERT", "LORIENTLANN BIHOUE", "LOUARGAT", "MERDRIGNAC", "MORLAIX_AERO", "NOIRMOUTIER EN", "PLEUCADEUC", "PLEYBERCHRIST SA", "PLOERMEL", "PLOUGUENAST", "PLOUMANAC'H", "PLOVAN", "POMMERITJAUDY", "PONTIVY", "PTE DE CHEMOULIN", "PTE DE PENMARCH", "PTE DU RAZ", "QUINTENIC", "ROSTRENEN", "SAINTCASTLEG", "SAINTJOACHIM_SAE", "SARZEAU SA", "SIBIRIL S A", "SPEZET", "ST BRIEUC", "STSEGAL S A", "THEIX", "TREGUNC", "VANNESSENE"]

The threshold is equal to 0.83

The signal is the least smooth at 2014-01-10 09:00:00

The signal is the smoothest at 2014-01-24 19:00:00

Question 5

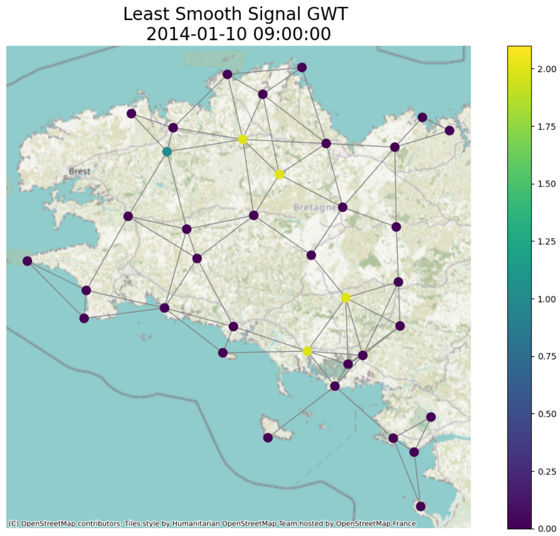
(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v, the vector $[W_f(1, v), W_f(2, v), \ldots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low/medium/high frequency:

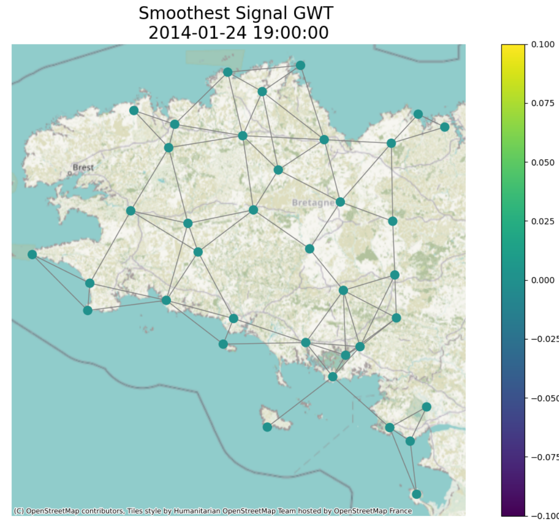
- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

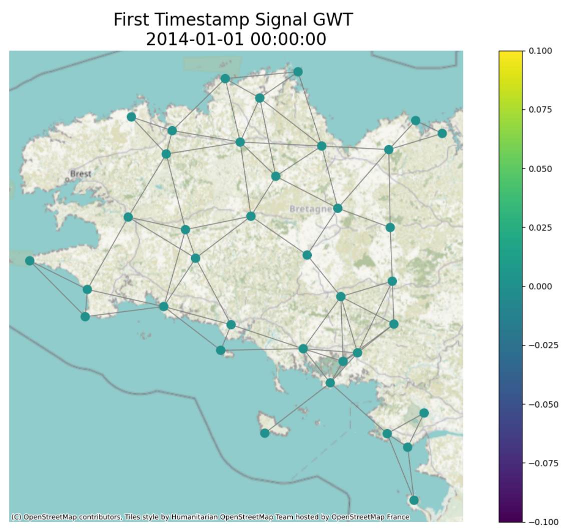
Answer 5



(a) Least smooth signal



(b) Smoothest signal



(c) First available timestamp

Figure 2: Classification of nodes into low/medium/high frequency

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

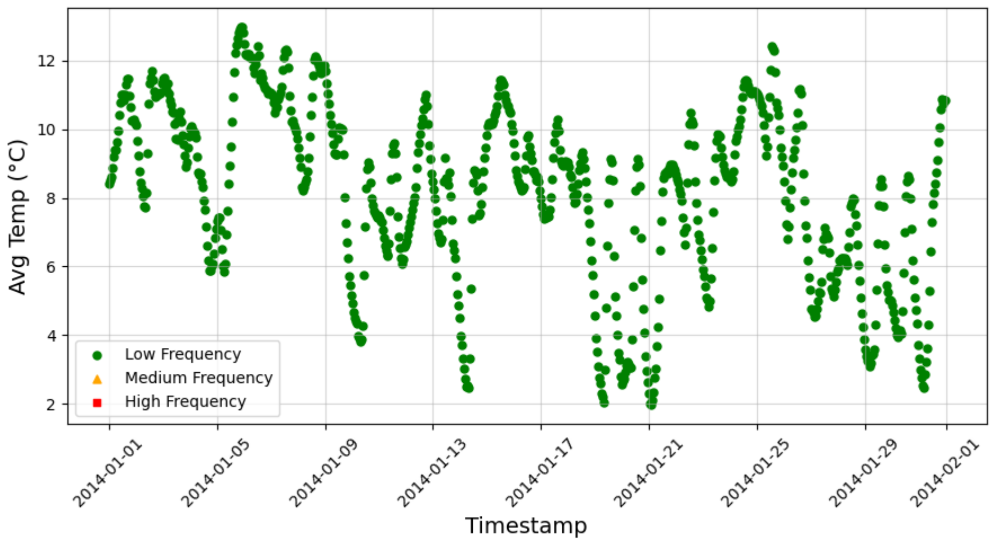


Figure 3: Average temperature. Markers' colours depend on the majority class.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G'.
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

- The Laplacian of H is expressed as:

$$L_H = L_G \otimes I_n + I_m \otimes L_{G'}$$

  where $\otimes$ denotes the Kronecker product, n is the number of vertices in G, and m is the number of vertices in G'.

- Let λ and μ be eigenvalues of L_G and $L_{G'}$ with corresponding eigenvectors a and b, respectively. Then, the eigenvalues and eigenvectors of L_H are:

$$\text{Eigenvalues: } \lambda + \mu, \quad \text{Eigenvectors: } a \otimes b$$

  Therefore, the spectrum of L_H is:

$$\text{Sp}(L_H) = \{\lambda + \mu \mid \lambda \in \text{Sp}(L_G), \mu \in \text{Sp}(L_{G'})\}$$

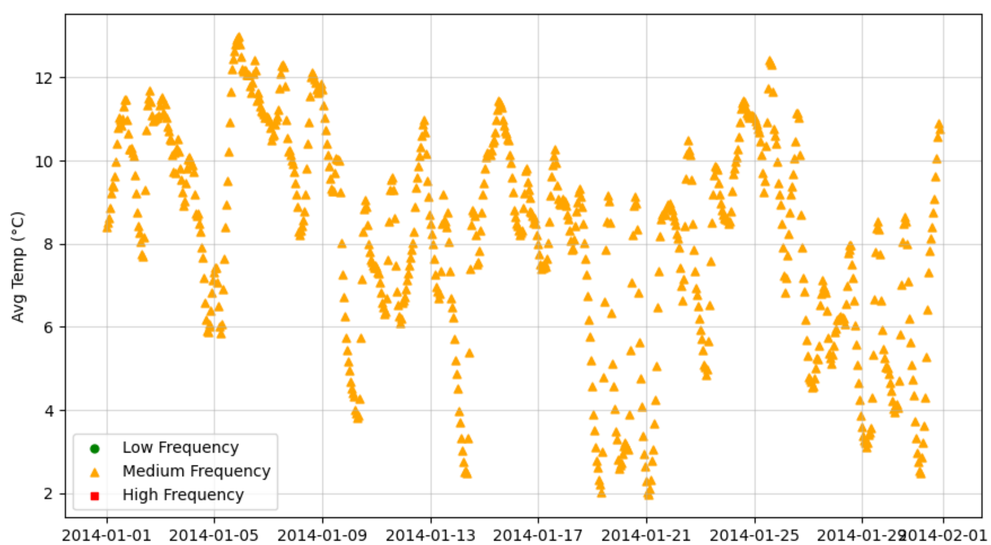


Figure 4: Average temperature. Markers' colours depend on the majority class.