Question 1. For each of the following functions, find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ (or $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$, etc.).

1. $f(x, y) = e^{xy} + \ln(x^2 + y^2)$
2. $g(x, y) = \int_x^y e^{t^2}\, dt - x^2 y$
3. $h(x, y) = \frac{x^2 y}{x^2+y^2} + \sin(xy)$

Question 2. Assume that the coefficients a_k are such that $\sum_{k=0}^{\infty} a_k x^k$ converges when $x = -2$ and diverges when $x = 4$.

For each of the following series, determine whether it converges or diverges:

1. $\sum_{k=0}^{\infty} a_k 3^k$
2. $\sum_{k=0}^{\infty} a_k$
3. $\sum_{k=0}^{\infty} a_k (-2)^k$
4. $\sum_{k=0}^{\infty} a_k (-5)^k$
5. $\sum_{k=0}^{\infty} a_k \left(\frac{1}{2}\right)^k$

In each case, determine if the series (A) Converges; (B) Diverges; (C) Cannot be determined from the given information. Explain your reasoning.

Question 3. Find the extrema of $f(x, y, z) = x + 2y + 3z$ constrained to the intersection of the two cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 2$.

Question 4. Evaluate the double integral

$$\iint_R \frac{x-y}{x+y}\, dA$$

where R is the region bounded by the lines $y = x$, $y = 3x$, $x + y = 2$, and $x + y = 6$.
Use the change of coordinates $u = \frac{y}{x}$ and $v = x + y$ in order to simplify the problem:

1. Sketch the region R
2. Determine the bounds of integration in the uv-plane
3. Find the inverse transformation to express x and y in terms of u and v
4. Compute the Jacobian
5. Transform the integrand $\frac{x-y}{x+y}$ using the new variables
6. Set up and evaluate the transformed integral

Question 5. Consider the vector field

$$\mathbf{F}(x, y) = (x^2 + 2y)\mathbf{i} + (2x^3 + y^2)\mathbf{j}.$$

Let C be the closed curve formed by the line segments connecting the points $(0, 0)$, $(1, 0)$, and $(0, 1)$ in that order.

1. Verify whether $\mathbf{F}$ is conservative.
2. Calculate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ using Green's theorem.

Question 6. Calculate the flux of the vector field

$$\mathbf{F}(x, y, z) = (x^2 + y)\mathbf{i} + (z - x)\mathbf{j} + (2y + z)\mathbf{k}$$

through the surface S defined by the paraboloid $z = 4 - x^2 - y^2$ for $z \geq 0$, oriented such that the normal vectors have positive z-component. Use the following steps:

1. Parametrize the surface S
2. Find the fundamental vector product associated to this parametric surface and verify its orientation
3. Set up the flux integral as a double integral over the parameter domain
4. Evaluate the integral