Introduction to Operations research
Notes and exercises

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Most exercises in this document are from our textbook "Hillier and Lieberman, Introduction to Operations Research". The exercises from the book can be recognized as they start with a number x.y-z (e.g. 3.1-8). The exercises are from the 9th edition of the book. Further exercises have been added. These exercises are mainly from old exams.

David Pisinger wrote the original version of the document, typing in the exercises from the book and wrote the note on the network simplex algorithm for the transportation problem. Morten Bondorf Gerdes and Christian Keilstrup Ingwersen translated the note on network simplex to English and translated Danish exercises to English. Stefan Ropke wrote most of the exercises that are not from Hillier and Lieberman.

Week 1, formulation of a linear program

Exercise 1

Peter is selling hot chocolate in the streets of Copenhagen. Peter offers three types of hot chocolate: *Premium*, *Superb* and *Elite*. He is able to sell the two first products for 20 DKK pr. deciliter and *Elite* can be sold for 70 DKK pr. deciliter. The two most important ingredients in the hot chocolate are chocolate plates and milk powder. The table below shows the amount of chocolate plates and milk powder that are required to produce one liter of hot chocolate.

	Premium	Superb	Elite
Chocolate plates	2	1	3
Bags of milk powder	1	2	4

Peter has 22 chocolate plates and 20 bags of milk powder and can not carry more than 10 liters of hot chocolate. He assumes that he is able to sell all of the hot chocolate that he produces. Peter wants to know how much chocolate he should produce and sell to maximize his profit.

1. Formulate a linear program that maximizes Peter's profit
2. Solve the LP using Julia. How many liters should Peter produce of each product?

Exercise 2

3.1-8. The WorldLight Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of \$1, and each unit of product 2, up to 60 units, gives a profit of \$2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model. What is the resulting total profit?

Exercise 3 (question b,c,d)

3.2-3. This is your lucky day. You have just won a \$10,000 prize. You are setting aside \$4,000 for taxes and partying expenses, but you have decided to invest the other \$6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any *fraction* of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction. Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

(b) Formulate a linear programming model for this problem.

(c) Use the graphical method to solve this model. What is your total estimated profit?

(d) Solve the problem using Julia.

Exercise 4

3.4-9. Ralph Edmund loves steaks and potatoes. Therefore, he has decided to go on a steady diet of only these two foods (plus some liquids and vitamin supplements) for all his meals. Ralph realizes that this isn't the healthiest diet, so he wants to make sure that he eats the right quantities of the two foods to satisfy some key nutritional requirements. He has obtained the following nutritional and cost information:

	Grams of Ingredient per Serving		
Ingredient	Steak	Potatoes	Daily Requirement (Grams)
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≤ 60
Cost per serving	\$4	\$2	

Ralph wishes to determine the number of daily servings (may be fractional) of steak and potatoes that will meet these requirements at a minimum cost.

(a) Formulate a linear programming model for this problem.

(b) Use the graphical method to solve this model.

(c) Solve the problem using Julia.

Week 2, Linear Programming: Simplex method

Exercise 1 (question e)

4.4-3. Consider the following problem.

Maximize
$$Z = 2x_1 + x_2$$
subject to
$$x_1 + x_2 \leq 40$$
$$4x_1 + x_2 \leq 100$$
and
$$x_1 \geq 0, x_2 \geq 0.$$

(e) Use hand calculations to solve this problem by the simplex method in tabular form.

Exercise 2 (question a)

4.7-4. Consider the following problem.

Maximize
$$Z = x_1 - 7x_2 + 3x_3$$
subject to
$$2x_1 + x_2 - x_3 \leq 4$$
$$4x_1 - 3x_2 \leq 2$$
$$-3x_1 + 2x_2 + x_3 \leq 3$$
and
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(a) Work through the simplex method step by step to solve the problem.

Exercise 3 - Minimum ratio test (from exam E2018)

In the tableau below we have illustrated a part of a simplex tableau. x_1 is about to enter the basis. Which variable should leave the basis (x_2, x_3, x_4, x_5, x_6 or x_7)?

b.v.	eq.	Z	x_1	...	RHS
Z	0	1	-1.5	...	10
x_2	1	0	3	...	0
x_3	2	0	2	...	1
x_4	3	0	1	...	2
x_5	4	0	0	...	3
x_6	5	0	-1	...	4
x_7	6	0	-2	...	5

Exercise 4 - Personnel Scheduling - abstract LP model

The personnel scheduling problem was defined on page 57 of the book and we looked at it during lecture 1. In this exercise we want to write up an abstract model for the personnel scheduling problem. We define the following notation:

- m is the number of time intervals that we care about (in the example from the book we have $m = 10$)
- n is the number of shifts that we can use (in the example from the book we have $n = 5$)

- b_i is the number of agents needed in time interval i ($i \in \{1, 2, \ldots, m\}$)
- c_j is the cost of assigning one agent to shift j ($j \in \{1, 2, \ldots, n\}$)
- a_{ij} is a parameter that is one if shift j is covering time interval i and 0 otherwise $i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, n\}$
- x_j is a decision variable. It counts the number of agents assigned to shift j ($j \in \{1, 2, \ldots, n\}$)

Using this notation

(a) write an abstract LP model for the Personnel Scheduling problem, using the notation defined above.

(b) implement the abstract model in Julia

(c) test the Julia model using the data

$$m = 3, n = 5, b = [2, 4, 3], c = [1500, 2000, 800, 1800, 1300], (a_{ij}) = \begin{bmatrix} 1 & 1 & & & \\ & 1 & 1 & 1 & \\ & & & 1 & 1 \end{bmatrix}$$

(d) try the Julia model using the larger dataset available on DTU inside (week2-ex3-dat.txt)

Exercise 5 - simplex with negative variable

In this exercise we solve a linear program with a variable that can attain negative values.

$$\max Z = x_1 + 3x_2$$

s.t.

(A)

$$\begin{aligned} x_1 + 2x_2 &\leq 0 \\ x_1 &\geq -7 \\ x_2 &\geq 0 \end{aligned}$$

- Show how the problem can be rewritten into a new LP that only contains variables greater or equal to 0 (see section 4.6 in book).
- Solve the new problem with the simplex method step by step.
- What is the solution to the original problem (A)?

Exercise 6 - Minimization and unrestricted variable

Consider the following LP

$$\min x_1 + 2x_2$$

subject to

$$\begin{aligned} -x_1 - x_2 &\leq 5 \\ 2x_1 + x_2 &\leq 6 \\ x_1 &\geq 0 \\ x_2 &\in \mathbb{R} \end{aligned}$$

(a) Rewrite the problem to an LP on the standard form (see chapter 4.6 in the book or the slides)

(b) Solve the resulting LP using the simplex method step by step.

(c) Draw the original problem in a two dimensional coordinate system. Does the drawing and your solution match?

(d) Now we make a small change to the second constraint so the LP becomes

$$\min x_1 + 2x_2$$

subject to

$$\begin{aligned} -x_1 - x_2 &\leq 5 \\ x_1 + 2x_2 &\leq 6 \\ x_1 &\geq 0 \\ x_2 &\in \mathbb{R} \end{aligned}$$

Rewrite the problem to an LP on the standard form and solve the resulting LP using the simplex method step by step.

(e) Draw the original problem in a two dimensional coordinate system. Does the drawing and your solution match?

Week 3, Linear Programming: Two-phase Simplex method

Exercise 1 (question a,c)

4.6-8. Consider the following problem.

Minimize

$$Z = 2x_1 + x_2 + 3x_3$$

subject to

$$5x_1 + 2x_2 + 7x_3 = 420$$
$$3x_1 + 2x_2 + 5x_3 \geq 280$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(a) Using the two-phase method, work through phase 1 step by step.

(c) Work through phase 2 step by step to solve the original problem.

Exercise 2 (exercise from exam: spring 2014)

Consider the following LP

Maximize

$$Z = 2x_1 + 4x_2 + 7x_3$$

subject to

$$4x_1 + 2x_2 + 2x_3 \leq 60$$
$$x_1 + 2x_2 + 2x_3 = 30$$
$$x_1 + 4x_2 + x_3 \geq 40$$

and

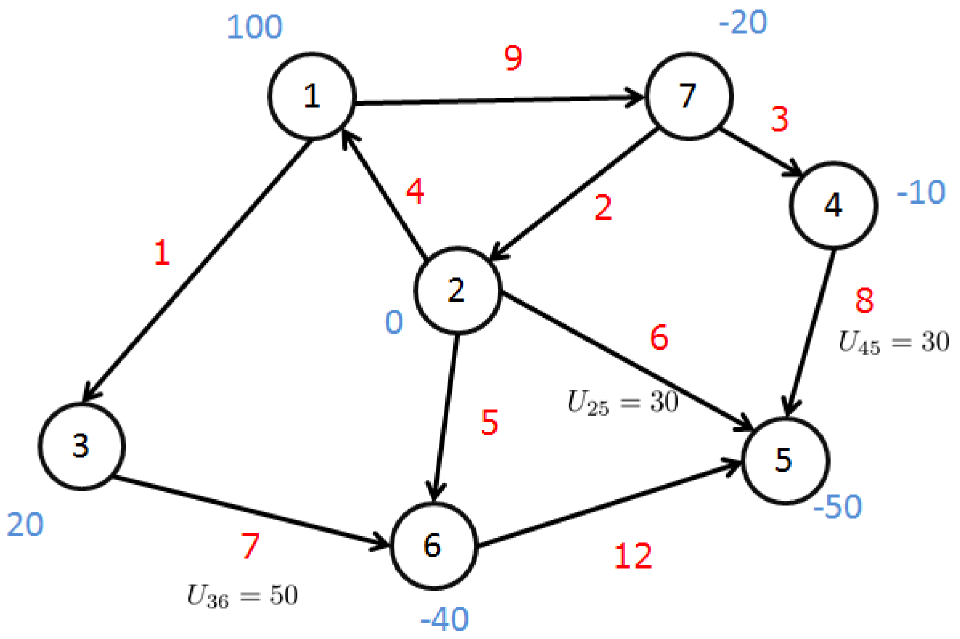
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(a) Rewrite to augmented form by introducing slack, surplus and artificial variables

(b) Solve the problem using the two-phase method

Exercise 3

Consider the following instance of the minimum cost flow problem. Numbers written in red are arc costs, numbers in blue are surplus / deficit at nodes and three arcs have a capacity as indicated by U_{ij} on the figure



- Solve the minimum cost flow problem defined by the figure (it is fine to use Julia).
- Draw the solution you obtained. Does the solution make sense - did you find a feasible flow.
- Management want to send at least as many units through arc (6,5) at as through arc (2,5). What constraint do you need to add to your model? Does this change the solution?

Exercise 4

10.6-3 A company will be producing the same new product at two different factories, and then the product must be shipped to two warehouses. Factory 1 can send an unlimited amount by rail to warehouse 1 only, whereas factory 2 can send an unlimited amount by rail to warehouse 2 only. However independent truckers can be used to ship up to 50 units from each factory to a distribution center, from which up to 50 units can be shipped to each warehouse. The shipping cost per unit for each alternative is shown in the following table (shown in red), along with the amounts to be produced at the factories and the amounts needed at the warehouses.

From \ To	Unit Shipping Cost			Output
	Distribution center	Warehouse 1	Warehouse 2	
Factory 1	3	7	—	80
Factory 2	4	—	9	70
Distribution center		2	4	
Demand		60	90	

- **(a)** formulate the network representation of this problem as a minimum cost flow problem
- **(b)** Formulate the linear programming model for this problem.
- **(c)** Solve the minimum cost flow problem

Week 4, Simplex using matrix computations

Exercise 1

5.2-2. Work through the simplex method in matrix form, step by step, to solve the following problem.

Maximize

$$Z=5x_1+8x_2+7x_3+4x_4+6x_5$$

subject to

$$2x_1+3x_2+3x_3+2x_4+2x_5\leq 20$$
$$3x_1+5x_2+4x_3+2x_4+4x_5\leq 30$$

and

$$x_j \geq 0, \quad j=1,2,3,4,5.$$

Exercise 2

5.3-3. Consider the following problem.

Maximize

$$Z= 6x_1+ x_2+2x_3$$

subject to

$$2x_1+2x_2+\tfrac{1}{2}x_3\leq 2$$
$$-4x_1-2x_2-\tfrac{3}{2}x_3\leq 3$$
$$x_1+2x_2+\tfrac{1}{2}x_3\leq 1$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

		Coefficient of:							
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	x_6	Right Side
Z	(0)	1				2	0	2	
x_5	(1)	0				1	1	2	
x_3	(2)	0				-2	0	4	
x_1	(3)	0				1	0	-1	

Use the fundamental insight presented in Sec. 5.3 to identify the missing numbers in the final simplex tableau. Show your calculations.

Exercise 3

5.3-5. Consider the following problem.

Maximize

$$Z=c_1x_1+c_2x_2+c_3x_3$$

subject to

$$x_1+ 2x_2+ x_3\leq b$$
$$2x_1+ x_2+ 3x_3\leq 2b$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Note that values have not been assigned to the coefficients in the objective function (c_1, c_2, c_3) and that the only specification for the right-hand side of the functional constraints is that the second one $(2b)$ be twice as large as the first (b).

Now suppose that you boss has inserted her best estimates of the values off c_1, c_2, c_3, and b without informing you and then has run the simplex method. You are given the resulting final simplex table below (where x_4 and x_5 are the slack variables for the respective functional constraints), but you are unable to read the value of Z^*.

			Coefficient of:					
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	x_5	Right Side
Z	(0)	1	$\frac{7}{10}$	0	0	$\frac{3}{5}$	$\frac{4}{5}$	Z^*
x_2	(1)	0	$\frac{1}{5}$	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	1
x_3	(2)	0	$\frac{3}{5}$	0	1	$-\frac{1}{5}$	$\frac{2}{5}$	3

(a) Use the fundamental insight presented in Sec. 5.3 to identify the values of (c_1, c_2, c_3) that was used.

(b) Use the fundamental insight presented in Sec. 5.3 to identify the value of b that was used.

(c) Calculate the value of Z^* in two ways, where one way usees your results from part (a) and the other way uses your results from part (b). Show your method for finding Z^*.

Exercise 4

5.1-10. Label each of the following statements about linear programming problems as true or false, and then justify your answer.

(a) If a feasible solution is optimal but not a CPF solution, then infinitely many optimal solutions exist.

(b) If the value of the objective function is equal at two different feasible points x^* and x^{**}, then all points on the line segment connecting x^* and x^{**} are feasible and Z has the same value at all those points.

(c) If the problem has n variables (before augmenting), then the simultaneous solution of any set of n constraint boundary equations is a CPF solution.

Week 5, Linear Programming: duality and sensitivity

Exercise 1 (question a)

6.1-4. Consider the following problem.

Maximize
$$Z = -x_1 - 2x_2 - x_3$$
subject to
$$x_1 + x_2 + 2x_3 \leq 12$$
$$x_1 + x_2 - x_3 \leq 1$$
and
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

(a) Construct the dual problem.

Exercise 2 (question f,g)

6.7-8. Consider the following problem.

Maximize
$$Z = c_1x_1 + c_2x_2$$
subject to
$$2x_1 - x_2 \leq b_1$$
$$x_1 - x_2 \leq b_2$$
and
$$x_1 \geq 0, x_2 \geq 0.$$

Let x_3 and x_4 denote the slack variables for the respective functional constraints. When $c_1 = 3, c_2 = -2, b_1 = 30$, and $b_2 = 10$, the simplex method yields the following final simplex tableau.

		Coefficient of:					
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	Right Side
Z	(0)	1	0	0	1	1	40
x_2	(1)	0	0	1	1	-2	10
x_1	(2)	0	1	0	1	-1	20

(f) Determine the allowable range for first c_1 and then c_2 using sensitivity analysis

(g) Determine the allowable range for first b_1 and then b_2 using sensitivity analysis

Exercise 3

6.4-1. Consider the following problem.

Maximize
$$Z = 5x_1 + 4x_2$$
subject to
$$2x_1 + 3x_2 \geq 10$$
$$x_1 + 2x_2 = 20$$
and
$$x_2 \geq 0 \quad (x_1 \text{unconstrained in sign}).$$

(a) Use the SOB method to construct the dual problem.

(b) Use Table 6.12 to convert the primal problem to our standard form given at the beginning of Sec. 6.1, and construct the corresponding dual problem. Then show that this dual problem is equivalent to the one obtained in part (a).

Exercise 4 (inspired by an exam question)

A bakery produces to types of cakes - called A and B. Cake A can be sold with a profit of 2 DKK and cake B can be sold with a profit of 3 DKK. To produce the cakes, flour and sugar are needed. The amount of flour and sugar in the cakes can be seen in the following table.

	A	B
Flour	1	2
Sugar	1	1

Every day it is possible to use 30 units of flour and 20 units of sugar. The following LP is solved to determine the maximum profit per day.

Maximize

$$Z = 2x_1 + 3x_2$$

subject to

$$x_1 + 2x_2 \leq 30$$
$$x_1 + x_2 \leq 20$$

and

$$x_1 \geq 0, x_2 \geq 0.$$

When solving the problem with the simplex method, slack variable x_3 and x_4 are added. The optimal tabular is given below.

		Coefficient of:					
Basic Variable	Eq.	Z	x_1	x_2	x_3	x_4	Right Side
Z	(0)	1	0	0	1	1	50
x_2	(1)	0	0	1	1	-1	10
x_1	(2)	0	1	0	-1	2	10

(a) The bakery wishes to introduce a new product that requires 2 units of flour and 3 units of sugar. What should the profit of the new product, at least be, to increase the total profit for the bakery?

Exercise 5

6.1-13. Consider the primal and dual problems in our standard form presented in matrix notation at the beginning of Sec. 6.1. Let y^* denote the optimal solution for this dual problem. Suppose that b is then replaced by $\overline{b}$. Let $\overline{x}$ denote the optimal solution for the new primal problem. Prove that

$$c\overline{x} \leq y^*\overline{b}.$$

Week 6, Modeling with integer variables

Exercise 1

11.1-3. A real estate development firm, Peterson and Johnson, is considering five possible development projects. The following table shows the estimated long run profit (net present value) that each project would generate, as well as the amount of investment required to undertake the project, in units of millions of dollars.

	Development Project				
	1	2	3	4	5
Estimated profit	1	1.8	1.6	0.8	1.4
Capital required	6	12	10	4	8

The owners of the firm, Dave Peterson and Ron Johnson, have raised \$20 million of investment capital for these projects. Dave and Ron now want to select the combination of projects that will maximize their total estimated long-run profit (net present value) without investing more than \$20 million

(a) formulate an integer programming model for this problem

(b) Use a computer to solve this model

Exercise 2

11.3-1. The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix. A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs).

	Product			
	1	2	3	4
Start-up cost	\$50,000	\$40,000	\$70,000	\$60,000
Marginal revenue	\$ 70	\$ 60	\$ 90	\$ 80

Let the continuous decision variables x_1, x_2, x_3, and x_4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

1. No more than two of the products can be produced.
2. Either product 3 or 4 can be produced only if either product 1 or 2 is produced.
3. Either $5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6,000$ or $4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6,000$.

(a) Introduce auxiliary binary variables to formulate a mixed BIP model for this problem.

(b) Use the computer to solve this model.

Exercise 3

11.3-4. The Toys-R-4-U Company has developed two new toys for possible inclusion in its product line for the upcoming Christmas season. Setting up the production facilities to begin production would cost \$50,000 for toy 1 and \$80,000 for toy 2. Once these costs are covered, the toys would generate a unit profit of \$10 for toy 1 and \$15 for toy 2.
The company has two factories that are capable of producing these toys. However, to avoid doubling the start-up costs, just one factory would be used, where the choice would be based on maximizing profit. For

administrative reasons, the same factory would be used for both new toys if both are produced. Toy 1 can be produced at the rate of 50 per hour in factory 1 and 40 per hour in factory 2. Toy 2 can be produced at the rate of 40 per hour in factory 1 and 25 per hour in factory 2. Factories 1 and 2, respectively, have 500 hours and 700 hours of production time available before Christmas that could be used to produce these toys.
It is not known whether these two toys would be continued after Christmas. Therefore, the problem is to determine how many units (if any) of each new toy should be produced before Christmas to maximize the total profit.

(a) Formulate an MIP model for this problem.

(b) Use the computer to solve this model.

Exercise 4

(This exercise is based on 12.3-2 from Hillier & Lieberman) Consider the following optimization problem

$$\max x_1 + 2x_2 + 3x_3$$

subject to

$$\begin{aligned} |x_1 - x_2| &= 0 \text{ or } 3 \text{ or } 6 \\ x_1 + 3x_2 + 5x_3 &\leq 10.7 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

obviously this problem is not an LP nor a MIP. However, the problem can be reformulated to a MIP problem by introducing one or more new integer variables as well as new constraints.

- Show how the problem can be reformulated as a MIP
- Solve the MIP in order to find the optimal solution to the problem

Exercise 5

(This exercise is based on 12.4-4 from Hillier & Lieberman) Consider the following integer nonlinear programming problem

$$\max Z = 4x_1^2 - x_1^3 + 10x_2^2 - x_2^4$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \\ x_1, x_2 &\in \mathbb{Z} \end{aligned}$$

This problem can be reformulated as an equivalent pure BIP problem (with linear objective function) with six binary variables y_{1j} and y_{2j} for $j = 1, 2, 3$ with y_{ij} having the following interpretation

$$y_{ij} = \begin{cases} 1 & \text{if } x_i = j \\ 0 & \text{otherwise} \end{cases}$$

- Formulate the equivalent BIP using the six binary variables
- Solve the BIP using a computer and find the optimal solution (x_1, x_2) to the original problem

Week 7, Modeling with integer variables

Exercise 1

As the leader of an oil-exploration drilling venture, you must determine the least-cost selection of five out of ten possible sites. Label the sites $S_1, S_2, S_3, \ldots, S_{10}$ and the exploration costs associated with each as $C_1, C_2, \ldots, C_{10}$. Regional development restrictions are such that:

- Evaluating sites S_1, and S_7 will prevent you from exploring site S_8
- Evaluating site S_3 or S_4 prevents you from assessing site S_5
- Of the group S_3, S_6, S_7, and S_8, only two sites may be assessed

Formulate an integer program to determine the minimum-cost exploration scheme that satisfies these restrictions

Exercise 2

Suppose that a mathematical model fits linear programming except for the restrictions that

- At least one of the following two inequalities holds

$$\begin{aligned} 3x_1 - x_2 - x_3 + x_4 &\leq 12 \\ x_1 + x_2 + x_3 + x_4 &\leq 15 \end{aligned}$$

- At least two of the following three inequalities holds

$$\begin{aligned} 2x_1 + 5x_2 - x_3 + x_4 &\leq 30 \\ -x_1 + 3x_2 + 5x_3 + x_4 &\leq 40 \\ 3x_1 - x_2 + 3x_3 - x_4 &\leq 60 \end{aligned}$$

Show how to reformulate these restrictions to fit an MIP model.

Exercise 3

Consider the following integer program:

$$\begin{aligned} \texttt{Maximize } Z &= x_1 + 5x_2 \\ x_1 + 10x_2 &\leq 20 \\ x_1 &\leq 2 \\ x_1, x_2 &\in \mathbb{Z}_+ \end{aligned}$$

1. Use a binary representation of the variables to reformulate this model as a Binary Integer Program
2. Solve the model using a computer and use the optimal solution to identify an optimal solution to the original problem

Exercise 4

The Fly-Right Airplane Company builds small jet planes to sell to corporations for the use of the executives. To meet the needs of these executives, the company's customers sometimes order a custom design of the air planes being purchased. When this occurs, a substantial start up cost is incurred to intiate production of these airplanes.

Fly-Right has recently received purchase requests from three customers with short deadlines. However, as the company's production facilities already are almost completely tied up filling previous orders, it will not be able to accept all three orders. Therefore a decision now needs to be made on the number of airplanes the company will agree to produce (if any) for each of the three customers.

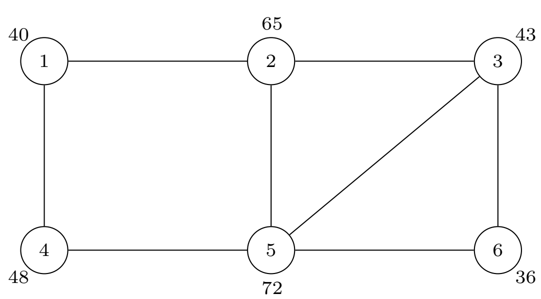
The table below gives the relevant information on each of the three customers. Each customer has a fixed start-up cost, paid just once irrespective of how many planes are produced for that customer. The marginal net revenue is received per plane produced. The third row gives the percentage of available capacity needed for each aircraft.The last row states the number of airplanes ordered by each customer (less will obviously be accepted).

	Customer		
	1	2	3
Start-up cost	\$3 million	\$ 2 million	0
Marginal net revenue	\$ 2 million	\$ 3 million	\$ 0.8 million
Capacity used per plane	20 %	40 %	20%
Maximum order	3 planes	2 planes	5 planes

1. Formulate an integer linear programming model that can be used to maximize Fly-Right's total profit (net revenue - start up costs)
2. Solve your model using a computer and state the number of airplanes produced for each customer

Exercise 5

The following map shows six intersections at which automatic traffic monitoring devices might be installed. A station at any particular node can monitor all the road links meeting that intersection. Numbers next to nodes reflect the monthly cost (in thousands of dollars) of operating a station at that location.



1. Formulate the problem of providing full coverage at minimum total cost as a set covering problem
2. Find the optimal solution to your model using a computer
3. Revise your formulation in Part 1) to obtain a binary integer programming that minimizes the number of uncovered road links while using at most two stations
4. Find the optimal solution to the revised problem using a computer

Exercise 6

Air Anton is a small commuter airline running six flights per day from New York City to surrounding resort areas. Flight crews are all based in New York, working flights to various locations and then returning on the next flight home. Taking into account complex work rules and pay incentives Air Anton schedulers have constructed the 8 possible work patterns detailed in the following table. Each row of the table indicates the flights that are covered in a particular pattern ($\times$) and the daily cost per crew (in thousands of dollars).

The company want to choose the minimum total cost collection of work patterns that cover all flights exactly once.

	Flight						
Pattern	1	2	3	4	5	6	Cost
1	-	$\times$	-	$\times$	-	-	1.40
2	$\times$	-	-	-	-	$\times$	0.96
3	-	$\times$	-	$\times$	$\times$	-	1.52
4	-	$\times$	-	-	$\times$	$\times$	1.60
5	$\times$	-	$\times$	-	-	$\times$	1.32
6	-	-	$\times$	-	$\times$	-	1.12
7	-	-	-	$\times$	-	$\times$	0.84
8	$\times$	-	$\times$	$\times$	-	-	1.54

1. Formulate the problem of providing full coverage at minimum total cost as a set partitioning problem
2. Find the optimal solution to your model using a computer

Week 8, Modeling with integer variables

Exercise 1

You are the marketing manager of a large company and are looking at a number of possible advertising campaigns in order to attract more customers. Six campaigns are possible, and they are detailed in the table below. Each campaign requires a certain investment (in millions of dollars) and will yield a certain number of new customers (in the thousands). At most 5 million dollars is available for the campaigns.

Campaign	Investment	Return
Superbowl half-time Adv.	3	80
Radio Adv. Campaign	0.8	20
Television (Non peak hour)	0.5	22
City Newspaper	2	75
Viral Marketing Campaign	0.05	4
Web advertising	0.60	10

1. Assuming it is possible to invest in fractions of a campaign, but not more than one of each, formulate a Linear Program that can be used to maximize the number of new customers.
2. Solve the problem again, this time using a greedy algorithm in which at each iteration you increase, as much as possible the value of the variable x_j that maximizes the ratio $\frac{p_i}{w_i}$, where p_i denotes the profit of investment campaign i and w_i refers to the investment required for campaign i. What do you observe? Can you prove that greedy is an optimal strategy in this case?
3. Assume now that you must invest in a campaign in its entirety or not at all. Update your model from part 1) to reflect this.
4. Solve your model with a computer
5. Solve this binary integer program again using the same greedy algorithm above. What do you observe?
6. You have been given a revised analysis of the marketing campaigns with following information

Campaign	Investment	Return
Superbowl half-time Adv.	1M	80
Radio Adv. Campaign	1.8M	20
Television (Non peak hour)	1.5M	22
City Newspaper	1.1M	75
Viral Marketing Campaign	2.2M	4
Web advertising	2M	10

Resolve your binary integer program with the computer and using your greedy algorithm. What do you observe?

Exercise 2

Consider the following binary integer program.

$$\begin{aligned} \texttt{Minimize}\ & 15x_1 + 18x_2 + 6x_3 + 20x_4 \\ \texttt{s.t.}\ & x_1 + x_4 \geq 1 \\ & x_1 + x_2 + x_4 \geq 1 \\ & x_2 + x_3 + x_4 \geq 1 \\ & x_i \in \{0,1\} \quad \text{for } i = 1,2,3,4 \end{aligned}$$

You would like to solve this problem using a greedy algorithm.

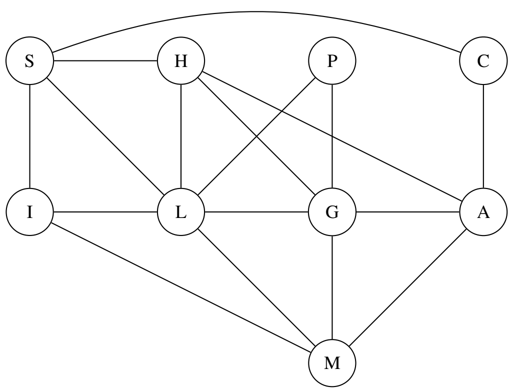
1. Explain why it seems reasonable to choose a free x_j to fix at the value one by picking the variable with the least ratio
2. Determine the solution obtained by the greedy algorithm
3. Solve the problem using a computer and comment on the result

Exercise 3

Suppose that you are responsible for scheduling times for lectures in a university. You want to make sure that any two lectures with a common student occur at different times to avoid a conflict. We could put the various lectures on a chart and mark with an $\times$ any pair that has students in common:

Lecture	A	C	G	H	I	L	M	P	S
(A)stronomy		$\times$	$\times$	$\times$			$\times$		
(C)hemistry	$\times$							$\times$	
(G)reek	$\times$			$\times$		$\times$	$\times$	$\times$	
(H)istory	$\times$		$\times$			$\times$			$\times$
(I)talian						$\times$	$\times$		$\times$
(L)atin			$\times$	$\times$	$\times$		$\times$	$\times$	$\times$
(M)usic	$\times$		$\times$		$\times$	$\times$			
(P)hilosphy			$\times$			$\times$			
(S)panish		$\times$		$\times$	$\times$	$\times$			

A more convenient representation of this information is a graph with one vertex for each lecture and in which two vertices are joined if there is a conflict between them



Now, we cannot schedule two lectures at the same time if there is a conflict, but we would like to use as few separate times as possible, subject to this constraint. How many different times are necessary? We can code each time with a color, for example 11:00-12:00 might be given the color green, and those lectures that meet at this time will be colored green. The no-conflict rule then means that we need to color the vertices of our graph in such away that no two adjacent vertices (representing courses which conflict with each other) have the same color.

One can apply the following greedy algorithm to color the graph

1. Color a vertex with color 1
2. Pick an uncolored vertex v. Color with the lowest-numbered color the has not been used on any previously colored adjacent vertices v. If all previously-used colors appear on vertices adjacent to v, then we must introduce a new color and number it.
3. Repeat the previous step until all vertices are colored.

Now, answer the following questions:

1. Using the set of colors {Green=1, Red=2, Blue=3, Yellow=4, and Cyan = 5}, color the vertices in the order G, L, H, P, M, A, I, S, C using the greedy algorithm above. How many colors do you need?
2. Using the set of colors {Green=1, Red=2, Blue=3, Yellow=4, and Cyan = 5}, color the vertices in the order A, I, P, M, S, C, H, L,G using the greedy algorithm above. How many colors do you need? Comment on the results to parts 1) and 2).

   The greedy approach is clearly not optimal. It provides solutions of different quality depending on which node is used to initialize the algorithm.
3. Formulate the graph coloring problem as an integer programming problem
4. Solve your model using a computer

Week 9, Modeling with integer variables IV

Exercise 1

Recall the marketing campaing problem from Week 8, Part 4), in which you had decide how many advertising campaigns (from a set of six possible) you would run, given a maximum budget of 5 million dollars, in order to maximize the number of new customers. Each campaign required a certain investment (in millions of dollars) and would yield a certain number of new customers (in the thousands). You were given the following data.

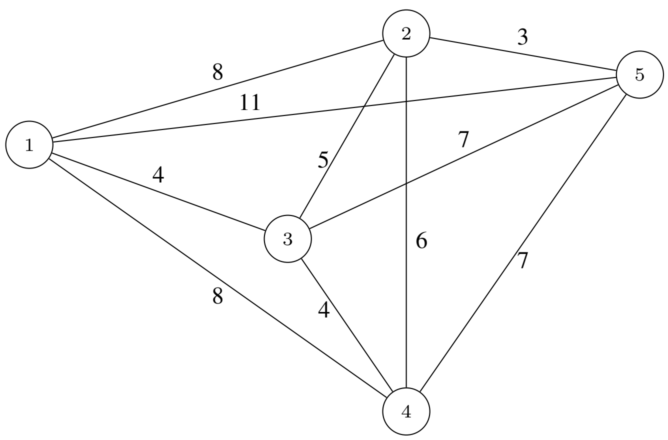
Campaign	Investment	Return
Superbowl half-time Adv.	3	80
Radio Adv. Campaign	0.80	20
Television (Non peak hour)	0.50	22
City Newspaper	2	75
Viral Marketing Campaign	0.05	4
Web advertising	0.6	10

You have applied a greedy heuristic and obtained a solution in which you will invest in all campaigns except that which involves advertising in the Superbowl half-time add. This stategy would lead to 131,000 new customers.

1. You decide to try and improve this solution by considering the neighbourhood of solutions obtained by removing one campaign and inserting an unused campaign. How many feasible neighbouring solutions are there?
2. Given this neighbourhood, is there an improving move?
3. Update your solution. Is it locally optimal? Explain your answer.

Exercise 2

Consider the Travelling Salesman Problem shown below, where City 1 is considered to be the home city.

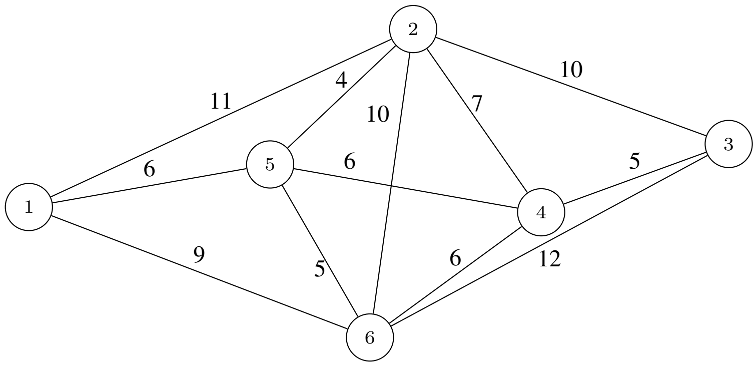


1. How many possible tours are there (excluding tours that are simply the reverse of others)?
2. Starting at City 1, apply the nearest neighbour heuristic to obtain a feasible solution. What is its objective value?

3. Consider all possible 2-opt edge swaps. How many lead to different feasible tours?
4. Given the solution $1-3-4-2-5-1$, perform a 2-opt swap on edges $(2,4)$ and $(1,5)$. What is the new tour, and what is its objective value?
5. Find the optimal solution using Julia. You might like to make use of the Julia code given in Lecture 9.

Exercise 3

Consider the Traveling Salesman Problem shown below, where City 1 is the home city.



1. Starting at City 1, apply the nearest neighbour heuristic to obtain a feasible solution. What is its objective value?
2. Depending on where you start, why might the nearest neighbour heuristic fail to generate a feasible solution in this case?
3. Consider a 3-opt exchange on edges $(2,4)$, $(1,6)$ and $(3,6)$. How many feasible new tours does this generate, and what are their respective objective values?
4. Find the optimal solution using Julia. You might like to make use of the Julia code given in Lecture 9. (**Hint:** introduce highly penalized edges to generate a complete graph). How does this compare to your best found tour?

Week 10, Total Unimodularity

Exercise 1

Consider the integer program:

$$
\begin{aligned}
\text{Min} \quad & 2.5x_1 + 7.9x_2 && (1)\\
\text{s.t.} \quad & x_1 \leq 5 && (2)\\
& x_2 \leq 4 && (3)\\
& x_2 \geq 2 && (4)\\
& x_1 + x_2 \geq 6 && (5)\\
& x_1, x_2 \quad \text{integer} && (6)
\end{aligned}
$$

(a) Does the feasible region have integer corner points? Hint: draw it on a graph.

(b) Is the constraint coefficient matrix totally unimodular? Hint: recall that for a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $ad - bc$.

(c) Repeat questions (a) and (b), but with the right-hand-side of constraint (3) changed to 4.5.

(d) Repeat questions (a) and (b), but with the coefficient of x_1 in constraint (5) changed to 2.

(e) What if the objective function coefficients are changed?

Exercise 2

Assume we have an assignment problem with three workers and three tasks. Each worker has to be assigned to exactly one task, and each task needs to be done by exactly one worker (any worker can be assigned to any task). Use the binary variable x_{ij} to denote whether or not worker i is assigned to task j, and write down the constraints.

(a) Is the constraint coefficient matrix totally unimodular? **Hint: Use slide 12 from the lecture.**

(b) Would the optimal solution to the LP relaxation be integral?

(c) Repeat questions (a) and (b), but now assume that some workers can do multiple tasks and some tasks require multiple workers.

Exercise 3

Are the following matrices totally unimodular or not:

1.
$$A_1 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

2.
$$A_2 = \begin{bmatrix} -1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

Week 11, Integer Programming, Chapter 12.6/12.7

Exercise 1

11.5-6. Label each of the following statements as True or False, and then justify your answer by referring to specific statements (with page citations) in the chapter.

(a) Linear programming problems are generally much easier to solve than Integer Programming (IP) problems.

(b) For IP problems, the number of integer variables is generally more important in determining the computational difficulty than is the number of functional constraints.

(c) To solve an IP problem with an approximate procedure, one may apply the simplex method to the LP relaxation problem and then round each noninteger value to the nearest integer. The result will be a feasible but not necessarily optimal solution for the IP problem.

Exercise 2

11.6-1. Use the Binary Integer Programming Branch-&-bound algorithm presented in Section 12.6 to solve the following problem interactively.

$$\begin{aligned} \texttt{Maximize}\ Z = 2x_1 - x_2 + 5x_3 - 3x_4 + 4x_5 & \\ \texttt{s.t.}\quad 3x_1 - 2x_2 + 7x_3 - 5x_4 + 4x_5 &\le 6 \\ x_1 - x_2 + 2x_3 - 4x_4 + 2x_5 &\le 0 \\ x_j \in \{0,1\}, \quad j = 1,2,3,4,5 & \end{aligned}$$

Hint: Use an LP solver to solve the subproblems you generate.

Exercise 3

11.7-2. Consider the following Integer Programming Problem.

$$\begin{aligned} \texttt{Maximize}\ Z &= -3x_1 + 5x_2 \\ \texttt{s.t.}\quad 5x_1 - 7x_2 &\ge 3 \\ x_j &\le 3 \quad j = 1,2 \\ x_j &\ge 0 \quad j = 1,2 \\ x_j &\in \mathbb{Z}_+ \quad j = 1,2 \end{aligned}$$

(a) Solve this problem graphically.

(b) Use the MIP branch-and-bound algorithm presented in Section 12.7 to solve this problem by hand. For each subproblem, solve its LP relaxation graphically.

(c) Use the binary representation for integer variables to reformulate this problem as a BIP problem.

(d) Use the BIP branch-and-bound algorithm presented in Section 12.6 to solve the problem as formulated in part (c) interactively.

	Generator			
	1	2	3	4
Operating Cost ($000s)	7	12	5	14
Output Power (Megawatts)	300	600	500	1600

Tabel 1: Exercise 4 Data

Exercise 4

River Power has four generators currently available for production and wishes to decide which to put on line to meet the expected 700-megawatt peak demand over the next several hours. The following table shows the cost to operate each generator (in thousands of dollars per hour) and their outputs in megawatts.

(a) Formulate this problem as a BIP.

(b) Use the Branch-&-bound algorithm to find an optimal solution to this problem. **Hint:** Use an LP solver to solve all subproblems that arise.

Week 12: Network Simplex for the transportation problem
David Pisinger

Translated by Morten Bondorf Gerdes and Christian Keilstrup Ingwersen

Hillier and Lieberman use a table when solving the transportation problem; however, in this course we will use a graphical method. In the following section we will go through the network simplex for the transportation problem.

Consider the the transportation problem for which the sources A_1, A_2, A_3 has to deliver respectively 4, 5 and 3 units while the destinations B_1, B_2, B_3 has to receive respectively 2, 6 and 4 units. The prices c_{ij} of sending a unit from source i to destination j is given in the following table.

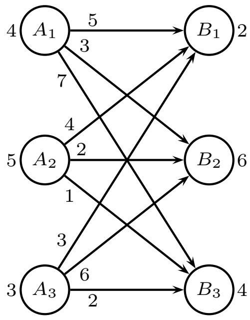
c_{ij}	B_1	B_2	B_3
A_1	5	3	7
A_2	4	2	1
A_3	3	6	2

If x_{ij} represent the amount transported from A_i to B_j, the transportation problem can be formulated as a LP model:

$$\begin{array}{lllllllllll}
\min & 5x_{11}+3x_{12}+7x_{13}+4x_{21}+2x_{22}+1x_{23}+3x_{31}+6x_{32}+2x_{33} \\
\text{s.t.} & -x_{11} & -x_{12} & -x_{13} & & & & & & & =-4\ (u_1) \\
& & & & -x_{21} & -x_{22} & -x_{23} & & & & =-5\ (u_2) \\
& & & & & & & -x_{31} & -x_{32} & -x_{33} & =-3\ (u_3) \\
& x_{11} & & & +x_{21} & & & +x_{31} & & & =\ 2\ (v_1) \\
& & x_{12} & & & +x_{22} & & & +x_{32} & & =\ 6\ (v_2) \\
& & & x_{13} & & & +x_{23} & & & +x_{33} & =\ 4\ (v_3) \\
& \multicolumn{10}{l}{x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33} \geq 0}
\end{array}$$

Note that when transporting units from a node, the demand is negative and when transporting units to a node, the demand is positive. This makes better definitions of the dual variables which are marked with red in parenthesis.

The problem can be illustrated graphically. The numbers on the edges from source to destination are the transportation cost pr. unit and the numbers on the sources and destinations are the supply and demand.



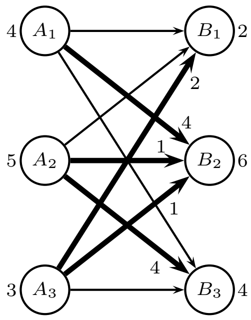
Greedy heuristic

When solving the transportation problem, an initial solution is needed. We will use a greedy heuristic to find the initial solution. We will consider the sources A_1, A_2, A_3 one at a time and choose to transport the cheapest units first to the destinations.

A_1 sends 4 units to B_2 with price 3. A_2 sends 4 units to B_3 with price 1 and 1 unit to B_2 with price 2. A_3 sends 2 units to B_1 with price 3 and 1 unit to B_2 with price 6.

This lead to the solution seen below. An edge from source i to destination j is marked bold if units are send

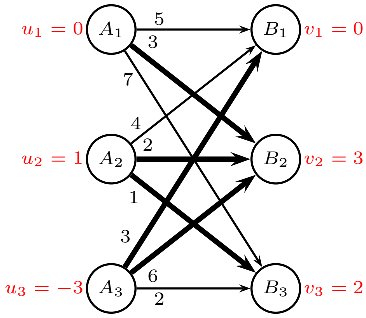
from source i to source j. Note that there exists 5 bold edges and that they form a tree - meaning that they are connected but do not form a cycle. The bold edges form a **basis solution**.
If there exist less than 5 bold edges (in this example), some extra edges transporting 0 units are introduced. It is important that these edges do not form a cycle but a connected graph.



The total price of the transportation problem is: $4 \cdot 3 + 1 \cdot 2 + 4 \cdot 1 + 2 \cdot 3 + 1 \cdot 6 = 30$.

Computation of dual variables

Choose source A_1 as the initial node and compute the distance, only using the bold edges, to all other nodes. The distance is defined as the price going from one node to another. The distance from A_1 to A_1 is 0. If we follow the directions of the edge (from source to destination), the distance is positive. If we go in the opposite direction (from destination to source), the distance is negative. The distances to the sources A_i are called u_i while the distance to the destinations B_j are called v_j. The figure blow shows the dual variables u_i and v_j.



Pivot operation

To improve the solution we consider edges with no flow in the current solution. The goal is to find a cycle that can improve the solution. Note that every edge with no flow, together with the edges in the basic solution form a cycle.
Edge (A_1, B_1) form the cycle $(A_1, B_1, A_3, B_2, A_1)$ which costs $5-3+6-3 = 5$. This can also be calculated as the reduced cost

$$\overline{c}_{11} = c_{11} + u_1 - v_1 = 5 + 0 - 0 = 5$$

Edge (A_1, B_3) form the cycle $(A_1, B_3, A_2, B_2, A_1)$ which costs $7-1+2-3 = 5$. This can also be calculated as the reduced cost

$$\overline{c}_{13} = c_{13} + u_1 - v_3 = 7 + 0 - 2 = 5$$

Edge (A_2, B_1) form the cycle $(A_2, B_1, A_3, B_2, A_2)$ which costs $4-3+6-2 = 5$. This can also be calculated as the reduced cost

$$\overline{c}_{21} = c_{21} + u_2 - v_1 = 4 + 1 - 0 = 5$$

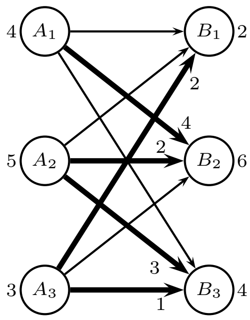
Edge (A_3, B_3) form the cycle $(A_3, B_3, A_2, B_2, A_3)$ which costs $2 - 1 + 2 - 6 = -3$. This can also be calculated as the reduced cost

$$\bar{c}_{33} = c_{33} + u_3 - v_3 = 2 - 3 - 2 = -3$$

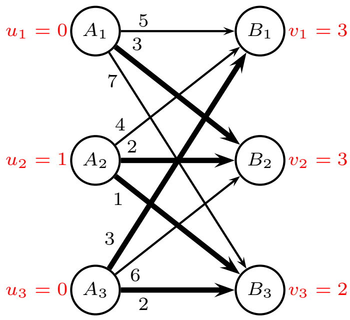
We choose the cycle with the most negative reduced cost ((A_3, B_3)) as this is the best improvement to the solution.
We now have to figure out the amount that can be send through the cycle. There is no limit on the edges going from the sources to the destinations. The limit for edges going from the destinations to the sources is the amount currently send as it is impossible to send an negative amount. The cycle $(A_3, B_3, A_2, B_2, A_3)$ has the following edges going from destinations to sources

$$\begin{array}{lll} (B_3, A_2) & \text{with flow} & 4 \\ (B_2, A_3) & \text{with flow} & 1 \end{array}$$

We choose the smallest flow going from destinations to sources. In this example we are able to send 1 unit through the cycle $(A_3, B_3, A_2, B_2, A_3)$. Every unit send through the cycle has a saving of 3 and the new solution is



Note that edge (A_3, B_2) is not bold anymore as there is no longer any flow on that edge. The bold edges yet again form a tree and the total transportation cost is $4 \cdot 3 + 2 \cdot 2 + 3 \cdot 1 + 2 \cdot 3 + 1 \cdot 2 = 27$.
We are now able to calculate the new dual variables as the distances from the source A_1 to all other nodes using the bold edges.



Stopping criteria

We want to improve the solution by finding a cycle with negative reduced cost.
Edge (A_1, B_1) form the cycle $(A_1, B_1, A_3, B_3, A_2, B_2, A_1)$ with reduced cost

$$\bar{c}_{11} = c_{11} + u_1 - v_1 = 5 + 0 - 3 = 2$$

Edge (A_1, B_3) form the cycle $(A_1, B_3, A_2, B_2, A_1)$ with reduced cost

$$\bar{c}_{13} = c_{13} + u_1 - v_3 = 7 + 0 - 2 = 5$$

Edge (A_2, B_1) form the cycle $(A_2, B_1, A_3, B_3, A_2)$ with reduced cost

$$\overline{c}_{21} = c_{21} + u_2 - v_1 = 4 + 1 - 3 = 2$$

Edge (A_3, B_2) form the cycle $(A_3, B_2, A_2, B_3, A_3)$ with reduced cost

$$\overline{c}_{32} = c_{32} + u_3 - v_2 = 6 + 0 - 3 = 3$$

As all reduced costs are positive, there exist no cycle that improves the solution and the algorithm terminates.

Proof of optimality

We have the primal and dual problem

$$\begin{array}{lll} \min & \sum_i \sum_j c_{ij} x_{ij} & \\ \text{st} & -\sum_j x_{ij} = -s_i \quad \forall i & (u_i) \\ & \sum_i x_{ij} = d_j \quad \forall j & (v_j) \\ & x_{ij} \geq 0 \quad \forall ij & \end{array} \qquad \begin{array}{lll} \max & -\sum_i s_i u_i + \sum_j d_j v_j & \\ \text{st} & -u_i + v_j \leq c_{ij} \quad \forall ij & (x_{ij}) \\ & u_i, v_j \text{ free} & \end{array}$$

Complementary slackness states that if x is primal feasible and u, v are dual feasible and the following applies

1. $u_i = 0 \quad \vee \quad -\sum_j x_{ij} = -s_i$
2. $v_j = 0 \quad \vee \quad \sum_i x_{ij} = d_j$
3. $x_{ij} = 0 \quad \vee \quad -u_i + v_j = c_{ij}$

then x, u, v are optimal.
The two first conditions are trivial as we have equality constraints in the transportation problem. The third condition requires

$$x_{ij} > 0 \quad \Rightarrow \quad c_{ij} + u_i - v_j = 0$$

But in the simplex algorithm for the transportation problem we calculate the dual variables as $v_j = u_i + c_{ij}$ for edges going from sources to destinations and $u_i = v_j - c_{ij}$ for edges going from destinations to sources. Now we need to make sure that x is primal feasible (this is ensured in each iteration) and that u, v are dual feasible. Feasibility for the dual variables are forced with the constraint $u_i - v_j \geq c_{ij}$ but this is exactly our stopping criteria and thereby dual feasibility is ensured.

Week 12, Transportation problem and Dual Simplex

Exercise 1

In this exercise we will consider a transportation problem with 3 sources and 3 destinations. The sources are named $A1$, $A2$ and $A3$ and the destinations are named $B1$, $B2$ and $B3$. The table below shows the transportation cost for one unit between the sources and the destinations. The table also shows the supply for each source and demand for each destination.

	B1	B2	B3	Supply
A1	3	1	8	18
A2	6	5	3	15
A3	5	2	8	12
Demand	20	12	13	

A feasible, but not optimal solution, is shown in the table below. The table shows the number of units send between sources and destinations.

From To	A1 B1	A1 B2	A2 B1	A2 B3	A3 B1
Amount	6	12	2	13	12

In this exercise you are supposed to make one iteration of the network simplex as described in "Network Simplex for the transportation problem"by David Pisinger. The starting point will be the solution above that we want to improve. The first step is to calculate the dual variables u_i (corresponding to the sources) and v_j (corresponding to the destinations) to the given solution. Which dual variables did you get?

A)

$$u_1 = 0, u_2 = -2, u_3 = -2$$
$$v_1 = 3, v_2 = 1, v_3 = 0$$

B)

$$u_1 = 0, u_2 = -3, u_3 = -2$$
$$v_1 = 3, v_2 = 1, v_3 = 0$$

C)

$$u_1 = 0, u_2 = -2, u_3 = -2$$
$$v_1 = 3, v_2 = 1, v_3 = 1$$

D)

$$u_1 = 0, u_2 = -2, u_3 = -2$$
$$v_1 = 3, v_2 = 1, v_3 = 2$$

E)

$$u_1 = 0, u_2 = -3, u_3 = -2$$
$$v_1 = 3, v_2 = 1, v_3 = 2$$

F)

$$u_1 = 0, u_2 = 0, u_3 = 0$$
$$v_1 = 3, v_2 = 1, v_3 = 1$$

Exercise 2

Consider the following transportation problem:

	Costumer			
Supplier	B_1	B_2	B_3	Supply
A_1	3	4	4	45
A_2	5	8	8	15
A_3	0	M	0	10
Demand	40	20	10	70

Transportation cost and supply/demand are specified in the above table. The value M indicates that it is not possible to transport from A_3 to B_2.

(a) Draw the problem as a network problem

(b) Use the greedy algorithm to find a solution.

(c) Use network simplex to find the optimal solution.

Exercise 3

8.1-2. The Childfair Company has three plants producing child push chairs that are to be shipped to four distribution centers. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance from each plant to the respective distributing centers is given to the right:

	distance			
	Distribution Center			
	1	2	3	4
1	800 miles	1,300 miles	400 miles	700 miles
Plant 2	1,100 miles	1,400 miles	600 miles	1,000 miles
3	600 miles	1,200 miles	800 miles	900 miles

The freight cost for each shipment is \$100 plus 50 cents per mile. How much should be shipped from each plant to each of the distribution centers to minimize the total shipping cost?

(a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

(b) Draw the network representation of this problem.

(c) Obtain an optimal solution.

Exercise 4 (question a)

8.1-4. The Versatech Corporation has decided to produce three new products. Five branch plants now have excess product capacity. The unit manufacturing cost of the first product would be \$41, \$39, \$42, \$38, and \$39 in Plants 1, 2, 3, 4, and 5, respectively. The unit manufacturing cost of the second product would be \$55, \$51, \$56, \$52, and \$53 in Plants 1, 2, 3, 4, and 5, respectively. The unit manufacturing cost of the third product would be \$48, \$45, and \$50 in Plants 1, 2, and 3, respectively, whereas Plants 4 and 5 do not have the capability for producing this product. Sales forecasts indicate that 700, 1,000, and 900 units of products 1, 2, and 3, respectively, should be produced per day. Plants 1, 2, 3, 4, and 5 have the capacity to produce 400, 600, 400, 600, and 1,000 units daily, respectively, regardless of the product or combination of products involved. Assume that any plant having the capability and capacity to produce them can produce any combination of the products in any quantity. Management wishes to know how to allocate the new products to the plants to minimize total manufacturing cost.

(a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

Exercise 5

7.1-2. Use the dual simplex method manually to solve the following problem.

Minimize

$$Z = 5x_1 + 2x_2 + 4x_3$$

subject to

$$3x_1 + x_2 + 2x_3 \geq 4$$
$$6x_1 + 3x_2 + 5x_3 \geq 10$$

and

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Exercise 6

7.1-1. Consider the following problem.

$$\max -x_1 - x_2$$

subject to

$$x_1 + x_2 \leq 8$$
$$x_2 \geq 3$$
$$-x_1 + x_2 \leq 2$$

(b) Use the dual simplex method manualy to solve this problem

(c) Trace graphically the path taken by the dual simplex method