Question 1. Find the Taylor series expansion around $x = 0$ for the following function and determine its radius of convergence:

$$f(x) = \frac{1}{1 + 2x^2}.$$

Write the first five non-zero terms of the series.

Question 2. Use the method of Lagrange multipliers to find the extrema of

$$f(x, y, z) = x + 2y + 3z$$

subject to the constraint $G = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 14\}$. Calculate the maximum and minimum values of f on G.

Question 3. Consider the following parametric curves:

$$\begin{aligned}
C_1 :&\quad \mathbf{r}_1(t) = (t, t^3, t^2), \quad t \in [0, 2] \\
C_2 :&\quad \mathbf{r}_2(t) = (\sin t, \cos t, t), \quad t \in [0, \pi] \\
C_3 :&\quad \mathbf{r}_3(t) = (e^{-t}, e^t, t), \quad t \in [0, 1] \\
C_4 :&\quad \mathbf{r}_4(t) = (t^2, t, 0), \quad t \in [0, 2]
\end{aligned}$$

1. Sketch each curve.
2. Calculate the arc length of each curve.

Hint: $\int \sqrt{1\frac{1}{4} + u^2}\, du = \frac{1}{4}u\sqrt{4u^2 + 1} - \frac{1}{8}\ln\left(\sqrt{4u^2 + 1} - 2u\right) + C.$

Question 4. Let D be the region bounded by the curves $y = x^2$, $y = 3x^2$, $x = 1$, and $x = 2$. Evaluate the double integral

$$\iint_D (x^2 + y^2)\, dx\, dy.$$

Sketch the region D and set up the integral in the appropriate order.

Question 5. Consider the vector field

$$\mathbf{F}(x, y) = (2x + y^2)\mathbf{i} + (x^2 + 3y)\mathbf{j}.$$

Let C be the closed curve formed by the line segments connecting the points $(0, 0)$, $(2, 0)$, $(2, 1)$, and $(0, 1)$ in that order.

1. Verify whether $\mathbf{F}$ is conservative.
2. Calculate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ using Green's theorem.

Question 6. Consider the surface S defined by $z = x^2 + y^2$ for $x^2 + y^2 \leq 4$ and the vector field

$$\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}.$$

Calculate the flux integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

where $\mathbf{n}$ is the unit normal vector pointing upward. *Hint: Parametrize the surface appropriately and compute the fundamental vector product.*