Mathematical Skills I: Group Project Proposal

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Working Title: "An Undergraduate Investigation into Elementary Category Theory, with applications in Pure Mathematics and Theoretical Computer Science." A brief index of likely reference works, and an example Ti*k*Z commutative diagram, can be found overleaf.

- **Matthew Drury.** "Axiomatic constructions of the general category; an investigation of functors, duality, pullbacks, and implications of the Yoneda lemma."

  MD will investigate and explore the foundational concepts of Category Theory, including a survey of its most basic axiomatic constructions, a discovery of concepts exclusively describable in category-theoretic language, such as the general isomorphism, and provide a framework for the contributions of *BB* and *OD*. Also comprising *MD*'s contribution will be various TEX-typeset commutative diagrams to succinctly visualise the relationships of objects and morphisms in simple categories. Although the respective contributions of each group member is expected to be reasonably distinct, *MD*'s section will likely intersect, to some degree, with the sections of *BB* and *OD*, due to its fundamentalist nature. An investigation of Sheaf Theory was also considered, in which a discussion of the role of sheaves, pre-sheaves, and profunctors in a CT-context would take place, however this was deemed overly ambitious.

- **Ben Brook.** "Some category-theoretic constructions of familiar structures in Abstract Algebra, Topology, and Set Theory."

  As done in many introductory texts on Category Theory, *BB* will aim to take prominent structures from Abstract Algebra, Topology, and Set Theory, and proceed to describe the core structures of their constructions through category-theoretic language. Although there are a plethora of examples in this direction—posets; based sets with point-preserving maps; preorders; monoids—some more exotic cases will also be covered, potentially on a type-theoretic stage. *BB*'s contribution will also, inevitably, involve extensive typesetting of commutative diagrams and general LATEX documents, in both article and Beamer contexts.

- **Oliver Dixon.** "Practical applications and examples of isomorphic relations to generalised descriptions of functional programming paradigms, with respect to λ-calculus."

  OD will review the most direct and modern application of Category Theory, with respect to formal systems of logic and functional programming whilst drawing upon tangential results from Theoretical Computer Science and, to a substantially lesser but still notable degree, applied Analytic Philosophy. There exist deep connexions between these fields and that of λ-calculus, and hence, the topics discussed will defer to this universal model where appropriate. There will be a slight intersection between the contributions of *OD* and *BB*, given the inherent theme of applicability; in particular, *OD* will introduce various categories pertaining to structures found in logical systems, such as the category of Deduction under some fixed formal logic.

Supporting Works of Reference

[Awo10] Steve Awodey. *Category Theory*. 2nd Ed. Oxford Logic Guides. Oxford, United Kingdom: Oxford University Press, 2010. ISBN: 978-0-199587360.

[Che22] Eugenia Cheng. *The Joy of Abstraction: An Exploration of Math, Category Theory, and Life*. Cambridge, United Kingdom: Cambridge University Press, 2022. ISBN: 978-1-108769389.

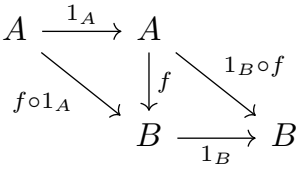
[Mic89] Greg Michaelson. *An Introduction to Functional Programming through Lambda Calculus*. International Computer Science Series. Wokingham, United Kingdom; Reading, Mass., USA: Addison–Wesley Pub. Co., 1989. ISBN: 0-201178125.

[Mac98] Saunders Mac Lane. *Categories for the Working Mathematician*. 2nd Ed. Graduate Texts in Mathematics. London, United Kingdom; New York, USA: Springer, 1998. ISBN: 0-387984038.

[Smi] Peter Smith. *Beginning Category Theory: A Gentle, Still Unfinished, Introduction*. Logic Matters. Publisher N/A. URL: `https://www.logicmatters.net/resources/pdfs/BeginCatTheory.pdf`.

[FS18] Brendan Fong and David I. Spivak. *Seven Sketches in Compositionality: An Invitation to Applied Category Theory*. 2018. DOI: `10.48550/ARXIV.1803.05316`.

An Example Commutative Diagram




```
\begin{tikzcd}
    A \arrow[r, "1_A"] \arrow[rd, "f\circ 1_A" swap] &
        A \arrow[d, "f"] \arrow[rd, "1_B\circ f"] \\
    & B \arrow[r, "1_B" below] & B
\end{tikzcd}
```

Figure 1: A commutative diagram, showing objects A, B, and the morphism $f : A \longrightarrow B$, with explicit identity morphisms. The corresponding LaTeX-Ti*k*Z source is shown to the right. Such diagrams will be abundant throughout the final dissertation and presentation.