Repeated Fair Allocation of Indivisible Items

Ayumi Igarashi,[1] Martin Lackner,[2] Oliviero Nardi[2] and Arianna Novaro[3]
[1]The University of Tokyo [2]DBAI, TU Wien [3]CES, University of Paris 1 (Panthon-Sorbonne)

Abstract

In practice, items are not always allocated once and for all, but often repeatedly. For example, when the items are recurring chores to distribute in a household. Motivated by this, we initiate the study of the repeated fair division of indivisible items.

Applications

- Fairly distributing household chores between a couple
- Allocating teaching duties to professors over the semesters
- Granting employees daily access to a common infrastructure

Repetition: Why Bother?

In the one-shot setting, we can't always find a **Proportional** (let alone Envy-Free) and **Pareto-Optimal** allocation. Our main goal:

"Can we guarantee better fairness and efficiency properties by looking at the repeated allocation of items?"

Main Idea

Suppose that we want to allocate a single item ▲ between two agents, 👤 and 👤. Problem:

👤	👤
▲	

not fair!

👤	👤

not efficient!

What if we share them over time?

	👤	👤
day 1	▲	
day 2		▲

Each day's allocation is not fair, but the overall allocation is!

Formal Model

We have n agents (👤, 👤, 👤, ...) that need to share some items (▲, ■, ★, ...). Agents have additive utilities:

	👤	👤	👤
▲	1	3	4
■	5	2	1
★	-3	-4	-2

We have k time-steps at our disposal. Example ($k = 3$):

	👤	👤	👤
day 1	■★	▲	
day 2	■		▲★
day 3	■		▲★

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement 101034440.

Axioms

An axiom can be satisfied overall (while looking globally at the whole bundle, over all time-steps) or per round (if it is satisfied individually by all time-steps).

- **Envy-freeness** (EF): No agent prefers someone else's bundle
- **Envy-freeness up to one item** (EF1): If an agent envies some other agent, we can eliminate envy by removing one item from the bundle of one of the two agents
- **Proportionality** (PR): Each agent receives at least $1/n$ of the value of the whole set of items
- **Pareto-optimality** (PO): We cannot find an allocation that is better for some agents, and worse for none

Results: General Case

Under certain conditions, envy-freeness is always achievable:

> *If k is a multiple of n, an **overall EF** allocation always exists.*

To achieve this, we can rotate the items at each time-step, e.g.:

	👤	👤	👤
day 1	▲	■	★
day 2	★	▲	■
day 3	■	★	▲

What about efficiency? Even if k is a multiple of n, an overall EF and PO allocation might not exist. Still:

> *If k is a multiple of n, an **overall PR** and **PO** allocation always exists.*

Results: Two-agent Case

For two agents, we have stronger fairness guarantees:

> *For two agents, if k is even, an **overall EF** and **PO** allocation always exists.*

What about the individual time-steps? We cannot have envy-freeness in every round. However:

> *For two agents, if k is even, an allocation which is **overall EF** and **EF1 per round** always exists.*

Can we additionally have efficiency? Not if $k > 2$, but:

> *For two agents, if $k = 2$, we can always find an **overall EF** and **PO** allocation that is **EF1 per round**.*

> *For two agents, if k is even, we can always find an **overall EF** and **PO** allocation that is **weakly EF1 per round**.*

Results: Variable Number of Rounds

What if the number of rounds is not known in advance? Via a connection to the randomised and divisible settings, we show:

> *For every utility profile, there is some k for which an **overall EF** and **PO** allocation that is **PROP[1, 1] per round** exists.*