Diagonalization

And Turing machines

Some infinities are bigger than others.

Oliver Chubet, November 4, 2024

The Natural Numbers $\mathbb{N}$

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, …

How many natural numbers are there?

The ***Even*** Natural Numbers $2\mathbb{N}$

$0, \cancel{1}, 2, \cancel{3}, 4, \cancel{5}, 6, \cancel{7}, 8, \cancel{9}, 10, \cancel{11}, 12, \ldots$

Is $\mathbb{N}$ ***larger than*** $2\mathbb{N}$***?***

Bijections *(or 1-1 Correspondence)*

How do you know if two sets are the same size without counting?

The Rational Numbers $\mathbb{Q}$

All numbers representable as fractions.

How many rational numbers are there?

The Real Numbers $\mathbb{R}$

All numbers on the real line.

How many real numbers are there?

The Power Set of Natural Numbers $\mathscr{P}(\mathbb{N})$

All subsets of the natural numbers.

How many sets does $\mathscr{P}(\mathbb{N})$ contain?

How many strings are there in $\{0,1\}^*$***?***

How many Turing machines exist?

How many languages exist over $\{0,1\}^*$***?***

There are more languages than possible Turing machines.