PHYS 608: Computational problem set 2

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1 Lorenz Attractor (Goldstein 11.6)

Examination of the solution of the three coupled equations

$$\frac{dx}{dt} = \sigma(y - x), \qquad \frac{dy}{dt} = rx - y - xy, \qquad \frac{dz}{dt} = xy - bz \tag{1}$$

for various values of the chaotic parameter r. In each of the following sections $\sigma = 10$, $b = 8/3$, $x(0) = 2$, $y(0) = 5$, and $z(0) = 5$.

1.1 $r = 0, 10, 20, 0 \leq t < 20$

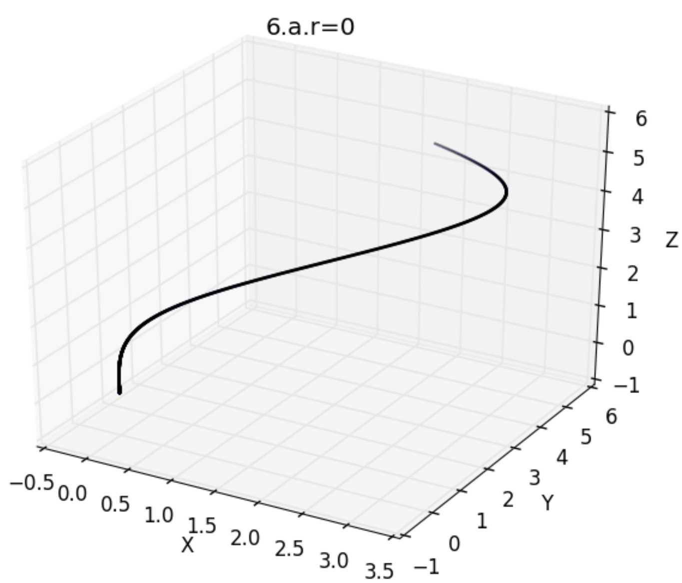

Fig.1. Lorenz attractor for $r = 0$.

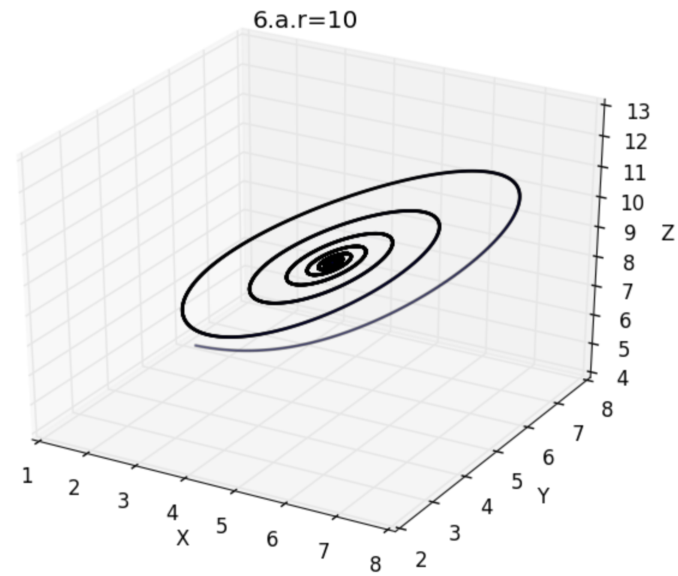

Fig.2 Lorenz attractor for $r = 10$.

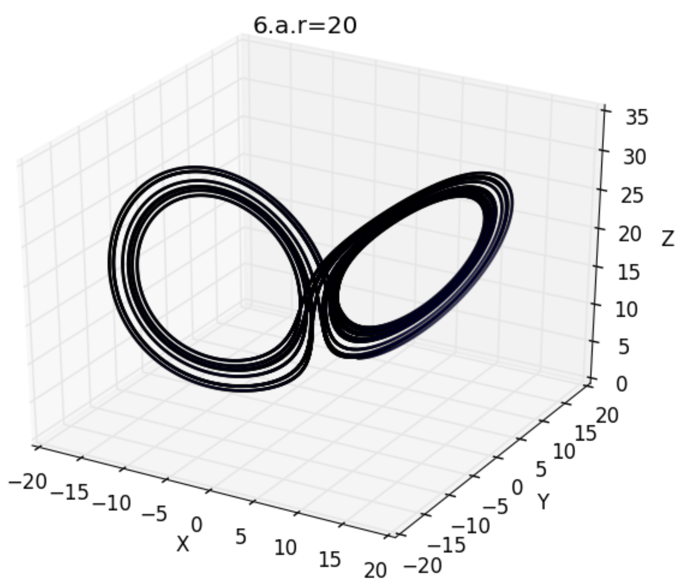

Fig.3 Lorenz attractor for $r = 20$.

1.2 $r = 28, 0 \leq t < 20$, where chaotic behavior set in for $t \approx 7$

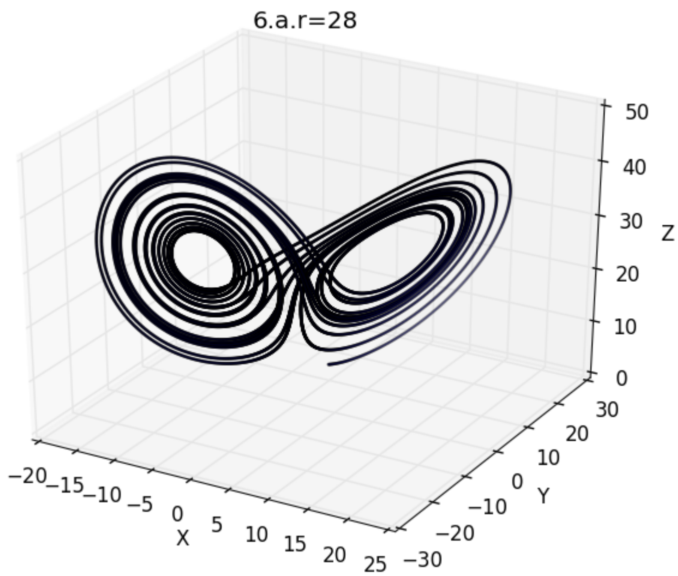

Fig.4 Lorenz attractor for $r = 28$.

2 Rossler attractor (Goldstein 11.7)

A nonphysical system which exhibits chaos. Solutions are found for the following system of coupled equations;

$$\frac{dx}{dt} = -(y+z) \qquad \frac{dy}{dt} = x + ay \qquad \frac{dz}{dt} = b + z(x-c) \tag{2}$$

2.1 $a = b = 0.2 \quad x(0) = -1, \quad y(0) = z(0) = 0$

Amplitude peak around $c = 5.7$:

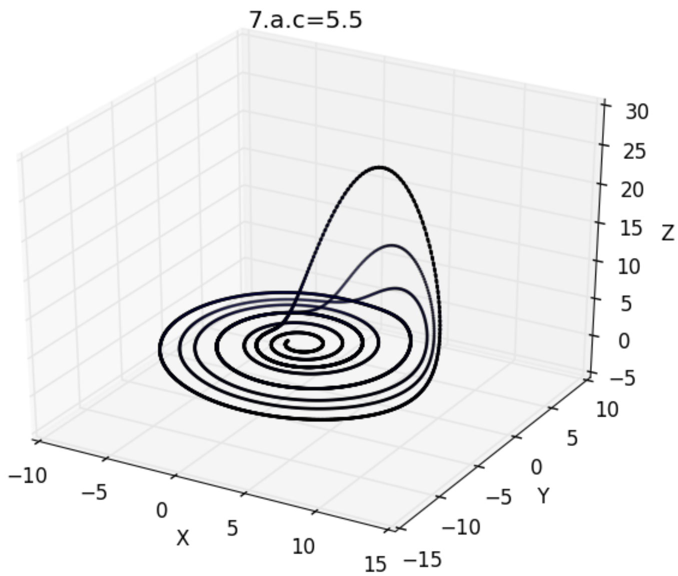

Fig.5 Rossler attractor for $c = 5.5$. Peak in Z around 16. 3 loops exiting xy plane.

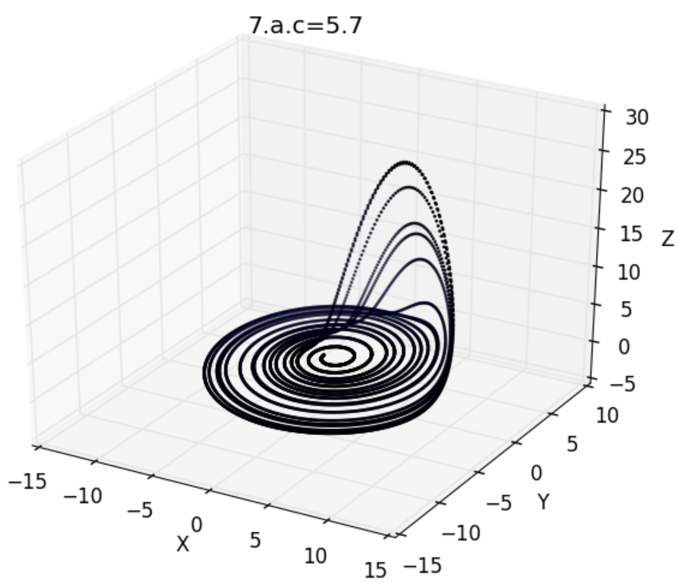

Fig.6 Rossler attractor for $c = 5.7$. Peak in Z around 19. 6 loops exiting xy plane.

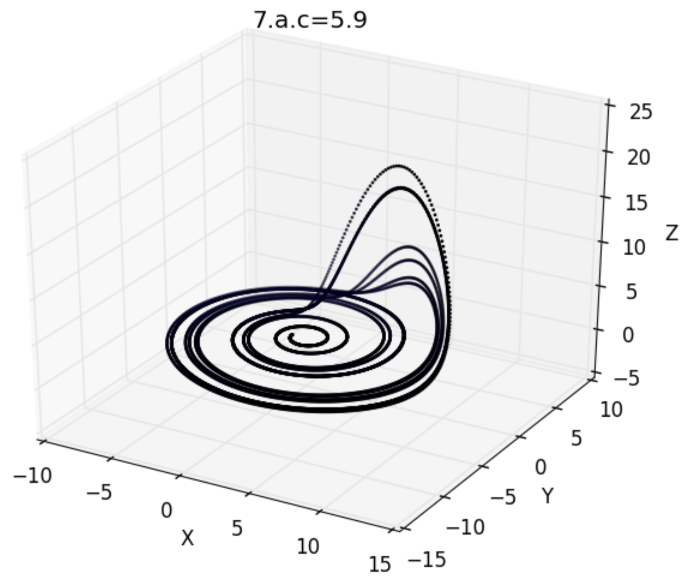

Fig.7 Rossler attractor for $c = 5.9$. Peak in Z around 14. 6 loops exiting xy plane.

2.2 $a = b = 0.2, \quad c = 5.7$

Variable number of limit cycles depending on initial conditions:

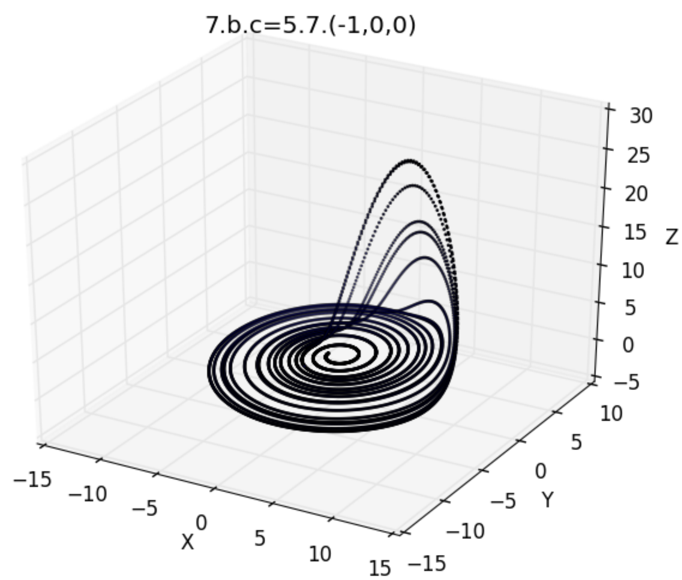

Fig.8 Rossler attractor for $c = 5.7$. $x(0) = -1, y(0) = z(0) = 0$. 6 loops exiting xy plane.

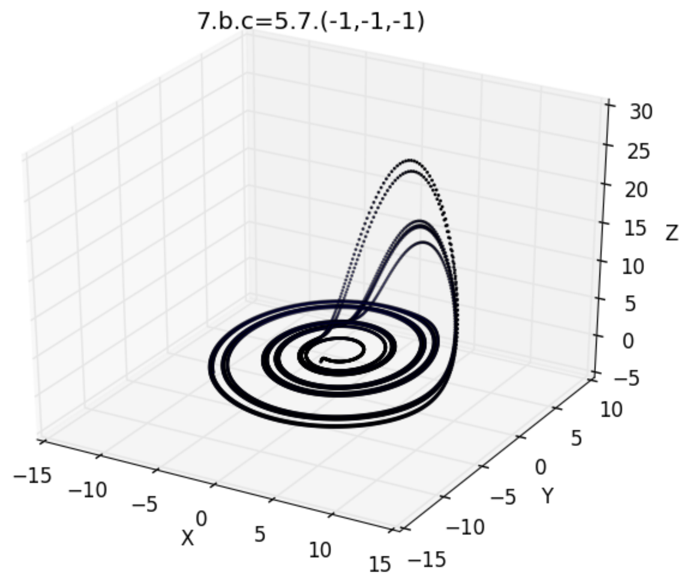

Fig.9 Rossler attractor for $c = 5.7$. $x(0) = -1, y(0) = z(0) = -1$. 5 loops exiting xy plane. Smaller number of unique paths in xy plane.

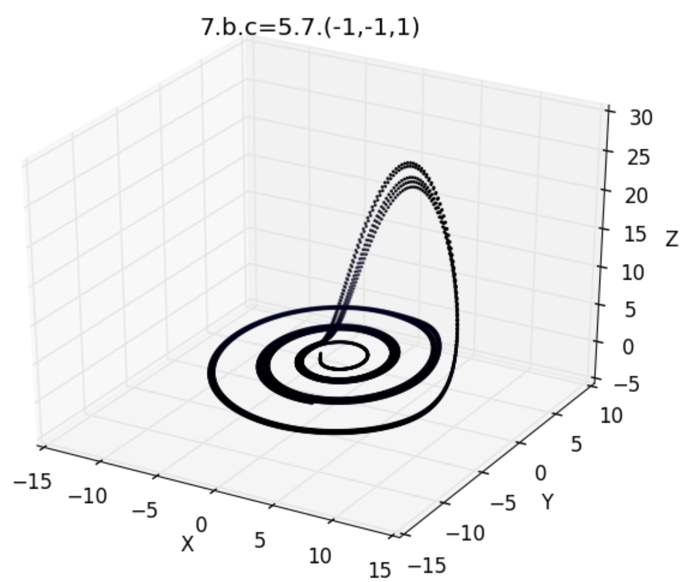

Fig.10 Rossler attractor for $c = 5.7$. $x(0) = -1, y(0) = -1, z(0) = 1$. 1 loop exiting xy plane. Only one major route out of the xy plane.

3 Forced damped oscillator (Duffing oscillator) (Goldstein 11.8)

The general forced damped oscillator equation is given by:

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \alpha x + \beta x^3 = F cos\omega t \tag{3}$$

3.1 Hysteresis of steady-state oscillations

I examine for $\alpha = 1$, $\beta = 0.2$, $\gamma = 0$, $F = 4.0$ and $[dx/dt]_{t=0} = 0$ and various values of $x_{t=0}$ and ω the relationship between the maximum value of the amplitude of oscillations to the driving frequency ω.

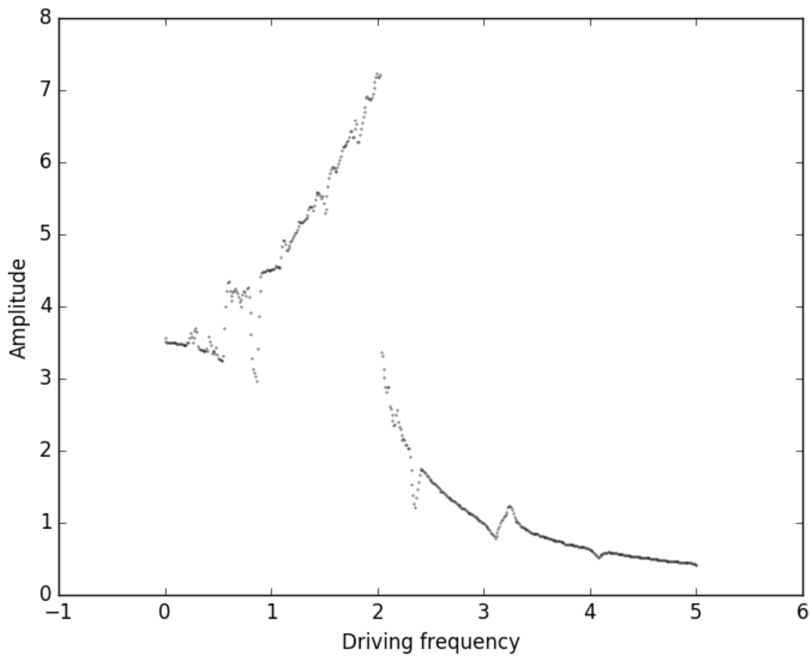

Fig.11 Duffing oscillator amplitude as a function of the driving frequency ω. A jump in amplitude occurs around Driving frequency $\omega = 2$.

3.2 Amplitude dependence on F for fixed ω

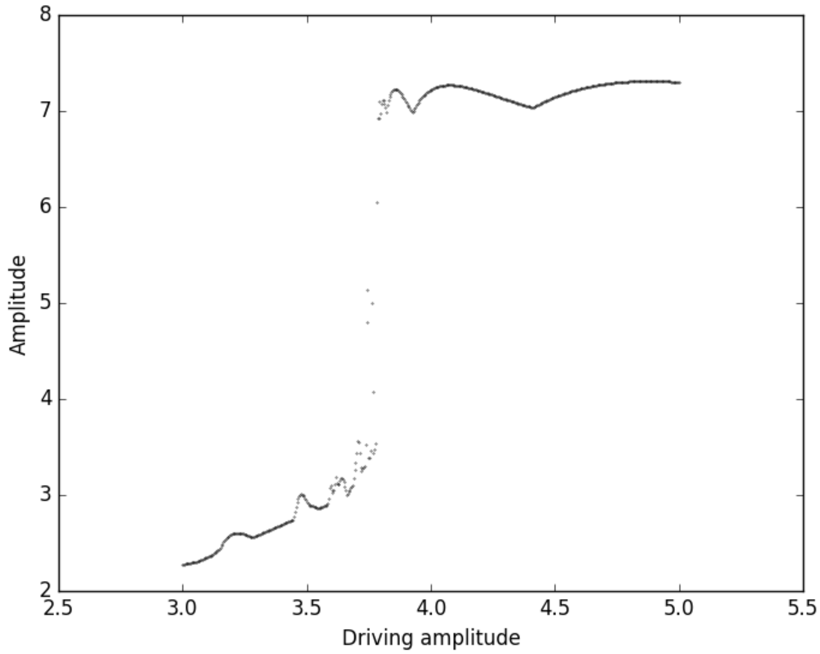

Fig.12 Duffing oscillator amplitude as a function of the driving amplitude F. A jump in the amplitude occurs around Driving amplitude $F = 3.8$.

4 Diffusion equation studied with random-aggregate fractal (Goldstein 11.12)

A particle is randomly placed in a grid and allowed to wander randomly until it either encounters another particle or exits the region. If the wandering particle encounters another previously fixed particle then it śtickst́o it. The fractal is śeededb́y an initial particle placed at the center of the grid. The fractal dimension of the resulting structure is then calculated as:

$$d = \frac{ln(N)}{ln(R_{min})} \tag{4}$$

where N is the number of particles and R_{min} is the minimum radius to enclose the entire structure.

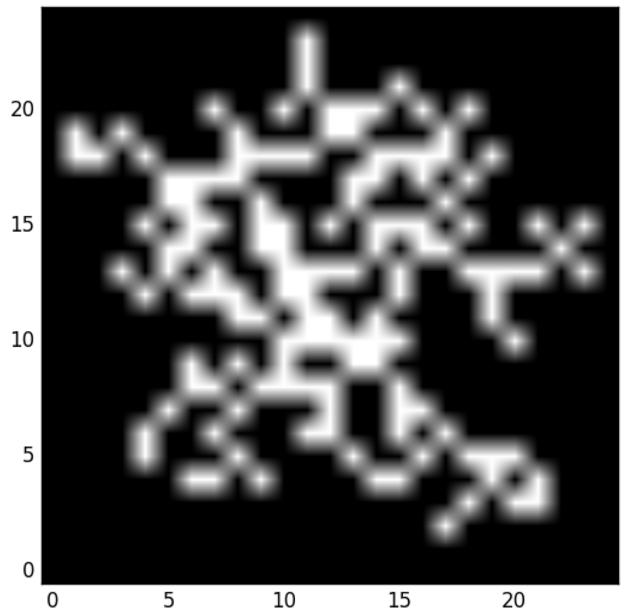

Fig.13 Snowflake fractal for grid size 25 by 25. Fractal dimension 1.817.

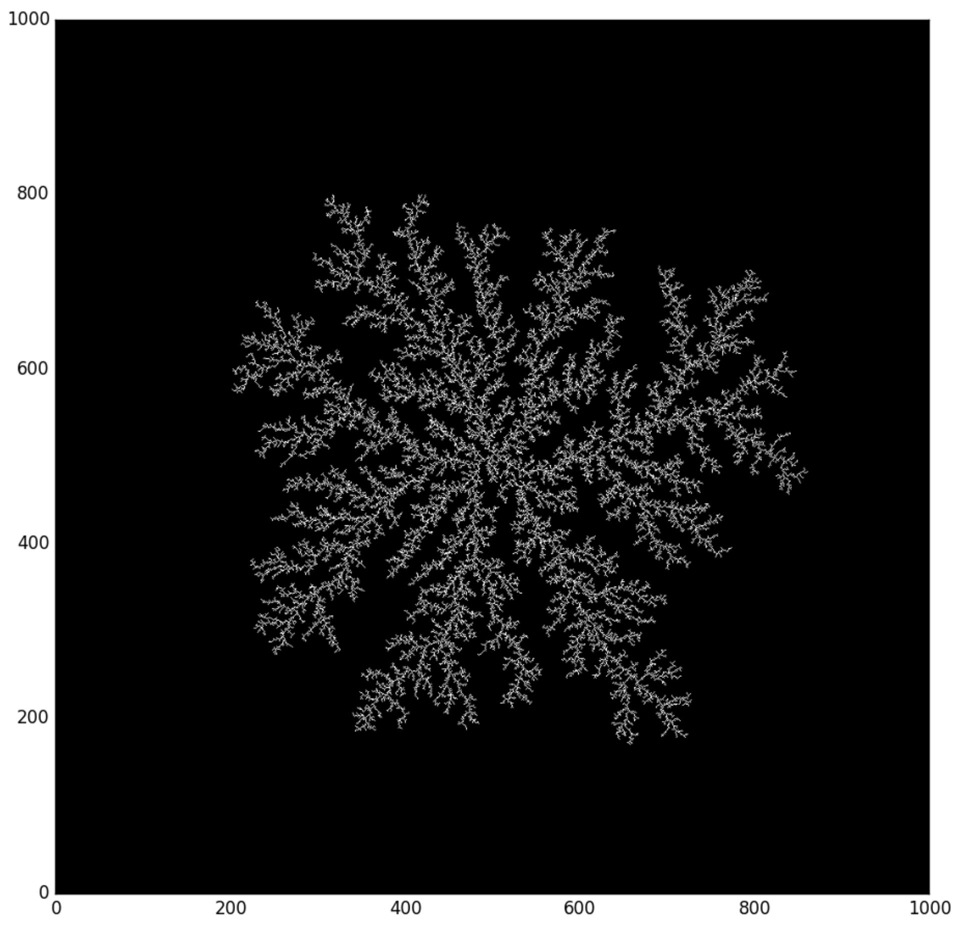

Fig.14 Snowflake fractal for grid size 1000 by 1000. Fractal dimension 1.607.