Repeated Fair Allocation of Indivisible Items

Ayumi Igarashi,[1] Martin Lackner,[2] Oliviero Nardi[2] and Arianna Novaro[3]

[1]The University of Tokyo [2]DBAI, TU Wien [3]CES, University of Paris 1 (Panthon-Sorbonne)

Abstract

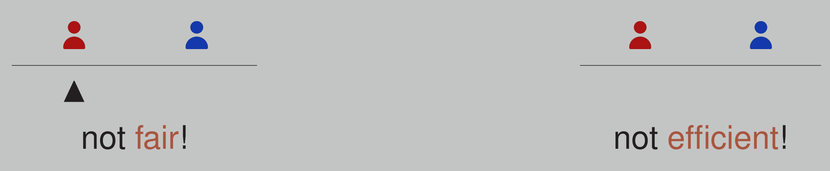
In practice, items are not always allocated once and for all, but often repeatedly. For example, when the items are recurring chores to distribute in a household. Motivated by this, we initiate the study of the repeated fair division of indivisible items.

Applications

- Fairly distributing household chores between a couple
- Allocating teaching duties to professors over the semesters
- Granting employees daily access to a common infrastructure

Main Idea

We want to allocate a single item ▲ between two agents, 🔴 and 🔵. Problem:



What if we share them over time?

Each day's allocation is not fair, but the overall allocation is!

Formal Model

We have n agents (🔴, 🔵, 🟠, ...) that need to share some items (▲, ■, ★, ...). Agents have additive utilities:

	🔴	🔵	🟠
▲	1	3	4
■	5	2	1
★	-3	-4	-2

We have k time-steps at our disposal. Example:

	🔴	🔵	🟠
day 1	■★	▲	
day 2	■		▲★
day 3	■		▲★

Axioms

An axiom can be satisfied overall (while looking globally at the whole bundle, over all time-steps) or per round (if it is satisfied individually by all time-steps).

- **Envy-freeness** (EF): No agent prefers someone else's bundle
- **Envy-freeness up to one item** (EF1): If an agent envies some other agent, we can eliminate envy by removing one item from the bundle of one of the two agents
- **Proportionality** (PR): Each agent receives at least $1/n$ of the value of the whole set of items
- **Pareto-optimality** (PO): We cannot find an allocation that is better for some agents, and worse for none

Repetion: Why Bother?

In the one-shot setting, we can't always find a **PR** (let alone EF) and **PO** allocation. Our main goal:

Can we guarantee better fairness and efficiency properties by looking at the repeated allocation of items?

Results: General Case

Under certain conditions, envy-freeness is always achievable:

*If k is a multiple of n, an **overall EF** allocation can always be found (in polynomial time).*

To achieve this, we can rotate the items at each time-step, e.g.:

	🔴	🔵	🟠
day 1	▲	■	★
day 2	★	▲	■
day 3	■	★	▲

What about efficiency? Even if k is a multiple of n, an overall EF and PO allocation might not exist. Still:

*If k is a multiple of n, an **overall PR** and **PO** allocation always exists.*

Results: Two-agent Case

For two agents, we have stronger fairness guarantees:

*For two agents, if k is even, an **overall EF** and **PO** allocation always exists.*

What about the individual time-steps? We cannot have envy-freeness in every round. However:

*For two agents, if k is even, an allocation which is **overall EF** and **EF1 per round** can always be found (in polynomial time).*

Can we additionally have efficiency? Not if $k > 2$, but:

*For two agents, if $k = 2$, we can always find an **overall EF** and **PO** allocation that is **EF1 per round**.*

Conclusions and Future Work

We have found that, by taking time into consideration, we might be able to distribute items more fairly and more efficiently.

We leave open a number of interesting questions, mainly about the complexity of finding fair and efficient allocations.

Contact Information

- igarashi@mist.i.u-tokyo.ac.jp
- lackner@dbai.tuwien.ac.at
- oliviero.nardi@tuwien.ac.at
- arianna.novaro@univ-paris1.fr