Question 1. For each of the following functions, find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ (or $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$, etc.).

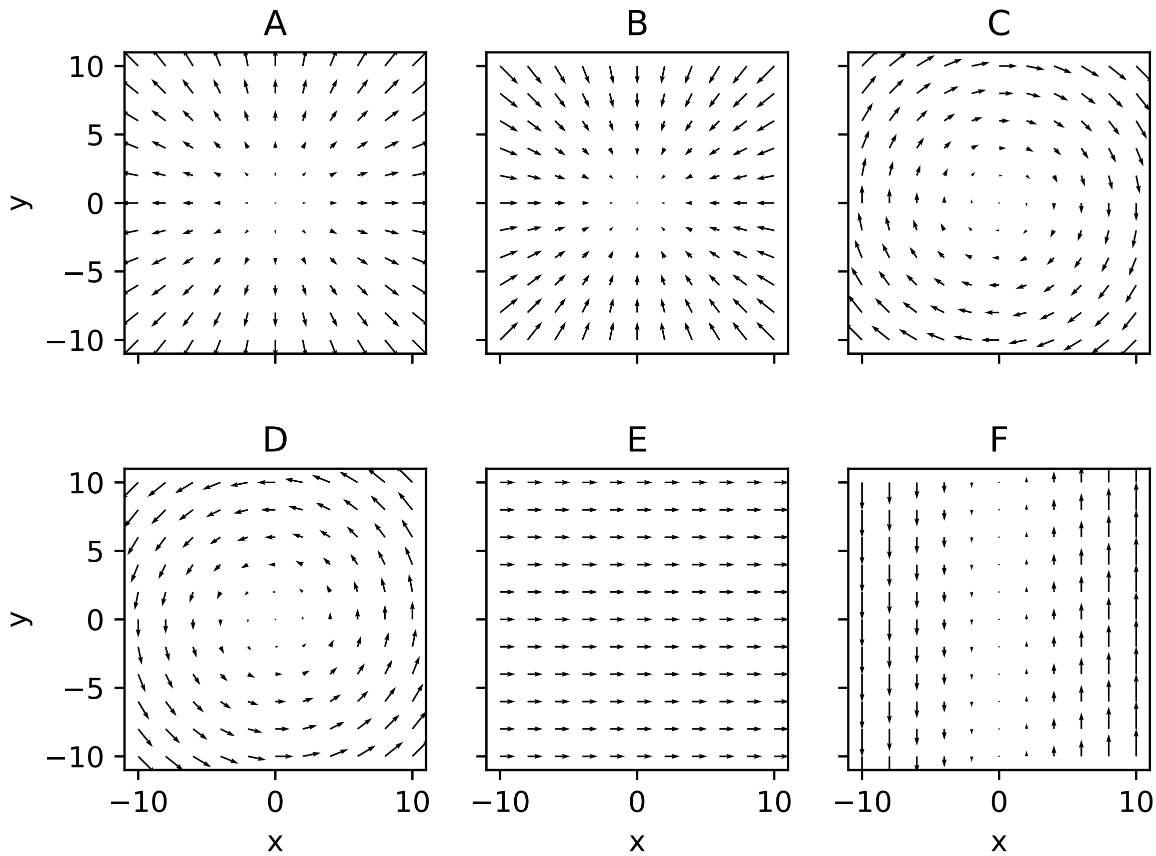
1. $f(x, y) = e^{x^2+y^2} \ln(xy)$
2. $g(x, y) = (x^2 + y^2)^{3/2}$
3. $h(x, y) = \int_0^{xy} e^{-t^2} dt$

Question 2. Find the extrema of $f(x, y, z) = x + 2y + 3z$ subject to the constraints $x^2 + y^2 = 1$ and $x + z = 2$. *Hint: Use the method of Lagrange multipliers.*

Question 3. Consider the following vector fields defined on $\mathbb{R}^2$:

$$\begin{aligned}
\mathbf{F}_1(x, y) &= x\mathbf{i} + y\mathbf{j} \\
\mathbf{F}_2(x, y) &= -x\mathbf{i} - y\mathbf{j} \\
\mathbf{F}_3(x, y) &= y\mathbf{i} - x\mathbf{j} \\
\mathbf{F}_4(x, y) &= -y\mathbf{i} + x\mathbf{j} \\
\mathbf{F}_5(x, y) &= \mathbf{i} \\
\mathbf{F}_6(x, y) &= x\mathbf{j}
\end{aligned}$$

1. Match each to one of the plots and briefly explain the logic/calculation for matching each (it is not required to give complete arguments).
2. Calculate the divergence, $\nabla \cdot \mathbf{F}_n(x, y)$ for each of the vector fields.



Question 4. Calculate the path integral $\int_C \mathbf{F} \cdot d\boldsymbol{\alpha}$ where $\mathbf{F}(x, y) = xy\mathbf{i} + x^2\mathbf{j}$ and C is the path from $(0, 0)$ to $(1, 1)$ along the curve $y = x^2$.

1. Parametrize the curve C
2. Express $\mathbf{F}$ and $d\boldsymbol{\alpha}$ in terms of the parameter
3. Set up and evaluate the path integral

Question 5. Evaluate the double integral using polar coordinates:

$$\iint_D \sqrt{x^2 + y^2}\, dA$$

where D is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Question 6. Consider the cone

$$V = \{(x, y, z) : x^2 + y^2 \leq 4, 0 \leq z \leq 2 - \sqrt{x^2 + y^2}\}$$

and the cylinder

$$W = \{(x, y, z) : (x - 1)^2 + y^2 \leq 1\}.$$

Let $D \subset \mathbb{R}^3$ be the subset of the cone V which is contained within the cylinder W. Calculate the volume of D.