Question 1

Not yet answered

Marked out of 3

Determine the outgoing flux of the vector field $\mathbf{F}(x, y) = y\ \mathbf{i} + x\ \mathbf{j} + z\ \mathbf{k}$ through S, the sphere centred in the origin of radius r. I.e., evaluate the flux integral $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\ dS = \frac{\boxed{\mathrm{a}}}{3}\pi\boxed{\mathrm{b}}^{\boxed{\mathrm{c}}}$ where $\hat{\mathbf{n}}$ denotes the outgoing normal vector of the surface S. The missing values/symbols are $\boxed{\mathrm{a}}$: ____, $\boxed{\mathrm{b}}$: ____, $\boxed{\mathrm{c}}$: ____.

Question 2

Not yet answered

Marked out of 1

Let S denote the portion of $x^2 + y^2 = 9 - z$ such that $0 \leq z \leq 8$. Consider the vector field $\mathbf{F}(x, y, z) = 2\ \mathbf{i} - 5\ \mathbf{j} + 3\ \mathbf{k}$. Evaluate the flux integral $\int_S \mathbf{F} \cdot \hat{\mathbf{n}}\ dS =$ ____ π. (Here $\hat{\mathbf{n}}$ denotes the unit normal pointing away from the z-axis.)

Hint: consider the solid $\{(x, y, z) : x^2 + y^2 \leq 9 - z, 0 \leq z \leq 8\}$ and use the divergence theorem.

Question 3

Not yet answered

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Let $V \subset \mathbb{R}^3$ be the set of (x, y, z) such that $x^2 + y^2 + z^2 \leq 25$ and $z \geq 3$. This solid is bounded by a closed surface S which is composed of two parts: Let S_1 denote the curved top part and let S_2 denote the planar part. Consider the vector field $\mathbf{F}(x, y, z) = xz\ \mathbf{i} + yz\ \mathbf{j} + \mathbf{k}$.

- Evaluate the surface integral $\iint_{S_2} \mathbf{F} \cdot \mathbf{n}\ dS =$ ____ π.
- Evaluate also the integral $\iiint_V \nabla \cdot \mathbf{F}\ dxdydz =$ ____ π.
- Using the divergence theorem (Gauss) $\iint_{S_1} \mathbf{F} \cdot \mathbf{n}\ dS =$ ____ π.

Question 4

Not yet answered

Marked out of 3

Consider the vector field $\mathbf{F}(x, y) = x^2y^2\ \mathbf{i} + ax\ \mathbf{j}$ where a is some parameter. Let Γ denote the closed curve formed by the portions of $x^2 + y^2 = 1$, $x = 1$, $y = x^2 + 1$ ($x, y \geq 0$). Let γ denote the path tracing out this curve in a counter clockwise direction.

Using Green's Theorem, compute the path integral. $\int_\Gamma \mathbf{F} \cdot d\gamma = a\left(\frac{\boxed{\mathrm{a}}}{3} - \frac{\pi}{\boxed{\mathrm{b}}}\right) - \frac{\boxed{\mathrm{c}}}{35}$. The missing values are $\boxed{\mathrm{a}}$: ____, $\boxed{\mathrm{b}}$: ____, $\boxed{\mathrm{c}}$: ____.

Hint: $\int_0^1 \sqrt{(1 - x^2)}\ dx = \frac{\pi}{4}$.

Question 5
Not yet answered
Marked out of 4

Let S denote the portion of the plane $7x+3y+4z=15$ that lies in the 1st octant ($x \geq 0, y \geq 0, z \geq 0$). A parametric representation of this surface is $S=\sigma(T)$ where $\sigma(u,v)=\left(u, v, \frac{\boxed{\text{a}}}{4}-\frac{\boxed{\text{b}}}{4}x-\frac{3}{4}y\right)$, $T=\{0 \leq x \leq \frac{15}{\boxed{\text{c}}}, 0 \leq y \leq 5-\frac{7}{3}\boxed{\text{d}}\}$.

The missing values/symbols are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______, $\boxed{\text{c}}$: ______, $\boxed{\text{d}}$: ______.

Question 6
Not yet answered
Marked out of 2

Evaluate the multiple integral, $\int_0^1 \left[\int_0^x \sqrt{x^2+y^2}\, dy\right] dx = \frac{1}{\boxed{\text{a}}}\left(\sqrt{2}+\ln(\boxed{\text{b}}+\sqrt{2})\right)$. The missing values are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______.

Hint: change to polar coordinates and use the integral $\int \sec^3 x\, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$.

Question 7
Not yet answered
Marked out of 5

Consider the vector field defined for $(x,y) \neq (0,0)$ as, $\mathbf{F}(x,y)=\frac{1}{x^2+y^2}(-y\ \mathbf{i}+x\ \mathbf{j})$.
Calculate the path integrals:

1. Let γ_1 denote the path anticlockwise around the circle of radius 1 centred at the origin, $\int \mathbf{F} \cdot d\gamma_1 =$ ______ π,
2. Let γ_2 denote the path anticlockwise around the circle of radius 2 centred at the origin, $\int \mathbf{F} \cdot d\gamma_2 =$ ______ π,
3. Let γ_3 denote the path anticlockwise around the square of side length 2 centred at the origin, $\int \mathbf{F} \cdot d\gamma_3 =$ ______ π,
4. Let γ_4 denote the path anticlockwise around the circle of radius 1 centred at $(1,0)$, $\int \mathbf{F} \cdot d\gamma_4 =$ ______ π,
5. Let γ_5 denote the path clockwise around the circle of radius 1 centred at the origin. $\int \mathbf{F} \cdot d\gamma_5 =$ ______ π.

Hint: calculate $\frac{\partial \mathbf{F}_2}{\partial x}-\frac{\partial \mathbf{F}_1}{\partial y}$.

Question 8
Not yet answered
Marked out of 3

Let S denote the portion of the surface $z=\frac{1}{2}y^2$ which satisfies $y \geq 0, y-x \leq 4, x+y \leq 4$. Compute the area of S.
$\text{Area}(S)=\frac{\boxed{\text{a}}}{3}\sqrt{\boxed{\text{b}}}+4\ln(\boxed{\text{c}}+\sqrt{17})+\frac{2}{3}$. The missing values are $\boxed{\text{a}}$: ______, $\boxed{\text{b}}$: ______, $\boxed{\text{c}}$: ______.

Question **9**

Not yet answered

Marked out of 1

Consider the parametric surface $S = \sigma(T)$ where $\sigma : (u,v) \to (u,v,uv)$ and $T = [0,1] \times [0,1]$. Evaluate the surface integral of the scalar field $f(x,y,z) = z(1+x^2+y^2)^{-1/2}$, $\iint_S f \; dS = \frac{1}{\boxed{\text{a}}}$. The missing value is $\boxed{\text{a}}$: ____.

Question **10**

Not yet answered

Marked out of 4

Consider the parametric surface $\sigma(u,v) = ((R + r\cos u)\sin v, (R + r\cos u)\cos v, r\sin u)$.

This has the Cartesian equation $(\sqrt{x^2+y^2} - R)^2 + z^2 = \boxed{\text{a}}^{\boxed{\text{b}}}$.

The major radius R is the distance from the center of the tube to the center of the torus and the minor radius r is the radius of the tube.

The fundamental vector product is $\frac{\partial \sigma}{\partial u} \times \frac{\partial \sigma}{\partial v} = r(\boxed{\text{c}} + r\cos u)\begin{pmatrix} \cos u \sin v \\ \cos u \cos v \\ \sin u \end{pmatrix}$. The norm of the above is,

$\left\| \left(\frac{\partial \sigma}{\partial u} \times \frac{\partial \sigma}{\partial v} \right)(u,v) \right\| = r(\boxed{\text{c}} + r\cos u)\boxed{\text{d}}$. The missing values/symbols are $\boxed{\text{a}}$: ____, $\boxed{\text{b}}$: ____, $\boxed{\text{c}}$: ____, $\boxed{\text{d}}$: ____.

Question **11**

Not yet answered

Marked out of 1

The torus $(\sqrt{x^2+y^2} - 2)^2 + z^2 = 1$ has surface area equal to ____ π^2.

Question **12**

Not yet answered

Marked out of 10

By calculating, prove the following identities (f, g are scalar fields, F, G are vector fields):

1. $\nabla \cdot (\nabla \times F) = 0$,
2. $\nabla(fg) = f\ \nabla g + g\ \nabla f$,
3. $\nabla(F \times G) = G \cdot (\nabla \times F) - F \cdot (\nabla \times G)$.

Recall that we already showed that $\nabla \times (\nabla f) = 0$ in Problem set 3, the first part above is a similar pattern.

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Question 13

Not yet answered

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Let the surface S be the portion of the sphere of radius 6 satisfying $x \leq 0, y \geq 0, z \geq 0$. Consider the vector field $\mathbf{F}(x, y, z) = \mathbf{i} + z\ \mathbf{j} + 6x\ \mathbf{k}$. and evaluate the flux integral $\iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\ dS$ where $\hat{\mathbf{n}}$ denotes the unit normal to the surface which points toward the origin.

One may assume facts about spherical coordinates without justification but the rest of the answer must have full details. Hint: the value of the integral is $9/4\pi + 45$.

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Question 14

Not yet answered

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Write the argument which proves that surface integrals are independent on choice of parametrization of the surface.

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