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CS 520 Final: Question 4 – GoatDiscoveryBot

a) Logically, this information can be modeled as follows:

$$InA \cup InB \cup InC$$

Probabilistically, this information can be modeled as:

$$P(InA) = P(InB) = P(InC) = \frac{1}{3}$$

b) Under the logical formulation, the bot will only take 'rational' moves. This means that it will never choose B for example if $\neg InB$ is in the Knowledge Base ($KB \cap InB$ is unsatisfiable). The limitation of the logical formulation is that if there are multiple choices and both of them are rational, it cannot deliver the concept of 'preference'. Therefore, at this stage, there is no obvious best choice, so the bot can only pick one location arbitrarily.

c) Under the probabilistic formulation, the bot will compare the probability of InA, InB and InC, and choose the one with the highest probability. At this stage, there is no obvious choice of best action.

d) Update the Knowledge Base with $\neg InB$, the KB is now:

$$InA \cup InC$$

e) Given

$$P(InA) = P(InB) = P(InC) = \frac{1}{3}$$

Update the probability formulation with:

If InA, CBMHBot can tell you either B or C (do not contain goat), so:

$$P(Select\ A, CBMHBot\ tells\ not\ B | InA) = \frac{1}{2}$$

If InB, CBMHBot would never tell you B (does not contain goat):

$$P(Select\ A, CBMHBot\ tells\ not\ B | InB) = 0$$

If InC, Select A, CBMHBot would have no choice but to tell you B (does not contain goat):

$$P(Select\ A, CBMHBot\ tells\ not\ B | InC) = 1$$

Apply law of total probability:

$$\begin{aligned} &P(Select\ A, CBMHBot\ tells\ not\ B) \\ &= P(Select\ A, CBMHBot\ tells\ not\ B | InA)P(InA) \\ &+ P(Select\ A, CBMHBot\ tells\ not\ B | InB)P(InB) \\ &+ P(Select\ A, CBMHBot\ tells\ not\ B | InC)P(InC) \\ &= \frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{1}{2} \end{aligned}$$

Update according to Bayes' Rule:

$$P(InA|Select\ A, CBMHBot\ tells\ not\ B)$$

$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InA)P(InA)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(InB|Select\ A, CBMHBot\ tells\ not\ B)$$

$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InB)P(InB)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{0 \times \frac{1}{3}}{\frac{1}{2}} = 0$$

$$P(InC|Select\ A, CBMHBot\ tells\ not\ B)$$

$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InC)P(InC)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

f) Under the logical formulation, query $\neg InB$ will not satisfy the KB, query either InA or InC will satisfy the KB. Therefore, the bot will not perform action 'Re-Select B', but it will treat both 'Re-Select A' and 'Re-Select C' as rational choices.

g) As is discussed in Question e), we could compare the updated probability of $P(InA|Select\ A, CBMHBot\ tells\ not\ B)$, $P(InB|Select\ A, CBMHBot\ tells\ not\ B)$, $P(InC|Select\ A, CBMHBot\ tells\ not\ B)$, which are $\frac{1}{3}, 0, \frac{1}{2}$. So, obviously there is a better chance that the goat is at C, and therefore 'Re-Select C' is the best choice.

h) Under logical formulation, having initially selected A, the bot can only tell that re-selecting B is a bad idea, but to stick with A or to change to select C seems to have no difference i.e. $KB \cap InA$, $KB \cap InC$ are both satisfiable. So, the bot could just stick with the initial choice and call it a day.

i) Under the probability formulation, the bot definitely would change to C. Because after knowing the fact that the bot initially selected A, the CBMHBot told him that B is not where the goat is at, the probability of the goat is at C is much higher than at A, according to the computation showed in the previous questions.

j) The ProbabilisticGoatDiscoveryBot is more successful. Comparing to the ProbabilisticGoatDiscoveryBot, the LogicalGoatDiscoveryBot: 1. Cannot formulate preference. All queries that satisfy the Knowledge Base are considered rational. 2. Is always prior. This means that it cannot take full use of the posterior knowledge to update its KB. The ProbabilisticGoatDiscoveryBot however, can update its KB utilizing the Bayes' theorem.

Bonus:

Given GBMHBot is biased:

$$P(Select\ A, CBMHBot\ tells\ not\ B|InA) = p$$

$$P(Select\ A, CBMHBot\ tells\ not\ B)$$
$$= P(Select\ A, CBMHBot\ tells\ not\ B|InA)P(InA)$$
$$+ P(Select\ A, CBMHBot\ tells\ not\ B|InB)P(InB)$$
$$+ P(Select\ A, CBMHBot\ tells\ not\ B|InC)P(InC)$$
$$= p \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{p+1}{3}$$

Therefore:

$$P(InA|Select\ A, CBMHBot\ tells\ not\ B)$$
$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InA)P(InA)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{p \times \frac{1}{3}}{\frac{p+1}{3}} = \frac{p}{p+1}$$

$$P(InB|Select\ A, CBMHBot\ tells\ not\ B)$$
$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InB)P(InB)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{0 \times \frac{1}{3}}{\frac{p+1}{3}} = 0$$

$$P(InC|Select\ A, CBMHBot\ tells\ not\ B)$$
$$= \frac{P(Select\ A, CBMHBot\ tells\ not\ B|InC)P(InC)}{P(Select\ A, CBMHBot\ tells\ not\ B)} = \frac{1 \times \frac{1}{3}}{\frac{p+1}{3}} = \frac{1}{p+1}$$

Because $p < 1$ is always true, the probability of the goat is at C, given you select A and CBMHBot tells you not B, is always higher. Therefore, you should always choose to re-select C.