Mass matrix integration

1. Plane stress mass matrix integration

From : Thomas, M., & Laville, F. (s.d.). Chapitre 14. La méthode des éléments finis, appliquée aux barres et aux poutres. Dans Simulation des vibrations mécaniques : par Matlab, Simulink et Ansys. (S.l.) : (s.n.).

The element mass matrix for x-y displacement in plane stress is given by

$$Me = \rho \int_V [N]^T [N] dV$$

With the Q4 element with 2 degrees of freedom shape functions :

$$N = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix}$$

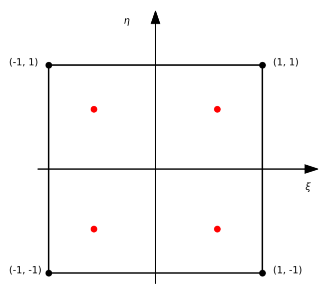
And W the plane stress mass tensor :

$$W = \begin{bmatrix} \rho t & 0 \\ 0 & \rho t \end{bmatrix}$$

For a planar element the integration becomes :

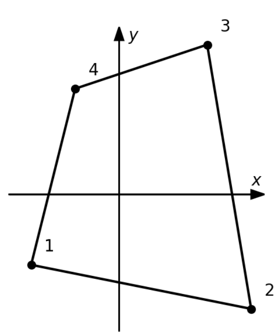
$$Me = \int_A [N]^T [W][N] dA$$

For the bilinear quadrialteral element Q4, the integration is done using 2nd order gauss quadrature. In order to integrate the shape functions of the real element, the corresponding integration points have to be transformed from the reference element with natural coordinates ξ and η (2nd order integration points are shown in red):



The integration points are equal to $\pm\frac{1}{\sqrt{3}}$ for both coordinates.

Here lets consider a real elements with nodes at arbitrary x and y coordinates :



The real coordinates of an integration point are given by the following formula :

$p_{x1} = N_1(\xi_1, \eta_1)x_1 + N_2(\xi_1, \eta_1)x_2 + N_3(\xi_1, \eta_1)x_3 + N_4(\xi_1, \eta_1)x_4$

$p_{y1} = N_1(\xi_1, \eta_1)y_1 + N_2(\xi_1, \eta_1)y_2 + N_3(\xi_1, \eta_1)y_3 + N_4(\xi_1, \eta_1)y_4$

The numerical integration of the element mass matrix can now be performed using the real coordinates integration points as follow:

$Me = \sum_{i=1}^{4} N'(p_{xi}, p_{yi}) \begin{bmatrix} h\rho & 0 \\ 0 & h\rho \end{bmatrix} N(p_{xi}, p_{yi})\ det(J(xi_i, \eta_i))$

In [ ]:

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