Series 6

1. In this exercise we investigate the connection between advertising expenditure and sales. Our example here is that of the annual advertising expenditure (`ADVERT`, in k$) for, and sales revenue (`SALES`, in k$) from, a particular brand of vegetable stock (Lydia Pinkham's Vegetable Compound, 1907 – 1960). These data are contained in the file `advert.dat`. Load and log-transform the data as follows:

```
> d.advert <- read.table("ftp://stat.ethz.ch/Teaching/Datasets/WBL/advert.dat",
   header = TRUE)
> ts.advert <- ts(log(d.advert[, "ADVERT"]), start = 1907)
> ts.sales <- ts(log(d.advert[, "SALES"]), start = 1907)
```

In the following problems, we will refer to `ts.advert` and `ts.sales` as $X_{1,t}$ and $X_{2,t}$, respectively.

a) First look at the time series plots. Are the series stationary?
R hint: you can either use the generic plot function (first line), or a specific time series plot function (second line):

```
> plot(ts.union(ts.sales, ts.advert), plot.type = "single")
> ts.plot(ts.sales, ts.advert)
```

(Unless the time series have been joined using `ts.union()`, the command `plot(ts.sales, ts.advert)` will plot both of them against each other. It is easy to see that they are correlated.)

b) Compute the first-order differences $Y_{1,t} := X_{1,t} - X_{1,t-1}$ (`ts.adv.d1`) and $Y_{2,t} := X_{2,t} - X_{2,t-1}$ (`ts.sal.d1`). Are these processes stationary?

c) Regard the transfer function model

$$Y_{2,t} = \sum_{j=0}^{\infty} \nu_j Y_{1,t-j} + E_t \, .$$

Describe this model in words. What key assumption is made? From a business point of view, what is an illustrative interpretation of this model?

d) Estimate the coefficients ν_j. Proceed as described below; see also the lecture slides. Make an interpretation of the model you estimate. What do you notice about this model?
Instructions:

- First we want to prewhiten the input series $Y_{1,t}$. Which model should you fit? Estimate its coefficients using the R function `arima()`.
- Compute D_t and Z_t for the transformed series.
  R hints: Suppose your R object with the ARMA model you used to fit $Y_{1,t}$ is called `r.fit.adv`. D_t then simply contains the residuals:

  ```
  > ts.D <- resid(r.fit.adv)
  ```

  To calculate Z_t, you can extract the AR (or possibly ARMA) coefficients of the fitted model and then use the R function `filter()`. For example, if `alpha` is a vector of AR coefficients, use

  ```
  > ts.Z <- filter(ts.sal.d1, c(1, -alpha), sides = 1)
  ```

  The argument `sides = 1` causes `filter()` to only take the past into account; cf. the help page.
  Since you should only consider time points present in both time series, use

  ```
  > ts.trans <- ts.intersect(ts.Z, ts.D)
  ```

- Draw the cross-correlogram for D_t and Z_t. Give an interpretation of them. What is the largest value? Is our assumption from part c) fulfilled?
  `R hint:`

  ```
  > acf(ts.trans, na.action = na.pass)
  ```

- Estimate the coefficients ν_j using the formula given in the lecture.
  R hints:
  The variance of the errors D_t can be found in `r.fit.adv$sigma2`. The command

  ```
  > acf(..., type = "covariance", na.action = na.pass)$acf[, 1, 2]
  ```

  also returns the cross-covariances between Y_t and Z_t.

2. In this exercise we shall look at the height of oceanic waves as measured at intervals of .25 seconds. The data can be found in `ocwave.dat`. Load them as follows:

```
> ts.ocwave <- ts(scan("http://stat.ethz.ch/Teaching/Datasets/WBL/ocwave.dat"),
   start = 1, frequency = 4)
```

a) Make a time series plot and comment on it.

b) Compute the periodogram with and without taper. Comment on your results.
R hints:

```
> spec.pgram(ts.ocwave, taper = ..., detrend = FALSE, demean = TRUE, log = "dB")
```

c) Smooth the tapered periodogram. Find the frequency for which the spectrum reaches its maximum and calculate the corresponding period. Check if your value makes sense by comparing it to the periodicity in the time series plot in a).

d) Fit an AR(p) model with a suitable order p. Use both the Yule-Walker and Burg methods. Compare the resulting prediction variances and the AR spectra between methods.
R hints:

```
> ocwave.yw <- ar.yw(ts.ocwave, aic = FALSE, order = ...)
> ocwave.burg <- ar.burg(ts.ocwave, aic = FALSE, order = ...)
```

The prediction variances can be found under `...$var.pred`. To facilitate the comparison, the spectra can be plotted in the same diagram:

```
> spec.ar(ocwave.burg, plot = TRUE, log = "dB")
> spec.ar(..., add = TRUE)
```

Preliminary discussion: Monday, May 22.