Question 1. Determine the Maclaurin series for the following indefinite integral:

$$\int \sqrt{1+x^3}\,dx.$$

It suffices to write the first four non-zero terms in the series. *Hint: first calculate the Taylor expansion of $\sqrt{y}$ around $y=1$.*

Question 2. Use the Lagrange multiplier method to find the extrema of

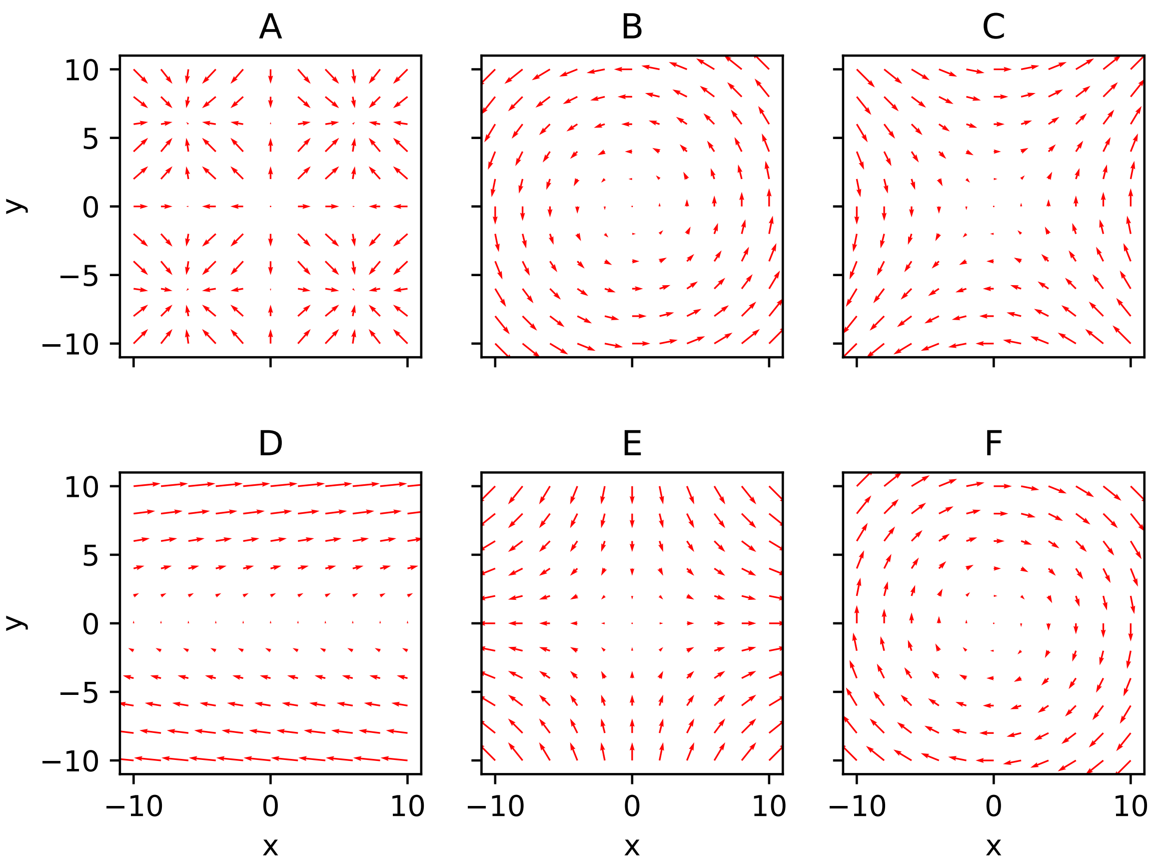
$$f(x,y) = 9x^2 + y^2 + 3$$

constrained to the set $G = \{(x,y) \in \mathbb{R}^2 : \frac{4}{9}x^2 + y^2 = 9\}$. Calculate the value of f at these extrema points.

Question 3. Consider the following vector fields defined on $\mathbb{R}^2$:

$$F_1(x,y) = y\mathbf{i} + \mathbf{j} \qquad F_2(x,y) = -y\mathbf{i} + x\mathbf{j}$$
$$F_3(x,y) = y\mathbf{i} + x\mathbf{j} \qquad F_4(x,y) = 2y\mathbf{i} - 2x\mathbf{j}$$
$$F_5(x,y) = 2x\mathbf{i} - 2y\mathbf{j} \qquad F_6(x,y) = \sin(x/2)\mathbf{i} + \sin(y/2)\mathbf{j}.$$

1. Match each to one of the plots and briefly explain the logic/calculation for matching each (it is not required to give complete arguments).
2. Calculate the divergence, $\nabla \cdot F_n(x,y)$ for each of the vector fields.



Question 1. Determine the Maclaurin series for the following indefinite integral:

$$\int \sqrt{1+x^3}\, dx.$$

It suffices to write the first four non-zero terms in the series. *Hint: first calculate the Taylor expansion of* $\sqrt{y}$ *around* $y = 1$.

Question 2. Use the Lagrange multiplier method to find the extrema of

$$f(x,y) = 9x^2 + y^2 - 2$$

constrained to the set $G = \{(x,y) \in \mathbb{R}^2 : \frac{4}{9}x^2 + y^2 = 4\}$. Calculate the value of f at these extrema points.

Question 3. Consider the following vector fields defined on $\mathbb{R}^2$:

$$F_1(x,y) = y\mathbf{i} + \mathbf{j} \qquad F_2(x,y) = -2y\mathbf{i} + 2x\mathbf{j}$$
$$F_3(x,y) = y\mathbf{i} + x\mathbf{j} \qquad F_4(x,y) = y\mathbf{i} - x\mathbf{j}$$
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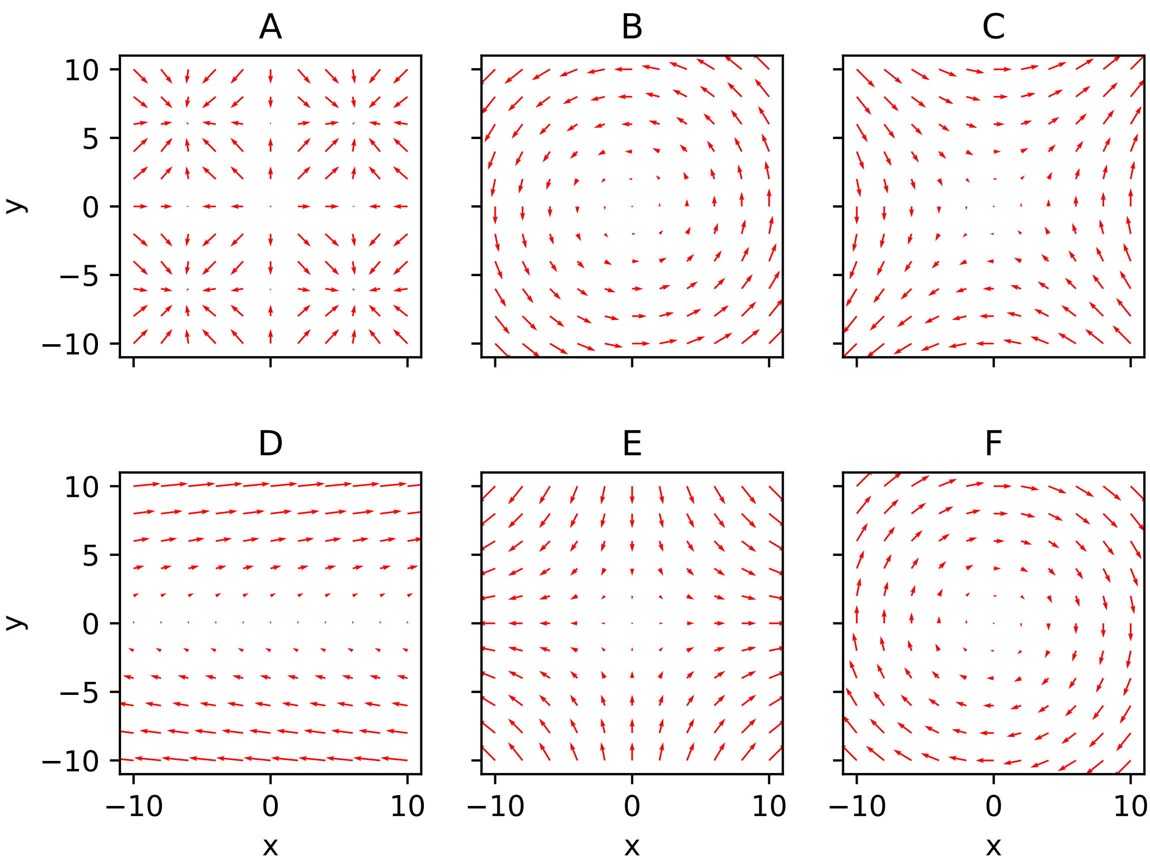

Question 4. Let

$$S = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0, x^2 + y^2 \leq 4, x^2 + y^2 - 2y \geq 0\}.$$

Sketch this set and then use polar coordinates to evaluate the integral

$$\iint_S x \, dxdy.$$

Hint: write the curve $x^2 + y^2 - 2y = 0$ in the form $x^2 + (y - 1)^2 =$? to sketch this curve.

Question 5. Let Γ denote the closed path along the portions of the curves $y = x^2, y = 1, x = 0$. Consider the vector field

$$F(x, y) = (5x^2 y - \sin x)\mathbf{i} + (x^3 + y \ln y)\mathbf{j}.$$

Compute the path integral of this vector field anticlockwise around the path. *Hint:* $\int t^3 \ln(t^2) \, dt = \frac{1}{4}t^4 \ln(t^2) - \frac{1}{8}t^4$, $\int t \ln(t) \, dt = \frac{1}{2}t^2 \ln(t) - \frac{1}{4}t^2$.

Question 6. Let $S = \sigma(T)$ be the parametric surface defined by $\sigma(u, v) = (u, v^2 - u, u + v)$, $T = [0, 2] \times [0, 1]$ and write $\mathbf{n}$ for the unit normal. Consider the vector field defined on $\mathbb{R}^3$ as

$$\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j}.$$

Evaluate the flux integral,

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS.$$

Question 4. Let

$$S = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0, x^2 + y^2 \leq 4, x^2 + y^2 - 2x \geq 0\}.$$

Sketch this set and then use polar coordinates to evaluate the integral

$$\iint_S y \, dxdy.$$

Hint: write the curve $x^2 + y^2 - 2x = 0$ in the form $(x - 1)^2 + y^2 =$? to sketch this curve.

Question 5. Let Γ denote the closed path along the portions of the curves $y = x^2, y = 1, x = 0$. Consider the vector field

$$F(x, y) = (4x^2 y - \sin x)\mathbf{i} + (x^3 + y \ln y)\mathbf{j}.$$

Compute the path integral of this vector field anticlockwise around the path. *Hint:* $\int t^3 \ln(t^2)\, dt = \frac{1}{4}t^4 \ln(t^2) - \frac{1}{8}t^4$, $\int t \ln(t)\, dt = \frac{1}{2}t^2 \ln(t) - \frac{1}{4}t^2$.

Question 6. Let $S = \sigma(T)$ be the parametric surface defined by $\sigma(u, v) = (u, v^2 - u, u + v)$, $T = [0, 2] \times [0, 1]$ and write $\mathbf{n}$ for the unit normal. Consider the vector field defined on $\mathbb{R}^3$ as

$$\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{k}.$$

Evaluate the flux integral,

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS.$$