

Repeated Fair Allocation of Indivisible Items

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Abstract

In practice, items are not always allocated once and for all, but often repeatedly. For example, when the items are recurring chores to distribute in a household. Motivated by this, we initiate the study of the **repeated fair division of indivisible items**.

Applications

- ▶ Fairly distributing household chores between a couple
- ▶ Allocating teaching duties to professors over the semesters
- ▶ Granting employees daily access to a common infrastructure

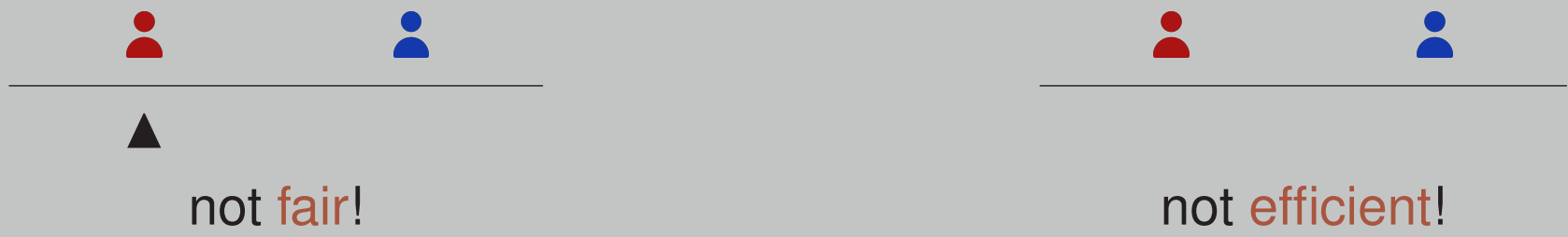
Repetition: Why Bother?

In the one-shot setting, we can't always find a **Proportional** (let alone Envy-Free) and **Pareto-Optimal** allocation. Our main goal:

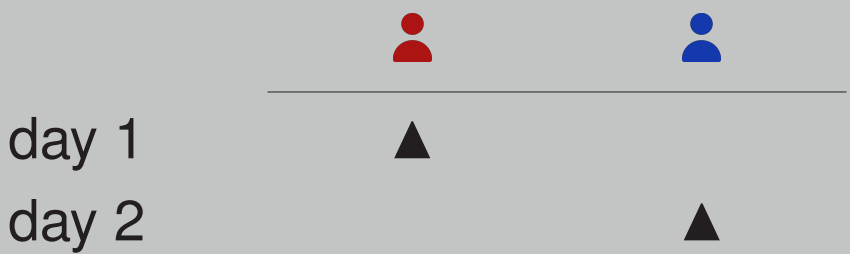
“Can we guarantee **better fairness** and **efficiency** properties by looking at the **repeated allocation** of items?”

Main Idea

Suppose that we want to allocate a single item \blacktriangle between two agents, red and blue . Problem:



What if we share them **over time**?



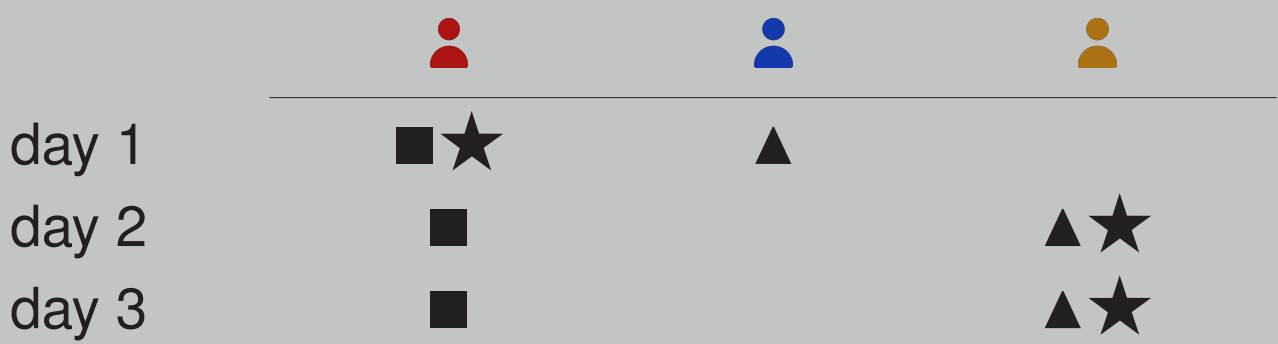
Each day's allocation is not fair, but the **overall allocation** is!

Formal Model

We have n agents (red , blue , yellow , ...) that need to share some items (\blacktriangle , \blacksquare , \star , ...). Agents have additive utilities:

	red	blue	yellow
\blacktriangle	1	3	4
\blacksquare	5	2	1
\star	-3	-4	-2

We have k time-steps at our disposal. Example ($k = 3$):



Axioms

An axiom can be satisfied **overall** (while looking globally at the whole bundle, over all time-steps) or **per round** (if it is satisfied individually by all time-steps).

- ▶ **Envy-freeness** (EF): No agent prefers someone else's bundle
- ▶ **Envy-freeness up to one item** (EF1): If an agent envies some other agent, we can eliminate envy by removing one item from the bundle of one of the two agents
- ▶ **Proportionality** (PR): Each agent receives at least $1/n$ of the value of the whole set of items
- ▶ **Pareto-optimality** (PO): We cannot find an allocation that is better for some agents, and worse for none

Results: General Case

Under certain conditions, envy-freeness is always achievable:

*If k is a multiple of n , an **overall EF** allocation always exists.*

To achieve this, we can rotate the items at each time-step, e.g.:

	red	blue	yellow
day 1	\blacktriangle	\blacksquare	\star
day 2	\star	\blacktriangle	\blacksquare
day 3	\blacksquare	\star	\blacktriangle

What about efficiency? Even if k is a multiple of n , an overall EF and PO allocation **might not exist**. Still:

*If k is a multiple of n , an **overall PR** and **PO** allocation always exists.*

Results: Two-agent Case

For two agents, we have stronger fairness guarantees:

*For two agents, if k is even, an **overall EF** and **PO** allocation always exists.*

What about the individual time-steps? We cannot have envy-freeness in every round. However:

*For two agents, if k is even, an allocation which is **overall EF** and **EF1 per round** always exists.*

Can we additionally have efficiency? **Not if $k > 2$** , but:

*For two agents, if $k = 2$, we can always find an **overall EF** and **PO** allocation that is **EF1 per round**.*

*For two agents, if k is even, we can always find an **overall EF** and **PO** allocation that is **weakly EF1 per round**.*

Results: Variable Number of Rounds

What if the number of rounds is not known in advance? Via a connection to the randomised and divisible settings, we show:

*For every utility profile, **there is some k** for which an **overall EF** and **PO** allocation that is **PROP[1, 1] per round** exists.*