2D Nonlinear Systems - Predator/Prey Model

Predator Prey Model

Consider the system given by

$$\dot{x} = f(x, y) \quad \text{where } f(x, y) = r\,x - a\,x\,y$$
$$\dot{y} = g(x, y) \quad \text{where } g(x, y) = b\,x\,y - k\,y$$

We would like to (1) conduct local analysis, and (2) graph the phase plane to validate our local analysis conclusions.

Local Analysis

```
In[•]:= f[x_, y_] = r x - a x y;
        g[x_, y_] = b x y - k y;

In[•]:= eqPts = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}]

In[•]:= DF[xs_, ys_] =
          {{D[f[x, y], x], D[f[x, y], y]}, {D[g[x, y], x], D[g[x, y], y]}} /. {x → xs, y → ys};

In[•]:= For[j = 1, j ≤ Length[eqPts], j++,
         Print[];
         Print[
          "================================================================================
            ================="];
         Print["At the equilibrium point ", eqPts[[j]],
          " the eigensystem is given by ", MatrixForm[Eigensystem[DF[x, y] /. eqPts[[j]]]]];
         Print[
          "================================================================================
            ================="];
         Print[];
        ]
```

Comparing the Local Analysis with the Phase Plane for specific parameter values

Let's first pick some specific parameter values so that we can plot the solution.

```
parameterVals = {b → 1, r → 2/3, k → 1, a → 1}
```

Now, let's create a loop that iterates over each of the equilibrium points. We will plot the eigenvectors

In[ ]:=

```
For[j = 1, j ≤ Length[eqPts], j++,
  esys = Eigensystem[DF[x, y] /. eqPts[[j]]];
  evPlots_j = ParametricPlot[
    esys[[2]] * s + Table[{x, y} /. eqPts[[j]] /. parameterVals, {k, 1, 2}], {s, -1, 1},
    PlotStyle → {Red, Thickness → .015},
    RegionFunction → Function[{u, v, vx, vy, n}, ((u - x)^2 + (v - y)^2) /. eqPts[[j]]) < .1]]];
EVPlot = Show[Table[evPlots_j, {j, 1, Length[eqPts]}]];
eqPtsPlot = ListPlot[{x, y} /. eqPts /. parameterVals,
   PlotMarkers → {Automatic, Scaled[.04]},
   PlotStyle → Black];
pplanePlot = StreamPlot[{f[x, y], g[x, y]} /. parameterVals, {x, -.5, 3}, {y, -.5, 2},
   FrameLabel → {"x", "y"},
   PlotLabel → "Phase Plane"];

Show[pplanePlot, EVPlot, eqPtsPlot]
```