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PHENOMENOLOGICAL α-α POTENTIALS

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Abstract: Phenomenological l-dependent α-α potentials are obtained for $l = 0$, 2 and 4 from fitting the relevant phase shifts for c.m. energies $\lessapprox 12$ MeV. For the nuclear part of the potentials, a superposition of repulsive and attractive Gaussian shapes was used. Following theoretical indications, we try to obtain fits with an l-independent attractive part having a longer range than the repulsive parts. Further, we attempt to obtain, at first, the attractive part from just the $l = 4$ phase shift in view of the effect of the large $l = 4$ centrifugal barrier in masking the inner repulsive part. It is, in fact, found possible to obtain acceptable potentials with a common attractive part. These potentials are, however, strongly l-dependent through the repulsive part which becomes weaker as l increases. For $l = 4$, only a quite weak repulsive part is found to be permissible. Our potentials are in good agreement with those recently obtained by Darriulat *et al.* The possible importance of the attractive tail of the α-α potential as a selective probe into just the central, spin-independent, isospin-independent part of the nuclear force is pointed out and discussed.

1. Introduction; Procedure for Determination of the Potentials

Phenomenological α-α potentials are of interest for applications to α-cluster models [1,2] ‡, for the correlation of experimental α-α scattering and for comparison with the results of theoretical studies of the interaction [3-6]. Most of these studies make use of nucleon-nucleon forces and the resonating-group-structure method with the α-particles constrained to be in the ground state.

The following are some general features indicated by the theoretical studies with the latter assumption. The interaction is made up of a direct and an exchange part; the former is an attractive local potential whereas the latter is non-local, effectively repulsive, and has a shorter range than the attractive part; for low energies ($\lessapprox 5$ MeV) the exchange part may be approximately simulated by a local but angular-momentum-dependent interaction [5]; for small α-α separations ($\lessapprox 2$ fm), only rather qualitative results are obtained since the direct and exchange contributions are similar in magnitude but of opposite sign and since, furthermore, the basis for the theoretical predic-

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‡ A very good account of the earlier work on the α-particle model of nuclei and references thereto has been given by Rosenfeld [1]). Refs. [2]) are concerned with applications to hypernuclei.

tions becomes quite uncertain. If mutual distortion of the α-particles is taken into account, then there is also a polarization potential (corresponding to the Van der Waals force in molecular interactions). This has been studied at low energies, in particular by Herzenberg and Roberts [6]), whose estimates indicate that its range is similar to that of the exchange part and that it is weak compared with the direct part (at any rate for separations greater than about 2.5 fm where such calculations may be expected to have some significance).

There is thus some interest in trying to determine phenomenological α-α potentials from the experimentally determined phase shifts, keeping these general features in mind. In particular, it is of interest to consider potentials which are in general l-dependent but local (for a given l) and which are the sum of an attractive and a repulsive part, the latter being of shorter range than the former.

Previous phenomenological investigations [7,8]), which however used rather restrictive potential shapes, found that a strong l-dependence was, in fact, needed to fit the $l = 0$, 2 and 4 phase shifts – even for c.m. energies $E \lessapprox 3$ MeV, i.e. even for just $l = 0$ and 2. A much less restrictive analysis has recently been made by Darriulat *et al.* [9]). In view of the fact that we use a distinct approach and also that our potentials were obtained independently before those of Darriulat *et al.* appeared, it is gratifying that our potentials are quite similar to theirs. The relations between our results and theirs is discussed further below.

We have considered the $l = 0$, 2 and 4 phase shifts (δ_0, δ_2 and δ_4, respectively) for c.m. energies $E \lesssim 12$ MeV where the use of effective local potentials has perhaps some justification [5]). Furthermore, for these energies the phase shifts are experimentally rather well determined [10]) and are real † and the phase shifts for $l > 4$ are effectively zero.

For our potentials, we have used the flexible four-parameter shape

$$V_{\alpha\alpha} = V_{\alpha\alpha}^{(\mathrm{N})} + V_{\mathrm{C}}, \tag{1}$$

where

$$V_{\alpha\alpha}^{(\mathrm{N})}(r) = V_{\mathrm{r}} \exp(-\mu_{\mathrm{r}}^2 r^2) - V_{\mathrm{a}} \exp(-\mu_{\mathrm{a}}^2 r^2) \tag{2}$$

is the nuclear part of the α-α potential; V_{r} and V_{a} are the strengths of the repulsive and attractive parts, respectively, and μ_{r} and μ_{a} the corresponding inverse ranges. For the Coulomb potential V_{C} we have mostly used just

$$V_{\mathrm{C}} = \frac{4e^2}{r}, \tag{3a}$$

but calculations were also made with [4])

$$V_{\mathrm{C}} = \frac{4e^2}{r} \operatorname{erf}\left(\frac{\sqrt{3}r}{2R_{\alpha}}\right). \tag{3b}$$

† The lowest reaction threshold is at $E = 17.365$ MeV for $^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Li} + \mathrm{p}$.

Expression (3b) includes the modification of the Coulomb interaction due to the finite size of the α-particles on the assumption that these have a Gaussian density distribution (for the proton centres) with an rms radius $R_\alpha = 1.44$ fm (ref. [11])).

Apart from the potential shapes used, our approach differs essentially from previous ones in that we try to obtain a fit with an l-independent attractive part and that we first consider δ_4. The reason for starting the analysis with $l = 4$ is that if we take the repulsive part of $V_{\alpha\alpha}^{(N)}$ to have a shorter range than the attractive part, then the large $l = 4$ centrifugal barrier is expected to mask, at least partially, the inner repulsive part – especially at the lower relevant energies where δ_4 becomes appreciable ($5.5 \text{ MeV} \lessapprox E \lessapprox 10 \text{ MeV}$). It is thus hoped largely to determine the attractive part from just the experimental $l = 4$ phase shift δ_4^{exp}.

Our approach is then to neglect, at first, any possible repulsive part for $l = 4$. Thus pairs of values (V_a, μ_a) are found from attempting to fit δ_4^{exp}. For each of these purely attractive potentials, a corresponding repulsive part is then obtained by fitting the experimental $l = 0$ phase shift δ_0^{exp}. A calculation of δ_2 for these $l = 0$ potentials then indicates whether any l dependence is needed between $l = 0$ and 2.

It is shown that it is in fact possible to choose a common attractive part for the $l = 0$, 2 and 4 potentials but that there is then a strong l dependence through the repulsive part. Furthermore, only a rather narrow range of purely attractive potentials gives an acceptable fit to δ_4^{exp}, and it then turns out that acceptable $l = 0$ and 2 potentials can be obtained only for these attractive parts.

One could, of course, always attempt to determine a common attractive part for $l = 0$ and 2 by trial and error and subsequently study $l = 4$. However, since the results for δ_4, especially at the lower relevant energies are expected to be most sensitive to just the attractive tail of $V_{\alpha\alpha}^{(N)}$, it seems reasonable to try to determine this part by starting with δ_4. The numerical calculation of the phase shifts made use of standard procedures.

2. Results and Discussion

Table 1 shows results obtained for δ_4 for various purely attractive potentials characterized by pairs of values (V_a, μ_a), where V_a is in MeV and μ_a in fm^{-1}. For each value of μ_a the value of V_a was chosen so as to give as tolerable a fit as possible to δ_4^{exp} (with emphasis on the lower relevant energies). Only results for $0.35 \text{ fm}^{-1} \leqq \mu_a \leqq 0.5 \text{ fm}^{-1}$ are shown since not even crude fits were possible much outside these limits. In fact, only potentials d_4 and e_4 give satisfactory fits to δ_4^{exp} for $E \lessapprox 11$ MeV. A purely attractive potential for $l = 4$ is thus seen to be fairly uniquely determined since d_4 and e_4 are in fact quite similar. For a given μ_a the strength V_a is determined to within less than about 5 MeV, as illustrated by the results for d_4' and e_4' shown in table 1.

For each of the purely attractive potentials a_4 to e_4, appropriate repulsive parts (V_r, μ_r) were then found by requiring the total potential to give a reasonable fit to

TABLE 1
The phase shift δ_4 (in degrees)

c.m. energy E (MeV)			6.15	7.6	8.9	10.2	10.9	11.45
Experimental phase shift δ_4^{exp}			3.0±1.5	5.2±2	16.2±2	27.7±2	47.0±2	59.2±2
Potentials ($V_r = 0$)								
Label	μ_a (fm^{-1})	V_a (MeV)						
a_4	0.35	30	4.7	9.2	14.3	20.0	23.2	25.7
b_4	0.42	150	4.3	9.5	16.3	24.9	29.9	33.8
c_4	0.45	190	4.3	10.5	20.4	35.1	44.4	51.8
d_4	0.475	130	2.45	6.3	13.9	31.8	50.4	70.0
d_4(3b)	0.475	130	2.6	6.3	14.1	33.0	52.9	73.5
d'_4	0.475	135	2.7	7.5	18.7	54.0	88.3	111.0
e_4	0.5	150	1.9	4.85	10.8	26.2	45.0	68.2
e_4 (3b)	0.5	150	1.9	4.9	11.1	27.8	48.7	74.4
e'_4	0.5	155	2.1	5.8	14.75	49.0	92.7	120.5

The phase shift δ_4 is calculated with use of eq. (3a) except for the values obtained with eq. (3b) as indicated.

δ_0^{exp}. These $l = 0$ potentials (a_0 to e_0) and the corresponding values of δ_0 are shown in table 2 and for d_0 and e_0 also in fig. 2.

The values of δ_2 obtained for the $l = 0$ potentials a_0 to e_0 are shown in table 3.

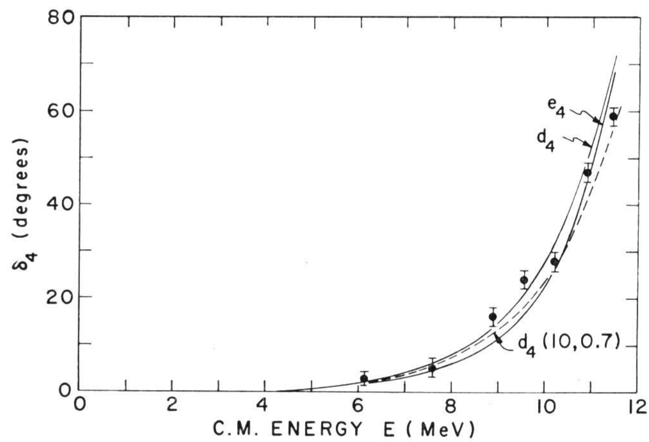

Fig. 1. The $l = 4$ phase shifts δ_4 as a function of the c.m. energy E for the purely attractive potentials d_4 and e_4 (table 1) and for the potential d_4 (10, 0.7). The latter (see table 4) has the repulsive part $V_r = 10$ MeV, $\mu_r = 0.7$ fm^{-1} in addition to the attractive part d_4. The experimental values are shown with error bars.

Comparison of tables 2 and 3 shows that, for these potentials, δ_0^{exp} and δ_2^{exp} cannot be fitted with a common l-independent potential. The $l = 0$ potentials are insufficiently attractive to reproduce δ_2^{exp}. Although the potential b_0 gives a better com-

TABLE 2

The phase shift δ_0 (in degrees)

c.m. energy E (MeV)					1	3	4.5	6.15	8.9	11.45	26.70	59.93
Experimental phase shift δ_0^{exp}					148 ± 1	87 ± 2	60 ± 4	29 ± 4	7 ± 2	-10.7 ± 2	-75.2 ± 2.4	-161.5 ± 6.3
	Potentials											
Label	μ_a (fm^{-1})	V_a (MeV)	μ_r (fm^{-1})	V_r (MeV)								
a_0	0.35	30	0.65	125	144.7	82.2	55.4	34.2	8.6	− 8.9		
b_0	0.42	150	0.55	325	152.9	86.3	55.6	30.5	−0.4	−22.0		
c_0	0.45	190	0.6	500	153.3	87·2	56.4	30.8	−1.1	−23.8		
d_0	0.475	130	0.7	500	148.6	87.0	58.6	35.0	5.4	−15.7	−91.6	−170.9
d_0(3b)	0.475	130	0.7	500	148.9	87.2	58.8	35.3	5.7	−15.4		
d'_0	0.475	130	0.7	475	152.2	90.8	62.3	38.7	9.2	−11.7	−86.8	−164.5
d''_0	0.475	130	0.8	1300	148.7	85.8	56.6	32.1	1.0	−21.45		
e_0	0.5	150	0.8	1050	151.7	90.8	62.4	38.6	8.4	−13.3	−94.1	−184.4
e_0(3b)	0.5	150	0.8	1050	152.0	91.1	62.6	38.8	8.6	−13.1		

The phase shift δ^0 is calculated with use of eq. (3a) except for the values obtained with eq. (3b) as indicated. Only energies $E \leqq 11.45$ MeV were used to obtain the potentials. The values of δ_0^{exp} for $E = 26.7$ MeV and 59.93 MeV are the real parts of the phase shifts.

TABLE 3

The phase shift δ_2 (in degrees)

c.m. energy E (MeV)					1	2	3.5	4.5	6.15	8.9	11.45	26.70	59.93
Experimental phase shift δ_2^{exp}					0 ± 0.1	10 ± 1.5	98 ± 9	116 ± 9	103 ± 8	104 ± 4	94 ± 2	$\pm47.9\pm1.7$	-16.0 ± 1.7
Potentials													
Label	μ_a (fm^{-1})	V_a (MeV)	μ_r (fm^{-1})	V_r (MeV)									
a_0	0.35	30	0.65	125	0.6	8.4	42.3	59.2	66.7	62.1	54.6		
b_0	0.42	150	0.55	325	0.9	16.8	85.0	94.0	89.0	74.2	61.0		
c_0	0.45	190	0.6	500	0.7	12.2	76.4	91.0	89.5	75.7	62.4		
d_0	0.475	130	0.7	500	0.3	4.3	28.6	52.0	70.4	70.0	61.5		
d''_0	0.475	130	0.8	1300	0.3	4.9	33.6	59.0	75.2	71.7	61.8		
e_0	0.5	150	0.8	1050	0.3	3.8	26.8	51.7	73.5	74.2	66.0		
d_2	0.475	130	0.7	320	0.5	10.2	98.6	115.0	114.1	103.1	92.5	47.8	-4.3
d_2(3b)	0.475	130	0.7	320	0.5	10.4	100.2	115.8	114.6	103.5	92.9		
e_2	0.5	150	0.8	640	0.4	8.7	96.5	116.2	116.1	105.0	93.9	44.2	-19.2
e_2(3b)	0.5	150	0.8	640	0.4	8.9	98.2	117.0	116.6	105.4	94.3		

The phase shift δ_2 is calculated with use eq. (3a) except for the values obtained with eq. (b) as indicated. For energies $E > 11.45$ MeV, see the remarks for table 2.

promise fit to both $\delta_0^{\rm exp}$ and $\delta_2^{\rm exp}$ than the compromise (step) potential of Van der Spuy and Pienaar [8]), it still does not reproduce $\delta_2^{\rm exp}$ at all well. This failure to obtain an

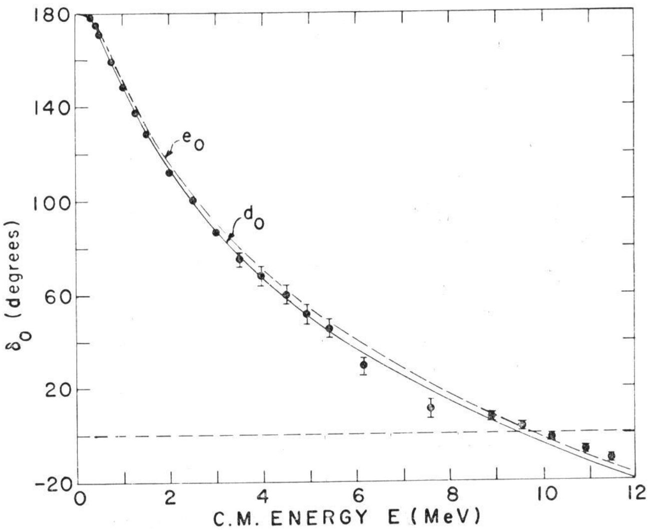

Fig. 2. The $l = 0$ phase shifts δ_0 as a function of the c.m. energy E for the potentials d_0 and e_0 (table 2). The experimental values are shown with error bars, except where the errors fall within the circles.

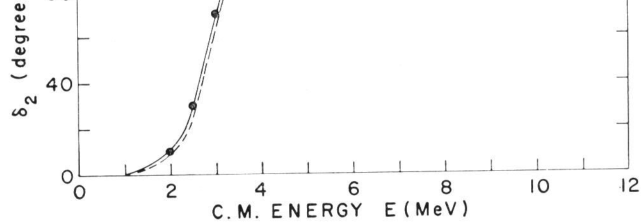

Fig. 3. The $l = 2$ phase shifts δ_2 as a function of the c.m. energy E for the potentials d_2 and e_2 (table 3). The experimental values are shown with error bars, except where the errors fall within the circles.

l-independent local potential for $l = 0$ and 2, as well as the result that the $l = 2$ potential must be more attractive than for $l = 0$, is consistent with and confirms the results of previous phenomenological studies [7-9]).

We then varied the repulsive part in an attempt to find acceptable $l = 2$ potentials with the *same attractive parts* as for the potentials in tables 1 and 2. It was not possible to obtain a reasonable fit for every such attractive part. In fact, acceptable $l = 2$

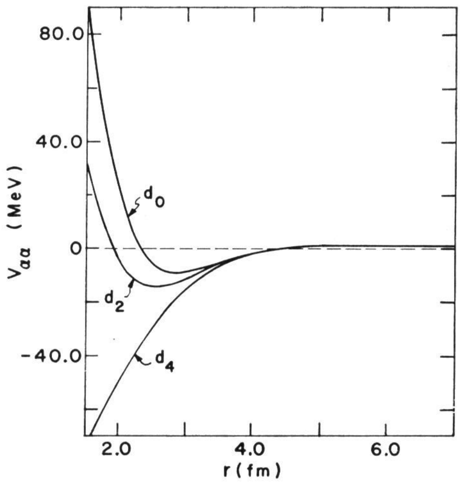

Fig. 4. The potentials d_0 ($l = 0$), d_2 ($l = 2$) and d_4 ($l = 4$) with inclusion of the Coulomb potential as functions of the α-α separation r.

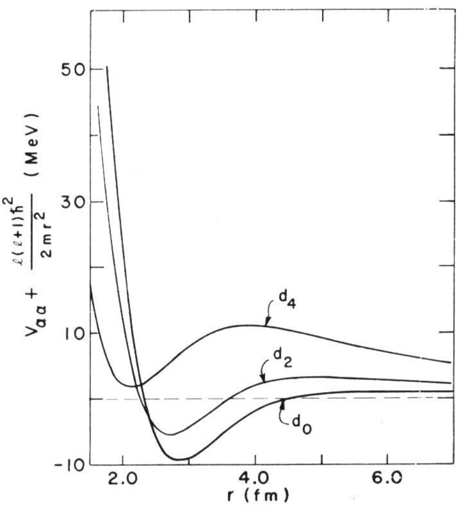

Fig. 5. The potentials d_0, d_2 and d_4 with inclusion of the Coulomb potential and the appropriate centrifugal barriers as functions of the α-α separation r.

potentials – the potentials d_2 and e_2 – could be obtained only for the two attractive parts d_4 and e_4. It is notable that these are in fact just the two $l = 4$ potentials that give a satisfactory fit to δ_4^{exp}. The values of δ_2 for d_2 and e_2 are shown in table 3 and also in fig. 3.

It is thus possible to obtain acceptable l-dependent potentials, for $E \lessapprox 11$ MeV, with a common attractive part which is obtained from considering δ_4. Figs. 2 and 3 show that the potentials d_0 and d_2 with the common attractive part d_4, i.e., $V_a = 130$ MeV, $\mu_a = 0.475\ \text{fm}^{-1}$, give perhaps marginally better fits to δ_0^{exp} and δ_2^{exp} than do the potentials e_0 and e_2 with the common attractive part e_4, i.e., $V_a = 150$ MeV, $\mu_a = 0.5\ \text{fm}^{-1}$. The potentials d_l with and without centrifugal barriers are shown in figs. 4 and 5, respectively. It is seen that one has a strongly l-dependent repulsive part which becomes rapidly less repulsive in going from $l = 0$ to $l = 4$. It should also be noted that the complete $l = 0$ potentials d_0 and e_0 are in fact almost identical for $r \gtrsim 3.0$ fm. The same is true for the complete $l = 2$ potentials d_2 and e_2.

The potential d_0 is very close to the potential $B(V_a = 160$ MeV, $\mu_a = 0.475\ \text{fm}^{-1}$, $V_r = 400$ MeV, $\mu_r = 0.635\ \text{fm}^{-1})$ which was considered for ${}^9_\Lambda$Be in ref. [2]). The two potentials also give almost identical fits to δ_0^{exp} (and somewhat better fits than the best $l = 0$ potential obtained by Van der Spuy and Pienaar [9])). In fact the binding energy B_Λ of ${}^9_\Lambda$Be was found to be quite sensitive to the details of the α-α potential and the experimental value of B_Λ was found to quite strongly favour the potential B as compared with potentials q and r which have the same attractive parts as B but the repulsive parts ($V_r = 300$ MeV, $\mu_r = 0.6\ \text{fm}^{-1}$) and ($V_r = 750$ MeV, $\mu_r = 0.7\ \text{fm}^{-1}$), respectively. The main difference between these potentials is that r has a more extended repulsive core and q a less extended one than does B (or equivalently d_0).

Tables 1–3 also show results obtained with the use of the modified Coulomb potential (3b). The use of this instead of (3a) is seen to have only a quite small effect which has very little significance for the determination of the potentials.

One may ask to what extent our approach uniquely determines the $l = 4$ potential and a common attractive part for the $l = 0$, 2 and 4 potentials. For example, can one find a common attractive part which is either the same or perhaps different from the ones (d_4 and e_4) found above and which is associated with an appreciable non-zero repulsive part for $l = 4$? In any case some repulsive part clearly can always be added to the acceptable purely-attractive $l = 4$ potentials d_4 and e_4; the relevant specific question in this connection is how strongly repulsive.

We note that the attractive parts corresponding to a_4 to c_4 are already ruled out by the requirement that δ_0^{exp} and δ_2^{exp} are to be fitted with a common attractive part. Further, we have also seen that a_4 to c_4 do not give a really acceptable fit to δ_4^{exp}. Since the addition of any repulsive part will reduce δ_4 for these attractive parts, the fit to δ_4^{exp} will be further worsened at the higher energies (in the range of energies we consider), although slightly improved at the lower energies. Thus, even with the inclusion of a repulsive part, the attractive parts a_4 to c_4 are not acceptable from a consideration of δ_4 alone, i.e., even if the requirement of a common attractive part for $l = 0$, 2 and 4 is relinquished.

For the acceptable potentials d_4 and e_4, the results of table 4 show that in fact only a quite weak repulsive part is permissible and that even allowing the presence of such a weak repulsion does not make the fit to δ_4^{exp} for $E \lessapprox 11$ MeV significantly better

TABLE 4

The phase shift δ_4 (in degrees)

c.m. energy E(MeV)					6.15	7.6	8.9	10.2	10.9	11.45	26.70	59.93
Experimental phase shift $\delta_4^{\exp}$					3.0 ± 1.5	5.2 ± 2	16.2 ± 2	27.7 ± 2	47.0 ± 2	59.2 ± 2	137.9 ± 1.5	130.3 ± 1.8
Potentials												
Label	μ_a (fm^{-1})	V_a (MeV)	μ_r (fm^{-1})	V_r (MeV)								
d_4	0.475	130		0	2.45	6.3	13.9	31.8	50.4	70.0	153.2	152.5
d_4(0.7, 10)	0.475	130	0.7	10	2.4	5.95	12.7	27.1	41.05	56.0	147.95	147.1
	0.475	130	0.7	15	2.35	5.8	12.2	25.4	37.8	51.1	145.4	144.5
	0.475	130	0.7	20	2.3	5.7	11.8	24.0	35.2	47.1	142.9	141.9
	0.475	130	0.7	40	2.2	5.3	10.6	20.3	28.4	36.7	133.2	131.7
e_4	0.5	150		0	1.9	4.85	10.8	26.2	45.0	68.2	159.9	160.8
	0.5	150	0.8	5	1.9	4.75	10.5	24.4	40.7	61.0	158.1	158.8
	0.5	150	0.8	10	1.9	4.7	10.1	23.0	37.3	55.1	156.3	156.9
	0.5	150	0.8	15	1.85	4.6	9.8	21.8	34.6	50.4	154.6	155.0
	0.5	150	0.8	20	1.8	4.5	9.6	20.8	32.4	46.4	152.9	153.0

The phase shift δ_4 is calculated with use of eq. (3a). For energies $E > 11.45$ MeV, see the remarks for table 2.

than the fits obtained with only the corresponding purely attractive potentials. Permissible limits for the $l = 4$ repulsive part are $V_r \lesssim 10$ MeV for $\mu_r = 0.7\ \mathrm{fm}^{-1}$ for the attractive part d_4 and $V_r \lessapprox 10$ MeV for $\mu_r = 0.8\ \mathrm{fm}^{-1}$ for e_4. The ranges μ_r^{-1} have been chosen to be the same as used for the corresponding $l = 0$ and 2 potentials. The strength of the permissible repulsive parts for $l = 4$ are thus very small compared with those for $l = 0$ and 2. The values of δ_4 for the attractive part d_4 with the addition of the repulsive part $V_r = 10$ MeV, $\mu_r = 0.7\ \mathrm{fm}^{-1}$ are also shown in fig. 1. It thus seems to be a significant result that a common attractive part for $l = 0$, 2 and 4 is fairly uniquely determined and that the repulsive part becomes rapidly weaker as one goes from $l = 0$ to $l = 4$.

It is clear from tables 1 and 4 that if the repulsive part for $l = 4$ were appreciably larger, then in order to obtain just the attractive tail from consideration of δ_4 one would have to restrict oneself to values of δ_4^{exp} at suitably low energies ($E \lessapprox 8$ MeV). Accurate values of δ_4^{exp} at such energies are thus of importance for a reliable determination of the attractive tail of $V_{\alpha\alpha}^{(\mathrm{N})}$. An important point is that such a determination of the attractive tail is independent of whether or not the shorter range repulsive part is assumed to be local (for a given l).

Some results for the phase shifts at higher energies ($E \gtrapprox 25$ MeV) are also given in tables 2–4. These may be compared with the corresponding values for the real parts of the experimental phase shifts [9]) which are also shown. Since our potentials are purely real (having been obtained at lower energies where the scattering is entirely elastic), such a comparison is not strictly justified and our phase shifts are to be regarded as an indication only. In view of this, our acceptable potentials are seen to give not unreasonable results at higher energies. Furthermore, as already mentioned, there is probably not too much justification for taking the effective local potentials obtained at low energies and using them at much higher energies.

It seems significant and is gratifying that although our approach to obtaining phenomenological α-α potentials is quite distinct from that used by Darriulat *et al.* [9]), our potentials are nevertheless quite similar to theirs. Darriulat *et al.* measured the differential elastic cross sections at c.m. energies between 26 MeV and 60 MeV and analysed their results in terms of complex phase shifts. These were then fitted with phenomenological potentials consisting of a sum of real repulsive, real attractive, and imaginary Saxon-Woods potentials. A different set of parameters for the real potentials was needed for each partial wave. In particular, they found that their real $l = 0$ potential is more repulsive than their real $l = 2$ potential and that for $l = 4$ the strength of the repulsive part was to be regarded as an upper limit, since if the repulsive part is removed the fit to δ_4^{exp} is still acceptable. Further, they found that the tail of the potential is almost the same for $l = 0$, 2 and 4. Furthermore, even in detail, our potentials are quite similar to those of Darriulat *et al.*, even though our analysis was made with different functional forms for the potential shapes.

It is also encouraging that our results are consistent with theoretical studies of the interaction. In this connection, the remarks made by Darriulat *et al.* [9]) apply also to our results – in particular, the comment that Shimodaya *et al.* [5]) have converted the non-local exchange part of their theoretical potential by the use of some approximations (valid at low energies) to local but l-dependent repulsive potentials which become weaker in going from $l = 0$ to $l = 4$. Further, they also have an outer attractive tail which is independent of l.

We remark that it seems quite promising that accurate values of $\delta_4^{\exp}$ at appropriately low energies ($\lessapprox$ 8 MeV) together with careful analyses may be able to elucidate the outer attractive tail of $V_{\alpha\alpha}^{(N)}$ in some detail. The same should be even more true for the higher phase shifts (e.g., δ_6 and δ_8).

If this phenomenological tail can be identified with the tail of the direct part of $V_{\alpha\alpha}^{(N)}$ arising from the nuclear forces (i.e., if polarization forces are sufficiently small in the region of this tail), then this identification would be of considerable interest for the nucleon-nucleon interaction. Thus because $T = 0$, $J = 0^+$ for the α-particle, only the central direct component V_0 (i.e., the term independent of both the spin and the isospin) in the nuclear force will contribute to the direct part of $V_{\alpha\alpha}^{(N)}$. All the other spin- and isospin-dependent terms will average out to zero †. The tail of $V_{\alpha\alpha}^{(N)}$ will thus pick out the attractive tail of V_0 alone in the nuclear force and is thus a selective probe into this particular component of the nuclear force.

These considerations are in fact entirely analogous to those given some time ago by Drell [12]) for the tail of the nucleon-nucleus optical potential for heavy nuclei. An advantage of considering the α-α interaction in this respect is that the α-particles are expected to be particularly rigid and hence the polarization forces to be relatively small, and further that the distribution of the neutrons and protons is expected to be very nearly the same and that this distribution is quite accurately known from electron-scattering experiments.

It is of interest to consider, more specifically, the relevance to a one-boson-exchange model for the nucleon-nucleon interaction [13]). Thus with this model the only bosons that will contribute to V_0 are a σ meson with $T = 0$, $J = 0^+$ and mesons with $T = 0$, $J = 1^-$. The lightest of the latter is the ω meson ($m_\omega = 5.6\ \mathrm{m}_\pi$) which gives rise to a short-range repulsion in V_0 whereas the former ($m_\sigma \approx 3\ \mathrm{m}_\pi$) will give rise to the required strong and relatively long-range attractive part of V_0. With a one-boson-exchange model, the attractive tail of $V_{\alpha\alpha}^{(N)}$ may thus hopefully be used to obtain information about m_σ as well as about the σNN coupling constant.

It is interesting to consider our results for μ_a as an illustration of these ideas. For the α-particle we take a wave function whose (normalized) space part is $(3/\pi a^2)^{\frac{9}{4}}$ $\exp(-3\sum_{i>j} r_{ij}^2/32a^2)$, where r_{ij} is the distance between the ith and the jth nucleons and where the sum is over all pairs of nucleons. This wave function gives a distribution for the nucleons which is proportional to $\exp(-r^2/a^2)$ and whose rms radius is ††

† The tensor force will give a small contribution because the α-particle wave function includes a small admixture of D states in addition to the dominant S-state [4]).

†† Such a Gaussian distribution gives an excellent fit to the electron scattering experiments (ref. [11])).

$R_\alpha = (\frac{3}{2})^{\frac{1}{2}}a$. Further, for the relevant part of the nuclear interaction, (i.e., for the long-range attractive part of V_0) we assume, for simplicity, an attractive Gaussian potential $V\exp(-r^2/a^2)$. The resulting direct part of $V_{\alpha\alpha}^{(N)}$ is then

$$16V\left(1+\frac{2a^2}{\alpha^2}\right)^{-\frac{3}{2}}\exp[-r^2/(2a^2+\alpha^2)].$$

If this is identified with the phenomenological shape used for the attractive tail of $V_{\alpha\alpha}^{(N)}$, which is proportional to $\exp(-\mu_a^2r^2)$, then one has $\mu_a^{-2} = 2a^2+\alpha^2$. With $a = 1.18\pm0.05$ fm (corresponding to a rms radius 1.45 ± 0.065 fm) and with our results for μ_a, i.e., $0.475\ \mathrm{fm}^{-1} \lessapprox \mu_a \leqq 0.5\ \mathrm{fm}^{-1}$, one obtains $0.97\ \mathrm{fm} \lessapprox \alpha \lessapprox 1.39$ fm. To obtain the corresponding mass of the exchanged $T = 0$, $J = 0^+$ boson, one can attempt roughly to relate α^{-1} to the range of a nucleon-nucleon Yukawa potential, proportional to $r^{-1}\exp(-\mu r)$, by obtaining the value of μ that gives the same intrinsic range b as the Gaussian interaction. Thus one has the relations [14]) $\alpha = b/\sqrt{2.06}$ and $\mu^{-1} = b/2.12$. One then obtains $0.66\ \mathrm{fm} \lessapprox \mu^{-1} \lessapprox 0.94$ fm. Although it is clear that not too much significance can be attached to these values of μ^{-1}, it is nevertheless noteworthy that these are significantly less than the one-pion range ($\mu_\pi^{-1} = 1.4$ fm) and are in fact close to the value corresponding to a mass $2m_\pi$.

For more precise values of μ and also for the σNN coupling constant, a more careful phenomenological analysis for the attractive tail of $V_{\alpha\alpha}^{(N)}$ would be required. Such an analysis would also have to use more realistic shapes for the attractive tail of the potential $V_{\alpha\alpha}^{(N)}$ (appropriate to a nucleon-nucleon Yukawa potential) and should also consider the possible effect of the polarization potential V_{pol}. The calculations of Herzenberg and Roberts [6]) indicate that, at least at low energies, V_{pol} is small in the region of the tail of $V_{\alpha\alpha}^{(N)}$. However, they used central nucleon-nucleon interactions whereas the most important long-range contribution to V_{pol} is expected to come from the strong and long-range tensor component of the one-pion-exchange potential. Nevertheless, they did make estimates for a Yukawa interaction with a (long) range of μ_π^{-1}. Furthermore, for the nucleon-nucleus case Drell [12]) estimated the contribution to V_{pol} from the one-pion-exchange tensor force and showed it to be small. It thus seems rather likely that the interesting (direct) part of the tail of $V_{\alpha\alpha}^{(N)}$ will not be obscured by the polarization potential, although a more realistic calculation of this would be desirable.

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