Turing machines

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A TM has…

- **States:** Q
- **Input alphabet:** Σ
- **Tape alphabet:** Γ
- **Transition function:** $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$
- **Start state:** q_0
- **Accept state:** q_{ACCEPT}
- **Reject state:** q_{REJECT}

Initial Setup

- The input is written on the tape starting on the leftmost position
- The rest of the tape is blank symbols $\sqcup$
- Start in q_0, the start state
- The head points to the leftmost position on the tape

<u>Initial Configuration</u>: $q_0\langle\textbf{input}\rangle \sqcup$

Computation

Each transition:

1. Read the tape
2. Write a symbol
3. Move the head left or right

$$\delta : \underset{\text{Current state}}{Q} \times \underset{\text{Read symbol}}{\Gamma} \to \underset{\text{Transition state}}{Q} \times \underset{\text{Write symbol}}{\Gamma} \times \underset{\text{Move head}}{\{L, R\}}$$

Read and accept…

$$\boxed{\begin{array}{l}\delta(q, \gamma) = (q, \gamma, R) \text{ for all } \gamma \neq \sqcup \\ \delta(q, \sqcup) = (q_{ACCEPT}, \sqcup, L)\end{array}}$$

Simulate the TM on input '`010000101`':

$q_0 \texttt{010000101} \sqcup$

$\texttt{0} q_0 \texttt{10000101} \sqcup$

…..

$\texttt{010000101} q_0 \sqcup$

$\texttt{01000010} q_{ACCEPT} \texttt{1} \sqcup$

Cross out the input and accept…

$$\delta(q, \gamma\ ) =$$

Accept even length strings

$$\delta(q, \gamma\ ) =$$

Turing Recognizable

- A language such that some TM **recognizes** it.
- Also called **recursively enumerable**

A is a **Turing recognizable** language.

Then there exists TM M such that M accepts a string a **iff** $a \in A$**.**

Not guaranteed to halt!

$$\delta(q, \gamma\ ) = \begin{cases} (q,\ 0,\ R) & \text{if } \gamma = 1 \\ (q,\ 1,\ R) & \text{if } \gamma = 0 \\ (q, \sqcup\ ,\ L) & \text{if } \gamma = \sqcup \end{cases}$$

Deciders

- A TM **halts** on a given input if it enters q_{ACCEPT} or q_{REJECT}.
- A TM that halts on every input is called a **decider.**
- Languages recognized by some decider are **decidable.**

B is a **decidable** language.

Then there exists TM M such that M accepts a string b **iff** $b \in B$

and rejects a string b **iff** $b \in B$.