Question 1

Not yet answered

Marked out of 3

Consider a thin triangular plate with vertices $(0,0)$, $(1,0)$, $(0,2)$ and with density $f(x,y) = 1 + 3x + y$. The mass is: $\boxed{\text{a}}/3$ and the centre of mass is: $(\boxed{\text{b}}/8, \boxed{\text{c}}/16)$. The missing values are $\boxed{\text{a}}$: ____, $\boxed{\text{b}}$: ____, $\boxed{\text{c}}$: ____.

Question 2

Not yet answered

Marked out of 1

A thin semicircular plate, $\{x^2 + y^2 \leq 1, y \geq 0\}$, has density proportional to the distance from the origin. The centre of mass is: $(0, \frac{\boxed{\text{a}}}{2\pi})$.

The missing value is $\boxed{\text{a}}$: ____.

Question 3

Not yet answered

Marked out of 1

Let A be the set bounded by $y = x^2$, $y = 2x^2$, $x = y^2$, $x = 3y^2$. Using the coordinate change determined by $u = \frac{y}{x^2}$, $v = \frac{x}{y^2}$, evaluate the multiple integral

$$\iint_A \frac{1}{x^2y^2}\ dxdy = \frac{2}{\boxed{\text{a}}}.$$

The missing value is $\boxed{\text{a}}$: ____.

Question 4

Not yet answered

Marked out of 1

Let $B = \{(x,y) : y \geq 0, 1 \leq x^2 + y^2 \leq 4\}$. Using polar coordinates, evaluate the multiple integral

$$\iint_B (3x + 4y^2)\ dxdy = \frac{\boxed{\text{a}}}{2}\pi.$$

The missing value is $\boxed{\text{a}}$: ____.

Question **5**

Not yet answered

Marked out of 4

Determine the value of $\boxed{\text{a}}$: ______ so that the vector field

$$\mathbf{F}(x,y,z) = (x^2+5\boxed{\text{a}}y+3yz)\ \mathbf{i}+(5x+3\boxed{\text{a}}xz-2)\ \mathbf{j}+((2+\boxed{\text{a}})xy-4z)\ \mathbf{k}.$$

is conservative and construct a potential φ such that $\mathbf{F} = \nabla\varphi$.

$$\varphi(x,y,z) = \frac{x^{\boxed{\text{b}}}}{3} + 3xyz + 5x\boxed{\text{c}} - 2\boxed{\text{d}}^2 - 2y.$$

The missing values/symbols are $\boxed{\text{b}}$: ______, $\boxed{\text{c}}$: ______, $\boxed{\text{d}}$: ______.

Question **6**

Not yet answered

Marked out of 2

Let $A = \{(x,y) : y \in [0,1], y \le x \le e^y\}$. Evaluate the multiple integral

$$\iint_A \sqrt{x}\ dxdy = \frac{\boxed{\text{a}}}{9}e^{3/2} - \frac{\boxed{\text{b}}}{45}.$$

The missing values are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______.

Question **7**

Not yet answered

Marked out of 2

Let B be the subset of $\mathbf{R}^2$ bounded by $y=0$, $y=x^2$, $x=1$. Evaluate the multiple integral

$$\iint_B x\cos y\ dxdy = \frac{1}{\boxed{\text{a}}} - \frac{\cos(\boxed{\text{b}})}{2}.$$

The missing values are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______.

Question **8**

Not yet answered

Marked out of 2

Let C be the triangle with vertices $(0,0)$, $(2,4)$, $(6,0)$. Evaluate the multiple integral

$$\iint_C ye^x\ dxdy = e^{\boxed{\text{a}}} - 9e^2 - \boxed{\text{b}}.$$

The missing values are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______.

Question **9**

Not yet answered

Marked out of 3

Swapping the order of integration,

$$\int_0^1 \left[\int_0^x f(x,y)\ dy\right]\ dx = \int_{\boxed{\text{a}}}^{\boxed{\text{b}}} \left[\int_{\boxed{\text{c}}}^1 f(x,y)\ dx\right]\ dy$$

where the missing symbols are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______, $\boxed{\text{c}}$: ______.

Question 10
Not yet answered
Marked out of 3

Swapping the order of integration,

$$\int_0^4 \left[\int_{y/2}^2 f(x,y)\ dx\right]\ dy = \int_{\boxed{a}}^{\boxed{b}} \left[\int_{\boxed{c}}^{2x} f(x,y)\ dy\right]\ dx$$

where the missing symbols are $\boxed{a}$: ____, $\boxed{b}$: ____, $\boxed{c}$: ____.

Question 11
Not yet answered
Marked out of 1

Evaluate the multiple integral

$$\int_0^1 \int_{\pi y}^{\pi} \frac{\sin x}{x}\ dxdy = \frac{\boxed{a}}{\pi}.$$

The missing value is $\boxed{a}$: ____.

Hint: the integrand is not integrable in elementary functions in x but changing the order of integration improves the situation.

Question 12
Not yet answered
Marked out of 3

Consider the parametric surface

$$\sigma(u,v) = uv\ \mathbf{i} + (1+3u)\ \mathbf{j} + (v^3+2u)\ \mathbf{k}.$$

The fundamental vector product associated to this parametric surface is

$$N(u,v) = \boxed{a}v^2\ \mathbf{i} + (\boxed{b}u - 3v^3)\ \mathbf{j} - \boxed{c}u\ \mathbf{k}.$$

The missing values are $\boxed{a}$: ____, $\boxed{b}$: ____, $\boxed{c}$: ____.

Question 13
Not yet answered
Marked out of 3

Consider the parametric surface

$$\sigma(u,v) = u\ \mathbf{i} + v\ \mathbf{j} + (u^2+3uv+v^2)\ \mathbf{k}.$$

The fundamental vector product associated to this parametric surface is

$$N(u,v) = -(\boxed{a}u + 3v)\ \mathbf{i} - (\boxed{b}u + 2v)\ \mathbf{j} + \boxed{c}\ \mathbf{k}.$$

The missing values are $\boxed{a}$: ____, $\boxed{b}$: ____, $\boxed{c}$: ____.

Question 14
Not yet answered
Marked out of 1

Let $A = [-1,1]\times[0,1]\times[0,2]$. Evaluate the triple integral

$$\iiint_A (xy - z^3)\ dxdydz = \boxed{a}.$$

The missing value is $\boxed{a}$: ____.

Question 15

Not yet answered

Marked out of 1

Let $B \subset \mathbb{R}^3$ be the set bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y = 1$, $y + z = 1$. Evaluate the triple integral

$$\iiint_B y \; dxdydz = \frac{1}{\boxed{\text{a}}}.$$

The missing value is $\boxed{\text{a}}$: .

Question 16

Not yet answered

Marked out of 1

Let $A \subset \mathbb{R}^3$ be the set bounded by the cylinder $x^2 + y^2 = 25$ and the planes $z = -1$, $z = 2$. Evaluate the triple integral

$$\iiint_A \sqrt{x^2 + y^2} \; dxdydz = \boxed{\text{a}}\pi.$$

The missing value is $\boxed{\text{a}}$: .

Hint: use cylindrical coordinates.

Question 17

Not yet answered

Marked out of 1

Let $B \subset \mathbb{R}^3$ be the subset of the octant $x, y, z \geq 0$ which is bounded by the spheres $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 4$. Evaluate the triple integral

$$\iiint_B x \; dxdydz = \frac{\boxed{\text{a}}}{16}\pi.$$

The missing value is $\boxed{\text{a}}$: .

Hint: use spherical coordinates.

Question 18

Not yet answered

Marked out of 10

Let $C = \{(x, y) : x, y \geq 0, x^2 + y^2 \leq 4, x^2 + y^2 - 2y \geq 0\}$. Using polar coordinates, evaluate the multiple integral

$$\iint_C x \; dxdy.$$

Give a fully justified solution. Hint: the answer is 2.

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Question 19

Not yet answered

Marked out of 10

The following three characterizations of conservative vector fields are equivalent:

- $F = \nabla\varphi$ for some scalar field φ,
- Path integrals of F do not depend on the path, only on the end points,
- Path integrals around closed paths are equal to zero.

Write the proof that these statements are equivalent (you may cite the two fundamental theorems of calculus of path integrals when required). Moreover, calculate that for any scalar field φ, $\nabla \times (\nabla\varphi) \equiv \mathbf{0}$.

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Question 20

Not yet answered

Marked out of 10

Prove that the vector path integral is independent of the choice of parametrization of the path.

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