Complexity comparison of Karatsuba, divide and conquer and grade school algorithms

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Problem statement

Algorithm's difficulty:

- Grade school multiplication: O(n^2)
- Divide and conquer multiplication: O(n^2)
- Karatsuba multiplication: O($n^{1.6}$)

Algorithm's principle:

- Grade school multiplication: $x \times y$
- Divide and conquer multiplication: $x \times y = \left(a \times 10^{n/2} + b\right) \times (c \times 10^{n/2} + d)$ =

= ac × 10^n + ad × $10^{n/2}$ + bc × $10^{n/2}$ + cd

- Karatsuba multiplication: $x \times y = \left(a \times 10^{n/2} + b\right) \times (c \times 10^{n/2} + d)$ =

= ac × 10^n + ad × $10^{n/2}$ + bc × $10^{n/2}$ + cd = ac × 10^n + (ad + bc) × $10^{n/2}$ + cd = ac × 10^n + cd + ((a+b)(c+d) - ac – bd) × $10^{n/2}$

I've decided to use std::string as a holder of our numbers. In my class Number the only field is string variable and multiplication table. I have a lot of overloadings, such as +, -, * and so on to make my project better to read and work with.

Useful data:

1. Karatsuba's algorithm
2. Divide and conquer algorithm
3. Grade school multiplication

Code

Structure:

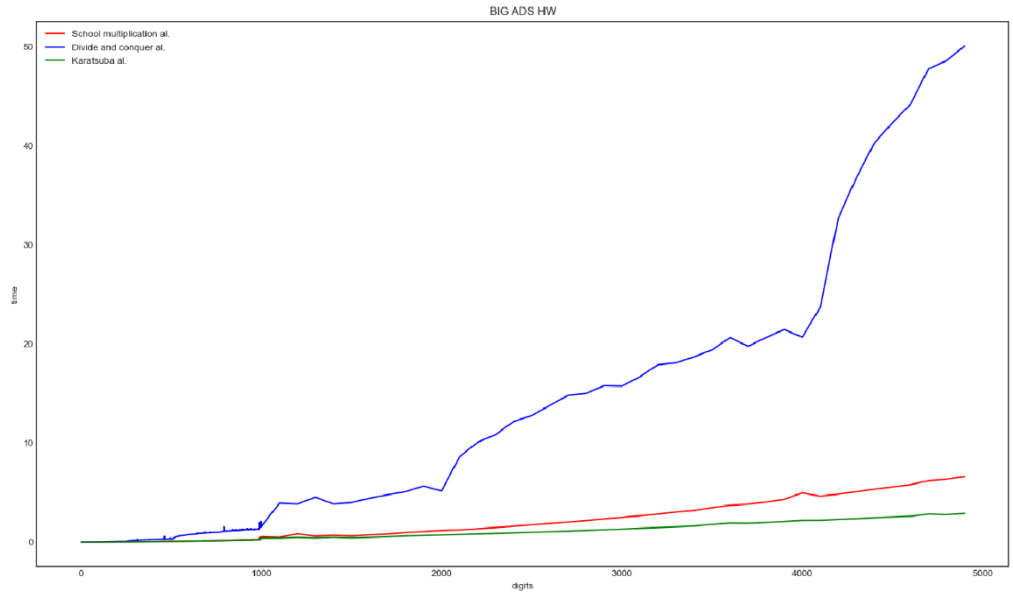
1. main.cpp
2. Multiplicator.cpp
3. Number.cpp
4. Multiplicator.h
5. Number.h

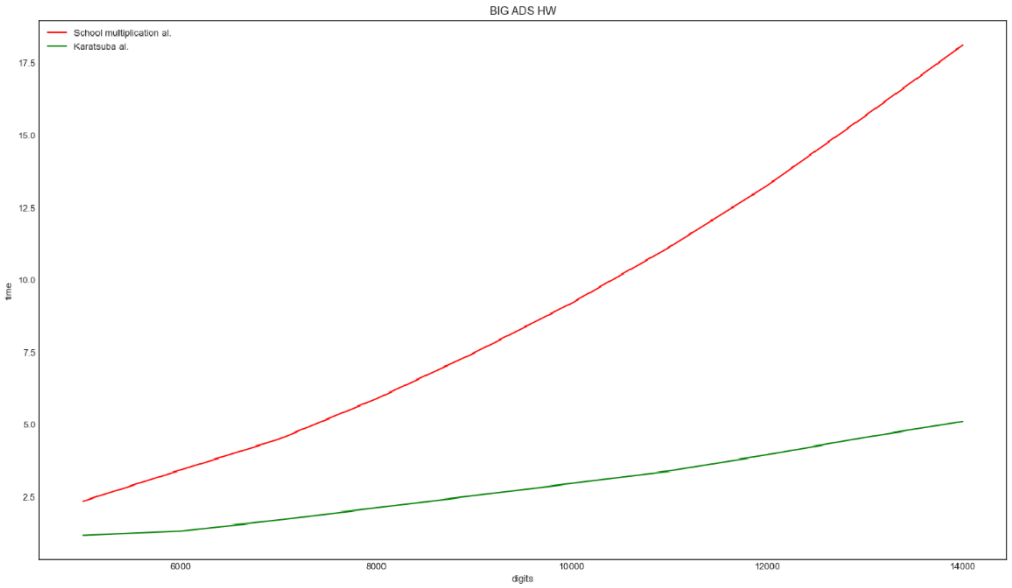
In header files I have only function headers and classes. The implementation of functions is situated in the .cpp files with the same name. In my recursive algorithms' implementations the base cases are all 1. I've tried to avoid any sort of multiplication, that is why I made the static second field (std::vector<std::vector<std::string>>). It is a multiplication table for 1-digit integers. Also, I use smart pointers working with polymorphic classes. In main.cpp I have two functions. Their main goals are to execute the algorithm 3 times and then write in csv file. Then by using python I build 3 graphs to represent asymptotic. Pandas and matplotlib helped me with the realization of graph.

URL:

https://github.com/olivan139/dsba-ads2020-hw1

Results





School multiplication al.

Divide and conquer al.

Karatsuba al.

As it can be seen from the graph divide and conquer algo is slower than grade school algo because of const factor. Recursion and copying objects makes it slower. Also, as it can be seen from the graph Karatsuba algorithm becomes faster on the interval from 500 to 1000. One can see this trend clearly starting from the point of 1500 or 5000 digits

Conclusion

To sum up, despite the fact that I faced a lot of problems during implementation I still like this task. The only thing I would improve is divide and conquer algorithm implementation in order to make it faster .