# Longitudinal Data Analysis

White Rose Social Sciences Doctoral Training Partnership

Thiago Oliveira

Lecturer in Criminology, University of Manchester

8 April 2025, University of Leeds

# Part II: causal inference with panel data

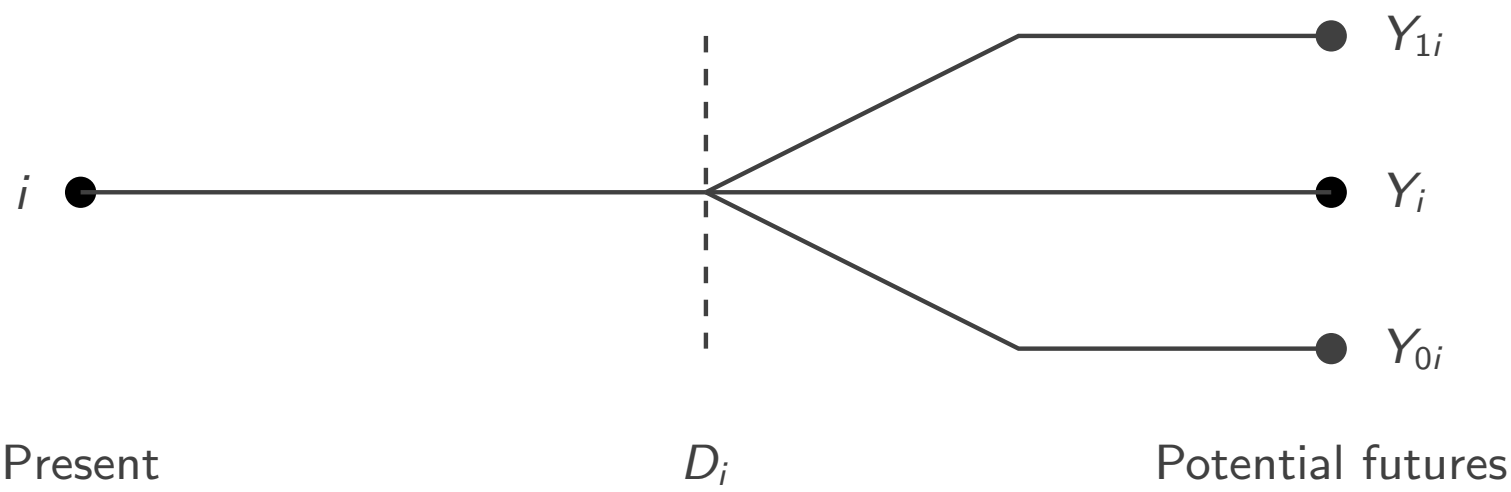
# Potential outcomes framework

# Causal inference and the potential outcomes framework

Goal in causal inference is to assess the causal effect of a treatment/exposure on some outcome

- ⇝ Does raising the minimum wage reduce employment?
- ⇝ Does housing assistance reduce homelessness?
- ⇝ Do body-worn cameras reduce police use of force?
- ⇝ Does voting by mail increase voter turnout?
- ⇝ Does exposure to misinformation reduce political trust??
- ⇝ ...

# Causal inference and the potential outcomes framework



# Causal inference and the potential outcomes framework

$Y_i$: Observed outcome variable of interest for unit $i$

## Potential outcomes

$Y_{0i}$ and $Y_{1i}$: Potential outcomes for unit $i$

$$Y_{\cdot i} = \begin{cases} Y_{1i} & \text{Potential outcome for unit } i \text{ with treatment} \\ Y_{0i} & \text{Potential outcome for unit } i \text{ without treatment} \end{cases}$$

$D_i$: Indicator of treatment intake for *unit* $i$

$$D_i = \begin{cases} 1 & \text{if unit } i \text{ received the treatment} \\ 0 & \text{otherwise.} \end{cases}$$

## Definition of causal effect

$$\delta_i = Y_{1i} - Y_{0i}$$

## Fundamental problem of causal inference

$\rightsquigarrow$ We cannot observe both potential outcomes for the same unit $i$!

# Causal inference and the potential outcomes framework

## Randomisation solves the problem!

Logic of randomised control trials

- ⇝ Randomly divide a sample in two groups
- ⇝ Because this was random, both groups are *on average* the same
- ⇝ Then apply the treatment/exposure to one group (the treatment group), but not the other (control group)
- ⇝ Because the exposure happened after the treatment assignment, the only difference between the two groups is the treatment/exposure
- ⇝ Therefore, any subsequently observed differences are attributable to the treatment/exposure
- ⇝ We randomisation, we can thus find the **average treatment effect**

# Causal inference and the potential outcomes framework

What if we cannot conduct an experiment?

- ⇝ Randomised Experiments
- ⇝ Observational Studies
  - Selection on observables
    - Regression
    - Matching
    - Weighting
  - Selection on unobservables
    - Difference-in-Differences and synthetic control
    - Instrumental Variables
    - Regression Discontinuity Designs

# Causal inference and the potential outcomes framework

- $\rightsquigarrow$ Causality is defined by potential outcomes, not by realised (observed) outcomes
- $\rightsquigarrow$ Observed association is neither necessary nor sufficient for causality
- $\rightsquigarrow$ Estimation of causal effects of a treatment (usually) starts with studying the assignment mechanism
- $\rightsquigarrow$ The goal is to mimic the features of a randomised experiment even if we don't have one
- $\rightsquigarrow$ When we don't have an RCT, our ability to make causal inferences often relies on making untestable assumptions about the assignment mechanism

$\Rightarrow$ Now let's see how we can leverage panel data to make causal inferences!

# Difference-in-differences

# Intuition of the difference-in-differences estimator

$\Rightarrow$ What if we use **time** in our favour?

- $\rightsquigarrow$ Collect data on $Y$ at two points in time: **before** and **after** the treatment/exposure/policy intervention
- $\rightsquigarrow$ Analyse the extent to which $Y$ **changes** in units that received the treatment
- $\rightsquigarrow$ Analyse the extent to which $Y$ **changes** in units that did NOT receive the treatment
- $\rightsquigarrow$ Compare the two **changes**

# Intuition of the difference-in-differences estimator

Some conceptual clarification to make our lives easier

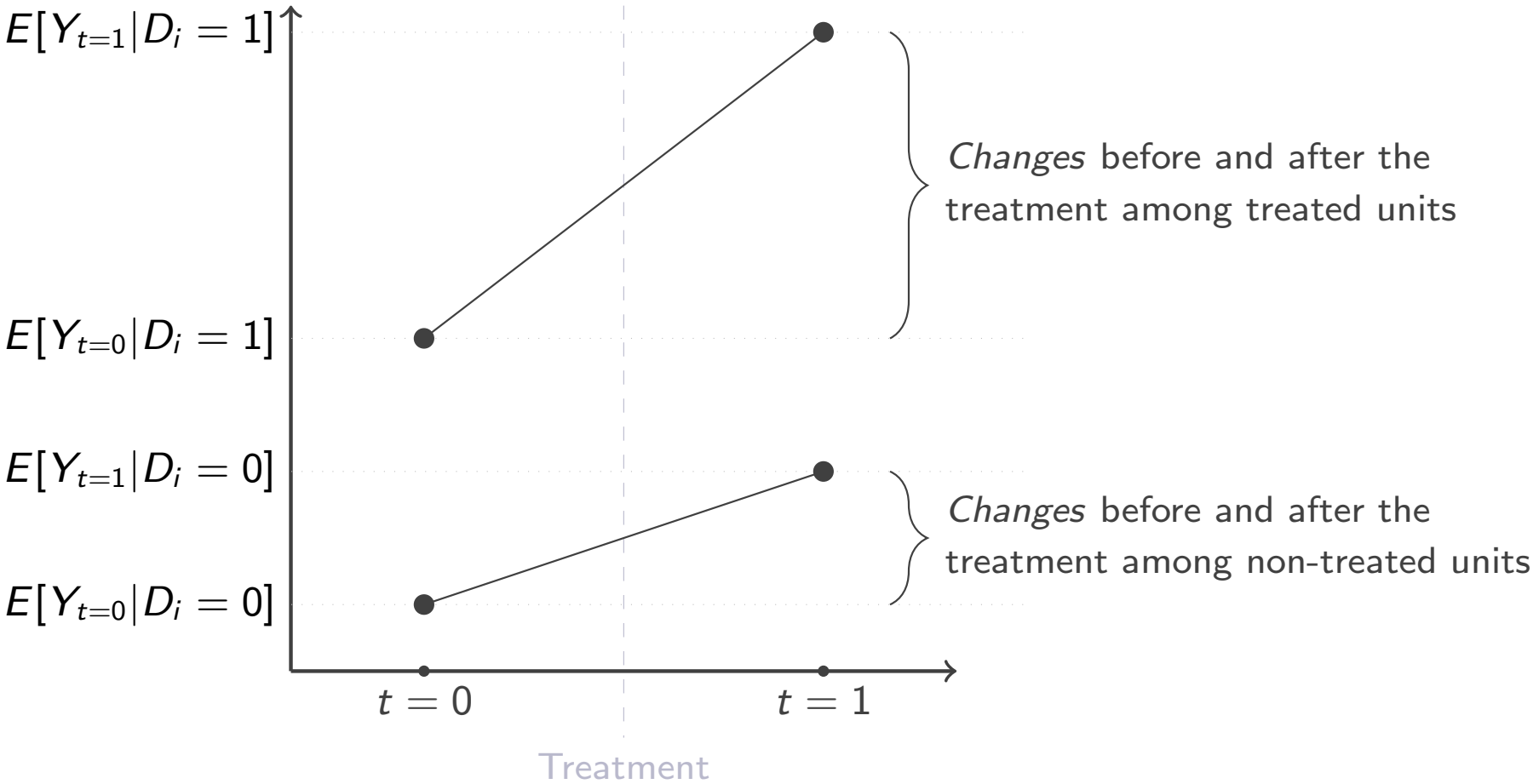
- $\rightsquigarrow$ Variation **between** units: **difference**
- $\rightsquigarrow$ Variation **within** units (over time): **changes**

$\Rightarrow$ We want to estimate the **difference in changes**
or (*difference-in-differences*)

- $\rightsquigarrow$ The difference between (a) changes in $Y$ before and after the intervention among treated units and (b) changes in $Y$ before and after the intervention among non-treated units is the **causal effect**!

(under some assumptions regarding those changes... Let's dive into it)

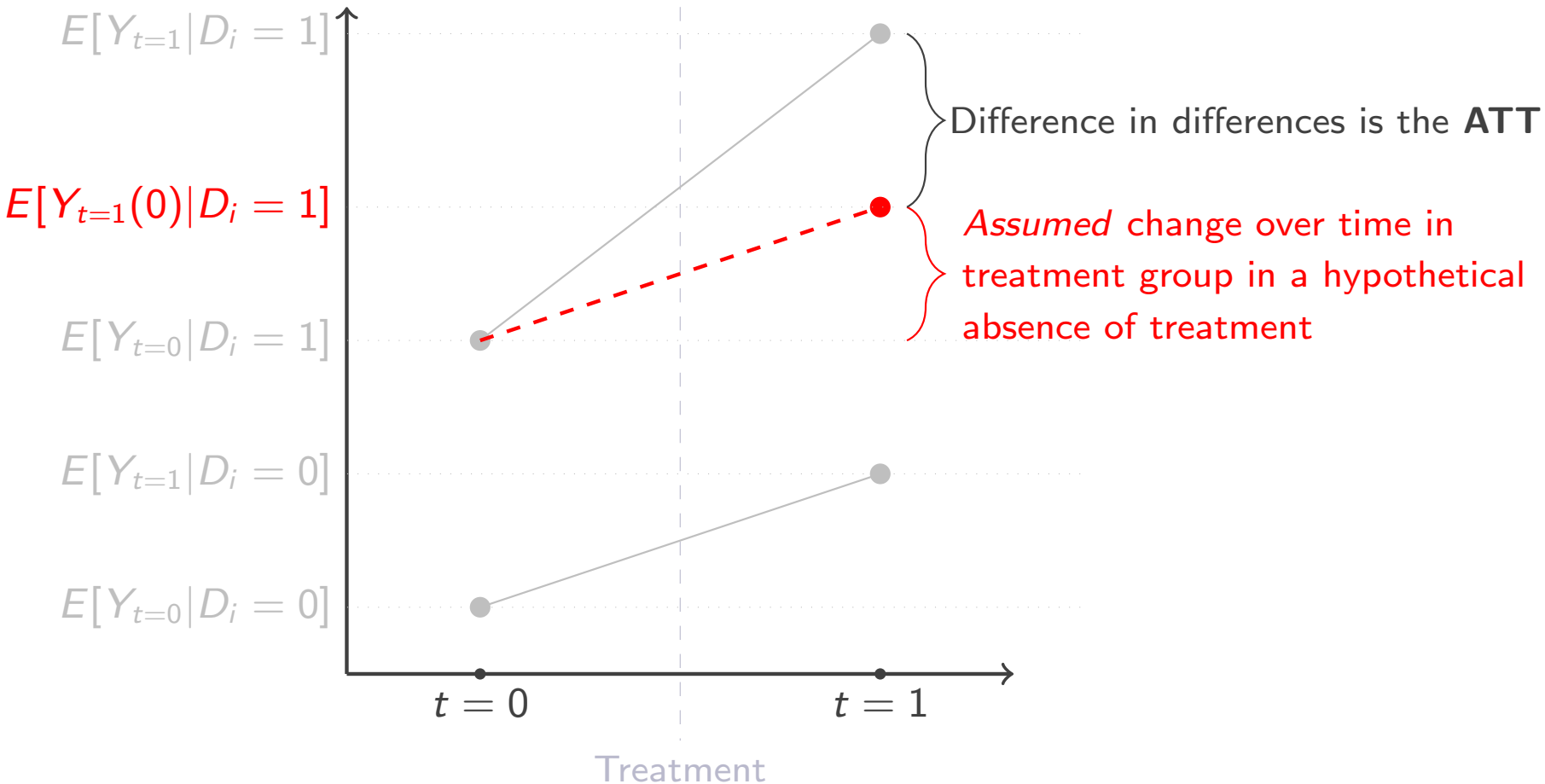
# Difference-in-differences setup




$\rightsquigarrow$ **Problem**: Missing potential outcomes: $E[Y_{i,t=1}(0)|D_i = 1]$ and $E[Y_{i,t=1}(1)|D_i = 0]$

# Difference-in-differences setup

**Strategy**: Use the change in the control group to *assume* $E[Y_{t=1}(0)|D_i = 1]$




**Assumption**: Trend over time would be the same for treatment and control

# Identification assumption

## Parallel trends

$\rightsquigarrow$ Had the treated units not received the treatment, they would have followed the same trend as the control units

## Difference-in-differences estimator

Difference in changes:

$$\delta_{ATT} = \Big\{\text{Changes in treatment group before and after treatment}\Big\} - \Big\{\text{Changes in control group before and after treatment}\Big\}$$

# Threats to validity

## Non-parallel trends

$\rightsquigarrow$ Very critical assumption: treatment units have similar trends to control units in the absence of treatment

$\rightsquigarrow$ Fundamental problem of causal inference: we cannot observe potential outcome under the control condition for treated units in the post-treatment period

$\Rightarrow$ What can we do?

*(more on that later. . . )*

- Careful assessment: is assuming parallel trends plausible?
- Estimate treatment effects at different time points (placebo tests)

# Using regression to estimate the difference-in-differences

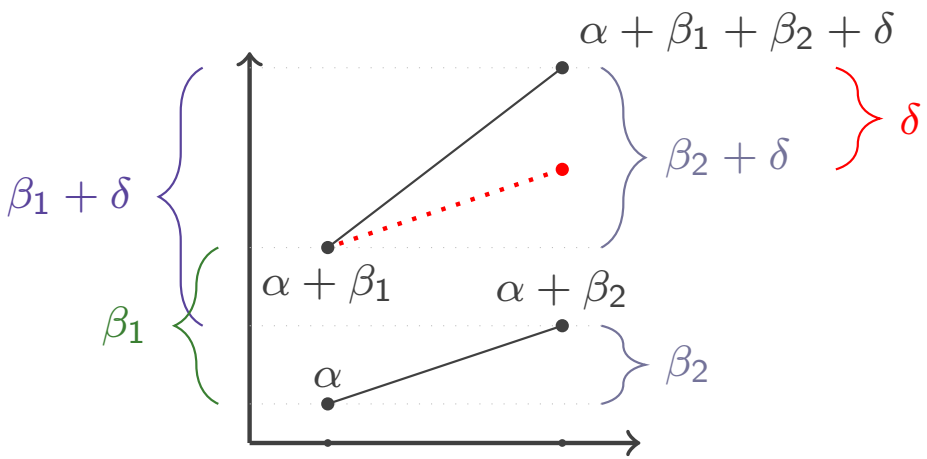
# Estimator (Regression 1)

*We can obtain the difference in differences using regression techniques.*

$$Y_i = \alpha + \beta_1 \cdot D_i + \beta_2 \cdot T_i + \delta \cdot (D_i \cdot T_i) + \varepsilon.$$

*We can see that:*

| $E[Y_i \mid D_i, T_i]$ | $T_i = 0$ | $T_i = 1$ | *Changes after - before* |
|---|---|---|---|
| $D_i = 0$ | $\alpha$ | $\alpha + \beta_2$ | $\beta_2$ |
| $D_i = 1$ | $\alpha + \beta_1$ | $\alpha + \beta_1 + \beta_2 + \delta$ | $\beta_2 + \delta$ |
| *Treated - control* | $\beta_1$ | $\beta_1 + \delta$ | $\delta$ |



# DiD: First differences estimator

## Estimator (Regression 2)

*With panel data we can use regression with first differences:*

$$\Delta Y_i = \alpha + \delta \cdot D_i + X'\beta + u,$$

*where* $\Delta Y_i = Y_i(1) - Y_i(0)$.

- *With two periods this gives the same result as other regressions*

# Advantages of the regression estimator

1. We can include covariates
   - Controlling for some covariates may increase precision
   - Time-varying covariates may strengthen the parallel assumptions
   - (add covariates cautiously! e.g., beware of post-treatment bias)
2. Easy to calculate standard errors
   - (though be careful about clustering)
3. Easy to extend to other types of treatment
   - (not just binary)

# Some limitations

>> This setup only works for the simplest scenario with two time periods

⇝ It doesn't make use more periods

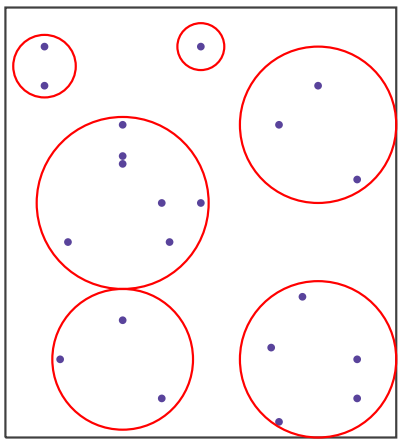
- Useful to make careful assessments of time trends

⇝ Sometimes different units are treated at different time points

# Difference-in-differences with multiple periods

# Intuition of fixed-effect regression

>> Assume a pool of structured data



$\rightsquigarrow$ Each dot represents a unit $i$

$\rightsquigarrow$ Each circle represents a group $j$

- Pooled approach
- Between approach
- Random Effects
- **Fixed Effects**

>> Focus on within-group variation

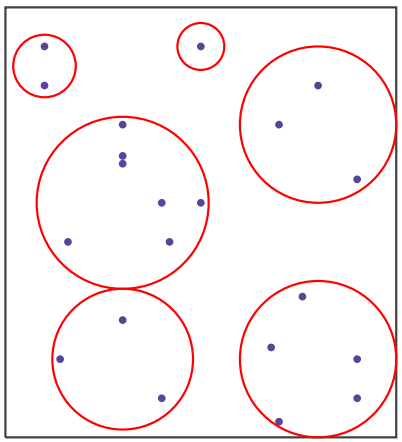
>> Implementation: dummy variables for each group $j$ ($\gamma_j$)

$$Y_{ij} = \gamma_j + \beta \cdot X_{ij} + \varepsilon$$

>> What about panel data?

# Fixed-effect regression with panel data

>> Assume a pool of structured data



⇝ Each dot represents a measure $t$

⇝ Each circle represents a unit $i$

>> Focus on **within-unit** variation

>> Implementation: dummy variables for each unit $i$ ($\gamma_i$)

$$Y_{it} = \gamma_i + \beta \cdot X_{it} + \varepsilon$$

>> What about time fixed-effect?

# DiD: Two-way fixed-effect regression

## Estimator (Regression with Multiple Time Periods)

*We can generalise to multiple groups/time periods using unit and period fixed-effects ('two-way' fixed-effect model):*

$$Y_{it} = \gamma_i + \alpha_t + \delta \cdot D_{it} + \varepsilon$$

- $\gamma_i$ *is a fixed-effect for units (dummy for each unit)*
- $\alpha_t$ *is a fixed-effect for time periods (dummy for each period)*
- $\delta$ *is the DiD estimate based on* $D_{it}$

## Very flexible approach

- we can replace $D_{it}$ with almost any type of treatment (not only binary)
- we can extend easily to multiple periods
- we can have units treated at different times
- we can estimate unit-specific time trends by including a unit-period interaction
  - $\rightsquigarrow$ useful when treatment occurs at different times for different units and there are slight deviations from parallel trends

# DiD: Two-way fixed-effect regression

## Why does two-way fixed-effect regression estimate the DiD?

- $\rightsquigarrow$ Unit FEs means that we are only using *within unit* variation in $Y$ to calculate the effect of $D$
  - i.e., *changes* over time!
  - This removes *all* time-constant confounders
- $\rightsquigarrow$ Time FEs means that we remove the effect of any changes to the response variable that affect all units at the same time
- $\rightsquigarrow$ $\hat{\delta} \rightarrow \hat{\delta}_{ATT}$ (*it might not be that simple...*)
  - It is hard to provide a visual inspection of the parallel trends assumption here as treatment may switch on at different time for different units
  - Nevertheless, we are still assuming that treated/control units would have evolved identically over time in absence of treatment

$>>$ Why not always use unit dummies?

- Fine in panel data, as we have same units at several points in time
- Not possible with repeated cross-section when we do not have the same units in each time period

# Some caution with two-way fixed-effect models

**Maxim Ananyev**
@maximananyev

A rare photo of an applied economist keeping up with the difference-in-differences literature

# Unit FEs and time-constant confounders

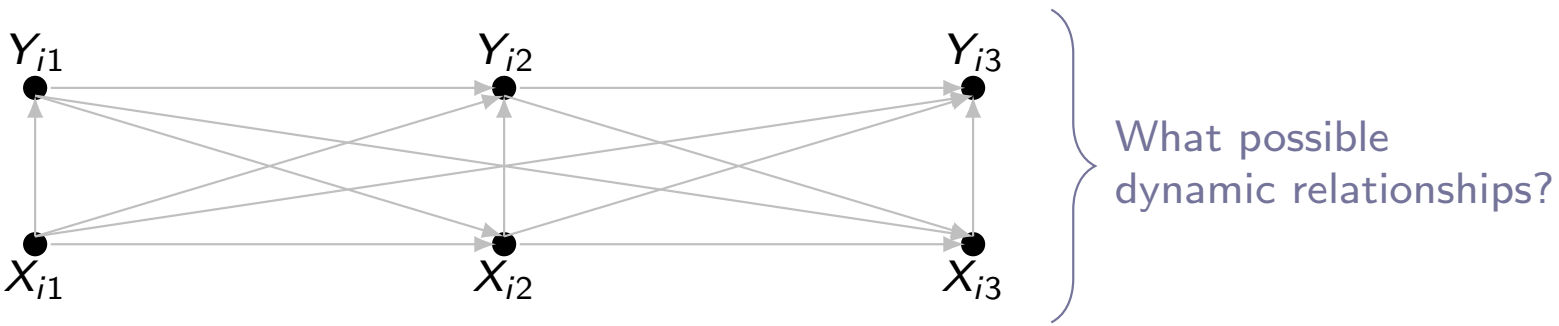
AJPS AMERICAN JOURNAL of POLITICAL SCIENCE

## When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

**Kosuke Imai** Harvard University
**In Song Kim** Massachusetts Institute of Technology

# Unit FEs and time-constant confounders

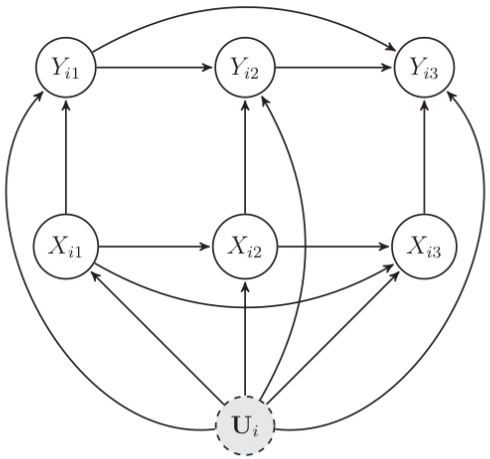
- ⇝ Imai & Kim (2019) show that unit FEs might not be that effective in adjusting for unobserved time-constant confounders
- ⇝ The issue is related to possible dynamic causal relationships



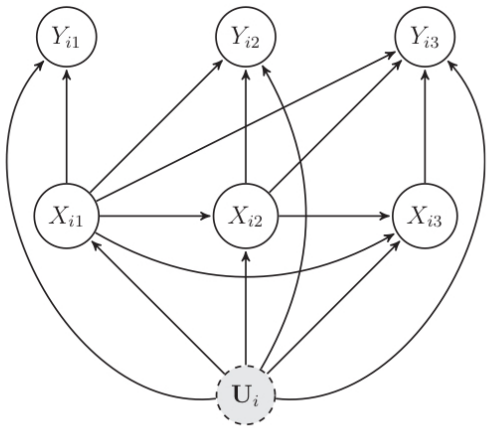

- ⇝ Some dynamic causal relationships compromise unit FEs

# Unit FEs and time-constant confounders

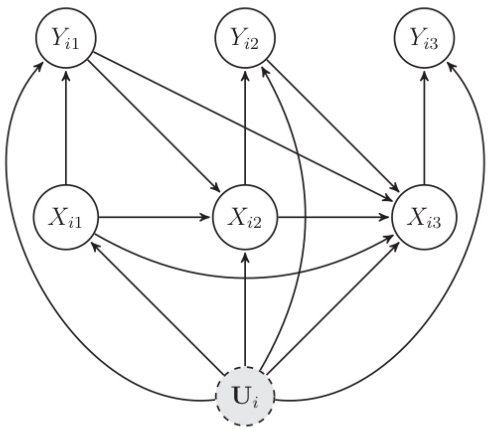
**FIGURE 2 Identification Assumptions of Regression Models with Unit Fixed Effects**



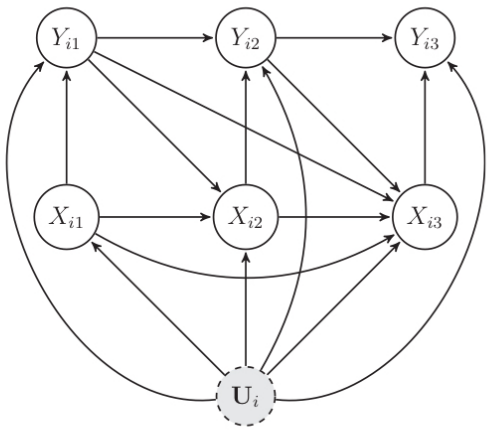

(a) past outcome affects current outcome




(b) past treatments affect current outcome




(c) past outcomes affect current treatment




(d) past outcomes affect both current outcome and treatment

- (1) Past outcome affects current outcome
- (2) Past treatments affect current outcome
- (3) Past outcomes affect current treatment
- (4) Past outcomes affect current outcome and treatment

# Unit FEs and time-constant confounders

## Key assumptions of unit fixed effects models

1. Past treatments do not directly influence current outcome
2. Past outcomes do not affect current treatment

# Summary

# Summary

- ⇝ Causal inference with observational data is really hard!
- ⇝ Longitudinal data can help, but it's not a silver bullet
  - Have a look at all assumptions involved
  - Parallel trends is an untestable assumption
- ⇝ This is a fast-changing topic. Keep up with the literature!
  - Callaway and Sant'Anna (2020); Callaway et al. (2021); Imai et al. (2021); Goodman-Bacon (2018); Imai and Kim (2019)
- ⇝ Now let's see how to estimate those models using R!
  - Find the lab notes here: thiagoroliveira/2-LDA-lab.html

# Thank you!

thiago.oliveira@manchester.ac.uk

ThiagoROliveira.com

@oliveiratr.bsky.social

# REFERENCES

Callaway, B., A. Goodman-Bacon, and P. H. Sant'Anna (2021). Difference-in-differences with a continuous treatment. *arXiv preprint arXiv:2107.02637*.

Callaway, B. and P. H. Sant'Anna (2020). Difference-in-differences with multiple time periods. *Journal of Econometrics*. Published online.

Goodman-Bacon, A. (2018). Difference-in-differences with variation in treatment timing. Technical report, National Bureau of Economic Research.

Imai, K. and I. S. Kim (2019). When should we use unit fixed effects regression models for causal inference with longitudinal data? *American Journal of Political Science 63*(2), 467–490.

Imai, K., I. S. Kim, and E. H. Wang (2021). Matching methods for causal inference with time-series cross-sectional data. *American Journal of Political Science*.