Error Analysis Exercise (EAX)

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1 First problem

We want to attain a specific activity with a precision of 1% from measurements at a rate of 1000 decays per 5 minutes of observation. The accuracy of an average value such as this is better by a factor of

$$\sqrt{N}. \tag{1}$$

This means that a precision P:

$$P = \sqrt{N}/N \tag{2}$$

is aquired when N samples are taken. If we are looking for a precision of 1% then we can calculate the require number of samples:

$$\begin{gathered} P = 0.01 = \sqrt(N)/N \rightarrow \\ N = 10,000\ samples. \end{gathered} \tag{3}$$

In order to gather this many samples at the prescribed rate of decays (1000 decays per 5 minutes), we must measure for approximately:

$$\begin{aligned} Time\ to\ sample &= \frac{10,000\ samples}{1,000\ samples/5\ min.} \\ &= 50\ minutes. \end{aligned} \tag{4}$$

2 Second Problem

We are given two measurements and their associated uncertainties:

$$A \pm \sigma_A \tag{5}$$

$$B \pm \sigma_B \tag{6}$$

Often we want to calculate the error associated with the combination of two or more quantities for which an uncertainty is known. The genral expression for the uncertainty of a quantity with functional dependence on several other quantities is:

$$\sigma_f^2 = \sum_{i=1}^{n} (\frac{\partial f}{\partial x_i})^2 \sigma_i^2. \tag{7}$$

For each of the following combinations of A and B, the corresponding error of the resulting quantity is calculated (the propagated error).

2.1 A+B

$$\sigma_{A+B}^2 = (1)^2 \sigma_A^2 + (1)^2 \sigma_B^2 \tag{8}$$

$$\sigma_{a+b} = \sqrt{\sigma_A^2 + \sigma_B^2} \tag{9}$$

This so called propagated error is used in the following way:

$$\begin{aligned} (A \pm \sigma_A) + (B \pm \sigma_B) &= (A + B) \pm \sigma_{a+b} \\ &= (A + B) \pm \sqrt{\sigma_A^2 + \sigma_B^2} \end{aligned} \tag{10}$$

2.2 A-B

$$\sigma_{A-B}^2 = (1)^2 \sigma_A^2 + (-1)^2 \sigma_B^2 \tag{11}$$

$$\sigma_{A-B} = \sqrt{\sigma_A^2 + \sigma_B^2} \tag{12}$$

2.3 2A+2B

$$\sigma^2_{2A+2B} = (2)^2\sigma^2_A + (2)^2\sigma^2_B \tag{13}$$

$$\sigma_{2A+2B} = \sqrt{4\sigma^2_A + 4\sigma^2_B} \tag{14}$$

2.4 A x B

$$\sigma^2_{A\times B} = (\sigma_B)^2\sigma^2_A + (\sigma_A)^2\sigma^2_B \tag{15}$$

$$\sigma_{A\times B} = \sqrt{2\sigma^2_A\sigma^2_B} \tag{16}$$

3 Third Problem

In this problem we are examining lists of normal distribution random numbers. The function we use in MATLAB is randn and it has true mean 0 and standard deviation 1.

3.1

If we sample the distribution N=5 times, we expect the mean to be 0 ± 0.2. (This is the known mean plus or minus the variance of the mean: S^2/N). The error on the mean is $\sigma_M = \sigma/\sqrt{N} = 1/\sqrt{5}$.

3.2

Using MATLAB, a list of N=5 normally distributed random numbers was generated with the following code:

```
A = randn(5,1);
M = mean(A)
S = std(A)
[N,L] = size(A)
Error_on_mean = S/sqrt(N)
```

Resulting in the following output:

```
M =
    0.2587
S =
    1.5219
Error_on_mean =
    0.6806
```

We get a mean of 0.2587, a standard deviation of 1.5219 and an error on the mean of 0.6806.

3.3

Now we want the mean, standard deviation, and errors on the means for M=1000 sets of N=5 measurements each, with a histogram of the distributions. To do this the following code was ran in MATLAB:

```
N1=5;
A1 = randn(5,1000);
Mean1 = mean(A1);
Std1 = std(A1);
Error_on_mean1 = S/sqrt(N1);
y1=histogram(Mean1)
```

Yielding the following outputs:

```
y1 =
  Histogram with properties:
             Data: [1x1000 double]
           Values: [3 9 31 41 78 143
   197 172 136 99
   51 26 12 2]
          NumBins: 14
         BinEdges: [-1.4000 -1.2000
      -1 -0.8000 -0.6000
      -0.4000 -0.2000 0
       0.2000 0.4000
       0.6000 0.8000
       1.0000 1.2000 1.4000]
         BinWidth: 0.2000
        BinLimits: [-1.4000 1.4000]
    Normalization: 'count'
        FaceColor: 'auto'
        EdgeColor: [0 0 0]
```