Homework 1: Lateral Inhibition

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1 Exercise 1

Let x_1, x_2, x_3 represent how many deciliters of each product A, B or C should be produced.
She can sell product A for 60kr per liter, product B for 70kr per liter and product C for 30kr per liter. This gives the objective function $Z = 6x_1 + 7x_2 + 3x_3$. The objective function says that for every dl of product A sold she will profit 6kr. If she sells 2dl of each product, she will earn $Z = 6 * 2 + 7 * 2 + 3 * 2 = 32$
Since the DTU student has 7 liters of Ethanol and product A uses 1dl, product B 2dl and product C 1dl, we have the constraint: $x_1 + 2x_2 + x_3 \leq 70$
She has 21 liters of apple juice. Product A uses 2dl, product B uses 2dl and product C uses 3dl, which gives the constraint $2x_1 + 2x_2 + 3x_3 \leq 210$
Lastly, she has 20 liters of Coca-Cola. Product A uses 3dl, product B uses 1dl and product C uses 1dl, which gives the constraint $3x_1 + x_2 + x3 \leq 200$
The constraints and objective function gives the following LP:

$$\begin{array}{ll} \text{Maximize} & Z = 6x_1 + 7x_2 + 3x_3 \\ \text{Subject to} & x_1 + 2x_2 + x_3 \leq 70 \\ & 2x_1 + 2x_2 + 3x_3 \leq 210 \\ & 3x_1 + x_2 + x_3 \leq 200 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

It is informed that for the optimal solution, she only makes product A and product B and that she does not use up all the apple juice. There are 3 constraints and so there must be 3 basic variables. Based on this information, it can be concluded that x_1, x_2 (product A and B) as well as the slack variable for the second constraint (apple juice) are basic variables. This is because x_1 and x_2 are non-zero in the solution and because the apple juice was not used up (the slack variable is also non-zero)

Initial system of equations:

B.V	Eq.	Z	x1	x2	x3	x4	x5	x6	RHS
Z	(0)	1	-6	-7	-3	0	0	0	0
x4	(1)	0	1	2	1	1	0	0	70
x5	(2)	0	2	2	3	0	1	0	210
x6	(3)	0	3	1	1	0	0	1	200

Using the fundamental insight described above, it is possible to write up the final tabeau. The basic variables are x_1, x_2 and x_5 and so:

$$c_B = [6, 7, 0] \qquad B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

The values are calculated in Julia like so:

Project 1.jl

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A=[1 2 1; 2 2 3; 3 1 1]
I=[1 0 0; 0 1 0; 0 0 1]
B=[1 2 0; 2 2 1; 3 1 0]
b=[70; 210; 200]
c=transpose([6,7,3])
cB=transpose([6,7,0])

cB*inv(B)*A-c
inv(B)*A
cB*inv(B)
inv(B)
cB*inv(B)*b
inv(B)*b
```

The optimal tableau becomes:

B.V	Eq.	Z	x1	x2	x3	x4	x5	x6	RHS
Z	(0)	1	0	0	1	3	0	1	0
x1	(1)	0	1	0	0.2	-0.2	0	0.4	66
x2	(2)	0	0	1	0.4	0.6	0	-0.2	2
x5	(3)	0	0	0	1.8	-0.8	1	-0.4	74

2 Exercise 2

end of the assignment