CS 331: Algorithms and Complexity (Fall 2014)

Unique numbers: 53001-53004

Assignment 1

Due on: Tuesday, September 9^{th}, at the start of class

Q1(a) (5 points) Sort the following list of functions in ascending order of growth rates. That is, if function $g(n)$ immediately follows function $f(n)$, then $f(n)$ should be order $O(g(n))$.

$$f_1(n) = 2^{2^n}$$
$$f_2(n) = 2^{n^2}$$
$$f_3(n) = n^{logn}$$
$$f_4(n) = n(logn)^3$$
$$f_5(n) = n^4$$
$$f_6(n) = 2^{2^{log(logn)}}$$

Q1(b) (5 points) Alice and Bob, two students of CS 331, were asked to provide algorithms for a simple program. The program is supposed to receive a stream of integers as input. After each input integer, the program must report the average of the integers it has seen so far. Alice and Bob came up with the following algorithms to do this simple task-

Alice's Algorithm

```
Initially sum is 0 and count of integers read is 0
While the input stream is not empty
    Read the integer i
    Add i to the sum
    Report average as sum / count
    Increase count by 1
Endwhile
```

Bob's Algorithm

```
Initially average is 0 and count of integers read is 0
While the input stream is not empty
    Read the integer i
    Modify average as (average * count + i) / (count + 1)
    Increase count by 1
    Report average
Endwhile
```

State whether Alice or Bob have provided correct algorithms. Also, either prove the algorithm to be right, or show how it is wrong. Proof should mathematically verify the correctness of the steps and invariants, and incorrect steps in the algorithms should be mentioned with rectifications.

Q2 (10 points) In a galaxy far far away, and not so long ago, the goverment introduced a strict policy that no couple can have more than one child. It turned out that most couples preferred sons to daughters and devised ways, often unscrupulous ones, to increase the probability of having a son. As a result, the number of men is now higher than the number of women. The government, as you might have guessed, considers public's personal lives also under their dominion, and now faces the problem of trying to match up the unequal number of men and women.

The government have created a website with a list of all eligible men and women, and asked each man to submit a complete list of women ordered according to his preference. Every woman must also do the same. The government will allow at most one partner for each man and exactly one partner for each woman (some men will obviously remain unmarried, and their genes will be lost forever in a very social darwinistic way).

The government have heard about the Gale-Shapley algorithm and are now thinking whether the same can be applied to solve their problem. However, just to maintain their 'altruistic' image, the government do not want people to express any displeasure with, or try to break up the matchups the government comes up with. Can you specify exactly how to modify the description of the stable marriage problem including the definition of instability, so that some men can remain unmarried; suggest necessary modifications to the GS algorithm; and prove the correctness of the solution?

Q3 Remember the 20 questions game? In that game, your friend thinks about something and you must guess what it is by asking at most 20 questions, such that every question has a yes/no answer. We shall consider some small variations of the game.

(a) (5 points) Show that if your friend thinks about a number between 1 and 1 million, then it is always possible to guess it using 20 questions.

(b) (5 points) Now, consider another case. In this variation, you are allowed to ask more than 20 questions if necessary, but if the answer to your question is 'no' then that counts as a strike. 3 strikes and you lose. Under this new rule, if your friend thinks about a number between 1 and 1 million, then what should be the strategy for asking questions so that you do not strike out, and at the same time use as few questions as possible (assume that players take turns in guessing, and in case neither player strikes out, the one who used fewer questions wins).