02157 Functional Programming

Lecture 1: Introduction and Getting Started

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WELCOME to
02157 Functional Programming

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The Functional Setting

A program *f* is a function

$$f : \mathit{Argument} \rightarrow \mathit{Result}$$

- *f* takes one argument and produces one result
- arguments and results can be composite values
- computation is governed by function application

Example: Insertion in an ordered list

```
insert(4, [1; 3; 7; 9]) = [1; 3; 4; 7; 9]
```

- the argument is a pair: (4, [1; 3; 7; 9])
- the result is a list: [1; 3; 4; 7; 9]
- the computation is guided by repeated function application

```
    insert(4, [1; 3; 7; 9])
⇝   1::insert(4, [3; 7; 9])
⇝   1::3::insert(4, [7; 9])
⇝   1::3::4::[7;9]
=   [1;3;4;7;9]
```

A simple problem solving technique

Solve a complex problem by

- partitioning it into smaller well-defined parts
- compose the parts to solve the original problem.

The main goal:

A program is constructed by
combining simple well-understood pieces

- A general technique that is natural in functional programming.

Insertion in an ordered list is a well-understood program.

- Can you use this in the construction of program for sorting a list?

A typed functional programming language

Supports

- Composite values like lists and trees
- Functions as ”first-class citizens”
- Recursive functions
- Patterns
- A strong polymorphic type system
- Type inference

A small collection of powerful concepts used to explain the meaning of a program

- binding
- environment
- evaluation of expressions

A value-oriented (declarative) programming approach where you focus on *what* properties a solution should rather than on the individual computation steps.

A typed functional programming language

does not support

- assignable variables, assignments
  `a[i] = a[i] + 1, x++, ...`
- imperative control statements `while i<10 do ...` `if b then a[i]=a[i]+1,...`
- object-oriented state-changing features `child.age = 5`
- ...

In imperative and object-oriented programming approaches focus is on *how* a solution is obtained in terms of a sequence of state changing operations.

About functional programming

Some advantages of functional programming:

- Supports fast development based on abstract concepts
  more advanced applications are within reach
- Supplements modelling and problem solving techniques
- Supports parallel execution on multi-core platforms

F# is as efficient as C#

Testimonials, quotes and real-world applications of FP languages:

- `http://fsharp.org`
- `homepages.inf.ed.ac.uk/wadler/realworld`

Functional programming techniques once mastered are useful for the design of programs in other programming paradigms as well.

Some functional programming background

- Introduction of λ-calculus around 1930 by Church and Kleene when investigating function definition, function application, recursion and computable functions. For example, $f(x) = x + 2$ is represented by $\lambda x.x + 2$.

- Introduction of the type-less functional-like programming language LISP was developed by McCarthy in the late 1950s.

- Introduction of functional languages with a strong type system like ML (by Milner) and Miranda (by Turner) in the 1970s.

- Functional languages (SML, Haskell, OCAML, F#, ...) have now applications far away from their origin: Compilers, Artificial Intelligence, Web-applications, Financial sector, ...

- Declarative aspects are now sneaking into "main stream languages"

- Peter Sestoft: Programming Language concepts, Springer 2012. Uses F# as a meta language.

Practical Matters (I)

- Textbook: Functional Programming using F#
  M.R. Hansen and H. Rischel. Cambridge Univ. Press, 2013.

  `www.imm.dtu.dk/~mrh/FSharpBook`

  Available at DTU's bookstore and library.

- F# (principal designer: Don Syme) is an open-source functional language. F# is integrated in the Visual Studio platform and with access to all features in the .NET program library. The language is also supported on Linux, MAC, . . . systems

- Look at `http://fsharp.org` concerning installations for your own laptops (Windows, Linux, Mac, Android, iPhone, . . .).
  Suggestions: Windows: Visual Studio and for Mac and Linux: Visual Studio Code

- Lectures: Friday 8.15 - 10:00 in Building 308, Auditorium 13

- Exercises classes: Friday 10:00 - 12:00 in Building 341
  Rooms 003, 015 and 019

Practical Matters: Mandatory assignments

- Four mandatory assignments. See course plan
- Three must be approved in order to participate in the exam.
- They do not have a role in the final grade.
- Groups of sizes 1, 2 and 3 are allowed.

Approvals of mandatory assignments from earlier years
DO NOT apply this year

Course context

- Prerequisites for 02157:
  - Programming in an imperative/object-oriented language
  - Discrete mathematics (previously or at this semester)

- 02157 is a prerequisite for 02141 Computer Science Modelling

- 02141 is a prerequisite for

- 02257 Applied Functional Programming January course

  – efficient use of functional programming in connection with activities at the M.Sc. programmes and in "practical applications".

  One project every week:
  - A Computer science application
  - A "Practical application".
  - A functional pearl.

Overview

Part 1 Getting Started:

- The interactive environment
- Values, expressions, types, patterns
- Declarations of values and recursive functions
- Binding, environment and evaluation
- Type inference

Main ingredients of F#

Part 2 Lists:

- Lists: values and constructors
- Recursions following the structure of lists
- Polymorphism

A value-oriented approach

GOAL: By the end of the day you have constructed succinct, elegant and understandable F# programs.

The Interactive Environment

```
2*3 + 4;;
val it : int = 10
```

⇐ Input to the F# system

⇐ Answer from the F# system

- The *keyword* `val` indicates a value is computed
- The *integer* 10 is the computed value
- `int` is the *type* of the computed value
- The *identifier* `it` names the (last) computed value

The notion *binding* explains which entities are named by identifiers.

`it` $\mapsto$ 10 reads: "`it` is bound to 10"

Value Declarations

A value declaration has the form: `let` *identifier* $=$ *expression*

```
let price = 25 * 5;;          ⇐ A declaration as input
val price : int = 125         ⇐ Answer from the F# system
```

The effect of a declaration is a binding: `price` $\mapsto$ 125

Bound identifiers can be used in expressions and declarations, e.g.

```
let newPrice = 2*price;;
val newPrice : int = 250

newPrice > 500;;
val it : bool = false
```

A collection of bindings

$$\left[\begin{array}{lll} \texttt{price} & \mapsto & 125 \\ \texttt{newPrice} & \mapsto & 250 \\ \texttt{it} & \mapsto & \mathrm{false} \end{array}\right]$$

is called an environment

Function Declarations 1: `let` $f\, x = e$

- x is called the *formal parameter*
- the defining expression e is called the *body* of the declaration

Declaration of the circle area function:

```
let circleArea r = System.Math.PI * r * r;;
```

- `System.Math` is a program library
- `PI` is an identifier (with type `float`) for π in `System.Math`

The type is automatically inferred in the answer:

```
val circleArea : float -> float
```

Applications of the function:

```
circleArea 1.0;; (* this is a comment *)
val it : float = 3.141592654

circleArea(3.2);; // A comment: optional brackets
val it : float = 32.16990877
```

`1.0` and `3.2` are also called *actual parameters*

Recursion. Example $n! = 1 \cdot 2 \cdot \ldots \cdot n$, $n \geq 0$

Mathematical definition: recursion formula

$$
\begin{aligned}
0! &= 1 && (i) \\
n! &= n \cdot (n-1)!, \quad \text{for } n > 0 && (ii)
\end{aligned}
$$

- $n!$ is defined recursively in terms of $(n-1)!$ when $n > 0$

Computation:

$$
\begin{aligned}
& 3! \\
= \;& 3 \cdot (3-1)! && (ii) \\
= \;& 3 \cdot 2 \cdot (2-1)! && (ii) \\
= \;& 3 \cdot 2 \cdot 1 \cdot (1-1)! && (ii) \\
= \;& 3 \cdot 2 \cdot 1 \cdot 1 && (i) \\
= \;& 6
\end{aligned}
$$

Function Declarations 2: `let` *rec f x* $=$ *e*

- the function *f* occurs in the body *e* of a *recursive declaration*

A recursive function declaration:

```
let rec fact n =
      if n=0 then 1                     (*  i *)
      else n * fact(n-1);;              (* ii *)
val fact : int -> int
```

Evaluation:

$$
\begin{array}{lllr l}
 & \texttt{fact}(3) & & & \\
\rightsquigarrow & 3 * \texttt{fact}(3-1) & & (ii) & [\texttt{n} \mapsto 3] \\
\rightsquigarrow & 3 * 2 * \texttt{fact}(2-1) & & (ii) & [\texttt{n} \mapsto 2] \\
\rightsquigarrow & 3 * 2 * 1 * \texttt{fact}(1-1) & & (ii) & [\texttt{n} \mapsto 1] \\
\rightsquigarrow & 3 * 2 * 1 * 1 & & (i) & [\texttt{n} \mapsto 0] \\
\rightsquigarrow & 6 & & &
\end{array}
$$

$e_1 \rightsquigarrow e_2$ reads: e_1 evaluates to e_2

- An environment is used to bind the formal parameter *n* to actual parameters $3, 2, 1, 0$ during evaluation

Patterns

A pattern is composed from identifiers, constants and the wildcard pattern _ using constructors (considered soon)

Examples of patterns are: `3.1`, `true`, `n`, `x`, `5`, `_`

- A pattern may match a value, and if so it results in an environment with bindings for every identifier in the pattern.
- The wildcard pattern `_` matches any value (resulting in no binding)

Examples:

- Value `3.1` matches pattern `x` resulting in environment: $[x \mapsto 3.1]$
- Value `true` matches pattern `true` resulting in environment $[\,]$
- Value `(1,true)` matches pattern `(x,y)` resulting in environment $[x \mapsto 1, y \mapsto \mathit{true}]$

Match expressions

A match expression e_m has the following form:

$$\begin{array}{l} \texttt{match}\ e\ \texttt{with} \\ \mid pat_1 \rightarrow e_1 \\ \quad\quad \vdots \\ \mid pat_n \rightarrow e_n \end{array}$$

A match expression e_m is evaluated as follows:

If

- v is the value of e and
- pat_i is the first matching pattern for v and env is the environment obtained from the pattern matching,

then

$$e_m \rightsquigarrow (e_i, env)$$

The environment env contains bindings for identifiers in pat_i

If no pattern matches v, then the evaluation terminates abnormally.

Example: Match on an integer

Let e_1 be given by:

```
match 3+5 with
 | 0 -> 45
 | n -> n * (n-1)
```

Evaluation:

$$
\begin{array}{ll}
 & e_1 \\
\rightsquigarrow & (\mathtt{n} * (\mathtt{n} - 1), [\mathtt{n} \mapsto 8]) \\
\rightsquigarrow & (8 * 7, [\mathtt{n} \mapsto 8]) \\
\rightsquigarrow & 56
\end{array}
$$

Example: Match on a pair

Let e_2 be given by:

```
match (3+5, 3<5) with
| (0, _)      -> 0
| (n,false)   -> -n
| (n,_)       -> 2*n
```

Evaluation:

$$\begin{array}{ll} & e_2 \\ \rightsquigarrow & (2 * \mathtt{n}, [\mathtt{n} \mapsto 8]) \\ \rightsquigarrow & (2 * 8, [\mathtt{n} \mapsto 8]) \\ \rightsquigarrow & 16 \end{array}$$

Example: Match expression in a declaration

Function declaration:

```
let rec fact n =
   match n with
   | 0 -> 1               (*  i *)
   | n -> n * fact(n-1)   (* ii *)
val fact : int -> int
```

Evaluation:

$$
\begin{array}{llll}
 & \mathtt{fact}(3) & & \\
\rightsquigarrow & 3 * \mathtt{fact}(3-1) & (ii) & [\mathtt{n} \mapsto 3] \\
\rightsquigarrow & 3 * 2 * \mathtt{fact}(2-1) & (ii) & [\mathtt{n} \mapsto 2] \\
\rightsquigarrow & 3 * 2 * 1 * \mathtt{fact}(1-1) & (ii) & [\mathtt{n} \mapsto 1] \\
\rightsquigarrow & 3 * 2 * 1 * 1 & (i) & [\mathtt{n} \mapsto 0] \\
\rightsquigarrow & 6 & &
\end{array}
$$

A match with a when clause and an exception:

```
let rec fact n =
   match n with
   | 0          -> 1
   | n when n>0 -> n * fact(n-1)
   | _          -> failwith "Negative argument"
```

Recursion. Example $x^n = x \cdot \ldots \cdot x$, n occurrences of x

Mathematical definition: recursion formula

$$
\begin{aligned}
x^0 &= 1 && (1)\\
x^n &= x \cdot x^{n-1}, \quad \text{for } n > 0 && (2)
\end{aligned}
$$

Function declaration:

```
let rec power(x,n) =
   match (x,n) with
   | (_,0) -> 1.0                  (* 1 *)
   | (x,n) -> x * power(x,n-1)     (* 2 *)
```

Patterns:

$(\_, 0)$ matches any pair of the form $(u, 0)$.

(x, n) matches any pair (u, i) yielding the bindings

$$\mathtt{x} \mapsto u, \mathtt{n} \mapsto i$$

Evaluation. Example: `power(4.0, 2)`

Function declaration:

```
let rec power(x,n) =
   match (x,n) with
   | (_,0) -> 1.0                        (* 1 *)
   | (x,n) -> x * power(x,n-1)           (* 2 *)
```

Evaluation:

$$
\begin{array}{lll}
 & \mathtt{power}(4.0, 2) & \\
\rightsquigarrow & 4.0 * \mathtt{power}(4.0, 2-1) & \text{Clause 2, } [\mathtt{x} \mapsto 4.0, \mathtt{n} \mapsto 2] \\
\rightsquigarrow & 4.0 * \mathtt{power}(4.0, 1) & \\
\rightsquigarrow & 4.0 * (4.0 * \mathtt{power}(4.0, 1-1)) & \text{Clause 2, } [\mathtt{x} \mapsto 4.0, \mathtt{n} \mapsto 1] \\
\rightsquigarrow & 4.0 * (4.0 * \mathtt{power}(4.0, 0)) & \\
\rightsquigarrow & 4.0 * (4.0 * 1) & \text{Clause 1} \\
\rightsquigarrow & 16.0 &
\end{array}
$$

Booleans

Type name `bool`

Values `false`, `true`

Operator	Type	
`not`	`bool -> bool`	negation

```
not true = false
not false = true
```

Expressions

e_1 && e_2	"conjunction $e_1 \wedge e_2$"
e_1 || e_2	"disjunction $e_1 \vee e_2$"

— are lazily evaluated, e.g.

```
1<2 || 5/0 = 1
⇝ true
```

Precedence: `&&` has higher than `||`

If-then-else expressions

Form:

$$\texttt{if}\ b\ \texttt{then}\ e_1\ \texttt{else}\ e_2$$

Evaluation rules:

$$\texttt{if}\ \mathrm{true}\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \quad \rightsquigarrow \quad e_1$$
$$\texttt{if}\ \mathrm{false}\ \texttt{then}\ e_1\ \texttt{else}\ e_2 \quad \rightsquigarrow \quad e_2$$

Notice:

- Both then and else branches must be present (because it is an expression that gives a value)
- either e_1 or e_2 is evaluated (but never both)
  provided that b terminates.

Strings

Type name `string`

Values `"abcd"`, `"  "`, `""`, `"123\"321"` (escape sequence for `"`)

Operator	Type	
`String.length`	`string -> int`	length of string
`+`	`string*string -> string`	concatenation
`= < <= ...`	`string*string -> bool`	comparisons
`string`	`obj -> string`	conversions

Examples

```
- "auto" < "car";
> val it = true : bool

- "abc"+"de";
> val it = "abcde": string
```

```
- String.length("abc"^"def");
> val it = 6 : int

- string(6+18);
> val it = "24": string
```

Types — every expression has a type $e : \tau$

Basic types:

	type name	example of values
Integers	`int`	~27, 0, 15, 21000
Floats	`float`	~27.3, 0.0, 48.21
Booleans	`bool`	true, false

Pairs:

If $e_1 : \tau_1$ and $e_2 : \tau_2$

then $(e_1, e_2) : \tau_1 * \tau_2$ pair (tuple) type constructor

Functions:

if $f : \tau_1$ -> τ_2 and $a : \tau_1$ function type constructor

then $f(a) : \tau_2$

Examples:

```
(4.0, 2):  float*int
power:  float*int -> float
power(4.0, 2):  float
```

$\star$ has higher precedence that ->

Type inference: `power`

```
let rec power (x,n) =
   match (x,n) with
   | (_,0) -> 1.0                      (* 1 *)
   | (x,n) -> x * power(x,n-1)         (* 2 *)
```

- The type of the function must have the form: τ_1 `*` τ_2 `->` τ_3, because argument is a pair.
- τ_3 = `float` because `1.0:float` (Clause 1, function value.)
- τ_2 = `int` because `0:int`.
- `x*power(x,n-1):float`, because τ_3 = `float`.
- multiplication can have

  `int*int -> int` or `float*float -> float`

  as types, but no “mixture” of `int` and `float`
- Therefore `x:float` and τ_1=`float`.

The F# system determines the type `float*int -> float`

Two alternative declarations of the power function:

```
let rec power pair = match pair with
                     | (_,0) -> 1.0
                     | (x,n) -> x * power(x,n-1);;
```

```
let rec power(x,n) = match n with
                     | 0  -> 1.0
                     | n' -> x * power(x,n'-1);;
```

Summary

- The interactive environment
- Values, expressions, types, patterns
- Declarations of values and recursive functions
- Binding, environment and evaluation
- Type inference

Breath first round through many concepts aiming at program construction from the first day.

We will go deeper into each of the concepts later in the course.

Overview of Part 2: Lists

- Lists: values and constructors
- Recursions following the structure of lists
- Polymorphism

The purpose of this lecture is to give you an (as short as possible) introduction to lists, so that you can solve a problem which can illustrate some of F#'s high-level features.

This part is *not* intended as a comprehensive presentation on lists, and we will return to the topic again later.

Lists

A list is a finite sequence of elements having the same type:

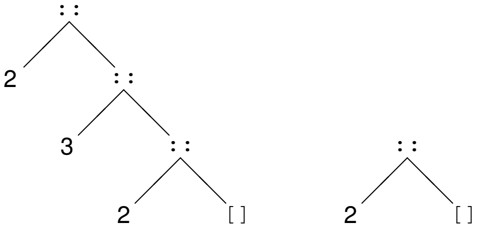
$[v_1; \ldots; v_n]$ ([] is called the empty list)

```
[2;3;6];;
val it : int list = [2; 3; 6]

["a"; "ab"; "abc"; ""];;
val it : string list = ["a"; "ab"; "abc"; ""]

[sin; cos];;
val it : (float->float) list = [<fun:...>; <fun:...>]

[(1,true); (3,true)];;
val it : (int * bool) list = [(1, true); (3, true)]

[[]; [1]; [1;2]];;
val it : int list list = [[]; [1]; [1; 2]]
```

Trees for lists

A non-empty list $[x_1; x_2; \ldots; x_n]$, $n \geq 1$, consists of

- a *head* x_1 and
- a *tail* $[x_2; \ldots; x_n]$



Graph for `[2;3;2]`   Graph for `[2]`

- `::` and `[]` are called constructors
  – they are used to construct and to decompose lists

`2::3::2::[]`   `2::[]`

Recursion on lists – a simple example

$$\texttt{suml [}x_1\texttt{;}x_2\texttt{; ... ;}x_n\texttt{]} = \sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n = x_1 + \sum_{i=2}^{n} x_i$$

Constructors are used in list patterns

```
let rec suml xs =
   match xs with
   | []       -> 0
   | x::tail -> x + suml tail;;
val suml : int list -> int
```

```
    suml [1;2]
⇝   1 + suml [2]         (x ↦ 1 and tail ↦ [2])
⇝   1 + (2 + suml [])    (x ↦ 2 and tail ↦ [])
⇝   1 + (2 + 0)          (the pattern [] matches the value [])
⇝   1 + 2
⇝   3
```

Recursion follows the structure of lists

A polymorphic list function (I)

The function remove(y,xs) gives the list obtained from xs by deleting every occurrence of y, e.g. $\texttt{remove}(2,[1;2;0;2;7]) = [1;0;7]$.

Recursion is following the structure of the list:

```
let rec remove(y,xs) =
   match xs with
   | []                 -> []
   | x::tail when x=y -> remove(y,tail)
   | x::tail           -> x::remove(y,tail);;
```

List elements can be of any type that supports equality

```
remove :  'a *  'a list ->  'a list when'a : equality
```

- `'a` is a *type variable*
- 'a : equality is a type constraint

The F# system infers the most general type for remove

A polymorphic list function (II)

- A type containing type variables is called a polymorphic type
- The remove function is called a polymorphic function.

```
remove : 'a * 'a list -> 'a list when 'a : equality
```

The function has many forms, one for each instantiation of 'a

Instantiating `'a` with `string`:

```
remove("a", [""; "a"; "ab"; "a"; "bc"]);;
val it : string list = [""; "ab"; "bc"]
```

Instantiating `'a` with `int`:

```
remove(2, [1; 2; 0; 2; 7]);;
val it : int list = [1; 0; 7]
```

Instantiating `'a` with `int list`:

```
remove([2], [[2;1]; [2]; [0;1]; [2]; [5;6;7]]);;
val it : int list list = [[2; 1]; [0; 1]; [5; 6; 7]]
```

Exploiting structured patterns: the isPrefix function

The function `isPrefix`(*xs*, *ys*) tests whether the list *xs* is a prefix of the list *ys*, for example:

$$\begin{array}{lcl} \texttt{isPrefix}([1;2;3],[1;2;3;8;9]) & = & \text{true} \\ \texttt{isPrefix}([1;2;3],[1;2;8;3;9]) & = & \text{false} \end{array}$$

The function is declared as follows:

```
let rec isPrefix(xs, ys) =
   match (xs,ys) with
   | ([],_)              -> true
   | (_,[])              -> false
   | (x::xtail,y::ytail) -> x=y && isPrefix(xtail, ytail);;

isPrefix([1;2;3], [1;2]);;
val it : bool = false
```

A each clause expresses succinctly a natural property:

- The empty list is a prefix of any list
- A non-empty list is not a prefix of the empty list
- A non-empty list (...) is a prefix of another non-empty list (...) if ...

Summary

- Lists
- Polymorphism
- Constructors (:: and [] for lists)
- Patterns
- Recursion on the structure of lists
- Constructors used in patterns to decompose structured values
- Constructors used in expressions to compose structured values

Blackboard exercises

- `memberOf`(x, ys) is true iff x occurs in the list ys
- `insert`(x, ys) is the *ordered list* obtained from the *ordered list* ys by insertion of x
- `sort`(xs) gives a ordered version of xs