Church-Turing Thesis

All “reasonable” model of computation are equivalent to Turing machines.

Oliver Chubet, October 30, 2024

Equivalent Models of Computation

A model of computation A is *equivalent* to a model of computation B if we can

1. **simulate every A-machine with a B-machine and**
2. **simulate every B-machine with an A-machine.**

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    - **Arden's Rule**

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    - Thomson's construction
    - Subset Construction
    - Arden's Rule
2. CFGs and PDAs
3. PDAs requiring empty stacks and PDAs that don't require empty stacks

Turing Complete

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A model of computation is Turing complete if it can

1. **simulate a Turing machine**
2. **be simulated by a Turing machine**

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For example, most programming languages are Turing complete.

Can we “upgrade” the Turing machine?

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- Adding more tapes
- Adding more heads
- Adding non-determinism
- Giving the head random access to the tape
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We do not know if there is a real model of computation that recognizes languages that are not Turing-recognizable.