Optimal FX Interventions with Limited Reserves*

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Preliminary Draft – Click here for the latest version

June 27, 2025

Abstract

We investigate the optimal time-consistent use of foreign exchange interventions (FXI) in a small open economy model driven by endowment and portfolio flow shocks. The model features endogenous FX market depth, which reflects limited risk-bearing capacity of FX traders, and optimal FXI policy is subject to a lower bound on FX reserves. In a competitive equilibrium, large capital flows increase conditional exchange rate volatility and make FX markets more shallow. Unlike in the unconstrained case, the central bank's optimal interventions are not solely targeted at offsetting fluctuations in the demand for currency but also incorporate a forward-looking element due to the risk of depleting reserves. We show that this consideration leads to optimal time-consistent FXI policy that responds less (more) than one-for-one to large (small) capital outflows. The policy delivers sizable welfare gains, exceeding those from simple FXI rules. Yet, these gains depend on sufficiently high initial FX reserves. When FX reserves are low, time inconsistency matters and commitment becomes more valuable.

JEL Codes: E44, F31, F32, F42

Keywords: Capital Flows, FX Interventions, Lower Bound on FX Reserves, Time-Consistent Policy

*The views expressed herein are those of the authors and should not be attributed to the IMF, its executive board, or its management. The authors would like to thank Rodrigo Alfaro, Sofia Bauducco, Mauricio Calani, Jose Gabriel Carreno, Martin Caruso Bloeck, Lawrence Christiano, Christopher Erceg, Bada Han, Oleg Itskhoki, Jesper Linde, Damian Moreno, Abolfazl Rezghi, as well as the participants of the UBC Macro-International Lunch Seminar, SNB Brown Bag, IMF MCM Policy Forum, seminar at Banco Central de Chile, and the 19th End-of-Year SEA conference for their helpful comments. Oliver Vogt is deeply grateful to his doctoral committee members Michael B. Devereux, Isha Agarwal, Viktoriya Hnatkovska, and Julien Bengui for their continuous support and guidance. He also thanks the Monetary Policy Modeling Unit of the IMF for hospitality during his 2023 summer internship.

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1 Introduction

Volatility in international capital flows is a significant concern for policymakers in small open economies, especially in a more shock prone environment amid rising geopolitical tensions, as noted in a recent speech by the IMF's First Deputy Managing Director Gopinath (2024). In shallow FX markets, such volatility exacerbates exchange rate fluctuations, distorting external financing conditions and impeding international risk-sharing. Foreign exchange interventions (FXI) are a key policy tool that potentially can be employed to mitigate these effects. Recent literature on open economy policy frameworks — such as Basu et al. (2020) and Adrian et al. (2022) — also emphasizes the useful role of FXI in stabilizing external financing conditions. However, when deciding on contemporaneous interventions, policymakers must consider the risk of running down FX reserves to uncomfortably low levels or even depleting them in the future, bringing intertemporal considerations to the forefront. This raises a number of questions about optimal implementation of FXI policies. How aggressively should the interventions offset current portfolio outflows, depending on whether these risks are expected to abate or intensify? Is the effectiveness of FXI state-dependent in such circumstances? What is the average level of FX reserves that should be held when conducting FXI optimally?

We address these and related questions by studying the optimal use of FXI in a model with endogenous FX market depth akin to Itskhoki & Mukhin (2023) and subject to a lower bound on FX reserves. In our model, which is otherwise a standard small open economy framework, FX markets are shallow (i.e., not perfectly elastic) due to the existence of risk-averse agents (dubbed as financiers), who are exposed to FX risk when intermediating cross-border flow of funds. This market structure gives rise to endogenous violations in the uncovered interest rate parity (UIP), with FX market depth depending negatively on conditional exchange rate volatility. As a result, when the exchange rate uncertainty is high, the economy becomes particularly vulnerable to volatile capital flows, including those reflecting exogenous non-fundamental swings in international investors' appetite for domestic currency.

In this environment, the central bank can facilitate international risk sharing and hence improve aggregate allocations by conducting FX interventions. As an illustration, consider an exogenous fall in demand for the domestic currency, which we refer to as a portfolio capital outflow shock. Everything else equal, financiers will accommodate it by selling foreign currency assets and buying domestic currency assets, which expands their net long FX exposure to the domestic currency. Since these agents are risk-averse, they need to be compensated for taking on this additional risk, so that the UIP premium increases and the domestic currency depreciates. In line with the literature that we build on (Gabaix & Maggiori, 2015; Itskhoki & Mukhin, 2021), FXI is effective since it can change the amount of funds intermediated by financiers. In our particular example of a capital outflow, a sterilized FX intervention, i.e., a purchase of domestic currency bonds financed by a sale of foreign currency bonds, lowers the long exposure of financiers to domestic currency, and hence mitigates the movement in the UIP premium.

Indeed, if this intervention policy can be conducted in an unconstrained fashion, it is optimal for the central bank to effectively take on the role of financiers since their intermediation is costly from the social welfare perspective. As a result, optimal FXI fully offsets portfolio flow shocks and eliminates ex-ante UIP deviations, resulting in the best possible allocation achievable in a world where only risk-free bonds can be traded, and which we will refer to as the first best. While the volatility of the exchange rate due to portfolio flow shocks is eliminated, optimal FXI policy still allows this variable to fluctuate, facilitating efficient expenditure switching in response to fundamental shocks.

However, if the level of FX reserves is constrained by an effective lower bound, meaning that the central bank cannot directly borrow in foreign currency, FXI policy may not be able to fully offset large capital outflows. Even if the monetary authority is unconstrained today, it has to take into account that FX reserves it holds may not be sufficient in all future states. One consequence is that FX markets become more shallow in the episodes of portfolio outflows, as the conditional exchange rate volatility is shaped to a larger degree by non-fundamental forces. The optimal FXI policy also becomes more nuanced. When intervening today, the central bank needs to take into account that it might run out of reserves in future periods. In other words, the intertemporal aspect becomes important for the optimal, second-best FXI policy conduct.

The primary contribution of our paper is an analytical and quantitative characterization of the optimal time-consistent FXI policy in a simple theoretical setup sketched out above. The reason we focus on this type of policy is that, as shown by Itskhoki & Mukhin (2023), the optimal plans under commitment become time inconsistent in the presence of an occasionally binding constraint on reserves. In contrast to that paper, which relies on a linear-quadratic approximation of the equilibrium conditions, we solve the exact nonlinear policy problem using a global and fully nonlinear solution algorithm.

This has important consequences for our results. From the positive perspective, our nonlinear model can capture time-variation in conditional exchange rate volatility, which increases (falls) during the periods of depreciation (appreciation). This pattern is consistent with empirical evidence on the behavior of currencies during risk-off episodes (De Bock & de

Carvalho Filho, 2015). On the normative part, our framework allows us to demonstrate that the key motive making the optimal time-consistent policy deviate from perfect stabilization of the UIP risk premium is driven by the desire to facilitate financiers' intermediation in the possibly constrained states in the future, thus relaxing the implicit borrowing limit faced by the economy as a whole. Interestingly, this is achieved by an aggressive, more than one-for-one offset of capital outflows if they are relatively mild and expected to abate. However, the focus shifts to maintaining a precautionary level of reserves when outflows become stronger, and the optimal policy chooses not to offset them completely, even if the current level of FX reserves is sufficient to achieve this goal. This precautionary motive crucially depends on endogenous FX market depth,[1] which the central bank can influence by "keeping the powder dry" and hoarding FX reserves so that they can be used to lean against future capital outflows. In the case of portfolio inflows, since the lower bound on FX reserves is less of a concern, the optimal policy resembles the first best, i.e., portfolio capital inflows are matched by buying FX reserves approximately one-for-one.

In the quantitative part of our analysis, we calibrate the model to Malaysia, a small open economy that actively uses FXI to manage the exchange rate. We back out the exogenous process for portfolio flows by applying the UIP condition from the model to the data. Our quantitative results confirm that the optimal time-consistent FXI reaction is more aggressive to small outflows than to large ones. We additionally find that, if FX interventions are conducted optimally, their effectiveness is state-dependent: FX purchases depreciate the real exchange rate by less than FX sales of the same magnitude. The reason is that FX purchases occur in periods of portfolio inflows, which are associated with relatively deep FX markets, while FX sales happen in episodes of portfolio outflows, when FX markets are endogenously shallower.

According to our model, while the standard deviation of the estimated portfolio flow process amounts to 4% of annual GDP, the average level of FX reserves in the optimal time-consistent policy regime is around 5% of GDP. This ensures that the unconditional probability of depleting FX reserves is limited, amounting to merely 2%. The policy also reduces the volatility of the exchange rate compared with the no-intervention regime, but does not fully stabilize this variable to preserve its expenditure switching role. Related to that, FX markets are also significantly deeper in the optimal policy regime, meaning that the economy is better insulated from exogenous swings in appetite for domestic currency.

Finally, we compute the welfare gains associated with adopting optimal time-consistent FXI

[1] In existing quantitative models (Itskhoki & Mukhin, 2021; Adrian et al., 2022; Davis et al., 2023, and others), FX market depth is a calibrated parameter and therefore assumed to be constant.

policy, assuming that the initial FX reserves correspond to their average value observed in our calibrated economy. The gains turn out to be sizable, amounting to 0.25% of annual consumption, coming fairly close to those associated with the first-best policy, which establishes their upper bound at 0.29%. These gains are also significantly bigger than those implied by a simple static FXI rule, which eliminates the UIP premium whenever it can, otherwise fully depleting the FX reserves. Importantly, much of the welfare benefit under the optimal time-consistent policy reflects transitional dynamics, in particular the gradual reduction of reserves that are initially excessive from the prespective of our model. When steady-state reserve levels are equalized across regimes at a lower optimal level, welfare differences narrow considerably. Since the lower bound on reserves generates time inconsistency, the optimal time-consistent policy may even underperform rule-based regimes.

Related Literature The theoretical foundation that gives rise to the effectiveness of FXI in open economy models like ours is the incorporation of gross asset positions and segmentation in international financial markets. Within a two-country dynamic general equilibrium framework, Devereux & Sutherland (2010) present an approximation method for characterizing time-varying equilibrium portfolios. Gabaix & Maggiori (2015) and Itskhoki & Mukhin (2021) provide microfounded models of a portfolio balance channel in currency markets, even though the very idea dates back to Kouri (1976). Similarly to Cavallino (2019), the role of FXI in our model is to address this friction by stabilizing the UIP deviations, thus smoothing inefficient fluctuations in external financing conditions. This view is consistent with the role of FXI in models with multiple policy tools such as Itskhoki & Mukhin (2023) and the IMF's work on the Intergrated Policy Framework (Basu et al., 2020; Adrian et al., 2022). Other related studies include Davis et al. (2023), Arce et al. (2019), Chang (2018), and Jeanne & Rancière (2011), who examine the macroprudential use of FXI to prevent sudden stops in emerging economies. Fanelli & Straub (2021) stress the forward guidance component of FX interventions and its time-inconsistency, while Babii et al. (2025) study global consequences of using this policy by a group of countries. Amador et al. (2020) and Cwik & Winter (2024) argue that FXI can function as an effective unconventional monetary policy tool when interest rates are at the effective lower bound (ELB) and appreciation pressures exist.

While arguably highly relevant in the policy debate, the constraints on FX reserve holdings, including their effective lower bound, have attracted only limited attention in theoretical literature. One of the very few exceptions is Basu et al. (2018), who examine the optimal use of FXI under limited reserves in a very stylized setup where the policy objective is to stabilize the exchange rate. Related to our work, they show how the lower bound on reserves renders the optimal policy time inconsistent and how simple intervention rules can achieve welfare gains relative to discretionary policies. The other key reference to us is a recent paper by Itskhoki & Mukhin (2023), who use a linear-quadratic framework to examine optimal FXI policy when FX reserve holdings are constrained, finding that it prescribes more aggressive interventions compared to the first best. We show in a fully non-linear framework that this motive is typically dwarfed by precautionary accumulation of reserves, which implies responding less than one-for-one to capital outflows.

The effectiveness of FXI in our model critically depends on FX market depth (i.e., the elasticity of currency demand), the estimation of which has been a key focus in the empirical literature (Chen et al., 2023; Hertrich & Nathan, 2023; Adler et al., 2019; Fratzscher et al., 2019; Blanchard et al., 2015; Fatum & M. Hutchison, 2003, among others). A major limitation for this literature is the availability of non-confidential, high-frequency data on capital flows (including FXI) and severe identification problems. Progress has been made in this regard by Adler et al. (2025), who provide a comprehensive FXI data set on a monthly and quarterly basis, on which we leverage when calibrating our model. Beltran & He (2024), Pandolfi & Williams (2019) and Broner et al. (2021) exploit exogenous changes in portfolio weights of benchmark indices of local-currency sovereign debt to infer the sensitivity of the exchange rate to capital flows. Maggiori (2022) discusses promising directions in this line of research.

Methodologically, our paper connects to the literature that computes decentralized and constrained-efficient equilibria in small open economies with occasionally binding constraints using global methods (Mendoza, 2010; Bianchi, 2011; Bianchi & Mendoza, 2018; Schmitt-Grohé & Uribe, 2016, 2021; Davis et al., 2023, and others). Similarly to Bianchi & Mendoza (2018), we rely on time iteration of the Euler equation and characterize the social planner equilibrium as optimal time-consistent policy.

Outline The remainder of this paper is structured as follows. Section 2 outlines the model. The optimal FXI problem is analyzed in Section 3. Section 4 presents and discusses the quantitative analysis. Finally, Section 5 concludes by summarizing the key findings.

2 Model

This section outlines a standard small open economy model with stochastic endowments, which we extend with a segmented international financial sector that consists of financiers, portfolio investors and the central bank as in Itskhoki & Mukhin (2023). We first describe the decentralized equilibrium in which the central bank does not engage in FXI, before characterizing the constrained-efficient equilibrium in which the central bank follows a discretionary, time-consistent FXI policy that faces a lower bound on FX reserves.

2.1 Outline

Utility Function Consider an economy that is populated by a large number of identical, infinitely-lived households with preferences described by the following utility function

$$\sum_{t=0}^{\infty} \mathbb{E}_0 \left[\beta^t u\left(C_{T,t}, C_{N,t}\right)\right],$$

where

$$u(C_{T,t}, C_{N,t}) = \frac{1}{1-\sigma}\left(\underbrace{\left[\alpha\left(C_{T,t}\right)^{\frac{\xi-1}{\xi}} + (1-\alpha)\left(C_{N,t}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}}_{C_t}\right)^{1-\sigma}.$$

Households derive utility from total consumption C_t that consists of tradable goods $C_{T,t}$ and nontradable goods $C_{N,t}$. Furthermore, $\beta \in (0,1)$ denotes the subjective discount factor, $1/\sigma$ is the intertemporal elasticity of substitution, ξ is the elasticity of substitution between tradable and nontradable goods, and $\alpha \in (0,1)$ controls the share of tradables in the total consumption basket.

Households' Budget Constraint Each period t, households receive stochastic endowments of tradable and nontradable goods, denoted by $Y_{T,t}$ and $Y_{N,t}$, respectively. The endowments are exogenous and follow a first-order Markov process. Furthermore, households have access to a one period local currency bond B_t that pays gross nominal interest rate R_t. A representative household's sequential budget constraint is then given by

$$P_{T,t}C_{T,t} + P_{N,t}C_{N,t} \leq P_{T,t}Y_{T,t} + P_{N,t}Y_{N,t} - B_t + B_{t-1}R_{t-1} + \Pi_{M,t} + \Pi_{F,t} + \Pi_{P,t}$$

where $\Pi_{M,t}$, $\Pi_{F,t}$ and $\Pi_{P,t}$ denote the profits of the central bank, financiers and portfolio investors, respectively. Note that this formulation of the budget constraint implies that the financial sector (i.e. financiers and portfolio investors) is fully domestically owned.

We assume that the law of one price holds for tradable goods and the price level abroad is normalized to unity so that the price of tradables equals the nominal exchange rate $P_{T,t} = \mathcal{E}_t$. As we explain below, the price of nontradables is normalized to unity ($P_{N,t} = 1$), so that $\mathcal{E}_t$ can be also interpreted as the relative price of tradable goods.

Households' Optimality The choice of B_t is characterized by the household Euler equation:

$$R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] = 1, \tag{1}$$

where Θ_{t+1} denotes the stochastic discount factor (SDF) of domestic households

$$\Theta_{t+1} = \beta \frac{u_1(C_{T,t+1}, C_{N,t+1})}{u_1(C_{T,t}, C_{N,t})} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\frac{1-\sigma\xi}{\xi}} \left(\frac{C_{T,t+1}}{C_{T,t}} \right)^{-\frac{1}{\xi}}.$$

In addition, combining the first-order conditions with respect to $C_{T,t}$ and $C_{N,t}$ allows us to obtain the equilibrium expenditure switching condition, which pins down the exchange rate

$$\mathcal{E}_t = \frac{u_1 \left(C_{T,t}, C_{N,t} \right)}{u_2 \left(C_{T,t}, C_{N,t} \right)} = \frac{\alpha}{1-\alpha} \left(\frac{C_{N,t}}{C_{T,t}} \right)^{\frac{1}{\xi}}. \tag{2}$$

Financiers Financiers intermediate funds by holding a zero capital portfolio of foreign currency bonds $B^*_{F,t}$ and local currency bonds $B_{F,t}$ such that $B_{F,t} + \mathcal{E}_t B^*_{F,t} = 0$. They exhibit mean-variance preferences of the form

$$\mathbb{E}_t \left[\Theta_{t+1} \tilde{R}^*_{t+1} B^*_{F,t} \right] - \frac{\omega}{2} var_t \left(\tilde{R}^*_{t+1} B^*_{F,t} \right).$$

Note that the SDF of domestic households Θ_{t+1} enters the objective function of financiers and $\omega > 0$ measures the (additional) degree of risk aversion of financiers. Moreover, R^* is the (constant) world interest rate and $\tilde{R}^*_{t+1} = R^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ denotes the carry trade return in foreign currency from period t to $t+1$. The first-order condition with respect to $B^*_{F,t}$ yields the following risk-augmented UIP condition

$$R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] - R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = \overbrace{-\omega \sigma_t^2 B^*_{F,t}}^{\text{Risk Sharing Wedge}}, \tag{3}$$

where $\sigma_t^2 = R_t^2 var_t \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right)$. The ex-ante UIP deviation, defined as excess return on domestic currency and which can be interpreted as the international risk sharing wedge, is driven by the amount of funds intermediated by financiers $B^*_{F,t}$ and the depth of FX markets $\omega \sigma_t^2 > 0$. The bigger the long exposure of financiers to domestic currency assets (i.e., the more negative is $B^*_{F,t}$), the bigger compensation they require in form of an excess return. Also note that a higher (lower) $\omega \sigma_t^2$ corresponds to shallower (deeper) FX markets, implying bigger sensitivity of the exchange rate to capital flows. Importantly, FX market depth is state-dependent in the sense that its variation comes from changes in conditional exchange rate volatility σ_t^2. Finally, the domestic currency profits of financiers in period t are given by $\Pi_{F,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{F,t-1}$.

Portfolio Investors Similar to financiers, portfolio investors hold a zero capital portfolio $(B^*_{P,t}, B_{P,t})$ such that $B_{P,t} + \mathcal{E}_t B^*_{P,t} = 0$. However, they are non-optimizing agents to the extent that they randomly buy (sell) foreign currency bonds $B^*_{P,t} > 0$ ($B^*_{P,t} < 0$) and sell (buy) domestic currency bonds $B_{P,t} < 0$ ($B_{P,t} > 0$). More precisely, $B^*_{P,t}$ is exogenous and follows a first-order Markov process. The profits of portfolio investors are $\Pi_{P,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{P,t-1}$.

Central Bank The monetary authority engages in sterilized FXI by adjusting its stock of foreign reserves $B^*_{M,t}$ and sterilization bonds $B_{M,t}$ such that $B_{M,t} + \mathcal{E}_t B^*_{M,t} = 0$. Crucially, this policy is subject to a non-negativity constraint on FX reserves (NNCR)

$$B^*_{M,t} \geq 0.$$

The central bank's profits from its bond portfolio are $\Pi_{M,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{M,t-1}$.

We assume that the central bank's interest rate policy is fully focused on stabilization of nontradable goods prices, which allows us to normalize their level to unity.[2]

Bond Market Clearing Overall, market clearing in the domestic bond market requires

$$B_{F,t} + B_t + B_{P,t} + B_{M,t} = 0$$

and defines the net foreign asset position in foreign currency as

$$B^*_t = B^*_{F,t} + B^*_{M,t} + B^*_{P,t}, \tag{4}$$

which implies $B^*_t = B_t/\mathcal{E}_t$ by domestic bond market clearing and the balance sheet equations of the financial sector.

Resource Constraint In equilibrium, consumption of nontradable goods must equal their endowment

$$C_{N,t} = Y_{N,t}. \tag{5}$$

Consolidating the household's budget constraint by inserting equation (5) and profits of the

[2]This assumption can be understood as capturing the traditional interest rate policy motive arising from nominal rigidities without explicitly modeling them. More specifically, if prices in the nontradable sector were sticky, the monetary policy that perfectly stabilizes them would implement a flexible price equilibrium.

financial sector yields the following economy-wide resource constraint

$$B_t^* - B_{t-1}^* R^* = Y_{T,t} - C_{T,t}. \tag{6}$$

It implies that, on aggregate, the domestic economy is borrowing in foreign currency at the world interest rate, which follows from the assumption of full domestic ownership of the financial sector.

2.2 Decentralized and Constrained-Efficient Equilibrium

Decentralized Equilibrium Having outlined the model, we now define the decentralized equilibrium, in which the central bank does not hold any FX reserves and hence does not conduct FX interventions.

Definition 1 (Decentralized Equilibrium without FX Interventions)**.** Given exogenous process $\{B_{P,t}^*, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and initial condition B_{-1}^*, a competitive equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$, bond positions $\{B_t^*, B_{F,t}^*\}_{t=0}^{\infty}$, and FXI policy $\{B_{M,t}^*\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize, implying (1), (2), and (3)
2. The central bank holds no FX reserves: $B_{M,t}^* = 0 \ \forall t$
3. Goods and bond markets clear, implying (5), (6), and (4)
4. Transversality condition on net foreign assets holds

$$\lim_{T\to\infty} \frac{B_T^*}{(R^*)^T} = 0.$$

Constrained-Efficient Equilibrium We next describe the problem of a central bank that optimally conducts FX interventions and we define the associated equilibrium. We assume that every period t the central bank chooses FX reserves $B_{M,t}^*$ in a discretionary fashion, not being able to credibly commit to future actions. It needs to be stressed that the intervention policy is fully time-consistent as the effects of the central bank's optimal plans on its future plans are taken into account. Consequently, the policy problem can be formulated as follows

$$\max_{\{C_{T,t},B_t^*,\mathcal{E}_t,R_t,B_{M,t}^*,\sigma_t^2\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[u(C_{T,t+s}, Y_{N,t+s})\right] \tag{7}$$

subject to the constraints

$$R_{t+s}\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right] = 1 \qquad \text{(Household Euler Equation)}$$

$$\mathcal{E}_{t+s} = \frac{u_1(C_{T,t+s}, Y_{N,t+s})}{u_2(C_{T,t+s}, Y_{N,t+s})} \qquad \text{(Expenditure Switching)}$$

$$B_{t+s}^* - B_{t+s-1}^* R^* = Y_{T,t+s} - C_{T,t+s} \qquad \text{(Resource Constraint)}$$

$$R^*\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right] = 1 + \omega\sigma_{t+s}^2\left(B_{t+s}^* - B_{M,t+s}^* - B_{P,t+s}^*\right) \qquad \text{(International Risk Sharing)}$$

$$\sigma_{t+s}^2 = R_{t+s}^2 var_{t+s}\left(\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right) \qquad \text{(Cond. Exchange Rate Volatility)}$$

$$B_{M,t+s}^* \geq 0 \qquad \text{(Constraint on Reserves)}$$

This allows us to arrive at the following definition of constrained-efficient equilibrium.

Definition 2 (Constrained-Efficient Equilibrium with FX Interventions)**.** Given the exogenous process $\{B_{P,t}^*, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and initial condition B_{-1}^*, a constrained-efficient equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$, bond positions $\{B_t^*, B_{F,t}^*\}_{t=0}^{\infty}$ and FXI policy $\{B_{M,t}^*\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize, implying (1), (2), and (3)

2. The central bank solves the policy problem (7)

3. Goods and bond markets clear, implying (5), (6), and (4)

4. Transversality condition on net foreign assets holds

$$\lim_{T\to\infty} \frac{B_T^*}{(R^*)^T} = 0.$$

3 Optimal FX Interventions

We proceed by analytically studying the optimal time-consistent use of FXI in the model described in Section 2. We first characterize the first best and next describe the key results obtained from the analysis of the second best policy. The proofs of all theorems and propositions presented in this section can be found in Appendix A.

First Best If households had direct, frictionless access to foreign currency bonds, the first-order condition associated with that asset would be

$$R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = 1. \tag{8}$$

It is straightforward to show that this condition holds in the constrained-efficient equilibrium without a NNCR as long as the financial sector is entirely owned by domestic households, which implies that the economy effectively borrows at the foreign interest rate. The associated optimal FXI policy is given by

$$B^*_{M,t} = B^*_t - B^*_{P,t}, \tag{9}$$

and it fully eliminates the international risk-sharing wedge. In particular, the central bank responds one-for-one to fluctuations in the demand for domestic currency, making costly intermediation provided by financiers redundant ($B^*_{F,t} = 0$).[3]

It is worth noting that strict stabilization of the UIP premium would no longer be optimal if we allowed the financial market participants to be at least partially owned by foreigners. In that case, the central bank would lean against capital flows less than one-for-one, thus opportunistically exploiting them to make profits on FX reserve management at the expense of agents living in the rest of the world. See Itskhoki & Mukhin (2023) and Adrian et al. (2022) for more discussion of this case.

Financial Conditions under Second Best In the existence of a NNCR, the central bank is generally unable to completely eliminate the risk sharing wedge in all periods as in the first best. One can easily imagine a situation in which the economy holds a relatively large amount of debt and/or is confronted with a considerable portfolio outflow such that $B_t - B^*_{P,t} < 0$, in which case the NNCR becomes relevant. Before we fully characterize the optimal FXI policy under such circumstances, it is useful to spell out two key implications of this equilibrium for the financial conditions faced by the small open economy. These are summarized in the following proposition

Proposition 1. *Consider an equilibrium in period t with a binding NNCR ($B^*_t - B^*_{P,t} < 0$) and where a portfolio outflow is associated with an improvement in the net foreign asset position ($\partial B^*_t / \partial B^*_{P,t} > 0$). Then, the equilibrium exhibits the following properties:*

(a) $\partial \sigma^2_t / \partial B^*_{P,t} > 0$: A portfolio outflow leads to an elevated conditional volatility of the

[3] A corollary is that the first best in an economy with portfolio shocks only is associated with no uncertainty about the exchange rate ($\sigma^2_t = 0$), i.e., a peg.

exchange rate.

(b) $\partial \mathbb{E}_t [\Theta_{t+1}] / \partial B^*_{P,t} < 0$*: A portfolio outflow leads to an expected decrease of the stochastic discount factor between periods* t *and* $t+1$*.*

The crucial relationship underpinning Proposition 1 is that we consider an economy in which portfolio outflows generate an improvement in the net foreign asset position. While this seems to be the plausible case, we provide the sufficient conditions in Appendix A for this relation to hold as well as the proof of Proposition 1. Intuitively, if the NNCR is binding, a portfolio outflow cannot be fully offset by selling FX reserves. Then, by the bond market clearing condition (4), for the net foreign assets position of the economy not to increase, all of the resulting imbalance would have to be absorbed by financiers. However, that implies tightening of the financial conditions as financiers require a higher premium when their exposure to domestic currency increases. This in turn incentivizes households to increase their savings, meaning an improvement in the country's net foreign assets position.

If the conditions specified in Proposition 1 apply, state-dependent FX market depth $\omega\sigma_t^2$ magnifies the impact of portfolio outflows on the risk sharing wedge. Intuitively, a portfolio outflow that cannot be fully offset by an FX intervention causes a positive UIP deviation not only through its impact on the amount of funds that must be intermediated by the financiers, but also by increasing the chances that the next period outflow will not be fully offset either. The latter implies an endogenous increase in the conditional volatility of the exchange rate, meaning higher riskiness of the balance sheets held by the financiers, for which they must be compensated. At the same time, a positive risk sharing (UIP) wedge means a tightening of borrowing conditions for households as they face a premium over the foreign interest rate. These tighter conditions imply lower consumption of tradable goods and hence a higher marginal utility today compared to tomorrow. As a result, the stochastic discount factor decreases.

Implicit Borrowing Limit One interesting consequence of a non-negativity constraint on FX reserves is that it imposes a lower bound on the country's net foreign assets, thus constituting an implicit borrowing limit that may constrain the optimal policy conduct. To see it, combine the international risk sharing condition (3) with the NNCR to obtain

$$B^*_t \geq B^*_{P,t} - \frac{1}{\omega\sigma_t^2}\left(1 - R^*\mathbb{E}_t[\Theta_{t+1}]\right) \equiv \Psi_t \tag{10}$$

We can hence reformulate the second best in terms of a consumption smoothing problem subject to this additional constraint.

The borrowing limit Ψ_t has two components. The first one is represented by exogenous portfolio outflows $B^*_{P,t}$, which increase Ψ_t and thus make the borrowing limit tighter. The second term corresponds to the domestic currency lending position by financiers (denominated in foreign currency), and it decreases Ψ_t whenever the NNCR is binding, i.e., when $R^*\mathbb{E}_t[\Theta_{t+1}] < 1$. Due to their mean-variance preferences, the financiers' position in equilibrium is a function of the expected excess return on domestic currency $1 - R^*\mathbb{E}_t[\Theta_{t+1}]$ and the risk factor $\omega\sigma_t^2$. If either the expected return increases and/or the risk factor decreases, financiers are willing to lend more in domestic currency, thereby relaxing the implicit borrowing limit faced by the optimal policy. Furthermore, note that optimal FX reserves simply reflect the difference between net foreign assets and the borrowing limit, i.e., $B^*_{M,t} = B^*_t - \Psi_t$. In particular, the borrowing limit is binding ($B^*_t = \Psi_t$) if and only if the central bank runs out of reserves ($B^*_{M,t} = 0$).

Value of Commitment and Time Inconsistency of Optimal Plans One important feature of the implicit borrowing limit given by equation (10) is that it is forward-looking. It depends on the conditional volatility of the exchange rate and on the stochastic discount factor that households use to evaluate future flows, both of which depend on expectations formulated at time t about events at time $t+1$. If the central bank could credibly commit to future FX interventions, it could steer these expectations in a way that relaxes the current period borrowing limit whenever it becomes binding. Such a policy would then resemble the "forward guidance" about the future path of the interest rates, which arises in the New Keynesian setup as a potentially powerful way to mitigate the consequences of the effective lower bound on the policy rate.

However, as in the New Keynesian analogy, committing to future interventions is not time consistent. Once new shocks materialize, it is optimal for the central bank to reoptimize its policy, possibly reneging on the promises made in the past. For this reason, in the remaining part of our analysis we will focus on discretionary but fully time-consistent FXI policy.

Intertemporal Tradeoffs under Second Best The first order condition associated with the second-best policy problem is given by

$$
\begin{aligned}
u_{1,t} = {} & \beta R^* \mathbb{E}_t \left[u_{1,t+1}\right] && (FB) \\
& + \lambda_t \left(1 - \frac{1 - R^* \mathbb{E}_t \left[\Theta_{t+1}\right]}{\omega(\sigma_t^2)^2} \frac{\partial \sigma_t^2}{\partial B_t^*} - \frac{R^*}{\omega \sigma_t^2} \mathbb{E}_t \left[\frac{\partial \Theta_{t+1}}{\partial B_t^*}\right]\right) && (\geq 0) \\
& - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{1 - R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right]}{\omega \left(\sigma_{t+1}^2\right)^2} \frac{\partial \sigma_{t+1}^2}{\partial B_t^*}\right] && (\geq 0) \\
& - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R^*}{\omega \sigma_{t+1}^2} \mathbb{E}_{t+1} \left[\frac{\partial \Theta_{t+2}}{\partial B_t^*}\right]\right] && (\leq 0)
\end{aligned}
\tag{11}
$$

where $u_{1,t} \equiv u_1 \left(C_{T,t}, Y_{N,t}\right)$ and λ_t denotes the Lagrange multiplier associated with the implicit borrowing constraint (10).[4]

Several observations are in order. First of all, if the lower bound on reserves is never binding ($\lambda_t = 0 \forall t$), the optimality condition (11) reduces to its first line, becoming equivalent to the first best (FB) given by equation (8), meaning that the central bank can perfectly eliminate the international risk sharing wedge. The second line in equation (11) corresponds to the case of insufficient FX reserves today. An economy that hits the NNCR at time t (so that $\lambda_t > 0$) experiences a tightening in financial conditions (UIP premium becomes positive), is forced to borrow less, and hence needs to restrict its consumption. Finally, the last two lines of equation (11) illustrate how the possibility of the NNCR becoming binding in the future ($\lambda_{t+1} > 0$ in some states) affects the allocations chosen by the optimizing central bank today, even if the current level of FX reserves is positive. These forward-looking motives of the optimal FXI policy reflect the fact that the central bank internalizes the effects of the economy's current savings on the future financial conditions arising from the presence of the implicit borrowing constraint.

Interestingly, there are two opposing forces at play here. The term in the third line arises as the economy's savings decisions in period t affect the future conditional exchange rate volatility σ_{t+1}^2. While each individual household takes this risk factor as given, the planner internalizes how it is affected by economy-wide savings. It can be shown that $\frac{\partial \sigma_{t+1}^2}{\partial B_t^*} < 0$, meaning that if the economy saves more today, the conditional exchange rate volatility decreases in the future. This is because a higher current net foreign assets position implies a lower FX intervention that is needed to prevent an increase in the international risk-sharing wedge for a given portfolio capital outflow in the future. As a result, the probability that such an outflow will not be fully neutralized due to insufficient FX reserves decreases, which implies a lower expected impact of non-fundamental shocks on the next period exchange rate.

[4] The complementary slackness conditions are $\lambda_t \geq 0,\ B_t^* - \Psi_t \geq 0,\ (B_t^* - \Psi_t)\lambda_t = 0$.

Since $1 - R^* \mathbb{E}_t [\Theta_{t+2}] > 0$ whenever FXI policy is constrained in period $t+1$ ($\lambda_{t+1} > 0$), the third line of equation (11) is positive, meaning that saving more today brings the benefit of a less binding implicit borrowing limit tomorrow due to compressed conditional exchange rate volatility.

On the other hand, the last line in equation (11) captures the effect of aggregate savings on the household stochastic discount factor between period $t+1$ and $t+2$, which financiers use to value their future profits. Again, an individual household does not take into account the effect of its intertemporal decisions on Θ_{t+2}. However, the central bank internalizes that higher savings in period t support tradable consumption in the future, thus increasing the aggregate stochastic discount factor in the next period. We hence have $\frac{\partial \Theta_{t+2}}{\partial B_t^*} > 0$. Note that higher Θ_{t+2} decreases the expected excess return on domestic currency $1 - R^* \mathbb{E}_t [\Theta_{t+2}]$, and hence financiers' expected profits, hampering their capacity to intermediate. As a result, the fourth line of equation (11) is negative, implying that saving more today involves the cost of tightening the implicit borrowing limit tomorrow by compressing financiers' expected profits.

Optimal Time-Consistent FXI under Second Best We have seen above that the optimizing central bank internalizes two opposing effects of aggregate savings, represented by the economy's net foreign assets position, on the implicit borrowing limit in the next period. The optimal FXI policy in period t is guided by whichever of the two motives dominates, as summarized in the following theorem

Theorem 1. *If the use of FXI is unconstrained in period t ($\lambda_t = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t [\lambda_{t+1}] > 0$), the optimal time-consistent FXI policy in period t is given by*

$$B_{M,t}^* = B_t^* - B_{P,t}^* + \frac{\beta R^*}{\omega \sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right], \tag{12}$$

which calls for a smaller (larger) intervention relative to the first best if the third term on the RHS is positive (negative).

Theorem 1 states that the optimal intervention in period t in anticipation of a binding NNCR in period $t+1$ can either be liquidity-providing, in the sense that the central bank decides to hold less reserves compared to the first best ($B_{M,t}^* < B_t^* - B_{P,t}^*$), or liquidity-absorbing to the extent that the central bank stocks more reserves ($B_{M,t}^* > B_t^* - B_{P,t}^*$). If the NNCR is expected not to bind in any possible states in period $t+1$, the optimal intervention in period t corresponds to the first best.

To grasp the intuition behind Theorem 1, recall that the friction in our model is the risk sharing wedge. While the wedge can always be completely eliminated in the first best, this is not feasible anymore in the second best. By definition, a period $t+1$ characterized by a binding NNCR is associated with negative intermediated funds $B^*_{F,t+1} = B^*_{t+1} - B^*_{P,t+1} < 0$ and consequently also a positive risk sharing wedge $-\omega\sigma^2_{t+1}B^*_{F,t+1} > 0$. The aim of the second best policy in this context is to relax the implicit borrowing limit by facilitating intermediation by financiers. Recall that the intermediated funds at time $t+1$ depend positively on the expected return $1 - R^*\mathbb{E}_{t+1}[\Theta_{t+2}]$ and negatively on the risk factor $\omega\sigma^2_{t+1}$. The central bank can affect both of these variables by engaging in FXI in period t. A liquidity-providing intervention $B^*_{M,t} < B^*_t - B^*_{P,t}$ can increase the expected return $1 - R^*\mathbb{E}_{t+1}[\Theta_{t+2}]$ and thereby increase financiers' capacity to lend to domestic households in the possibly constrained period $t+1$. Conversely, a liquidity-absorbing intervention $B^*_{M,t} > B^*_t - B^*_{P,t}$ can decrease the conditional volatility of the exchange rate σ^2_{t+1}, making the financiers more willing to lend.

Precautionary Accumulation of FX Reserves The presence of a liquidity-absorbing intervention is the key difference between our analysis and that recently offered by Itskhoki & Mukhin (2023). In contrast to our global approach to the optimal policy problem, their analytical framework relies on the first-order approximation of the equilibrium system. This brings substantial gains in terms of tractability but leaves only the liquidity-providing motive of deviating from the first best whenever the central bank might run out of FX reserves in the future. As a result, if FXI is unconstrained at period t but possibly so in the future, their model implies intervening more than one-for-one to a capital outflow, thus resulting in a *negative* UIP risk premium.[5]

The liquidity-absorbing intervention motive that is captured by our analysis works in the opposite direction and has a precautionary flavor. It relies on the negative impact of conditional exchange rate volatility on FX market depth. Any given portfolio outflow shock has less of an effect on the risk sharing wedge in the constrained period $t+1$ if FX markets are deeper, which happens when the conditional exchange rate volatility σ^2_{t+1} is lower. Crucially, and as already discussed above, entering the constrained period with larger net foreign assets dampens the conditional exchange rate volatility. Unlike households, the central bank internalizes this general equilibrium effect. By accumulating reserves in period t beyond what the first best suggests, the central bank forces the economy to save more on aggregate. This, in turn, makes it less likely that a given portfolio capital outflow in period $t+1$ will lead to depletion of FX reserves, which would then increase the loading of non-fundamental forces

[5] See Theorem 2 in Itskhoki & Mukhin (2023).

on exchange rate risk. Overall, a liquidity-absorbing intervention $B^*_{M,t} > B^*_t - B^*_{P,t}$ can be interpreted as a "keeping powder dry" strategy as it leads to precautionary accumulation of FX reserves. By doing this, the central bank can smooth the international risk sharing wedge over time by compressing the UIP premium in the constrained period $t+1$ at the cost of increasing it in period t.

Whether the liquidity-absorbing motive dominates the liquidity-providing motive is described by the following proposition

Proposition 2. *Suppose the use of FXI is unconstrained in period t ($\lambda_t = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t[\lambda_{t+1}] > 0$). Then, a precautionary level of reserves exists (i.e., the optimal level of reserves is higher relative to the first best) if and only if*

$$R^*\mathbb{E}_t\left[\Theta_{t+2}\right] < \frac{2}{3} - \frac{\mathbb{C}ov_t\left(\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right), R^*\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]\right)}{\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\right]}, \tag{13}$$

which is more likely to hold if the (expected) stochastic discount factor between the potentially constrained period $t+1$ and period $t+2$ is lower.

According to Proposition 2, the liquidity-absorbing motive prevails if the stochastic discount factor is expected to drop sufficiently in the possibly constrained period $t+1$. In other words, if households are expected to experience a particularly severe drop in tradable consumption when the economy hits the implicit borrowing limit (the NNCR becomes binding), it is optimal for the central bank to hold more reserves relative to the first best (i.e., absorb liquidity). By doing so, it can make the potential crisis less severe if it unfolds.

Naturally, the severity of a crisis hinges critically on the potential scale of portfolio outflows. Accordingly, we now state the following proposition

Proposition 3. *Suppose the use of FXI is unconstrained in period t ($\lambda_t = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t[\lambda_{t+1}] > 0$). Then, a sufficiently large (small) expected future portfolio capital outflow $B^*_{P,t+1}$ calls for an optimal intervention that offsets the contemporaneous portfolio outflow $B^*_{P,t}$ less (more) than one-for-one.*

Proposition 3 is obtained by combining Proposition 1, Theorem 1 and Proposition 2. It has stark policy implications. The central bank should provide liquidity and intervene aggressively on the FX market in response to contemporaneous portfolio capital outflows only if future outflows are anticipated to be relatively mild. In contrast, absorbing liquidity and maintaining a precautionary level of reserves is appropriate when severe future portfolio outflows are expected.

A straightforward corollary follows if portfolio capital outflows $B^*_{P,t}$ are driven by a positively autocorrelated AR(1) process. Then, future capital outflows are likely to be high (low) if they are high (low) today. As a result, and to the extent possible with the current stock of reserves, optimal FXI policy should respond more than one-for-one to small capital outflows, but less than one-for one if they are sufficiently large. What exactly "sufficiently" means in this context is a quantitative question, the answer to which depends on the model parameterization and the initial net foreign asset position. We will explore this in our quantitative analysis presented below.

4 Quantitative Analysis

In this section, we begin by describing the calibration of the model presented above and then analyze its quantitative implications, highlighting the key insights in remarks.

4.1 Calibration

We calibrate the model to Malaysia, a small open emerging economy with a central bank that is an active user of FXI. We think of Malaysia as a fitting case to explore through the lens of our model since its central bank uses FXI continuously to stabilize the exchange rate, especially in periods of large and volatile two-way capital flows (Aziz, 2019). More specifically, we calibrate the model to data from 2010 to 2023, a period in which the Malaysian ringgit operated under a managed float regime and FX interventions were used when, quoting Bank Negara Malaysia (BNM, 2025), "the ringgit market movements are not orderly and to ensure there is enough liquidity in the banking system". While this episode is characterized by relatively stable economic conditions and no major domestic financial crisis, the central bank frequently intervened in FX markets in both directions.[6]

The parameter values of the model are listed in Table 1. We calibrate the model at a quarterly frequency. The annualized world interest rate is 4% and the relative risk aversion is set to $\sigma = 2$, both parameter values are standard in the small open economy literature.

[6]Based on data from Adler et al. (2025), the standard deviation of quarterly spot FX interventions by the BNM between 2010 and 2023 amounts to 1.5% of annual GDP. Interventions were roughly balanced over this period, with 27 quarters of net purchases and 29 quarters of net sales.

Table 1: Calibration

Description	Value	Source/Target
World interest rate, quarterly	$R^* = 1.01$	Standard value DSGE-SOE
Relative risk aversion	$\sigma = 2$	Standard value DSGE-SOE
Elasticity of substitution of T-NT goods	$\xi = 0.83$	Conservative value, Bianchi (2011)
Weight on traded goods in CES aggregator	$\alpha = 0.39$	Malaysia's economy
Subjective discount factor, quarterly	$\beta = 0.9871$	NFA-GDP ratio, Malaysia's economy
Financiers' risk aversion	$\omega = 28$	FX market depth, Davis et al. (2023)

Regarding the crucial parameter guiding the elasticity of substitution between tradable and nontradable goods ξ, we choose a value $\xi = 0.83$ that is at the upper bound of the empirical estimates in the literature following Bianchi (2011). We calibrate the remaining structural parameters α, β and ω to Malaysia's economy. In particular, we set the weight on traded goods in the CES aggregator equal to the average share of the tradable component in GDP, which is 39%. Furthermore, the value of the subjective discount factor β targets the observed private NFA-to-GDP ratio and the financiers' risk aversion ω is set to imply an average FX market depth $\omega\overline{\sigma}^2 = 0.05$ in the observed economy, consistent with Davis et al. (2023). Both of these targeted moments crucially depend on the FXI regime. For example, a higher level of reserves increases the net foreign asset position and an FXI regime that dampens real exchange rate volatility is associated with deeper FX markets in the model. Since Malaysia is a frequent user of FXI, we need to take its FXI regime into account when calibrating β and ω. In Appendix B, we provide details on this part of the calibration. Crucially, it implies a reasonable reaction of the exchange rate to capital flows, including FX interventions.[7]

For the endowment part of the exogenous driving forces in our model $\{Y_{T,t}, Y_{N,t}\}$, we follow the standard methodology in the literature and use the cyclical components of tradable and nontradable GDP at constant prices, retrieved from the BNM statistics. We classify agriculture, mining and quarrying and manufacturing as tradables and treat the rest of GDP as nontradable. To obtain the cyclical components, we remove a cubic trend and seasonality from the natural logarithm of tradables and nontradables following Schmitt-Grohé & Uribe (2016).

To estimate a process of the exogenous portfolio outflows $\{B^*_{P,t}\}$, we first calculate an empirical measure of the risk sharing wedge (or ex-ante UIP deviation) as it is defined in our

[7]More specifically, if simulate the data from our baseline model and regress the log change in the exchange rate on portfolio outflows (net of FXI and expressed as percent of annual GDP), controlling for endowment shocks, we obtain a coefficient of around 0.6. This number increases to 1.3 if we use data from the decentralized equilibrium (without FXI) but drops to 0.4 under optimal time-consistent policy.

model

$$\widehat{RSW}_t = \widehat{\Theta}_{t+1} \left(\hat{R}_t^* - \hat{R}_t \frac{\hat{\mathcal{E}}_t}{\widehat{\mathbb{E}_t \left[\mathcal{E}_{t+1} \right]}} \right), \tag{14}$$

where $\hat{R}_t^*$ is the effective federal funds rate, $\hat{R}_t$ denotes the BNM overnight policy rate, $\hat{\mathcal{E}}_t$ is the USD to Malaysian Ringgit (MYR) spot rate, and $\widehat{\mathbb{E}_t \left[\mathcal{E}_{t+1} \right]}$ is the Bloomberg composite one-quarter ahead forecast of the USD/MYR exchange rate. Furthermore, our measure of the stochastic discount factor is given by $\widehat{\Theta}_{t+1} = \beta \left(\hat{C}_{t+1} / \hat{C}_t \right)^{\frac{1-\sigma\xi}{\xi}} \left(\hat{C}_{T,t+1} / \hat{C}_{T,t} \right)^{-\frac{1}{\xi}}$ where we compute $\hat{C}_t$ and $\hat{C}_{T,t}$ using the cyclical components of tradable and nontradable GDP (see above) as well as the quarterly trade surplus retrieved from the IMF IFS.[8] Next, we make use of the international risk sharing condition of the model in order to back out the exogenous process for portfolio outflows

$$\hat{B}_{P,t}^* = \hat{B}_t^* - \hat{B}_{M,t}^* - \frac{\widehat{RSW}_t}{\omega \hat{\sigma}_t^2}, \tag{15}$$

where we obtain the quarterly measure of Malaysia's NFA position $\hat{B}_t^*$ from the IMF IFS, the quarterly proxy for Malaysia's FXI from Adler et al. (2025), and use the three month implied USD/MYR volatility from Bloomberg to retrieve $\hat{\sigma}_t^2$.[9]

We then incorporate the exogenous states in the model as an AR(1) process $\mathbf{s}_t = \rho \mathbf{s}_{t-1} + \varepsilon_t$ where $\mathbf{s}_t = \left[\log Y_{T,t}, \log Y_{N,t}, \sinh^{-1} \left(B_{P,t} - \overline{B}_P \right) \right]'$ are obtained as described above for the period 2010:Q1 to 2023:Q4.[10] The error term $\varepsilon_t = [\varepsilon_{T,t}, \varepsilon_{N,t}, \varepsilon_{P,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\mathbf{V}$ while ρ is a 3×3 matrix consisting of the autocorrelation terms

$$\mathbf{V} = \begin{bmatrix} 0.0005447 & 0.0005911 & 0.0019075 \\ 0.0005911 & 0.0008851 & 0.0013138 \\ 0.0019075 & 0.0013138 & 0.1727534 \end{bmatrix}, \quad \rho = \begin{bmatrix} 0.8213977 & -0.3171368 & -0.0201376 \\ 0.2110661 & 0.3794069 & -0.0260989 \\ -0.650205 & -0.1477713 & 0.4799129 \end{bmatrix}$$

Without loss of generality, we normalize the mean of the endowment processes to one while the mean of portfolio outflows is $\bar{B}_P = -3.09$, indicating that Malaysia experiences portfolio inflows on average. Furthermore, the unconditional standard deviations of the exogenous

[8]Note that the SDF $\widehat{\Theta}_{t+1}$ in (14) is computed using realized consumption in period $t+1$, whereas the SDF in (3) is expressed in terms of expected consumption for that period. We adopt this simplification because data on expected consumption is unavailable.

[9]Since Adler et al. (2025) only provide FX interventions data, we additionally use IMF IFS data to account for the level of FX reserves needed to compute $\hat{B}_{M,t}^*$.

[10]Note that we remove the mean $\overline{B}_P$ from the portfolio flow process and apply the inverse hyperbolic sine transformation due to negative values.

driving forces are $\sigma_{Y_T} = 0.029$, $\sigma_{Y_N} = 0.037$ and $\sigma_{B_P^*} = 0.521$. The tradable and nontradable endowment process are highly positively correlated ($\sigma_{Y_T,Y_N} = 0.795$) while portfolio outflows are weakly negatively correlated with the endowments ($\sigma_{B_P^*,Y_T} = -0.036$ and $\sigma_{B_P^*,Y_N} = -0.147$). This weak correlation between portfolio outflows and the endowment process is a manifestation of the well-documented exchange rate disconnect, i.e., the fact that exchange rates are to a large extent driven by factors other than the fundamentals of the economy.

The exogenous state variables S are discretized into a first-order Markov process with four grid points for both of the endowment processes $\{Y_T, Y_N\}$ and eight grid points for the portfolio flow process B_P^* while the endogenous state variable B^* contains 1000 grid points. Finally, we solve the decentralized equilibrium and the constrained planner's problem using time iteration on the Euler equation.

4.2 Policy Functions

Figure 1 shows the policy functions for the small open economy's net foreign assets (NFA) position B^*, separately for the decentralized equilibrium without FXI as shown by the blue solid line and the constrained planner's solution (optimal FXI) as shown by the red dashed line. For low levels of the current NFA position, the economy increases its NFA position next period while the opposite is true for a relatively high current NFA position. As a result, both lines intersect the 45 degree line and a stationary equilibrium exists in both economies. Intuitively, the financiers' risk aversion gives rise to an upward-sloping supply of funds to the domestic economy which limits the amount of external borrowing in equilibrium. In addition, Figure 1 reveals that the constrained planner's policy function lies above that characterizing the decentralized equilibrium, indicating the importance of FX interventions for the economy's steady state NFA position.

To further shed light on the characteristics of optimal FXI policy, Figure 2 shows the optimal level of FX reserves B_M^* as a function of portfolio outflows B_P^*, distinguishing by states with a relatively low (solid blue) and high (dashed red) NFA position. The corresponding first best policy is depicted by the black solid lines. The presented policy functions confirm the key analytical results derived in Section 3. More specifically, for a relatively high NFA position and moderate outflows, the optimal policy offsets portfolio outflows approximately one-for-one as dictated by the first best policy. This is because, if FXI is conducted optimally, a relatively high NFA position is associated with a considerable level of FX reserves, implying that the lower bound on reserves is less relevant in this scenario and, as a result, the reaction to portfolio flows resembles the first best. In contrast, the lower bound on reserves becomes

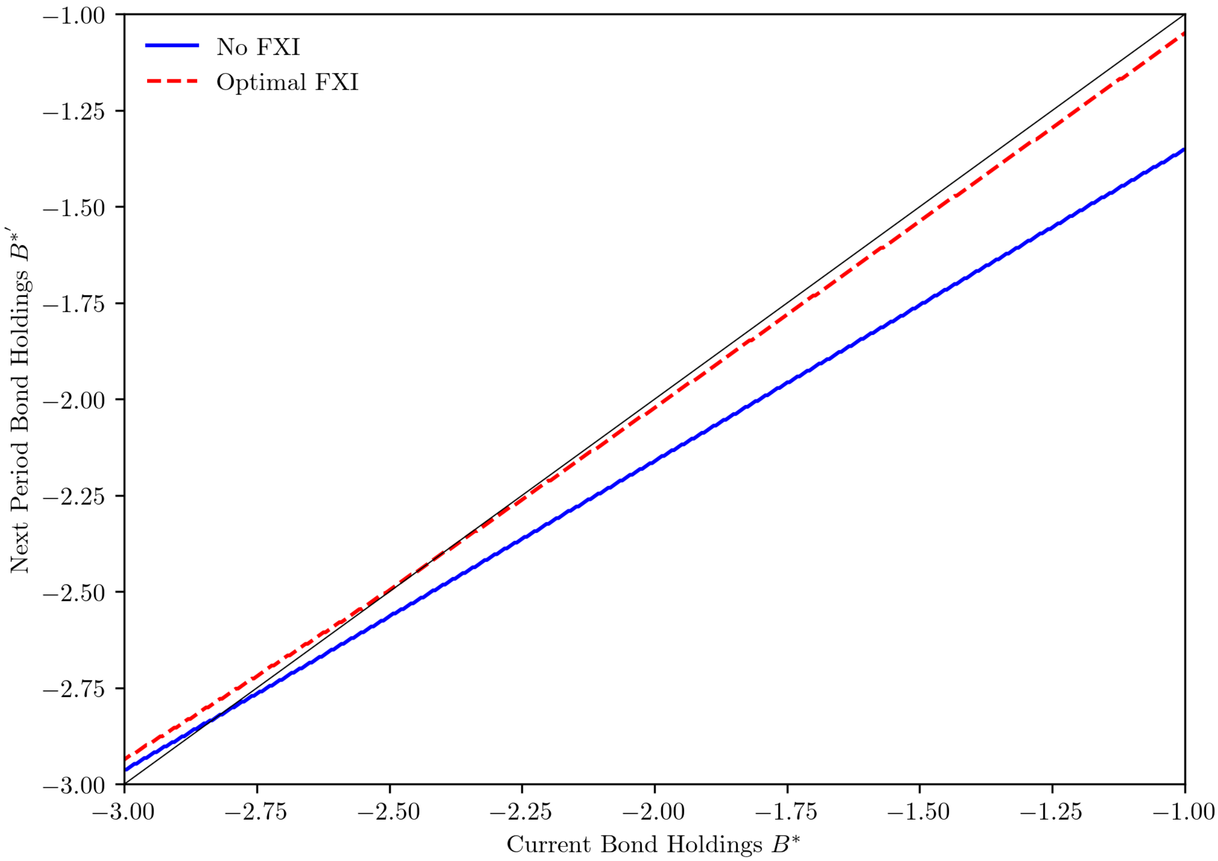

Figure 1: Policy function for net foreign assets B^*

Notes: The figure shows the policy function for bond holdings of the competitive equilibrium without FXI (blue solid line) and the constrained planner's solution (red dashed line) for the exogenous state $\{Y_T, Y_N, B_P^*\} = \{1.0, 1.0, -2.6\}$.

relevant if the economy finds itself in a state with relatively low NFA, in which case FX reserves are also low and hence there is non-negligible risk of their depletion in the next period. As described in Section 3, the optimal policy is offsetting portfolio outflows more than one-for-one if they are relatively mild. Yet, this reaction turns out to be quantitatively very small. In contrast, as suggested by Proposition 3, optimal policy is guided by maintaining a precautionary stock of reserves, which implies intervening less than one-for-one, if outflows are more severe. This leads us to our first remark which is closely related to Proposition 3.

Remark 1. Conditional on the net foreign asset position, the degree to which optimal time-consistent FXI policy offsets contemporaneous portfolio outflows diminishes as their magnitude increases.

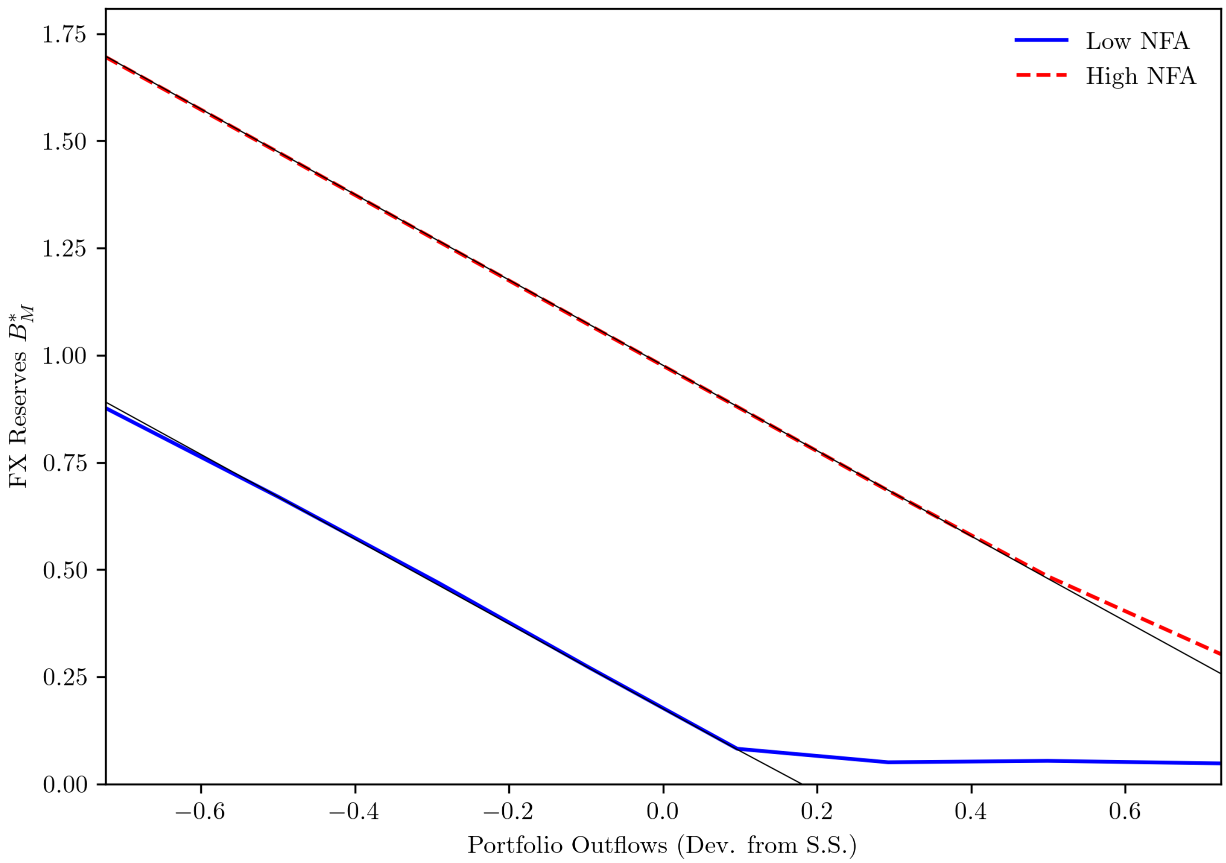

Figure 2: Policy function for FX reserves

Notes: The figure shows the policy function for FX reserves of the constrained planner's solution conditional on relatively high current NFA (red dashed line) and relatively low NFA (blue solid line). The black solid lines depict the corresponding policy functions under the first best.

4.3 Ergodic Implications

We next compare the ergodic distributions of key macroeconomic variables in an economy without FXI to those implied by the optimal use of FXI. To this end, we first generate a long sequence (2,000,000 periods) of the exogenous variables $\{Y_{T,t}, Y_{N,t}, B^*_{P,t}\}_{t=1}^{2\times10^6}$ based on the estimated Markov process as described in Section 4.1. Using this exogenous sequence and the policy functions, we then compute the equilibrium variables for the economy without FXI and with optimally conducted FXI.

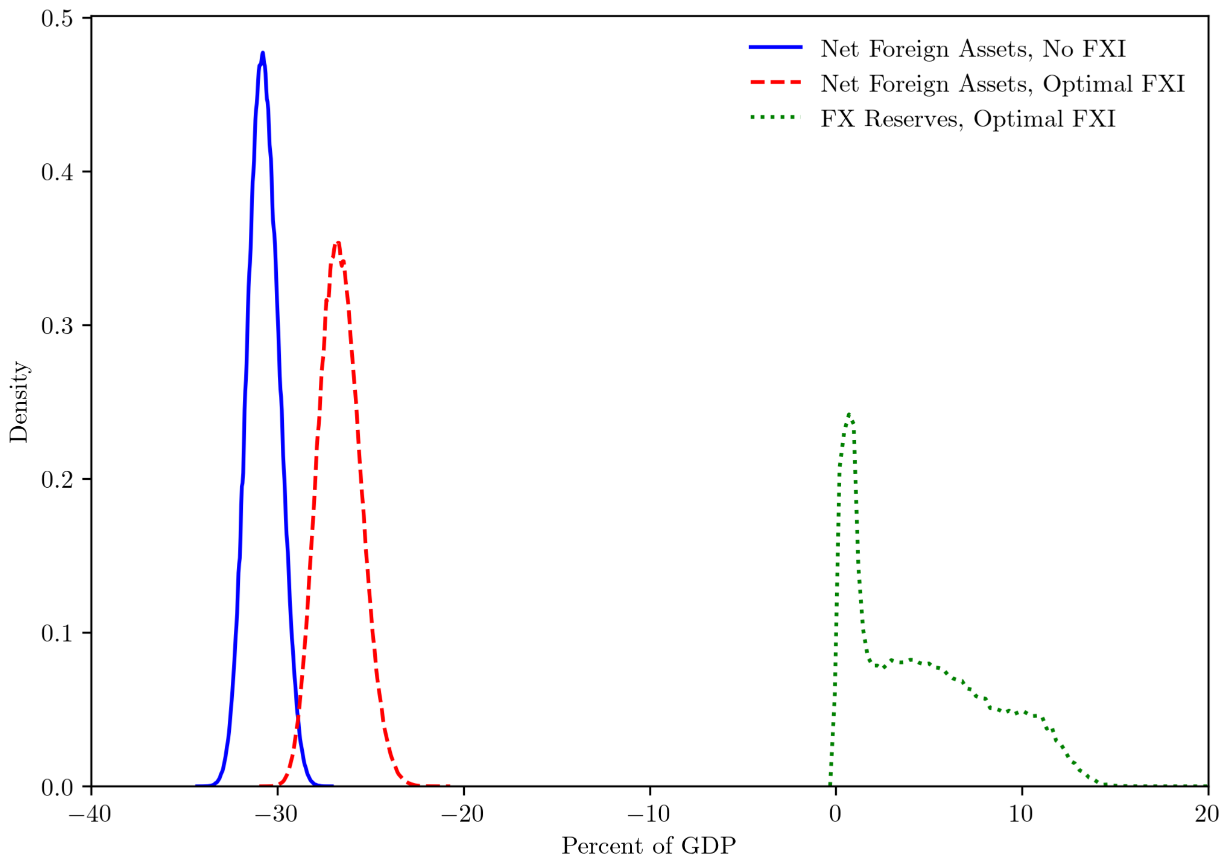

Figure 3: Ergodic distribution of net foreign assets and FX reserves

Notes: The figure shows the densities for the net foreign asset positions and FX reserves obtained by simulating the model for 2×10^6 quarters.

Figure 3 shows the unconditional distribution of the net foreign asset position under these two alternative policies, as well as of FX reserves under the optimal FXI policy. Strikingly, the optimal use of FXI changes significantly the steady state of the economy: it is saving more on average, resulting in a higher net foreign asset position. The reason is that active use of FXI implies a positive average level of FX reserves, which means that the demand for foreign currency is higher. To accommodate it, equilibrium on the bond market (4) requires either lower borrowing by households (higher NFA B_t^*) or increased intermediation by financiers (higher $-B_{F,t}^*$). In our model, both happen. NFA improves markedly, but not one-for-one with FX reserve holdings, indicating that FX market conditions also become easier, allowing financiers to expand their balance sheets.

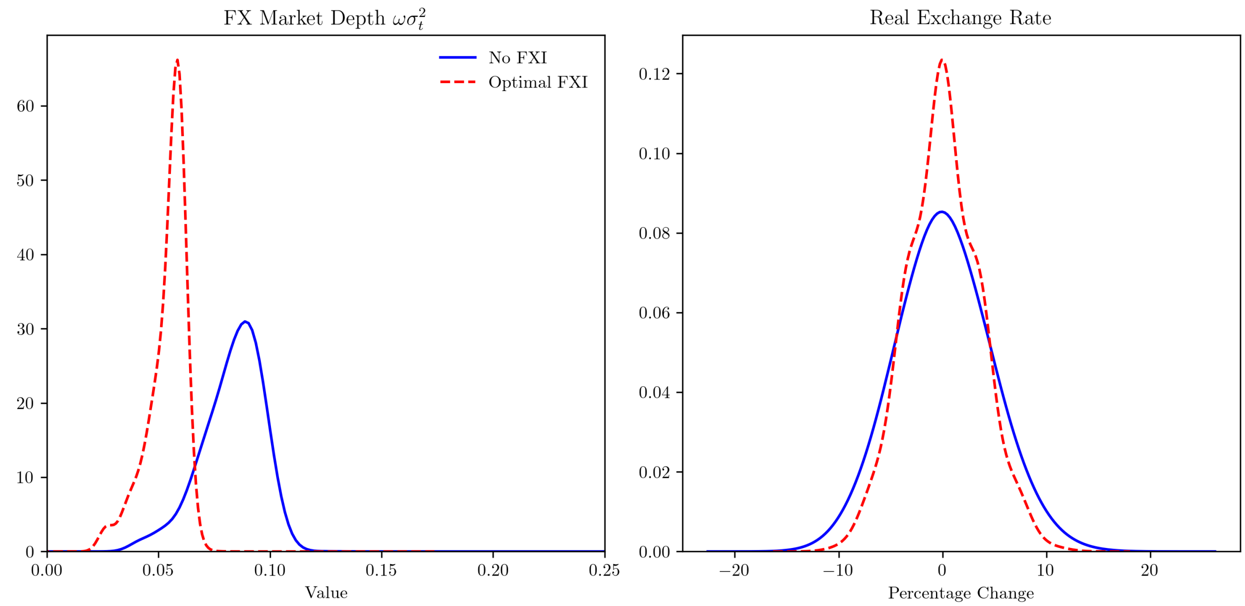

Figure 4: FX market conditions

Notes: The plot shows the densities for the FX market depth and the real exchange rate obtained from a simulation based on 2×10^6 quarters. The real exchange rate is defined as $\mathcal{Q}_t = \left[\alpha^\xi + (1-\alpha)^\xi \left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}$.

This is indeed possible, because FX markets become deeper, as illustrated in Figure 4. It shows the unconditional distribution of FX market depth $\omega\sigma_t^2$ (left panel) and the real exchange rate (right panel) under the two considered FXI regimes. Optimal FXI policy has a clearly stabilizing effect on the exchange rate. This is because, by counteracting portfolio flows, the central bank reduces the non-fundamental part of the exchange rate fluctuations that are inefficient from the planner's perspective. However, it does allow for efficient exchange rate fluctuations originating from endowment shocks, which explains why there is still sizable exchange rate variability even in the optimal policy regime. We can also see that, by reducing exchange rate risk, the optimal policy has a favorable effect on FX market depth.

The following remark summarizes these key steady state characteristics of the optimal FXI policy.

Remark 2. Compared to a no-FXI equilibrium, the stochastic steady state under the optimal time-consistent FXI policy is characterized by:

(a) a precautionary level of FX reserves and thus higher net foreign asset position,

(b) deeper FX markets as the central bank acts as an FX liquidity provider.

4.4 Dynamics

After examining the ergodic properties of equilibrium with and without FXI, we now shift our focus to the model's dynamics. The subject of interest in this regard are episodes of portfolio outflows or inflows and how they affect the economy, depending on whether FXI is used or not. To characterize an episode of portfolio capital outflows, we use the stochastic simulations described above, gathering all non-overlapping subsamples of a length of 17 quarters in which the economy experiences the peak outflow in the middle period. We then compute the average paths of relevant macroeconomic variables over all these extracted subsamples. We proceed analogously when analyzing the episodes of portfolio capital inflows.

Figure 5 depicts a typical episode of portfolio outflows computed as described above. In particular, the peak outflow amounts to about 8% of GDP and corresponds to period $t = 0$, with the presented window spanning from 8 quarters before ($t = -8$) to 8 quarters after ($t = 8$). In the absence of FX interventions (solid blue lines), a fall in the demand for domestic currency tightens external financial conditions and weakens the exchange rate. A higher UIP premium limits the extent to which households can protect their real spending by borrowing from abroad (the NFA as a percentage of GDP decreases only moderately), and so consumption contracts sharply.[11] Moreover, portfolio outflows make FX markets shallower as illustrated by the bottom right panel of Figure 5. To understand the intuition, recall that market depth is inversely related to the risk borne by financiers, which is captured by the conditional exchange rate volatility σ_t^2. Holding their foreign currency borrowing fixed, a depreciation of the domestic currency scales up the (expected) payoff on their domestic currency position. This proportional increase magnifies both the mean and the variance of percentage returns — just as leverage would — thereby raising the required risk compensation.[12]

If the central bank follows optimal time-consistent FXI policy (dashed red lines), the reaction of FX reserves mirrors that of portfolio outflows, although the offset is not full. At the peak of

[11] The NFA to GDP ratio falls only because the exchange rate depreciation outweighs a small increase in the foreign currency value of the NFA $B^*_{P,t}$.

[12] This point is closely related to Proposition 1(a).

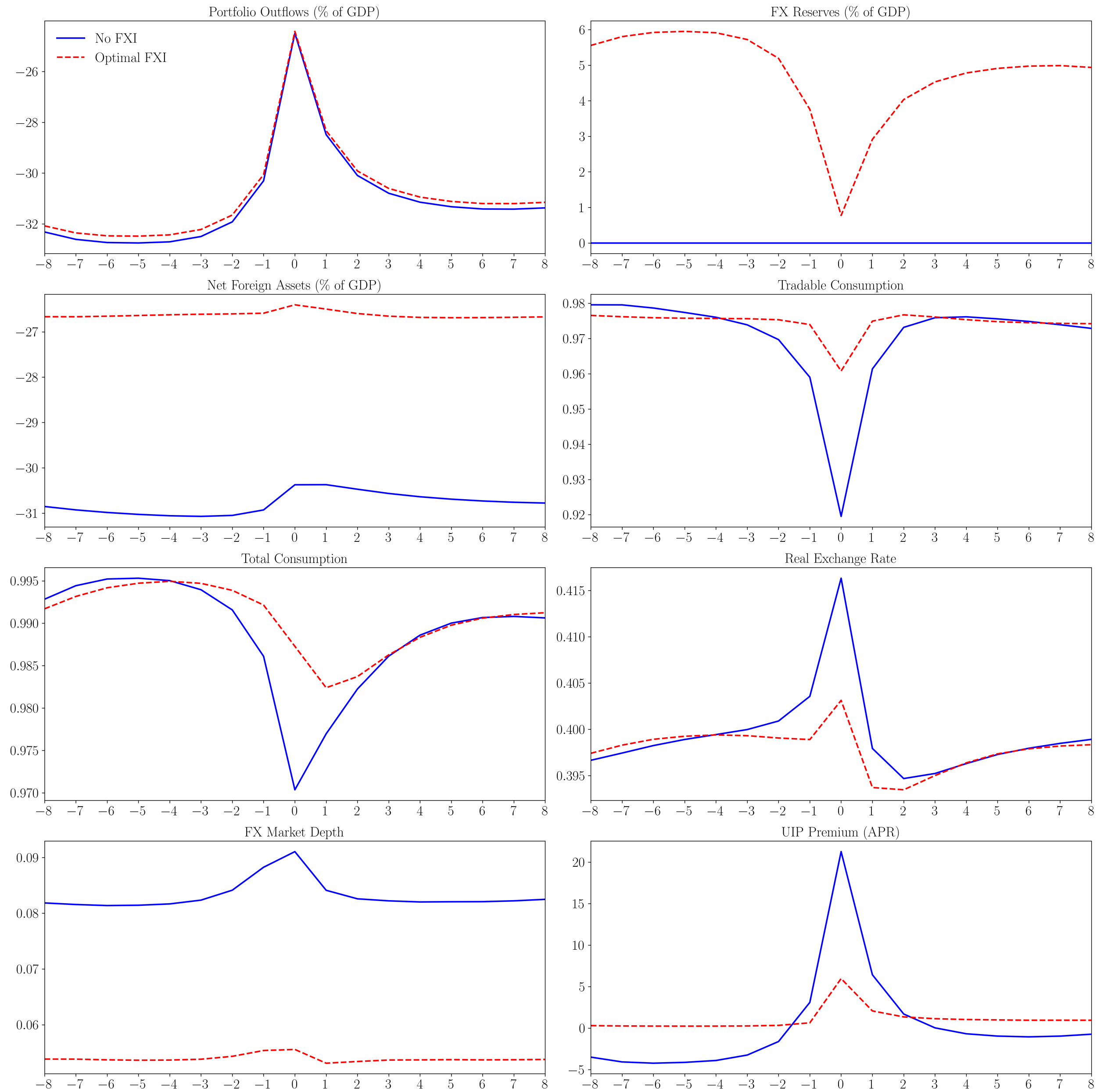

Figure 5: Portfolio outflow episode

Notes: The figure depicts the average paths of selected variables during an episode of portfolio outflows. One period corresponds to one quarter, and period 0 coincides with the peak outflow.

portfolio outflows in period $t = 0$, the central bank sells reserves worth around 5% of GDP, choosing not to run them down completely. Nevertheless, this policy helps stabilize the economy to the extent that it significantly reduces the exchange rate depreciation and limits the deterioration in external financing conditions. As a result, the fall in consumption is greatly reduced. Besides stabilizing the exchange rate, the optimal FXI policy also prevents a spike in FX market depth. In line with the explanation offered above, this mainly reflects that the intervention makes the currency stronger. However, preventing an increase in FX market tightness is also partially achieved by following a "keeping powder dry" strategy, i.e., intervening less than one-for-one. In particular, rather than depleting its FX reserves in period $t = 0$, thus better stabilizing the economy at the time when portfolio capital outflows culminate, the central bank chooses to keep their stock positive. By doing this, the monetary authority is better prepared to offset a possible further capital outflow next period, thus reducing the non-fundamental component of the exchange rate risk. This in turn increases the ability of risk-averse financiers to intermediate funds, thus making FX markets deeper.

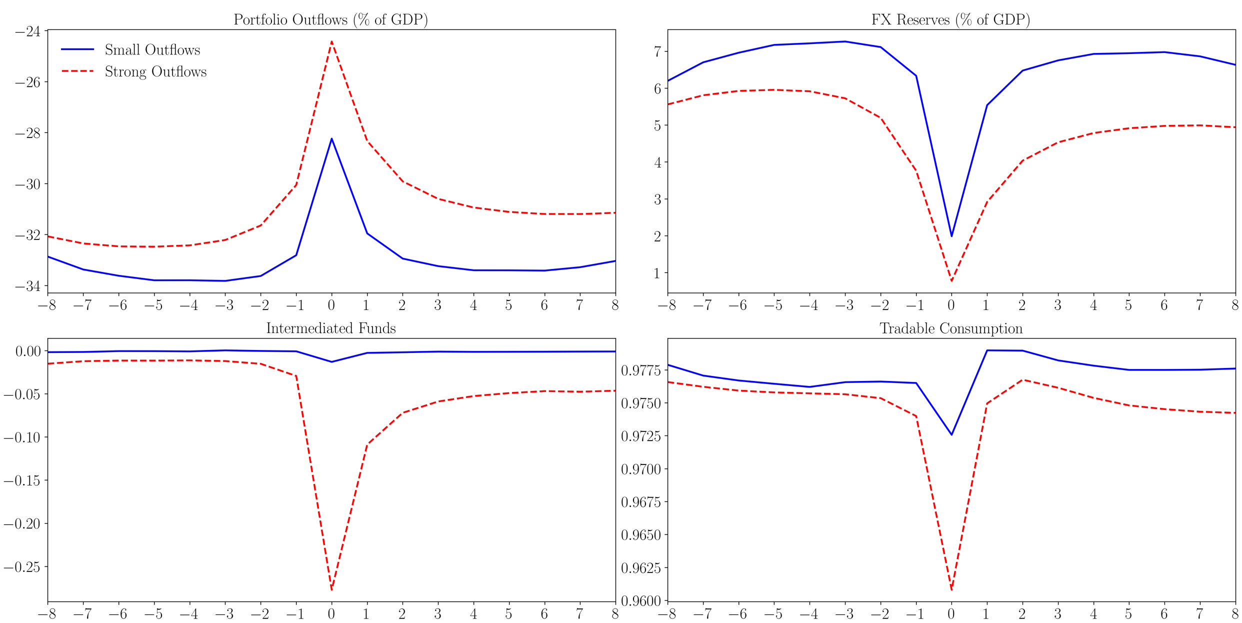

Figure 6: Optimal response to small and large portfolio outflows

Notes: The figure depicts the average paths of selected variables during episodes of small and large portfolio outflows. One period corresponds to one quarter and period 0 coincides with the peak outflow.

The extent to which the optimal FXI policy offsets contemporaneous portfolio outflows depends on the size of (expected) portfolio outflows. While this offset is partial in Figure 5, it can also be more aggressive, as Proposition 3 states. We illustrate this point in Figure

6, where we distinguish between episodes of relatively small (blue solid lines) and large (red dashed lines) portfolio outflows. While the depicted episodes of strong portfolio outflows are around four percentage points larger in terms of GDP than the episodes of relatively small outflows, the optimal FXI reaction is similar, implying a bigger offset. This asymmetry is reflected in the amount of funds intermediated by financiers $B^*_{F,t}$. For strong portfolio outflows, financiers increase their long exposure to domestic currency as portfolio outflows are only partially absorbed by central bank FX sales. This effect on financiers' balance sheets almost vanishes in the case of small portfolio outflows. Since the central bank's FX sales nearly fully offset the portfolio outflows, the net demand for domestic currency is essentially unchanged.

While the previous figures were concerned with portfolio outflows, Figure 7 depicts the average responses in the model during episodes of portfolio inflows. The central bank offsets contemporaneous portfolio inflows almost one-for-one and is therefore able to follow the first best policy more closely. The reason being that the lower bound on reserves becomes less relevant in episodes of inflows and, as a result, the policy response approaches the unconstrained case. Optimal policy effectively stabilizes the economy during inflow episodes and mutes the real exchange rate appreciation.

Note that, unlike portfolio outflows, portfolio inflows deepen FX markets. A corollary of this observation is that FX interventions are more effective in episodes of portfolio outflows than inflows as FX markets are generally shallower in the former case. In other words, the central bank that follows the optimal FXI policy gets more bang for the buck while selling reserves than when purchasing them. This can be also seen by comparing Figures 5 and 7, where the central bank conducts FX sales (purchases) worth 4.8% (6.6%) of GDP between periods $t = -8$ and $t = 0$. However, the real exchange rate at the peak of outflows in period $t = 0$ is 3.5% more appreciated under optimal FXI compared to the decentralized equilibrium, while it is around 3.4% weaker in the case of inflows. Therefore, in our capital outflow and inflow episodes, FX sales are around 44% more powerful than FX purchases in terms of their impact on the real exchange rate. We highlight this state-dependency of FXI effectiveness in the following remark.

Remark 3. Under the optimal time-consistent FXI policy, FX sales are more effective than FX purchases due to shallower FX markets.

Finally, we address the question to what extent FX interventions should be used to manage fundamental shocks, represented in our model by stochastic endowments. Figure 8 presents an average episode of endowment decreases. Starting with the case of no FXI (solid blue lines), the economy experiences a sharp contraction in consumption, which is only partially

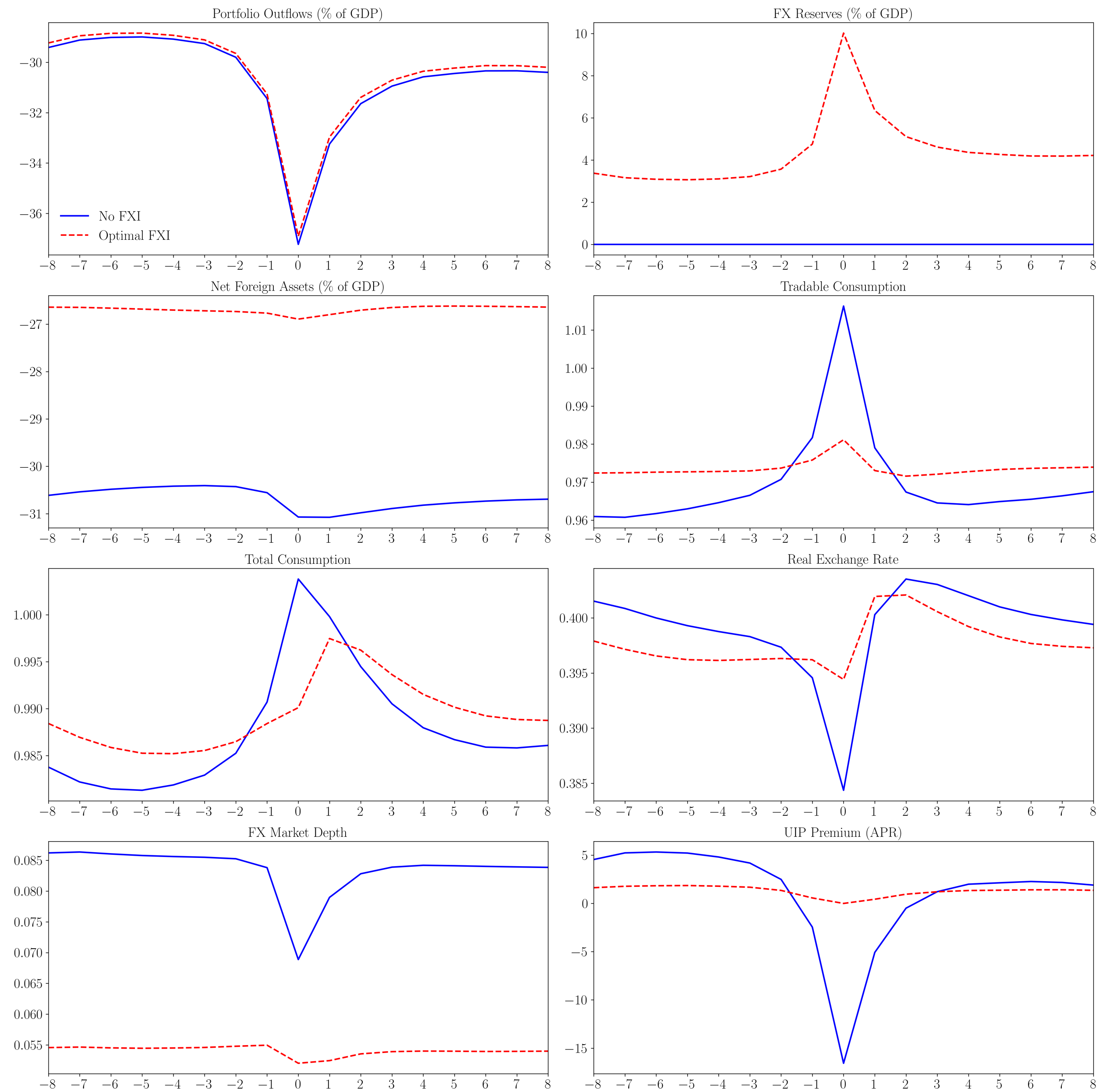

Figure 7: Portfolio inflow episode

Notes: The figure depicts the average paths of selected variables during an episode of portfolio inflows. One period corresponds to one quarter and period 0 coincides with the peak inflow.

cushioned by increased borrowing from abroad. The real exchange rate appreciates because the fall in nontradable endowment (and thus nontradable consumption) dominates the decrease in tradable consumption.[13] Since the exogenous processes for endowments and portfolio outflows are slightly negatively correlated (see Section 4.1), portfolio capital flees the country.

If FXI is conducted optimally, it is focused on stabilizing the international risk-sharing wedge, which otherwise would reflect the fluctuations in the portfolio capital and the country's net foreign asset position. However, the policy is not effective in limiting the fall in consumption that is driven by fundamental forces. Its effect on the exchange rate is also small as exchange rate adjustment is desirable to facilitate efficient expenditure switching. We summarize these observations in the following remark.

Remark 4. In response to fundamental shocks, the optimal time-consistent FXI policy focuses on stabilizing the international risk-sharing wedge, otherwise allowing the exchange rate to float.

4.5 Welfare Implications

So far, we have seen how optimal FXI policy is beneficial in stabilizing the economy by reducing excessive exchange rate volatility arising due to cross-border capital flows. In this section, we compute the welfare implications of this policy and compare them to other possible policy regimes.

As is standard in the literature, we express welfare gains in consumption equivalence units. Formally, let κ be the additional fraction of consumption that households in the benchmark economy b would have to receive to make them indifferent to living in an economy with alternative policy p. Given our assumptions on the utility functional, κ_p can be computed as

$$\kappa_p = \left(\frac{\tilde{V}_p\left(\overline{B}^*\right)}{\tilde{V}_b\left(\overline{B}^*\right)} \right)^{\frac{1}{1-\sigma}} - 1, \tag{16}$$

where $\tilde{V}_i$ is the household lifetime utility under policy regime $i \in b, p$, conditional on the initial bond holdings that coincide with their steady state value in the benchmark economy $\overline{B}^*$, and averaged over the stationary distribution of the exogenous state process. Appendix C provides derivation of this welfare measure. Since the measure conditions on the benchmark

[13] Figure C.1 in the Appendix shows episodes in which the tradable endowment falls but the nontradable endowment increases, in which case the real exchange rate depreciates.

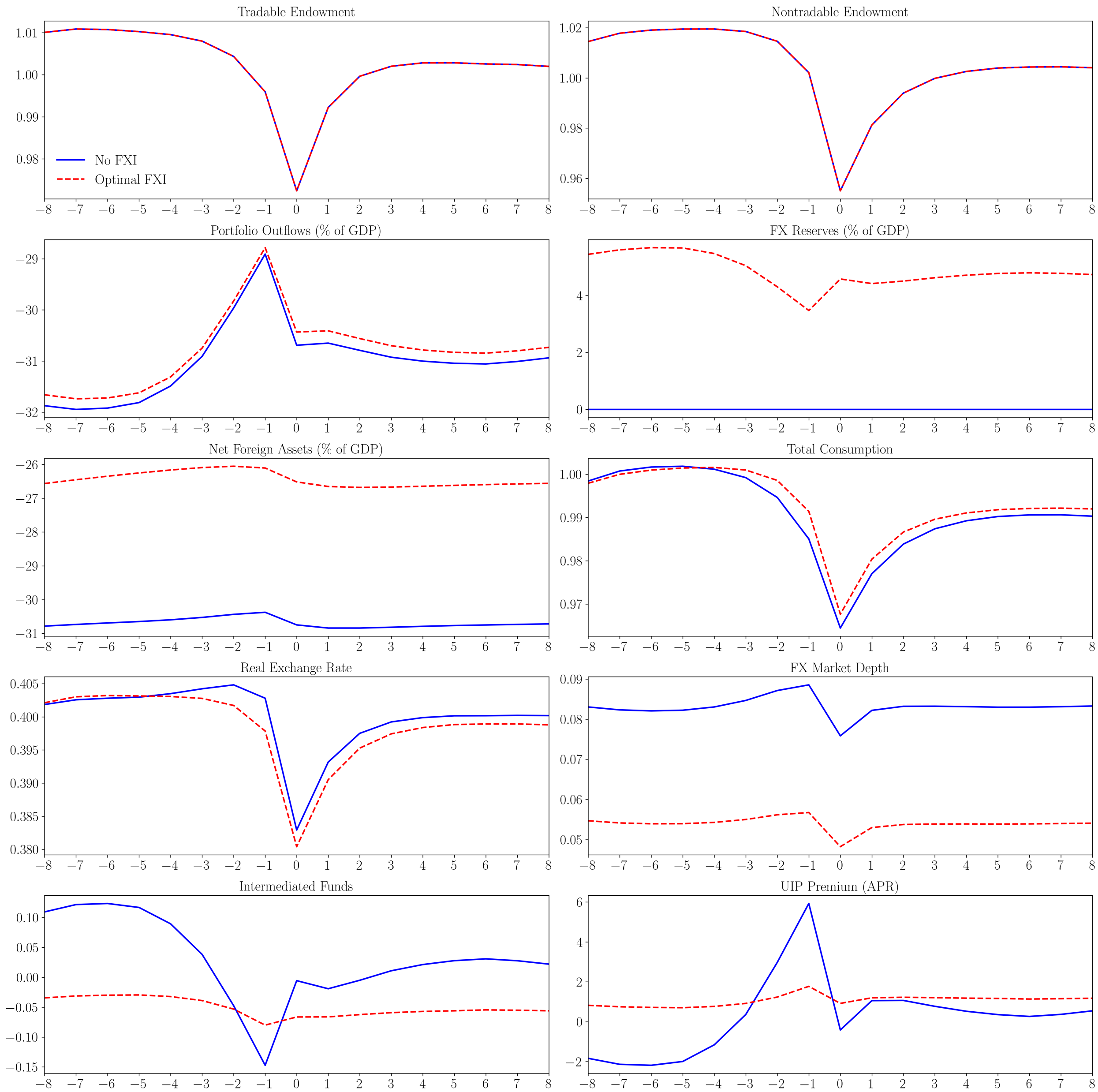

Figure 8: Episodes of negative endowment shocks

Notes: The figure depicts the average paths of selected variables during episodes of negative endowment shocks. One period corresponds to one quarter and period 0 coincides with the peak of the negative endowment shock.

economy's steady state, the welfare gain κ_p takes into account the transition dynamics to the alternative policy regime, and thus is not simply based on an unconditional comparison of the two steady states.

As the benchmark economy, we take the version of the model calibrated to Malaysia, which accounts for the observed FX interventions conducted by Bank Negara Malaysia. We compare it to several alternative FXI policy regimes. These include the optimal time-consistent FXI policy as described in Section 3 and two policies in which the central bank commits to a simple FXI rule respecting the non-negativity constraint on reserves. The reason for considering simple rules is that the optimal intervention formula (12) is very complicated and includes expectations, which may make it difficult to implement.[14] Finally, to provide an upper bound on welfare gains, including those achievable with commitment, we consider a policy representing the first best.

More specifically, to solve for the first best policy, we assume that FXI is characterized by equation (9) and thereby the central bank is able to borrow in FX reserves. The associated equilibrium is stationary since households are impatient ($\beta R^* < 1$) and there is a lower bound on the net foreign asset position guaranteeing that the transversality condition is satisfied.[15] As regards the simple rules, the first one is inspired by the first best policy, with the crucial distinction that the central bank is subject to the NNCR

$$B^*_{M,t} = \max\left(B^*_t - B^*_{P,t}, 0\right). \tag{17}$$

Note that this policy, which we will refer to as the UIP rule, eliminates the international risk-sharing wedge whenever feasible. The second simple rule that we consider, referred to as the portfolio flow rule, offsets only the exogenous portfolio flow position

$$B^*_{M,t} = \max\left(-B^*_{P,t}, 0\right). \tag{18}$$

These two FXI rules are interesting for two reasons. First, as outlined in Section 3, the optimal time-consistent policy resembles the UIP rule, but deviates from it by taking into account the possibility of the NNCR in the future. A quantitative comparison of these two policy regimes helps us evaluate the significance of these forward-looking considerations. Second, comparing the UIP rule to the portfolio flow rule allows us to assess the welfare implications of stabilizing two components of the international risk-sharing wedge: the en-

[14] That feature of the optimal time-consistent FXI policy in our model is shared with the optimal tax on debt formula that decentralizes the planner's allocation in models with collateral constraints, see, e.g., Bianchi & Mendoza (2018).

[15] For more details, see the recursive representation of the first best provided in Appendix C.

dogenous one associated with the net foreign asset position B_t^* versus the exogenous one represented by the portfolio capital position $B_{P,t}^*$.

Table 2 reports the welfare gains for the four policy regimes described above. Moving to the optimal time-consistent FXI policy improves welfare by 0.25% of consumption. This comes quite close to the maximum achievable gains of 0.29% represented by the first best. The rule-based policy regimes also enhance welfare, albeit to a lesser extent, with the UIP and portfolio flow rules yielding welfare gains of 0.15% and 0.05%, respectively. Overall, the optimal use of FXI significantly outperforms the considered rules and is only slightly worse compared to the first best. This highlights that the forward-looking element in the optimal time-consistent FXI policy is quantitatively important and that the scope for further gains achievable with commitment is limited. However, as we explain later, the latter part of this conclusion crucially depends on the initial level of FX reserves.

Table 2: Welfare Implications of Alternative FXI Policies

	Portfolio Rule	UIP Rule	Time-Consistent Optimal FXI	First Best
Welfare Gain	0.045	0.150	0.248	0.288

Notes: The welfare gain is computed according to formula (16) and expressed in percentages. The UIP and portfolio flow rules are defined by equations (17) and (18), respectively.

To better understand the differences between the considered FXI policies, it is useful to examine their ergodic implications. Table 3 presents a selection of first and second moments obtained from long stochastic simulations.[16] One point to emphasize is that the average stock of FX reserves varies significantly across the different policy regimes. This further translates into the differences in the net foreign assets position as FX reserves, by increasing the net demand for foreign currency, partially crowd out borrowing by domestic households.

We have already seen in section 4.3 that optimal time-consistent FXI policy results in moderately positive FX reserves. Due to the precautionary motive, their average level is higher than under the closely related UIP rule, but the difference is small. However, this is enough to drastically reduce the frequency of episodes in which FX reserves are depleted. FX markets are also deeper and the volatility of the risk-sharing wedge is substantially reduced if FXI is conducted optimally rather than following a simple UIP rule.

[16]Figure C.2 in Appendix C compares how the economy behaves under the different policy regimes during episodes of portfolio outflows.

Table 3: Unconditional Moments under Alternative FXI Regimes

	No FXI		Optimal FXI		UIP Rule		Portfolio Flow Rule		First Best	
Variable Name	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Consumption	0.988	0.025	0.990	0.020	0.990	0.019	1.000	0.022	0.987	0.023
Net Foreign Assets	-0.307	0.008	-0.266	0.011	-0.271	0.017	-0.006	0.008	-0.337	0.007
FX Reserves	0.000	0.000	0.046	0.038	0.041	0.038	0.301	0.043	-0.029	0.044
Real Exchange Rate	0.399	0.014	0.398	0.012	0.398	0.013	0.390	0.011	0.400	0.011
FX Market Depth	0.083	0.013	0.054	0.008	0.058	0.009	0.047	0.008	0.042	0.010
UIP Premium	0.007	0.127	0.012	0.028	0.012	0.032	0.012	0.012	0.000	0.000
Reserve Depletion Freq.	–		0.020		0.168		0.000		–	

Notes: The moments are obtained by simulating each FXI regime over 2×10^6 quarters. NFA and FX reserves are expressed as a fraction of annual GDP. The risk sharing wedge is annualized.

If the FXI rule responds only to the portfolio component of the UIP premium, FX reserves are very high. This simply reflects the fact that, according to our calibration, there is positive appetite for domestic currency on average ($B^*_{P,t}$ is typically negative) while the economy's net foreign assets position (i.e, the part of the UIP premium that the rule ignores) is negative as domestic households are relatively impatient ($\beta R^* < 1$). The high level of reserves implies that the risk of depleting them is virtually nil. In consequence, exchange rate volatility is much lower and FX markets are deeper compared to the optimal policy. This finally translates into smaller fluctuations in the international risk sharing wedge, even though the rule does not respond to its endogenous component associated with the NFA. However, these stabilization gains come at a cost of low international borrowing (the steady state NFA position is nearly balanced), which is the main reason why this policy is associated with relatively small welfare gains.

In the first best, the central bank does not need to accumulate precautionary FX reserves since its use of FXI is unconstrained. In fact, it typically borrows from abroad in order to accommodate the borrowing needs of domestic households, so that the country's NFA position is lower than in other FXI regimes. The first best policy perfectly eliminates the risk-sharing wedge but does not stabilize the exchange rate much beyond what the portfolio rule achieves, reflecting the motive to allow for efficient expenditure switching in response to fundamental shocks. As a result, FX markets are less shallow than under any other of the considered policy, but not perfectly deep.

One complication in interpreting the welfare gains reported in Table 2 is that the various policy regimes imply different steady states. Recall that the initial NFA position used to compute welfare gains, as defined in (16), corresponds to the observed economy. This economy features a relatively high level of reserves (30% of annual GDP) which translates into a similarly elevated NFA position of about 2% of annual GDP. In the context of our model, these reserve levels are excessive. Consequently, the gradual reduction of reserves during the transition contributes positively to the welfare gains reported in Table 2. Figure 9 illustrates this point by comparing the value functions under different policy regimes as a function of the initial NFA position.

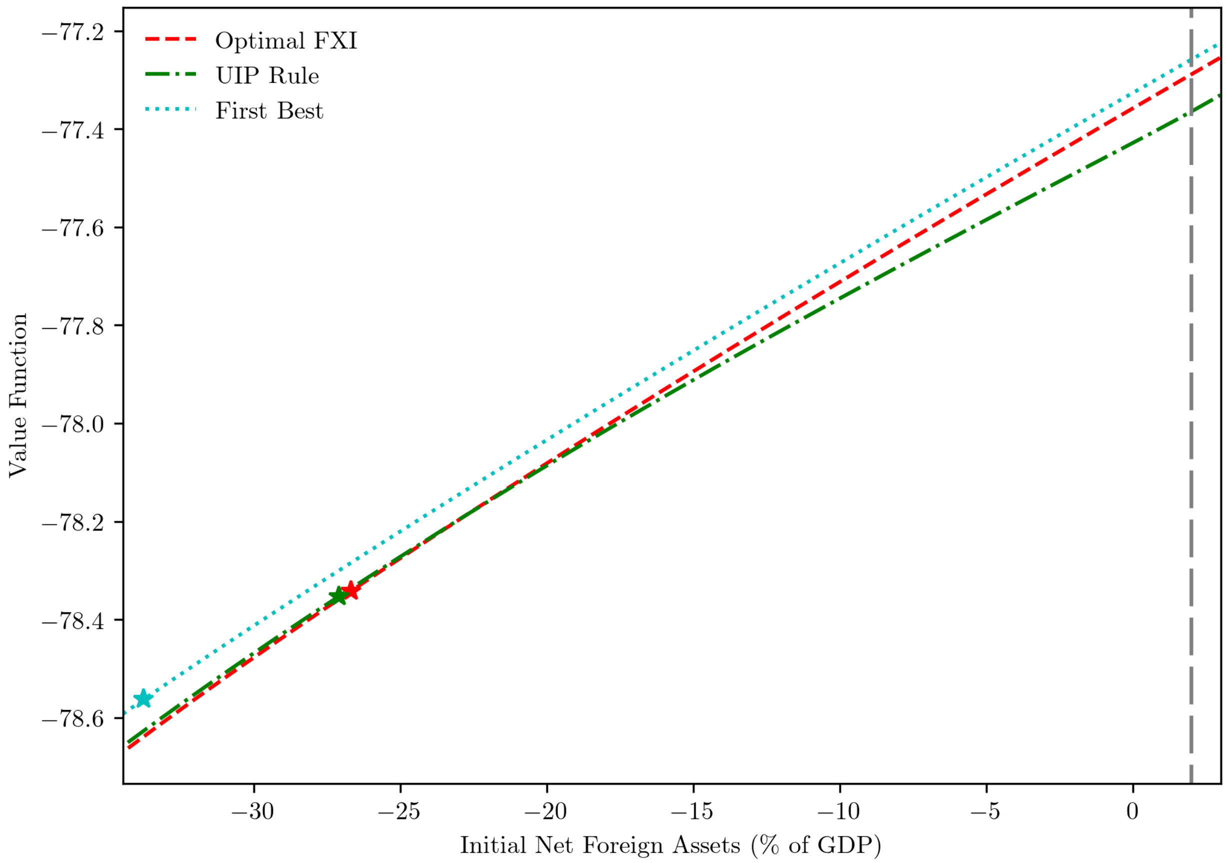

Figure 9: Value Functions of Alternative FXI Policies

Notes: The figure depicts value functions $\tilde{V}_i$ of alternative policy regimes. The vertical dashed line corresponds to the net foreign asset position (or bond holdings) in the stochastic steady state of the calibrated economy. The stars indicate the net foreign asset positions in the stochastic steady state of the respective alternative FXI policies.

To isolate the welfare effects beyond those arising from transitional reserve decumulation, we conduct a comparison where policy regimes share a common steady-state NFA position. Specifically, we redefine the benchmark economy to be the optimal time-consistent policy and adjust the stock of FX reserves in each alternative policy regime to match the steady-state NFA position under this benchmark. The alternative regimes we consider are: (1) the

UIP rule defined in (17), modified with a constant term to achieve the desired NFA level,[17] (2) a regime with a fixed stock of FX reserves and no interventions, referred to as "no FXI", and (3) a policy aimed at smoothing the real exchange rate, referred to as the RER rule, which is defined as

$$B^*_{M,t} = \max\left(\gamma_{RER} + \phi\left(\frac{\overline{\mathcal{Q}} - \mathcal{Q}_t}{\overline{\mathcal{Q}}}\right), 0\right) \tag{19}$$

where $\gamma_{RER} > 0$ is set to ensure the steady-state NFA matches that under the optimal time-consistent policy, $\overline{\mathcal{Q}}$ is the steady-state real exchange rate, and $\phi \geq 0$ guides how strongly the central bank purchases (sells) FX reserves when the real exchange rate is stronger (weaker) than in steady state. In the RER rule, ϕ is chosen to maximize welfare.[18]

Table 4: Welfare Costs of FXI Rules relative to optimal time-consistent policy

	No FXI	RER Rule	UIP Rule
Welfare Cost	0.029	0.011	-0.003

Notes: The welfare cost is expressed in percentages and computed as $-\kappa_p$ in formula (16) where the benchmark economy b is the optimal time-consistent policy. All considered FXI rules have identical NFA positions in steady state as described in the text. The RER rule is defined by equation (19).

Table 4 presents the welfare costs of the various FXI rules relative to the optimal time-consistent policy with identical steady-state NFA positions across regimes. Notably, the welfare differences reported here are substantially smaller than those in Table 2, underscoring the significant role played by the average level of FX reserves in shaping welfare outcomes. Under the optimal time-consistent policy, welfare expressed in consumption equivalence units is approximately 3 basis points higher than under a regime with no interventions. Furthermore, the (optimized) RER rule has a welfare cost of around a basis point relative to the optimal time-consistent policy. The RER rule may seem appealing to policymakers, as it relies on readily observable variables. However, its simplicity prevents it from distinguishing between inefficient exchange rate volatility caused by portfolio flow shocks and efficient volatility driven by endowment shocks. In contrast, the UIP rule does make this distinction and delivers slightly higher welfare than the optimal time-consistent policy. This outcome reflects the presence of time inconsistency in the model: the optimal time-consistent policy does not coincide with the optimal policy under commitment.

As discussed in Section 3, a key feature of the fully optimal FXI policy is a "forward guidance"

[17]The adjusted rule is $B^*_{M,t} = \max(\gamma_{UIP} + B^*_t - B_{P,t^*}, 0)$ where γ_{UIP} is the added constant term.

[18]Note that the "no FXI" regime corresponds to the special case where $\phi = 0$.

mechanism that requires commitment. Specifically, under commitment, the optimal policy prescribes lower FX reserves in response to reserve depletion in the previous period. This promise is not possible under time consistency, which becomes a limitation when reserves are near their lower bound. Indeed, this constraint is relevant in our setting, as average FX reserves under the time-consistent policy are only around 5% of GDP.

The following remark concludes our analysis of the welfare implications of alternative FXI regimes.

Remark 5. The optimal time-consistent FXI policy delivers robust welfare gains over simple rule-based alternatives, with performance close to the first best. However, a large part of these gains stem from the gradual decumulation of initially excessive FX reserves in the benchmark economy. Once this transitional effect is removed by equalizing steady-state NFA across regimes, the welfare differences narrow significantly. Furthermore, a rule targeting ex-ante UIP deviations can outperform the time-consistent policy, highlighting the importance of time inconsistency and commitment.

5 Conclusions

This paper analyzes the optimal use of foreign exchange intervention (FXI) in a small open economy with endogenous FX market depth and a lower bound on FX reserves. We find that optimal time-consistent FXI can effectively reduce exchange rate volatility caused by portfolio flow shocks, thereby stabilizing deviations from uncovered interest parity (UIP) and improving market depth. The optimal response hinges on the anticipated path of portfolio flows: when outflows are moderate and expected to ease, aggressive intervention is optimal, whereas in the face of larger, likely persistent outflows, the desire to preserve a precautionary reserve level dominates.

Our quantitative analysis further reveals that the effectiveness of FXI is state-dependent in the optimal policy regime. Specifically, FX purchases tend to have a lower impact on the exchange rate as they occur during periods of capital inflows and deeper FX markets, while FX sales are relatively more effective as they take place amid portfolio outflows and shallower markets. We find that the optimal time-consistent FXI policy is associated with substantial welfare gains, at least if the economy starts out with a sufficiently high level of FX reserves. Committing to FXI rules that aim to smooth UIP deviations improves welfare, albeit to a lesser extent. However, when reserves are relatively low, this result reverses as time inconsistency, stemming from the lower bound on reserves, becomes more pronounced.

Overall, our analysis provides a tractable quantitative framework to analyze the use of FXI. It highlights the importance of a precautionary motive behind reserve accumulation and state dependency in the conduct of FXI policy. Future research could build on this quantitative framework and incorporate additional financial and nominal rigidities to see how their presence modifies the optimal use of FXI.

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Appendix

A Proofs

Proof of Proposition 1 By the resource constraint, note that $\partial B_t^*/\partial B_{P,t}^* = -\partial C_{T,t}/\partial B_{P,t}^*$. We proceed by proving the two properties of the proposition. Starting with point (a), the conditional exchange rate volatility can be rewritten as:

$$\sigma_t^2 = R_t^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \left(\frac{1}{\mathbb{E}_t\left[\Theta_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]}\right)^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right),$$

where we have used the Euler equation in the second line. The partial derivative with respect to consumption in period t is given by

$$\frac{\partial\sigma_t^2}{\partial C_{T,t}} = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right) = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\sigma_t^2.$$

Now, take the partial derivative with respect to $B_{P,t}^*$ and use the previous equations to get the result:

$$\frac{\partial\sigma_t^2}{\partial B_{P,t}^*} = \frac{\partial\sigma_t^2}{\partial C_{T,t}}\frac{\partial C_{T,t}}{\partial B_{P,t}^*} = 2\overbrace{\left(\frac{u_{11,t}}{u_{1,t}}\right)}^{<0}\sigma_t^2\overbrace{\left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)}^{<0} > 0$$

Turning to property (b), suppose by contradiction that $\frac{\partial\mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*} \geq 0$, which then would imply

$$\begin{aligned}\frac{\partial\mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B_{P,t}^*} &= \frac{\partial\mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial C_{T,t}}\frac{\partial C_{T,t}}{\partial B_{P,t}^*} = \left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\Theta_{t+1}\right]\frac{\partial C_{T,t}}{\partial B_{P,t}^*} \geq 0 \\ &\Rightarrow \frac{\partial C_{T,t}}{\partial B_{P,t}^*} \geq 0 \Rightarrow \frac{\partial B_t^*}{\partial B_{P,t}^*} \leq 0.\end{aligned}$$

However, this violates our assumption $\frac{\partial B_t^*}{\partial B_{P,t}^*} > 0$, and thus we must have $\frac{\partial\mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*} < 0$. ■

Derivation of $\partial B_t^*/\partial B_{P,t}^*$ Take the partial derivative with respect to $B_{P,t}^*$ of the IRS condition

$$\omega\frac{\partial\sigma_t^2}{\partial B_{P,t}^*}\left(B_t^* - B_{P,t}^*\right) + \omega\sigma_t^2\left(\frac{\partial B_t^*}{\partial B_{P,t}^*} - 1\right) = R^*\frac{\partial\mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B_{P,t}^*}.$$

Next insert the expressions for $\frac{\partial \sigma_t^2}{\partial B_{P,t}^*}$ and $\frac{\partial \mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*}$ from the proof of Proposition 1 to obtain

$$2\omega\left(\frac{u_{11,t}}{u_{1,t}}\right)\sigma_t^2\left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)\left(B_t^*-B_{P,t}^*\right)+\omega\sigma_t^2\left(\frac{\partial B_t^*}{\partial B_{P,t}^*}-1\right)=-R^*\left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\Theta_{t+1}\right]\frac{\partial B_t^*}{\partial B_{P,t}^*},$$

and insert the IRS condition $R^*\mathbb{E}_t\left[\Theta_{t+1}\right]=1+\omega\sigma_t^2\left(B_t^*-B_{P,t}^*\right)$ on the RHS to get

$$2\omega\left(\frac{u_{11,t}}{u_{1,t}}\right)\sigma_t^2\left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)\left(B_t^*-B_{P,t}^*\right)+\omega\sigma_t^2\left(\frac{\partial B_t^*}{\partial B_{P,t}^*}-1\right)=-\left(-\frac{u_{11,t}}{u_{1,t}}\right)\frac{\partial B_t^*}{\partial B_{P,t}^*}\left(1+\omega\sigma_t^2\left(B_t^*-B_{P,t}^*\right)\right).$$

After rearranging, we have

$$\omega\sigma_t^2\frac{\partial B_t^*}{\partial B_{P,t}^*}\left[\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_t^*-B_{P,t}^*\right)+\frac{1}{\omega\sigma_t^2}\right)+1\right]=\omega\sigma_t^2.$$

Finally, rewrite the equation as an expression for $\frac{\partial B_t^*}{\partial B_{P,t}^*}$

$$\frac{\partial B_t^*}{\partial B_{P,t}^*}=\frac{1}{1-\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_{P,t}^*-B_t^*\right)-\frac{1}{\omega\sigma_t^2}\right)},$$

and therefore the sufficient condition for $\frac{\partial B_t^*}{\partial B_{P,t}^*}>0$ is given by

$$\frac{\partial B_t^*}{\partial B_{P,t}^*}>0\Leftrightarrow\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_{P,t}^*-B_t^*\right)-\frac{1}{\omega\sigma_t^2}\right)<1.$$

Proof of Theorem 1 The Lagrangian of the second-best can be written as

$$\begin{aligned}\mathcal{L}=\mathbb{E}_t\sum_{s=0}^{\infty}\beta^{t+s}\Bigg[&u\left(B_{t+s-1}^*R^*-B_{t+s}^*+Y_{T,t+s},Y_{N,t+s}\right)\\&-\lambda_{t+s}\left(B_{P,t+s}^*+\frac{1}{\omega\sigma_{t+s}^2}\left(R^*\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right]-1\right)-B_{t+s}^*\right)\Bigg].\end{aligned}$$

The first-order condition with respect to B_t^* yields (11) in the main text

$$\begin{aligned}&u_{1,t}-\lambda_t\left(1-\frac{\partial\sigma_t^2}{\partial B_t^*}\frac{-1}{\omega(\sigma_t^2)^2}\left(R^*\mathbb{E}_t\left[\Theta_{t+1}\right]-1\right)-\frac{R^*}{\omega\sigma_t^2}\mathbb{E}_t\left[\frac{\partial\Theta_{t+1}}{\partial B_t^*}\right]\right)\\=&\beta R^*\mathbb{E}_t\left[u_{1,t+1}\right]-\beta\mathbb{E}_t\left[\lambda_{t+1}\frac{\partial\sigma_{t+1}^2}{\partial B_t^*}\frac{-1}{\omega\left(\sigma_{t+1}^2\right)^2}\left(R^*\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]-1\right)\right]-\beta\mathbb{E}_t\left[\lambda_{t+1}\frac{R^*}{\omega\sigma_{t+1}^2}\mathbb{E}_{t+1}\left[\frac{\partial\Theta_{t+2}}{\partial B_t^*}\right]\right].\end{aligned}$$

Note that σ^2_{t+1} can be written as

$$\begin{aligned}\sigma^2_{t+1} &= R^2_{t+1} var_{t+1}\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right) \\ &= \left(\frac{1}{\mathbb{E}_{t+1}\left[\Theta_{t+2}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right]}\right)^2 var_{t+1}\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right) \\ &= \frac{\left(u_{1,t+1}\right)^2}{\left(\mathbb{E}_{t+1}\left[\beta\frac{u_{1,t+2}}{\mathcal{E}_{t+2}}\right]\right)^2} var_{t+1}\left(\frac{1}{\mathcal{E}_{t+2}}\right),\end{aligned}$$

which yields the following partial derivative with respect to B^*_t

$$\frac{\partial\sigma^2_{t+1}}{\partial B^*_t} = 2R^*\left(\frac{u_{11,t+1}}{u_{1,t+1}}\right)\sigma^2_{t+1} < 0. \tag{A.1}$$

Furthermore, the partial derivative of the stochastic discount factor Θ_{t+2} with respect to B^*_t is given by

$$\frac{\partial\Theta_{t+2}}{\partial B^*_t} = R^*\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\Theta_{t+2} > 0. \tag{A.2}$$

Next, insert (A.1) and (A.2) into the first-order condition derived above to obtain

$$\begin{aligned}u_{1,t} &= \beta R^*\mathbb{E}_t\left[u_{1,t+1}\right] \\ &\quad - \beta R^*\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)2\left(B^*_{t+1} - B^*_{P,t+1}\right)\right] - \beta R^*\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\frac{R^*}{\omega\sigma^2_{t+1}}\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]\right].\end{aligned}$$

Insert the period $t+1$ IRS condition $\frac{R^*}{\omega\sigma^2_{t+1}}\mathbb{E}_{t+1}\left[\Theta_{t+2}\right] = \frac{1}{\omega\sigma^2_{t+1}} + \left(B^*_{t+1} - B^*_{P,t+1}\right)$ into the third term on the RHS of the equation above to get

$$\begin{aligned}u_{1,t} = \beta R^*\mathbb{E}_t\Bigg[u_{1,t+1} &- \lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)2\left(B^*_{t+1} - B^*_{P,t+1}\right) \\ &- \lambda_{t+1}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(\frac{1}{\omega\sigma^2_{t+1}} + \left(B^*_{t+1} - B^*_{P,t+1}\right)\right)\Bigg].\end{aligned}$$

Simplifying and rewriting yields

$$1 - R^*\mathbb{E}_t\left[\Theta_{t+1}\right] = \frac{\beta R^*}{u_{1,t}}\mathbb{E}_t\left[\lambda_{t+1}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(3\left(B^*_{P,t+1} - B^*_{t+1}\right) - \frac{1}{\omega\sigma^2_{t+1}}\right)\right].$$

Finally, plugging in the period t IRS condition $R^*\mathbb{E}_t\left[\Theta_{t+1}\right] = 1 + \omega\sigma^2_t\left(B^*_t - B^*_{P,t} - B^*_{M,t}\right)$

and rearranging gives

$$B^*_{M,t} = B^*_t - B^*_{P,t} + \frac{\beta R^*}{\omega\sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega\sigma_{t+1}^2} \right) \right],$$

which completes the proof. ■

Proof of Proposition 2 By Theorem 1, the optimal level of reserves is higher relative to the first best if and only if

$$\begin{aligned}
& B^*_{M,t} > B^*_t - B^*_{P,t} \\
\Leftrightarrow & \frac{\beta R^*}{\omega\sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega\sigma_{t+1}^2} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(-3B^*_{F,t+1} - \frac{1}{\omega\sigma_{t+1}^2} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(\omega\sigma_{t+1}^2 B^*_{F,t+1} + \frac{1}{3} \right) \right] < 0,
\end{aligned}$$

where we have made use of $B^*_{F,t} = B^*_{t+1} - B^*_{P,t+1}$. Next, use the IRS condition $R^*\mathbb{E}_{t+1}[\Theta_{t+2}] - 1 = \omega\sigma_{t+1}^2 B^*_{F,t+1}$ to obtain

$$\begin{aligned}
& \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(R^*\mathbb{E}_{t+1}[\Theta_{t+2}] - \frac{2}{3} \right) \right] < 0 \\
\Leftrightarrow \; & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right] \mathbb{E}_t \Big[R^*\mathbb{E}_{t+1}[\Theta_{t+2}] \Big] + cov \left(\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^*\mathbb{E}_{t+1}[\Theta_{t+2}] \right) \\
& \qquad < \frac{2}{3} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right].
\end{aligned}$$

Finally, divide by $\mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]$ on both sides and use the law of iterated expectations to get

$$R^*\mathbb{E}_t[\Theta_{t+2}] < \frac{2}{3} - \frac{cov \left(\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^*\mathbb{E}_{t+1}[\Theta_{t+2}] \right)}{\mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]},$$

which completes the proof. ■

The third term on the RHS of (12) is positive if $-3B^*_{F,t} - 1/\omega\sigma_t^2 > 0$ for periods t in which $\lambda_t > 0$ (and $B^*_{M,t} = 0$), where we have used $B^*_{F,t} = B^*_t - B^*_{P,t}$. This condition can be rewritten as

$$\omega\sigma_t^2 B^*_{F,t} < -\frac{1}{3}. \tag{A.3}$$

Furthermore, the modified UIP condition (3) and Euler equation (1) can be combined to

$$R^{*}\mathbb{E}_{t}\left[\Theta_{t+1}\right]-1=\omega\sigma_{t}^{2}B_{F,t}^{*}.$$

Use the equation above and (A.3) to obtain:

$$R^{*}\mathbb{E}_{t}\left[\Theta_{t+1}\right]<\frac{2}{3}.$$

Derivation of the Absolute Risk Aversion The marginal utility with respect to tradables is given by

$$u_{1,t}=\alpha C_{t}^{-\sigma}\left(\frac{C_{t}}{C_{T,t}}\right)^{\frac{1}{\xi}}=\alpha C_{t}^{\frac{1-\sigma\xi}{\xi}}C_{T,t}^{-\frac{1}{\xi}}.$$

Taking the derivative with respect to tradables again yields

$$u_{11,t}=\alpha C_{t}^{\frac{1-\sigma\xi}{\xi}}C_{T,t}^{-\frac{1}{\xi}}\left(\frac{1-\sigma\xi}{\xi}C_{t}^{-1}\frac{\partial C_{t}}{\partial C_{T,t}}-\frac{1}{\xi}C_{T,t}^{-1}\right)=\alpha C_{t}^{\frac{1-\sigma\xi}{\xi}}C_{T,t}^{-\frac{1}{\xi}}\left(\frac{1-\sigma\xi}{\xi}C_{t}^{-1}\alpha\left(\frac{C_{t}}{C_{T,t}}\right)^{\frac{1}{\xi}}-\frac{1}{\xi}C_{T,t}^{-1}\right),$$

which allows us to characterize the absolute risk aversion as

$$\frac{-u_{11,t}}{u_{1,t}}=\frac{\sigma\xi-1}{\xi}\alpha C_{t}^{\frac{1-\xi}{\xi}}C_{T,t}^{-\frac{1}{\xi}}+\frac{1}{\xi}C_{T,t}^{-1}.$$

Derivation of the Price Index Rewrite the intratemporal optimality condition as

$$C_{T,t}=\left(\frac{\alpha}{1-\alpha}\right)^{\xi}C_{N,t}\mathcal{E}_{t}^{-\xi}.$$

Define the price index P_t as the price of aggregat consumption good C_t. Making use of the equation above, we can then write

$$\left(\frac{\alpha}{1-\alpha}\right)^{\xi}C_{N,t}\mathcal{E}_{t}^{1-\xi}+C_{N,t}=P_{t}C_{t}\Leftrightarrow C_{N,t}=\frac{P_{t}C_{t}}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_{t}^{1-\xi}+1\right]}$$

and similarly

$$C_{T,t}\mathcal{E}_{t}+C_{T,t}\left(\frac{\alpha}{1-\alpha}\right)^{-\xi}\mathcal{E}_{t}^{\xi}=P_{t}C_{t}\Leftrightarrow C_{T,t}=\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_{t}^{-\xi}\frac{P_{t}C_{t}}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_{t}^{1-\xi}+1\right]}.$$

Next, plug these expressions into the consumption aggregator to obtain

$$
\begin{aligned}
C_t &= \left[\alpha\left(C_{T,t}\right)^{\frac{\xi-1}{\xi}} + (1-\alpha)\left(C_{N,t}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \\
&= \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_t^{1-\xi}+1\right]}\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}(1-\alpha)\,\mathcal{E}_t^{1-\xi}+(1-\alpha)\right]^{\frac{\xi}{\xi-1}} \\
&= P_t C_t\left[\alpha^{\xi}\mathcal{E}_t^{1-\xi}+(1-\alpha)^{\xi}\right]^{\frac{1}{\xi-1}},
\end{aligned}
$$

which implies that the price index is given by

$$
P_t = \left[\alpha^{\xi}\mathcal{E}_t^{1-\xi}+(1-\alpha)^{\xi}\right]^{\frac{1}{1-\xi}}.
$$

Derivation of the Real Exchange Rate Define the real exchange rate as

$$
\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t}.
$$

Using the expression for the domestic price index and the fact that the price level abroad is normalized to unity, we obtain the following expression for the real exchange rate

$$
\mathcal{Q}_t = \left[\alpha^{\xi}+(1-\alpha)^{\xi}\left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}.
$$

B Calibration Details

Observed FXI regime While we do not know the exact FXI reaction function of the Bank Negara Malaysia, we can account for their FXI policy in the model by treating the observed FX interventions $\hat{B}_{M,t}$ as an exogenous process similar to portfolio outflows $\hat{B}_{P,t}$. For this purpose, we define a new variable that is the sum of portfolio outflows and FX interventions $\hat{B}_{PM,t} = \hat{B}_{P,t} + \hat{B}_{M,t}$ and replace $B_{P,t}$ by $B_{PM,t}$ as the third exogenous state variable. We can make this simplification since the only difference between $B_{P,t}$ and $B_{M,t}$ from the model's perspective, namely that $B_{P,t}$ is exogenous and $B_{M,t}$ is a policy variable, ceases to exit. Given the changed exogenous state variables, we estimate an AR(1) process $\tilde{\mathbf{s}}_t = \tilde{\rho}\tilde{\mathbf{s}}_{t-1} + \tilde{\varepsilon}_t$ where $\mathbf{s}_t = \left[\log Y_{T,t}, \log Y_{N,t}, \sinh^{-1}\left(B_{PM,t} - \overline{B}_{PM}\right)\right]'$ and $\overline{B}_{PM} = 0.24$. The error term $\tilde{\varepsilon}_t = [\tilde{\varepsilon}_{T,t}, \tilde{\varepsilon}_{N,t}, \tilde{\varepsilon}_{PM,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\tilde{\mathbf{V}}$ and $\tilde{\rho}$ is a 3×3 matrix consisting of the autocorrelation terms

$$\tilde{\mathbf{V}} = \begin{bmatrix} 0.0005258 & 0.0005685 & -0.000789 \\ 0.0005685 & 0.0008582 & -0.002274 \\ -0.000789 & -0.002274 & 0.1716685 \end{bmatrix} \quad \tilde{\rho} = \begin{bmatrix} 0.829771 & -0.414713 & -0.024469 \\ 0.220326 & 0.2561583 & -0.031291 \\ -1.15058 & -0.708053 & 0.441873 \end{bmatrix}.$$

In the data, FX interventions are negatively correlated with portfolio outflows at $\sigma_{B_P^*,B_M^*} = -0.3077$, which is directionally consistent with the optimal policy.

Calibration of the subjective discount factor β In the model with an exogenous state process that accounts for the observed FXI regime (see above), we choose β such that the model matches the mean of the observed net foreign asset position as a percentage of annual GDP, which is 2%. The value of the subjective discount factor that matches this moment is $\beta = 0.9871$.

Calibration of the financiers' risk aversion ω The target for the calibration of the financiers' risk aversion is an average FX market depth of $\omega\overline{\sigma}^2 = 0.05$ in the observed economy. This condition holds for $\omega = 28$. Note that changing the financiers' risk aversion not only has implications for the endogenous variables in the model but is also associated with a change in the estimated (exogenous) portfolio outflow process $B_{P,t}^*$ since ω enters the risk sharing wedge that we use to calculate these flows. As is evident from (15), a higher financiers' risk aversion ω makes FX markets more shallow, magnifying the effect of portfolio outflows on the risk sharing wedge. Therefore, a given level of fluctuations in the risk sharing wedge can either be explained by more volatile portfolio outflows and deeper FX markets or less volatile portfolio outflows and more shallow FX markets.

C Simulation Details

Derivation of the Consumption Equivalence Let κ be the additional fraction of consumption that households in the benchmark economy b will have to receive to make them indifferent to the economy with the alternative policy p

$$
\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left((1+\kappa)C_t^b\right)\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left(C_t^p\right)\right] \\
\Leftrightarrow &\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left((1+\kappa)C_t^b\right)^{1-\sigma}\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^p\right)^{1-\sigma}\right] \\
\Leftrightarrow &(1+\kappa)^{1-\sigma} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^b\right)^{1-\sigma}\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^p\right)^{1-\sigma}\right].
\end{aligned}
$$

Next use

$$
V_i\left(B^*, S\right) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left(C_t^i\right)\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^i\right)^{1-\sigma}\right]
$$

for $i \in \{b, p\}$ to obtain

$$
\kappa = \left(\frac{V_{ap}\left(B^*, S\right)}{V_b\left(B^*, S\right)}\right)^{\frac{1}{1-\sigma}} - 1.
$$

We weigh the value functions with the stationary distribution of the exogenous state process

$$
\tilde{V}_i\left(B^*\right) = \sum_{S \in \mathcal{S}} \psi_S V_i\left(B^*, S\right),
$$

where $\mathcal{S}$ is the set of all exogenous states S and ψ_S is the probability of state S according to the stationary distribution, with $\sum_{S \in \mathcal{S}} \psi_S = 1$. Finally, we impose the steady state net foreign asset position of the benchmark economy $B^* = \overline{B}^*$ as the initial condition to get

$$
\kappa = \left(\frac{\tilde{V}_{ap}\left(\overline{B}^*, \mathcal{S}\right)}{\tilde{V}_b\left(\overline{B}^*, \mathcal{S}\right)}\right)^{\frac{1}{1-\sigma}} - 1.
$$

Solving for the First Best Given exogenous states $S = \{Y_T, Y_N, B_P^*\}$, the unconstrained planner solves

$$
V(B^*, S) = \max_{B^{*\prime}} \left\{u(R^* B^* + Y_T - B^{*\prime}, Y_N) + \beta \mathbb{E}_S V(B^{*'}, S')\right\},
$$

subject to the market clearing conditions and a lower bound on bond holdings $\underline{B}^*$

$$
\underline{B}^* \leq B^{*\prime} \leq R^* B^* + Y_T.
$$

Note that we also imposed the same lower bound $\underline{B}^*$ when computing all other policy regimes, but it is never binding in these cases since the economy is already subject to a stricter implicit borrowing limit associated with the non-negativity constraint on FX reserves. In the first best, the latter constraint does not exist since the central bank is allowed to take a negative position in foreign currency bonds.

Additional Simulations Figure C.1 presents the paths of selected macroeconomic variables in an average episode of a fall in tradable endowment.

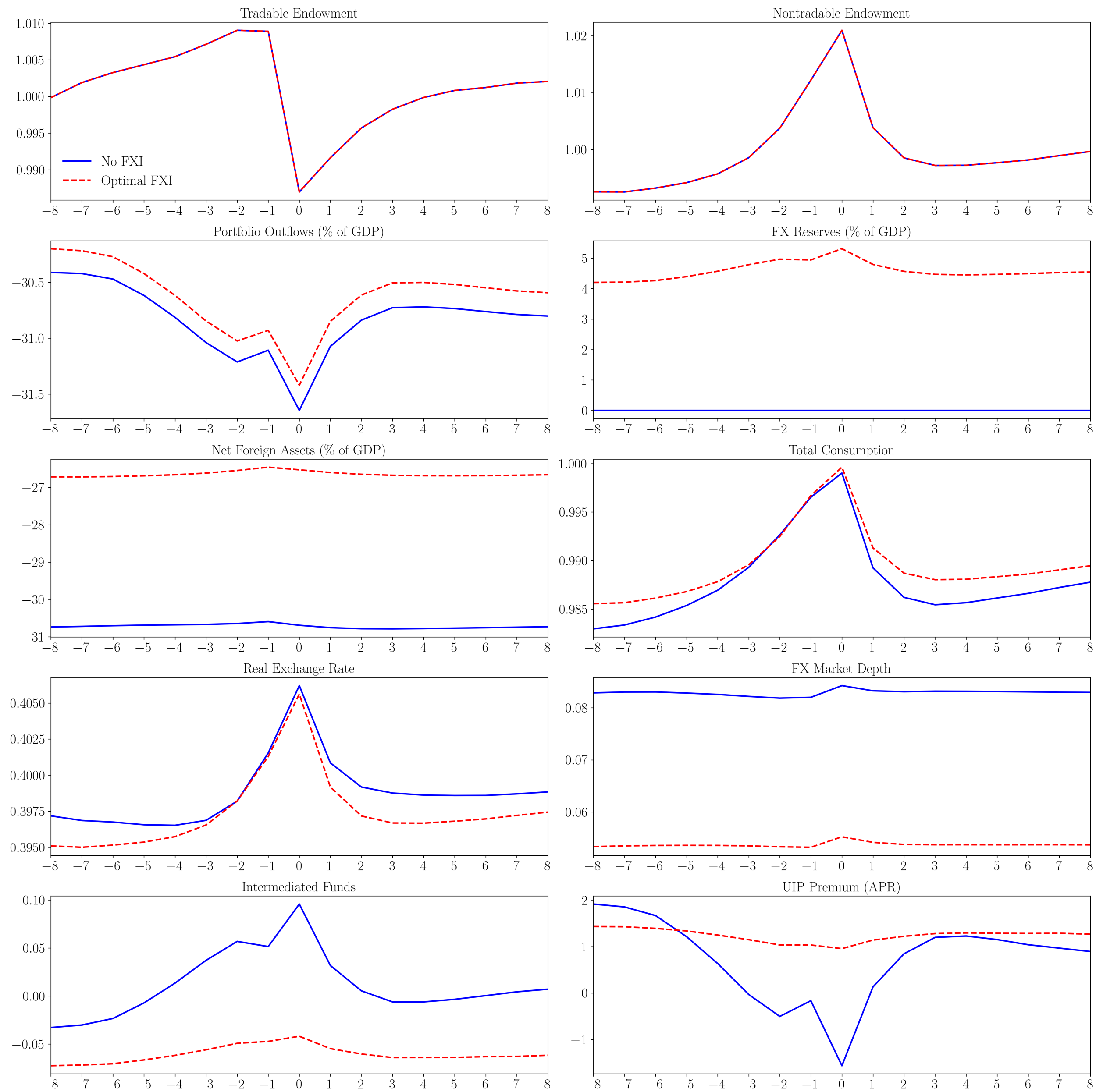

Figure C.1: Episodes of negative endowment shocks in the tradable sector

Notes: The figure depicts the average response in the model in episodes in which tradable endowment decreases relative to nontradable endowment. One period corresponds to one quarter and period 0 coincides with the tradable endowment trough.

Figure C.2 compares the outcomes in a capital outflow episode as considered in Figure 5 across the policies discussed in Section 4.5.

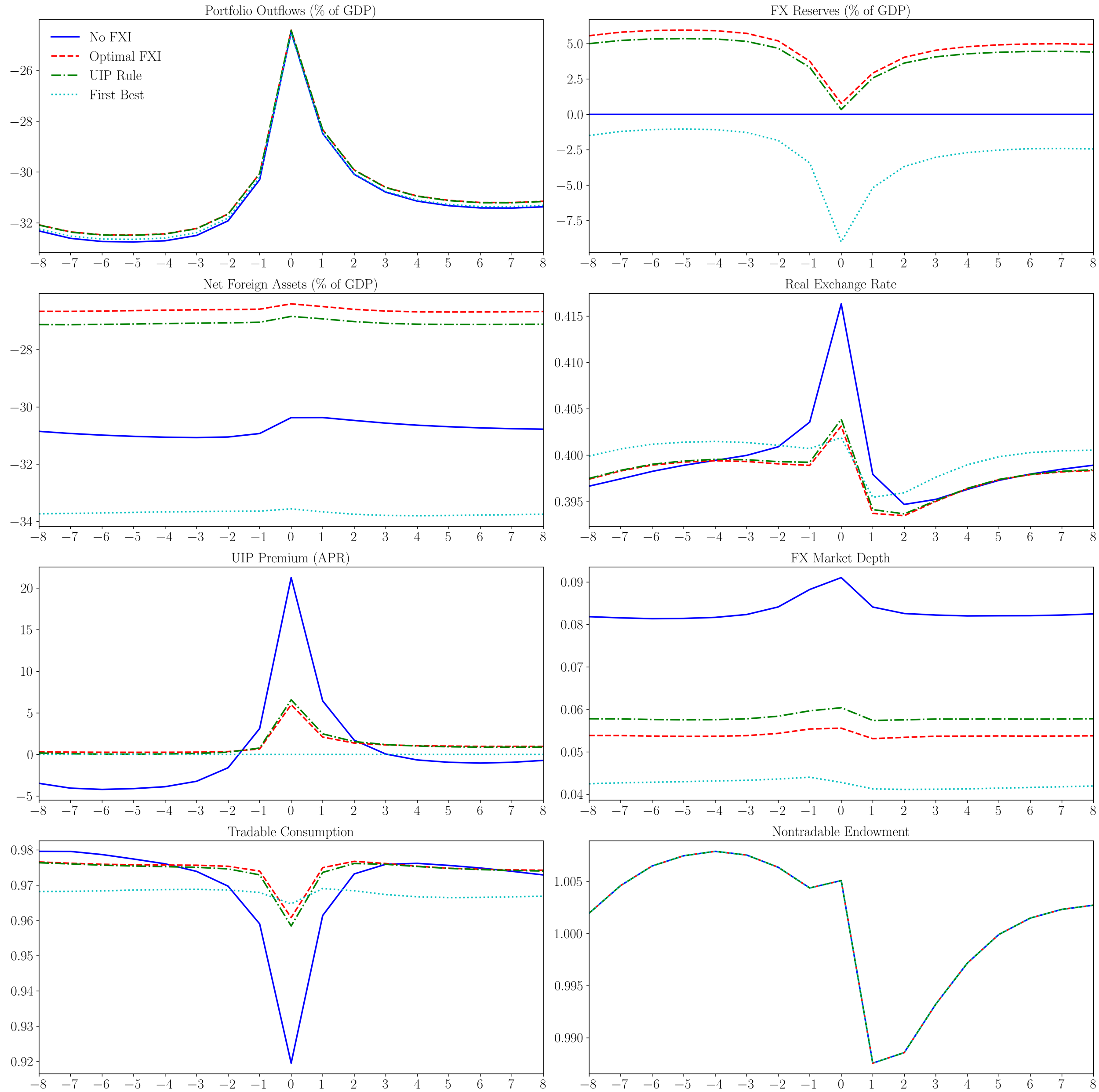

Figure C.2: Episodes of portfolio outflows in different policy regimes

Notes: The figure depicts the average response in the model in episodes of portfolio outflows for different policy regimes. One period corresponds to one quarter and period 0 coincides with the peak outflow.