Question 1. Find and classify the stationary points of the following function:

$$f(x,y) = \exp\left(\frac{x^2}{3} - 2xy + 2y^3\right).$$

Question 2. Let S denote the region bounded by the curves $y = 2x$, $y = x$, $xy = 2$, $xy = 3$. Sketch this set and then evaluate the integral

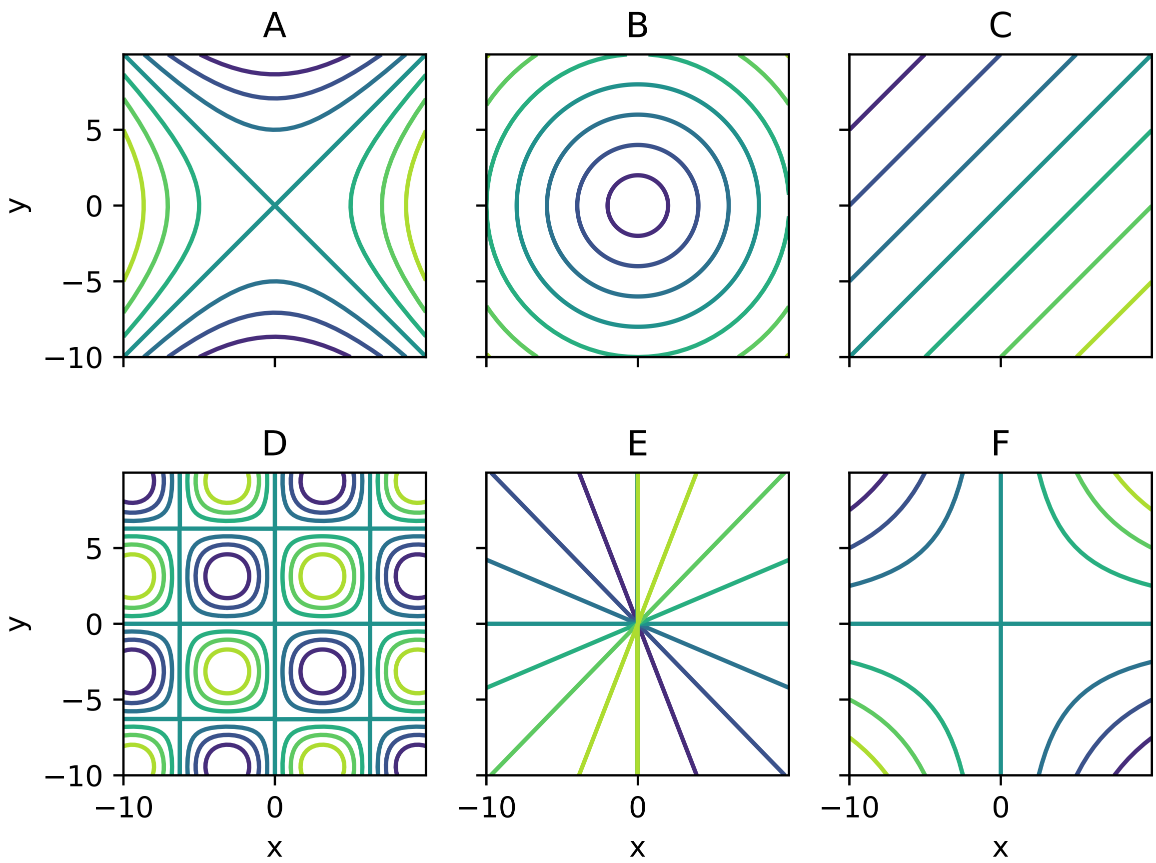
$$\iint_S \frac{x^5y^5}{x^3y^3+1}\, dxdy.$$

In order to evaluate the integral, use the change of coordinates $u = xy$, $v = y/x$.

Question 3. Consider the level sets of the following scalar fields:

$$f_1(x,y) = \sin(x/2)\sin(y/2) \qquad f_2(x,y) = xy$$
$$f_3(x,y) = \cos(\sqrt{x^2+y^2}) \qquad f_4(x,y) = \sqrt{y/x}$$
$$f_5(x,y) = \arctan(x-y) \qquad f_6(x,y) = x^2 - y^2.$$

1. Match each to one of the plots and briefly explain the logic/calculation for matching each (it is not required to give complete arguments).
2. For each of the scalar fields, calculate the gradient $\nabla f(x,y)$.



Question 4. Let $\boldsymbol{\alpha}$ be the closed path composed of three line segments with vertices $(0,0)$, $(0,2)$, $(-2,0)$. Consider the vector field defined on $\mathbb{R}^2$ by

$$\mathbf{F}(x,y) = \begin{pmatrix} x^2y + y \\ x - 1 \end{pmatrix}.$$

Verify Green's theorem for the line integral $\int \mathbf{F} \cdot d\boldsymbol{\alpha}$ by (1) computing the line integral directly and (2) using Green's Theorem to compute the line integral.

Question 5. consider the following sequence of functions:

$$f_n(x) = nx(1-x^2)^{2n}, \quad x \in [-1,1].$$

1. Describe f_n: Is it odd? Even? Neither? What is the value of $f_n(-1)$, $f_n(0)$, $f_n(1)$?
2. What is the maximum and minimum of $f_n(x)$ for $x \in [0,1]$?
3. Sketch the function for a few choices of n;
4. Determine the pointwise and uniform convergence of this sequence: On what set does it converge pointwise? What function does it converge to? On what set does it converge uniformly?
5. Calculate both sides of the following equality in order to show that it is false:

$$\lim_{n\to\infty} \int_0^1 f_n(x)\, dx = \int_0^1 \lim_{n\to\infty} f_n(x)\, dx.$$

Question 6. Consider the surface $S = \{x^2+y^2+z^2 = 4, x \geq 0, y \geq 0, z \geq 0\}$ and the vector field

$$\mathbf{F}(x,y,z) = \mathbf{i} + 3z\,\mathbf{j} + 2x\,\mathbf{k}.$$

Evaluate the flux integral

$$\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$$

where $\hat{\mathbf{n}}$ denotes the unit normal to the surface which points away from the origin.