In-Class NullCline Examples

Example #1

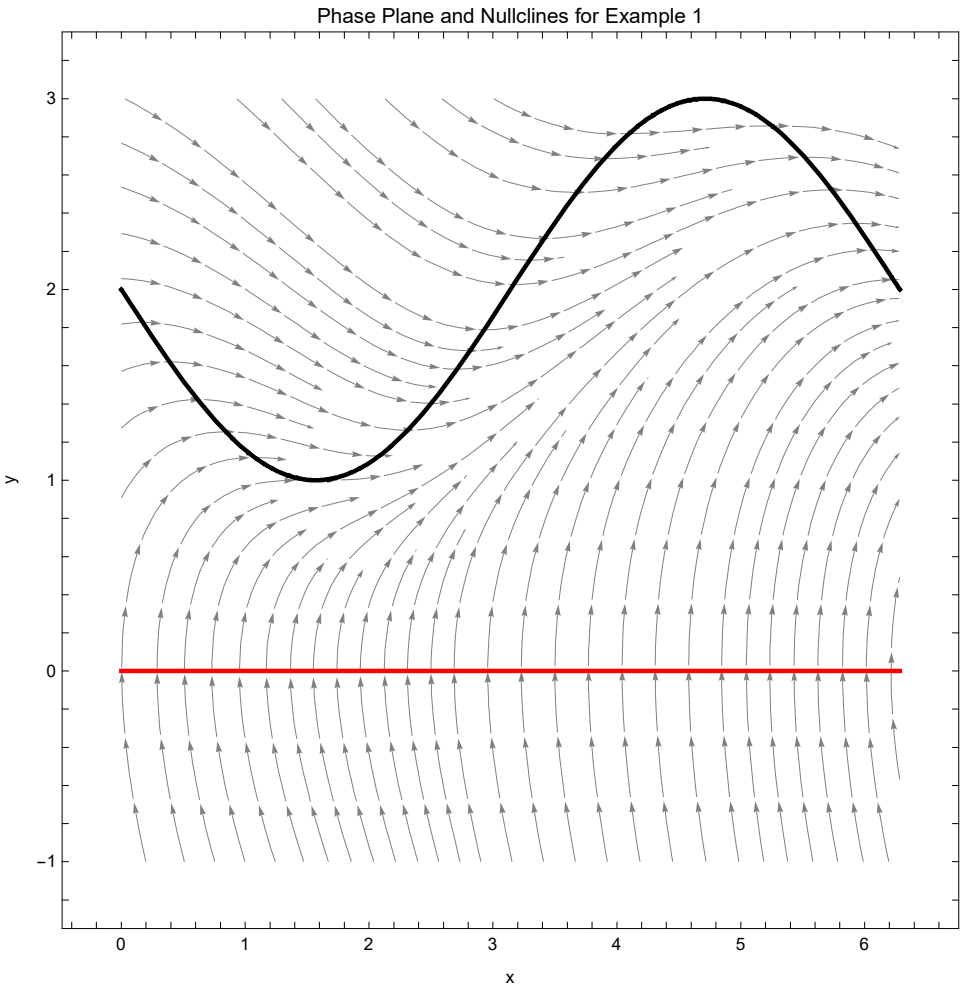
Let's begin by considering the following system.

$$\dot{x} = y$$
$$\dot{y} = 2 - y - \sin(x), \quad x \in [0, 2\pi), \; y \in \mathbb{R}$$

Let's plot the phase-plane along with the nullclines. Here, the x-nullcline is given by y = 0, and the y nullcline is given by y = 2 - sin(x).

In[4]:=
```
f = y;
g = 2 - y - Sin[x];
p1 = StreamPlot[{f, g}, {x, 0, 2 π}, {y, -1, 3}, ImageSize → 500,
  StreamStyle → Gray, StreamPoints → 60, StreamScale → 0.05];
p2 = Plot[{0, 2 - Sin[x]}, {x, 0, 2 π},
   PlotStyle → {{Red, Thickness → 0.005}, {Black, Thickness → 0.005}}];
Show[p1, p2, FrameLabel → {"x", "y"}, PlotLabel →
  "Phase Plane and Nullclines for Example 1"]
```

Out[7]=



Example #2

Now, let's consider the system given by

$$\dot{x} = x^2 - 1$$
$$\dot{y} = a\left(x^2 - 1\right) - x\,y, \qquad x, y \in \mathbb{R},\ a \geq 0.$$

We can construct a manipulate to plot the phase plane and nullclines as we vary the parameter a.

Here, the x-nullclines are given by $x = \pm 1$, and the y-nullclines are given by $y = 0$, $x = 0$ if $a = 0$, **or** y = $a(x^2 - 1)/x$ if $a > 0$.

In[16]:=
```
f2 = x^2 - 1;
g2[a_] = a (x^2 - 1) - x y;
eqPts = Solve[{f2 == 0, g2[a] == 0}, {x, y}];
Manipulate[
 p1 = StreamPlot[{f2, g2[a]}, {x, -2, 2}, {y, -2, 2},
   ImageSize → 500, StreamStyle → Gray, StreamPoints → 40, StreamScale → 0.05];
 If[a > 0,
  p2 = ParametricPlot[{{x, a (x^2 - 1) / x}, {-x, -a (x^2 - 1) / x}}, {x, (-1 + Sqrt[1 + a^2])/a, 2},
    PlotStyle → {{Red, Thickness → 0.005}}, PlotRange → {-3, 3}],
  p2 = ParametricPlot[{{0, t}, {t, 0}}, {t, -2, 2},
    PlotStyle → {{Red, Thickness → 0.005}}]];
 p3 = ParametricPlot[{{1, t}, {-1, t}}, {t, -2, 2},
   PlotStyle → {{Black, Thickness → 0.005}}];
 eqPtsPlot = ListPlot[{x, y} /. eqPts,
   PlotMarkers → {Automatic, Scaled[.02]},
   PlotStyle → Black];
 Show[p1, p2, p3, eqPtsPlot, PlotRange → {-2, 2}, FrameLabel → {"x", "y"},
  PlotLabel → "Phase Plane and Nullclines for Example 2"],
 {a,
  0,
  .2}]
```

Out[19]=