Reanalysis of $\alpha+\alpha$ scattering and the β-delayed α spectra from ^{8}Li and ^{8}B decays

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(Received 3 January 2002; published 9 May 2002)

We reanalyzed the Wilkinson-Alburger delayed-α spectra from ^{8}Li and ^{8}B decay, accounting for lepton-recoil broadening that had been neglected in previous analyses. This substantially improved the quality of the fits. The ^{8}Li and ^{8}B delayed-α spectra are now consistently described by the same ^{8}Be final-state continuum. In our analysis, which did not invoke a low-lying intruder state, the discrepancy between the final-state continua inferred from the delayed-α spectra and from the $L=2$ $\alpha+\alpha$ phase shifts is much less than found by previous authors. This largely resolves discrepancies noted by Barker and Warburton; the remaining differences may be artifacts of the assumption that the β-decay matrix elements are independent of excitation energy. Our analysis of the Wilkinson-Alburger data is consistent with results from the recent coincidence study of Ortiz *et al.*, which gives additional confidence in using delayed-α spectra to infer the spectrum of ^{8}B neutrinos that dominate the counting rate of many solar neutrino detectors.

DOI: 10.1103/PhysRevC.65.055502 PACS number(s): 23.40.−s, 23.60.+e, 27.20.+n

I. INTRODUCTION

The broad, low-lying $J^\pi=2^+$ structure in ^{8}Be is of considerable interest for at least two reasons:

(1) The ^{8}B neutrino spectrum, which dominates the counting rates of the Kamiokande, Super-Kamiokande, and SNO solar neutrino detectors, depends on the shape of this ^{8}Be final-state continuum. To understand if neutrino mixing distorts the solar neutrino spectrum one must have good confidence in the undistorted spectrum shape.

(2) A controversy exists over shape of the 2^+ continuum in ^{8}Be. Barker [1] argued that to get a consistent R-matrix fit to both the $L=2$ $\alpha+\alpha$ phase shifts and the β-delayed α spectra from ^{8}Li and ^{8}B decay data one needs to introduce *two* low-lying 2^+ states where only a single level was expected. This raises the possibility of unknown systematic errors in the experimental results, perhaps calibration errors in the delayed-α spectra, a point that has been discussed previously [2]. Warburton [3] responded to Barker's point by arguing that the low-lying intruder state was an artifact of Barker's choice of a large (about 7 fm) matching radius; he showed that one could fit both the $\alpha+\alpha$ data and the β-delayed α spectra without a low-lying intruder state if one used a more conventional matching radius of 4.5 fm. However, a problem remained. The width and position of the lowest ^{8}Be 2^+ level extracted from the β-decay data disagreed with those extracted from elastic scattering—the β-decay results gave excitation energies and widths that differed from the $\alpha+\alpha$ scattering data by ~100 keV and ~270 keV, respectively [3]. Furthermore, the energies and widths of the 3 MeV level inferred from ^{8}Li and ^{8}B decays differed by ~30 and ~70 keV, respectively.

In this brief paper we largely resolve this problem by including an effect that was neglected in previous analyses: lepton-recoil broadening of the delayed-α spectra. Including lepton-recoil yields substantially better fits to the β-decay data and gives an excitation energy of the lowest 2^+ state in ^{8}Be in complete agreement with the value inferred from $\alpha+\alpha$ scattering data with no need for a low-lying intruder state. However, the widths of the 3 MeV state inferred individually from β decay and from elastic scattering still disagree by about 180 keV. This residual difference could be an artifact of the assumption that the Gamow-Teller (GT) matrix element of a broad level is not truly constant over a several MeV range of excitation energy.

II. DELAYED-α SPECTRA

A. Data and analysis

We reanalyzed the Wilkinson-Alburger [4] ^{8}Li and ^{8}B thin-catcher delayed-α spectra tabulated in Ref. [5], as well as the thick-catcher data tabulated in Ref. [3]. Pulse-height spectra were converted into dN/dE distributions, taking into account the Jacobian from the nonlinear energy calibration given in Refs. [3,5]. We analyzed the β-delayed α spectra using Warburton's version of the one-channel many-level R-matrix formalism as cited in Eqs. (9) and (12)–(15) of Ref. [3] (corrected for misprints [6]). The statistical rate function $f(E_\beta)$ was evaluated using the prescription of Wilkinson and Macefield [7]. Following Warburton, the final-state continuum was decomposed into three physical levels, the broad 3 MeV state and the narrow doublet at 16.626 and 16.922 MeV, plus a "background level" at $E_x=37$ MeV. The energy and width of the lowest level, and the width of the background level, were treated as adjustable parameters, as were the GT matrix elements feeding the 3 MeV level (M_1), the $T=0$ component of doublet (M_{2+3}), and the background state (M_4). The energies and widths of the 16 MeV doublet levels were fixed at their accepted values [8].

B. Lepton-recoil broadening

Lepton-recoil broadening plays an exceptionally large role in ^{8}Li and ^{8}B decay because of the large energy releases in the β decays and the small mass of the daughter nucleus. In addition, the e-ν-α triple correlation A for these GT de-

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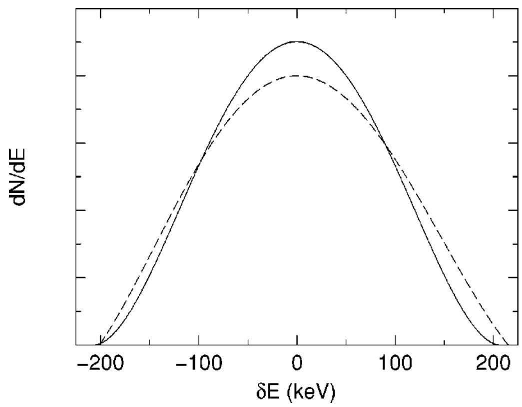

FIG. 1. Lepton-recoil effect for ^{8}B decay to ^{8}Be at $E_x = 3.0$ MeV. The solid curve is a calculation using the formalism of Ref. [9]. Our approximation given in Eqs. (1) and (3) cannot be distinguished from the solid curve. Wilkinson's approximation (Ref. [10]) is shown by the dashed line.

cays is unusually strong. The $\alpha+\alpha$ final state has $L=2$, $M=0$ where the z axis is defined by the α-particle momentum. This implies that the leptons must be in an $J=1$, $M=1$ state, and that therefore $A=-1$. Warburton [3] argued that lepton recoil could be neglected when extracting the ^{8}Be level parameters. We show below that, in fact, lepton broadening has a significant effect on the level parameters extracted from the delayed-α spectra.

Figure 1 shows the predicted broadening effect, calculated using the formalism of Ref. [9] which treats the leptons relativistically, the slow-moving daughter nonrelativistically, and neglects terms that are second-order in the recoil velocity. Note that, with these approximations, the broadening function is symmetric, has a vanishing slope at the end points, and has unit area. This suggests a simple and general approximation for the broadening effect

$$\frac{dn}{dE}(x) = \frac{15}{16\,T_{\max}}(1-2x^2+x^4)$$

$$x = \delta E/T_{\max}, \quad -1 \leqslant x \leqslant +1, \tag{1}$$

which has a full-width at half-maximum

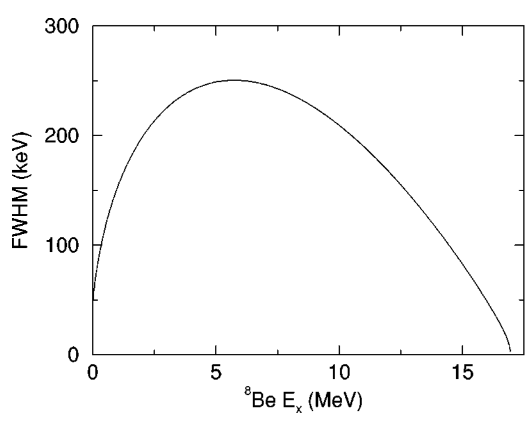

FIG. 2. FWHM of the lepton-recoil broadening in ^{8}B as a function of the final-state excitation energy.

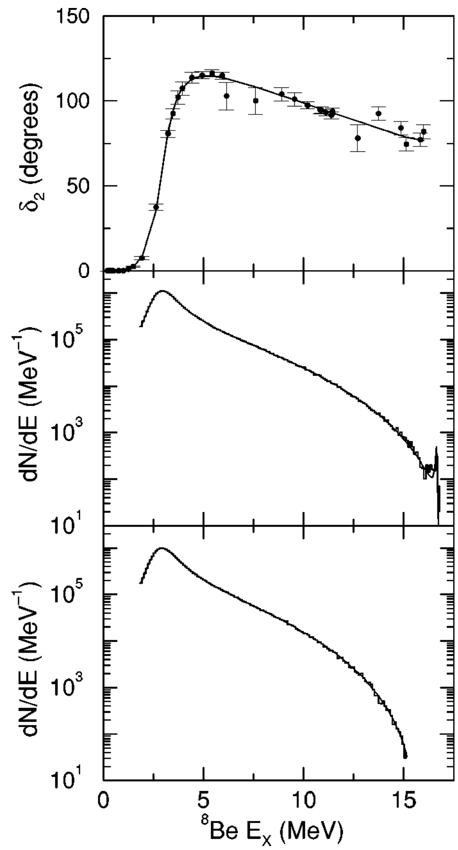

FIG. 3. Individual R-matrix fits. Top panel: $L=2$ $\alpha+\alpha$ phase shifts. Center panel: ^{8}B delayed-α spectrum. Bottom panel: ^{8}Li delayed-α spectrum.

$$\mathrm{FWHM} = 2T_{\max}(1-1/\sqrt{2})^{1/2}. \tag{2}$$

The maximum recoil shift of an α particle in mass 8 decays is

$$T_{\max}(E_x) = \sqrt{W_0^2-1}\,\frac{m_e}{M}\sqrt{2Qmc^2\frac{M-m-Q}{M-Q}}, \tag{3}$$

where E_x is the excitation energy of the ^{8}Be daughter state, m_e, m, and M are the electron, α-particle, and ^{8}Be masses, $W_0(E_x)$ is the β end point total energy in units of $m_e c^2$, and $Q = E_x + 91.88$ keV. Figure 1 also shows the recoil broadening approximation for mass 8 decays developed by Wilkinson and co-workers [10] and cited by Warburton [3]. Their approximation is invalid because it neglects the triple correlation, implicitly assuming $A=-1/3$. Note that the FWHM of the lepton-recoil broadening in ^{8}B decay, shown in Fig. 2, is significantly greater than that shown in Fig. 9 of Ref. [3].

For completeness, we give the corresponding approximation for the lepton-recoil broadening of any pure Fermi transition that must have $A=+1$. In this case, the slope at the end points is not zero, but the curvature vanishes at $x=0$, so that

$$\frac{dn}{dE}(x) = \frac{5}{8T_{\max}}(1-x^4) \tag{4}$$

TABLE I. R-matrix parameters of the ^{8}Be 2^+ continuum inferred from $\alpha+\alpha$ scattering, and from ^{8}Li and ^{8}B β decay. Quantities without errors were held fixed in the analyses. Energies and widths are in MeV; GT matrix elements are in units of μ_N. Coulomb functions were evaluated at a radius of 4.5 fm. Errors are formal fitting errors and do not include contributions from systematic uncertainties in the energy calibration, etc.

Param.	$\alpha+\alpha$	^{8}Li	^{8}B	Combined
E_1	3.049 ± 0.023	3.045 ± 0.001	3.062 ± 0.001	3.055 ± 0.002
Γ_1	1.395 ± 0.037	1.579 ± 0.002	1.568 ± 0.002	1.577 ± 0.003
M_1		-0.1595 ± 0.0002	-0.1505 ± 0.0001	-0.1597 ± 0.0002^a -0.1507 ± 0.0002^b
E_2	16.626	16.626	16.626	16.626
Γ_2	0.108	0.108	0.108	0.108
E_3	16.922	16.922	16.922	16.922
Γ_3	0.074	0.074	0.074	0.074
M_{2+3}		2.783 ± 0.041	2.426 ± 0.023	2.736 ± 0.043^a 2.437 ± 0.037^b
E_4	37.0	37.0	37.0	37.0
γ_4^2	5.35 ± 0.14	6.41 ± 0.07	5.78 ± 0.05	6.00 ± 0.08
M_4		-0.198 ± 0.007	-0.142 ± 0.004	-0.192 ± 0.008^a -0.145 ± 0.007^b
χ^2/ν	0.64	1.56	1.33	2.15

[a]GT matrix element for ^{8}Li decay.
[b]GT matrix element for ^{8}B decay.

with

$$\mathrm{FWHM}=2T_{\max}/\sqrt[4]{2}. \tag{5}$$

C. Results

We first fitted the Wilkinson-Alburger data by folding the R-matrix spectrum shape with a Gaussian resolution function with FWHM=34 keV as quoted in Ref. [3] and obtained level parameters very close to those found by Warburton with reduced χ^2 values of 2.41 (ν=130), 2.55 (ν=136), 2.41 (ν=147), and 1.98 (ν=149) for the ^{8}Li thin-catcher, ^{8}Li thick-catcher, ^{8}B thin-catcher, and ^{8}B thick-catcher data, respectively. Then we included lepton-recoil broadening by convolving the above fitting function with the broadening given by Eqs. (1) and (3). These fits gave significantly different values for the position and width of the 3 MeV level, and substantially improved reduced χ^2 values of 1.60, 1.28, 1.37, and 1.22. Next, we replaced the Gaussian resolution function with a more realistic shape consisting of a Gaussian folded with a pair of low-energy tails,

$$R(E,E')=\sum_{i=1}^{2}\frac{A_i}{2\lambda_i}\exp\left(\frac{(E-E')}{\lambda_i}+\frac{\sigma^2}{2\lambda_i^2}\right)\mathrm{erfc}\left(\frac{E-E'+\sigma^2/\lambda_i}{\sqrt{2}\sigma}\right), \tag{6}$$

where σ=FWHM/(8 ln2), λ_i is an exponential decay length, and erfc is the complement of the incomplete error function. The normalization coefficients are $A_1=1/(1+f)$ and $A_2=f/(1+f)$ with f the relative area of tail 2 compared to tail 1. The shape parameters had FWHM=34 keV, and tails consistent with spectra we obtained from a spectroscopic-grade ^{148}Gd source. This resolution function improved the χ^2 values slightly to 1.56, 1.25, 1.33, and 1.21, but the fitting parameters were essentially unchanged. Our results from fitting the thin-catcher data including recoil broadening and a realistic response function are shown in Fig. 3 and tabulated in Table I. The thick-catcher results are similar to those from the thin-catcher data. The differences between the isospin-mirror GT matrix elements are not unexpected and presumably reflect differing radial overlaps, etc., arising from Coulomb (and possibly other charge-symmetry violating) effects.

Finally, we found that the quality of the fits of the delayed-α data depended very weakly on the width of the background level. We could retain good fits to the delayed-α data and bring the $\alpha+\alpha$ phase shift results (discussed below) into better agreement with the β-decay data if we fixed the reduced width of the background level at the average of the individual best-fit values from the β-decay and phase-shift analyses.

III. $L=2$ $\alpha+\alpha$ PHASE SHIFTS

A. Data and analysis

We analyzed essentially the same data set as Barker and Warburton. Phase shifts for 0.4 MeV$\leq E_{\mathrm{lab}}\leq$3.0 MeV were taken from Heydenburg and Temmer [12]; we assigned uncertainties of 0.1° for points up to 1.5 MeV, and 0.5° for the higher points. Phase shifts for 3.84 MeV$\leq E_{\mathrm{lab}}\leq$11.88 MeV were taken from Tombrello and Senhouse [13]. Data for 12.3 MeV$\leq E_{\mathrm{lab}}\leq$22.9 MeV were taken from Nilson *et al.* [14], while phase shifts for 20 MeV$\leq E_{\mathrm{lab}}\leq$31.67 MeV were obtained from Fig. 5(b) of Bredin *et al.* [15]. The 32 MeV point of Bacher *et al.* [16] was also included.

The R-matrix description (Refs. [1,11]) for the $L=2$ $\alpha+\alpha$ phase shifts is

$$\delta_2(E) = \tan^{-1}\frac{P_2(E)}{\left[\sum_\lambda \gamma_\lambda^2/(E_\lambda - E)\right]^{-1} - S_2(E) + B_2} - \phi_2, \tag{7}$$

where ϕ_2 is the hard-sphere phase shift. We fitted the phase-shift data with the same four *R*-matrix levels used for the delayed-α spectra. Again, all level parameters were held fixed at their established values except for the energy and width of lowest 2^+ state and the width of the background level.

B. Results

We orignally followed Warburton and fitted the phase-shift data with six levels: the four used in the analysis of the delayed-α data plus known states [8] at 20.10 and 22.2 MeV that were not needed to fit the β-decay data as they have very little GT strength. Our best-fit parameters differed from those quoted by Warburton; we cannot explain this discrepancy as we could reproduce Barker's results. We found that the 20 and 22 MeV levels had little effect on the phase shifts and obtained equally good fits to the phase-shift data [$\chi^2/\nu = 0.64$, $\nu = 31$, $P(\chi^2,\nu) = 94\%$] using only the four levels needed for the delayed-α data. The best fit, shown in Fig. 3, gave a background-level width that was close to the β-decay value, but the 3 MeV level width was about 180 keV lower than our β-decay value. However, the width of the 3 MeV level was strongly correlated with the reduced width, γ_4^2, of the background level. When γ_4^2 was fixed to the average of the values from the three independent fits to the ^{8}Li, ^{8}B, and phase-shift data, the phase-shift data gave a 3 MeV level width consistent with the β-decay value; χ^2 of the scattering data fit worsened but was still acceptable [$\chi^2/\nu = 1.25$, $P(\chi^2,\nu) = 16\%$]. Results from a combined fit to the ^{8}Li and ^{8}B thin-catcher delayed-α spectra plus the $L = 2$ phase shifts, constrained to contribute equally to the total χ^2, are also shown in Table I. The residual discrepancies between the extracted β-decay and elastic-scattering level parameters could well be due to the simplifying assumption that β-decay matrix elements to broad levels are independent of excitation energy. The importance of radial overlaps in GT matrix elements is illustrated by the observed difference (see Table I) in the isospin-mirror GT matrix elements in ^{8}Li and ^{8}B decays.

IV. CONCLUSION

Because of the impact of the ^{8}B solar neutrino spectrum on neutrino oscillation scenarios, it is essential to have an accurate prediction for the laboratory spectrum of ^{8}B neutrinos. Therefore, the detailed shape of the very broad final-state continuum in ^{8}Be must be known to high accuracy. We argue that it is better, in principle, to infer the continuum shape from delayed-α spectra, rather than from β spectra as was done in Ref. [2]. β-spectrum-shape measurements are notoriously subject to systematic errors from scattering in the source, the detector, and the rest of the apparatus. Furthermore, the β spectra must be corrected for radiative effects, and the broad final-state distribution must be unfolded from an even broader β spectrum. Unfortunately the existing delayed-α spectra have inconsistent energy calibrations as shown in Ref. [2], but this can readily be improved in future experiments; we have undertaken such a measurement that will be reported elsewhere.

Delayed-α data can be obtained either as singles spectra, as exemplified by the Wilkinson-Alburger data, or as summed coincidence spectra recently pioneered by Ortiz *et al.* Each method has its advantages and its disadvantages. In the summed coincidence experiment of Ortiz et al. [17,18], a strong magnetic field swept out the β's and the summed energies of the two coincident α's were measured. This eliminated the β background and the first-order energy-smearing effect of lepton recoil, but introduced energy-

TABLE II. Recommended *R*-matrix description of the final-state distribution in ^{8}B decay. Coulomb functions are calculated at a matching radius of 4.5 fm. Unless otherwise noted, central values and uncertainties are the average and half the difference of this work and Ref. [18].

Parameter	Recommended value
E_1 (Mev)	3.041 ± 0.030^{a}
Γ_1 (MeV)	1.561 ± 0.008
M_1 (μ_N)	-0.149 ± 0.002
E_2 (MeV)	16.626 ± 0.003^{b}
Γ_2 (MeV)	0.1081 ± 0.0005^{b}
E_3 (MeV)	16.922 ± 0.003^{b}
Γ_3 (MeV)	0.0740 ± 0.0004^{b}
M_{2+3} (μ_N)	2.525 ± 0.099
E_4 (MeV)	37.0
γ_4^2 (MeV)	5.95 ± 0.17
M_4 (μ_N)	-0.162 ± 0.020

[a]Based on energy calibration given in Ref. [3].
[b]From Ref. [8].

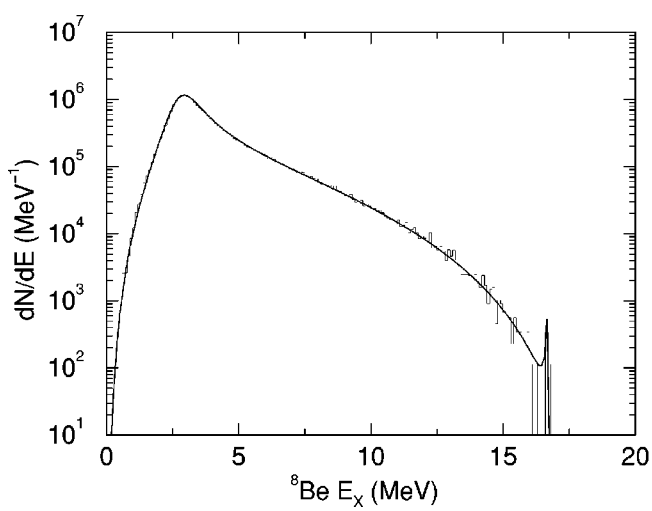

FIG. 4. Comparison of the final-state continuum in ^{8}B decay from our analysis of the Wilkinson-Alburger data and from the summed coincidence data of Ref. [18]. The data of Ref. [18] have been shifted upward by 40 keV.

dependent distortions of the spectrum from charge-state fractionation and lepton recoil which had to be accounted for by Monte Carlo simulations. The singles method is experimentally straightforward, but one must account for lepton-recoil effects when extracting the final-state distribution from the delayed-α spectra. We developed a simple approximation that gives an accurate description of the lepton-recoil smearing. This largely resolved a long-standing puzzle regarding the shapes of the singles β-delayed α spectra from ^{8}Li and ^{8}B decays. We find that, after lepton-recoil effects are included, the inconsistency between Wilkinson-Alburger delayed-α spectra and the $L=2$ $\alpha+\alpha$ phase shifts is much smaller than previously thought and may be due to a simplifying assumption made in analyzing the β-decay data. This weakens the argument for a low-lying intruder state and gives confidence in the procedure of using the β-delayed α data to determine the spectrum of ^{8}B neutrinos.

We recommend the *R*-matrix parameters given in Table II, based on this work and Refs. [17,18], as the best available description of the final-state distribution in ^{8}B decay. Figure 4 compares our results to the data from Refs. [17,18]. It is gratifying that 1.57 MeV width of the 3 MeV level we extracted from the Wilkinson-Alburger singles data agrees well with the 1.55 MeV width from the summed coincidence study of Refs. [17,18]. Our resonance energy is about 40 keV higher than that quoted in Ref. [18] but this could reflect uncertainties in the absolute energy calibration (given as ± 30 keV in Ref. [4] and not stated in Refs. [17,18]). Still higher-quality delayed-α data will be forthcoming, and should determine the laboratory neutrino spectrum so precisely that will not be a significant source of uncertainty in the analysis of future solar-neutrino results.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy. We thank John Schiffer for a helpful comment.

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