

**Question 1**

Not yet answered

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Consider the following sets:

- $A = \{(x, y) \in \mathbb{R}^2 : x \neq y^2\},$
- $B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 > 9\},$
- $C = (-3, -2) \cup (-2, 3),$
- $D = \{(x, y) \in \mathbb{R}^2 : 0 < x^2 + y^2 < 1\}.$

For each function choose the set which corresponds to the domain of that function.

$$f(x, y) = \frac{x-3y+7}{x-y^2}$$

Choose...

$$f(x, y) = (\log(1 - x^2 - y^2), x, (x^2 + y^2)^{-1})$$

Choose...

$$f(x) = ((x - 2)/(x + 2), \log(9 = x^2))$$

Choose...

$$f(x, y, z) = \log(x^2 + y^2 + z^2 - 3^2)$$

Choose...

**Question 2**

Not yet answered

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Calculate the partial derivatives of the function

$$f(x, y) = \int_x^y \cos t^2 \, dt.$$

Choose the correct answers from the following:

- a.  $\cos(x)$
- b.  $\cos(y^2)$
- c.  $-\cos(x^2)$
- d.  $\sin(x)$
- e.  $\sin(y^2)$
- f.  $-\sin(x^2)$

$$\frac{\partial f}{\partial x}(x, y) = ?$$

Choose...

$$\frac{\partial f}{\partial y}(x, y) = ?$$

Choose...

**Question 3**

Not yet answered

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Assume that the coefficients  $a_k$  are such that  $\sum_{k=0}^{\infty} a_k x^k$  converges when  $x = -2$  and diverges when  $x = 3$ .

$$\sum_{k=0}^{\infty} a_k 7^k$$

Choose...

$$\sum_{k=0}^{\infty} a_k$$

Choose...

$$\sum_{k=0}^{\infty} a_k (-4)^k$$

Choose...

$$\sum_{k=0}^{\infty} a_k (-3)^k$$

Choose...

## Question 4

Not yet answered

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Consider the sequence of maps,

$$f_n(x) = \arctan(nx), \quad x \in \mathbb{R}.$$

1. The sequence converges pointwise on  $\mathbb{R}$ ?
2. The sequence converges pointwise on  $[1, \infty)$ ?
3. The sequence converges uniformly on  $\mathbb{R}$ ?
4. The sequence converges uniformly on  $[1, \infty)$ ?
5. For each  $n$ , the function  $f_n$  is continuous on  $\mathbb{R}$ ?
6. The limit function is continuous on  $\mathbb{R}$ ?
7. The limit function is constant on  $(-1, 1)$ ?
8. The limit function is constant on  $(0, \infty)$ ?
9. The limit function is constant on  $(-\infty, 0)$ ?
10. The sequence converges pointwise to an unbounded function?

## Question 5

Not yet answered

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Calculate the gradient of the function

$$f(x, y) = \arctan \frac{x+y}{x-y}.$$

$$\nabla f(x, y) = \left( -\frac{\boxed{a}}{x\boxed{c}+y\boxed{c}}, \frac{\boxed{b}}{x\boxed{c}+y\boxed{c}} \right)$$

The missing symbols are:  $\boxed{a}$ : ,  $\boxed{b}$ : ,  $\boxed{c}$ : .

## Question 6

Not yet answered

Marked out of 4

Let

$$f(s, t) = \sqrt{s+t},$$

$$g(x, y) = xy\mathbf{i} + \frac{x}{y}\mathbf{j}.$$

Compute the gradient of the composition of the two functions:

$$\nabla(f \circ g)(x, y) = \left( \frac{1}{\boxed{a}} \sqrt{\frac{\boxed{b}}{xy^2+x}} \left( y + \frac{1}{y} \right), \frac{1}{\boxed{a}} \sqrt{\frac{\boxed{b}}{xy^2+x}} \left( \boxed{c} - \frac{\boxed{d}}{y^2} \right) \right)$$

The missing symbols are:  $\boxed{a}$ : ,  $\boxed{b}$ : ,  $\boxed{c}$ : ,  $\boxed{d}$ : .

## Question 7

Not yet answered

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Write as a Maclaurin series the following indefinite integral:

$$\int \sin(x^2) \, dx.$$

The Maclaurin series of the integral is:

$$C + \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k + \boxed{\text{b}})(2k+1)!} x^{\boxed{\text{a}}k+3}$$

where the missing coefficients are  $\boxed{\text{a}}$ : ,  $\boxed{\text{b}}$ : .

## Question 8

Not yet answered

Marked out of 4

The Jacobian matrix of the function

$$f(x, y) = e^{2x+y} \mathbf{i} + \cos(x + 2y) \mathbf{j}$$

is

$$\mathbf{J}f(x, y) = \begin{pmatrix} \boxed{\text{a}} \exp(2x + y) & \boxed{\text{b}} \exp(\boxed{\text{c}}x + y) \\ -\sin(x + 2y) & \boxed{\text{d}} \sin(x + 2y) \end{pmatrix}$$

where the missing coefficients are  $\boxed{\text{a}}$ : ,  $\boxed{\text{b}}$ : ,  $\boxed{\text{c}}$ : ,  $\boxed{\text{d}}$ : .

## Question 9

Not yet answered

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Find the Maclaurin series (Taylor series around the point  $x_0 = 0$ ) of the following function and compute the radius of convergence:

$$f(x) = \log \frac{1+x}{1-x}.$$

The Maclaurin series is:

$$\sum_{k=0}^{\infty} \frac{\boxed{\text{a}}}{2k + \boxed{\text{b}}} x^{2k+1}$$

where the missing coefficients are  $\boxed{\text{a}}$ : ,  $\boxed{\text{b}}$ : . The radius of convergence is  $R =$  . Hint: use the expansion of  $\log(1+t)$ .

## Question 10

Not yet answered

Marked out of 2

The partial derivatives of the function

$$f(x, y) = \sqrt{3x + y^2},$$

at the point  $(x_0, y_0) = (1, 2)$  are

$$\frac{\partial f}{\partial x}(1, 2) = \frac{\boxed{\text{b}}}{2\sqrt{7}}, \quad \frac{\partial f}{\partial y}(1, 2) = \frac{2}{\sqrt{\boxed{\text{a}}}},$$

where the missing coefficients are  $\boxed{\text{a}}$ : ,  $\boxed{\text{b}}$ : .

**Question 11**

Not yet answered

Marked out of 2

The partial derivatives of the function

$$g(x, y, z) = ye^{x+yz},$$

at the point  $(x_0, y_0, z_0) = (0, 1, -1)$  are

$$\frac{\partial g}{\partial x}(0, 1, -1) = e^{\boxed{\text{a}}}, \quad \frac{\partial g}{\partial x}(0, 1, -1) = \boxed{\text{b}},$$

$$\frac{\partial g}{\partial x}(0, 1, -1) = e^{\boxed{\text{a}}}$$

where the missing coefficients are  $\boxed{\text{a}}$ : ,  $\boxed{\text{b}}$ : .

**Question 12**

Not yet answered

Marked out of 5

Consider the power series:

$$\sum_{k=2}^{\infty} (-1)^k \frac{x^k}{3^k \log k}.$$

The radius of convergence is  $R =$   and the set of convergence is the interval  $I =$   ,  .

**Question 13**

Not yet answered

Marked out of 5

Consider the power series:

$$\sum_{k=0}^{\infty} k^2 (x - 4)^k.$$

The radius of convergence is  $R =$   and the set of convergence is the interval  $I =$   ,  .

**Question 14**

Not yet answered

Marked out of 3

Identify and classify the stationary points of the function

$$f(x, y) = x^2y + x^2 - 2y.$$

There are  local minima,  local maxima and  saddle point(s).

**Question 15**

Not yet answered

Marked out of 3

Identify and classify the stationary points of the function

$$f(x, y) = xye^{-x/5-y/6}.$$

There are  local minima,  local maxima and  saddle point(s).

## Question 16

Not yet answered

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Find the Taylor series of the function  $f(x) = \frac{1}{x}$  about the point  $x_0 = 1$  and determine the radius of convergence of the series.

The Taylor series is:

$$\sum_{k=0}^{\infty} (\boxed{a})^k (x-1)^{\boxed{b}},$$

where the missing coefficients is  $\boxed{a}$ :  and the missing symbol is  $\boxed{b}$ : . The radius of convergence is  $R =$  .

## Question 17

Not yet answered

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Let  $p$  be a positive integer. Find, as a function of  $p$ , the radius of convergence of

$$\sum_{k=0}^{\infty} \frac{(k!)^p}{(pk)!} x^k.$$

Select one:

- ☐ a. 0
- ☐ b.  $\infty$
- ☐ c. 1
- ☐ d.  $1/p$
- ☐ e.  $p!$
- ☐ f.  $p^p$

## Question 18

Not yet answered

Marked out of 10


Consider the sequence of functions

$$f_n(x) = \frac{nx}{1+n^3x^3},$$




defined for  $x \in [0, \infty)$ .


- Sketch the functions for a couple of choices of  $n$ ,
- Determine the limit function,
- Determine the sets of pointwise and uniform convergence.

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## Question 19


Not yet answered

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Sketch the following sets and determine the interior, the closure and the boundary. Identify if the sets are open, closed, connected, convex or bounded.


- $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 < y < 1\}$ .
- $B = (\{x^2 + y^2 \leq 4\} \setminus ([-1, 1] \times \{0\})) \cup ((-1, 1) \times \{3\})$ .
- $C = \{(x, y) \in \mathbb{R}^2 : |y| > 2\}$ .

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## Question 20

Not yet answered


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Determine the Taylor polynomial of order two for the function




$$f(x, y) = \cos x \cos y$$


at the point  $(x_0, y_0) = (0, 0)$ .

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