Mathematical Analysis 2

Call 4 – 23/07/2024

Part 1 – 10:00-11:30

Do **not** turn this sheet over until instructed to do so.

Name: [ ],
Mat: [ ] Seat: **A**

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Respond to the questions with **fully and clearly argued solutions**. If there is something that you are uncertain about, do the best you can and demonstrate your knowledge of the relevant concepts.

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Question 1. Prove by induction the following statements:

1. $\sum_{k=1}^{n} k = \frac{1}{2}n(n+1);$
2. $\sum_{k=1}^{n} (2k-1) = n^2;$
3. $n! > 2n$ whenever $n \geq 4$.

Question 2.

1. Consider the surface $x^2y = 4ze^{x+y} - 35$. Verify that the point $(3, -3, 2)$ lies in the surface. Write the surface in the form of a level set and hence calculate the normal vector and the tangent plane at this point.
2. Find the point(s) on the surface $6x^2 + y^2 - 3z^2 = 4$ where the tangent plane to the surface is parallel to the plane given by $2x + 7y - z = 6$.

Question 3. Use the Lagrange multipliers method to find the maximum and minimum values of $f(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.

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Part 2 – 11:45-13:15

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Question 4. Let $\boldsymbol{\alpha}$ be the closed path composed of three line segments with vertices $(0,0)$, $(0,3)$, $(-3,0)$. Consider the vector field defined on $\mathbb{R}^2$ by

$$\mathbf{F}(x,y) = \begin{pmatrix} xy^2 + x^2 \\ 4x - 1 \end{pmatrix}.$$

Verify Green's theorem for the line integral $\int \mathbf{F} \cdot d\boldsymbol{\alpha}$ by (1) computing the line integral directly and (2) using Green's Theorem to compute the line integral.

Question 5. Let V denote the region between the two planes $x + y + z = 2$and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$. Evaluate $\int\int\int_V z \, dxdydz$.

Question 6. Evaluate the surface integral $\int\int_S (2y)\, dS$ where S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.

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Name: **Duru Balci**,
Mat: **'0298794'** Seat: **1**

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Name: **Chaima Bensmail**,
Mat: **'0274444'** Seat: **2**

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Name: **Elvin Mursalov**,
Mat: **’0273121’** Seat: **3**

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Name: **Deepshika Naresh Thakur**,
Mat: **'0285115'** Seat: **4**

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Name: **Raja Anique Ahmad**,
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Name: **Ojas Gangwal**,
Mat: **'0282813'** Seat: **6**

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- You may choose to leave the test early but only after receiving confirmation from an invigilator.

Question 4. Let $\boldsymbol{\alpha}$ be the closed path composed of three line segments with vertices $(0, 0)$, $(0, 3)$, $(-3, 0)$. Consider the vector field defined on $\mathbb{R}^2$ by

$$\mathbf{F}(x, y) = \begin{pmatrix} xy^2 + x^2 \\ 4x - 1 \end{pmatrix}.$$

Verify Green's theorem for the line integral $\int \mathbf{F} \cdot d\boldsymbol{\alpha}$ by (1) computing the line integral directly and (2) using Green's Theorem to compute the line integral.

Question 5. Let V denote the region between the two planes $x + y + z = 2$and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$. Evaluate $\int\int\int_V z\, dxdydz$.

Question 6. Evaluate the surface integral $\int\int_S (2y)\, dS$ where S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.