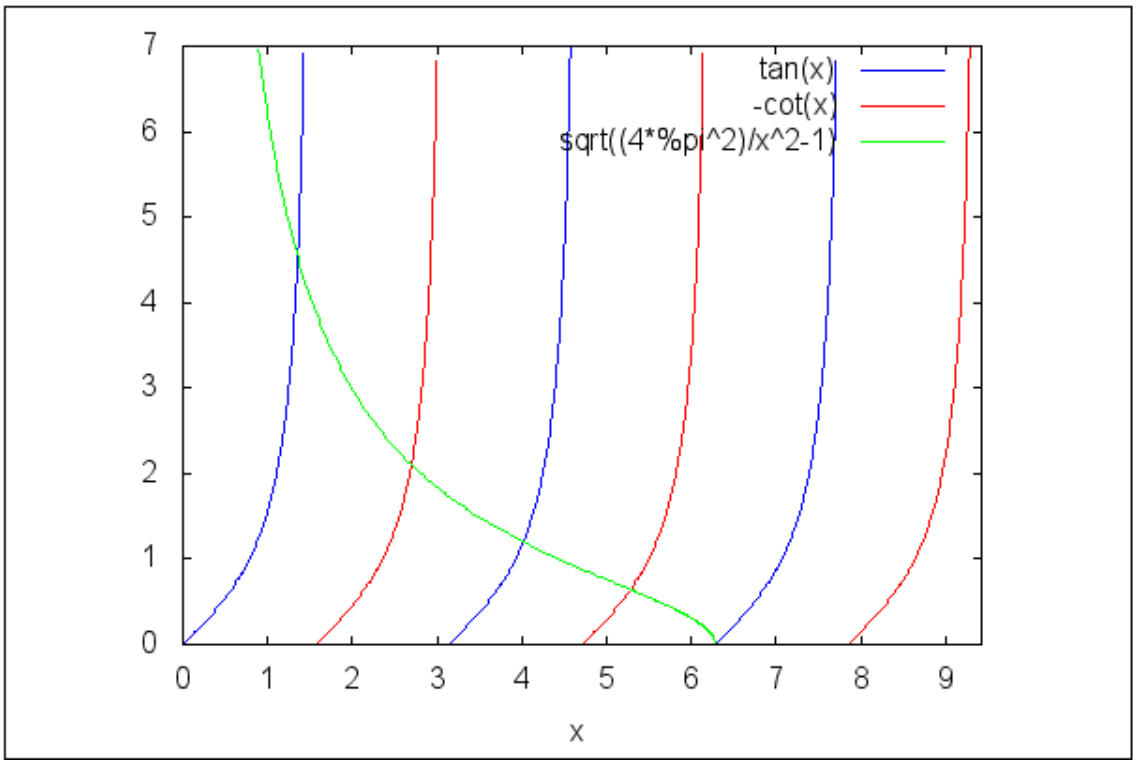
Physics 137A

1 Problem 6 from Chapter 2: The finite square well ii.

Here are the plots of tan(theta), tan(theta+90deg.)=-cot(theta), and theta_naught^2/theta^2-1)^(1/2) with theta_naught set to 2pi in order that there be just barely 5 bound states (this value was found by simply solving the last of these three equations for f(2pi)=0).

```
--> wxplot2d([tan(x), tan(x+(%pi/2)), (((2*%pi)^2/x^2)-1)^(1/2)],
    [x,0,3*%pi], [y,0,7])$
plot2d: some values were clipped.
cot: argument 0.0 isn't in the domain of cot.
plot2d: expression evaluates to non-numeric value somewhere in plotting range.
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plotting range.
plot2d: some values were clipped.
```

(%t1)



Here I have numerically solved for the solutions of the graphs by guessing the interval where the functions cross from the graph above. The values are in radians.

```
(%i2) find_root(tan(x)-((2*%pi)^2/x^2-1)^(1/2), x, 1.1, 1.5);
(%o2) 1.353653308326061
```

```
(%i3) find_root(tan(x)-((2*%pi)^2/x^2-1)^(1/2), x, 3.8, 4.1);
```

```
(%o3) 4.018465064366159
```

```
(%i4) find_root(tan(x)-((2*%pi)^2/x^2-1)^(1/2), x, 6.1, 6.5);
(%o4) 6.283185307179589
```

```
(%i5) find_root(-cot(x)-((2*%pi)^2/x^2-1)^(1/2), x, 2.5, 2.7);
(%o5) 2.697799621202854
```

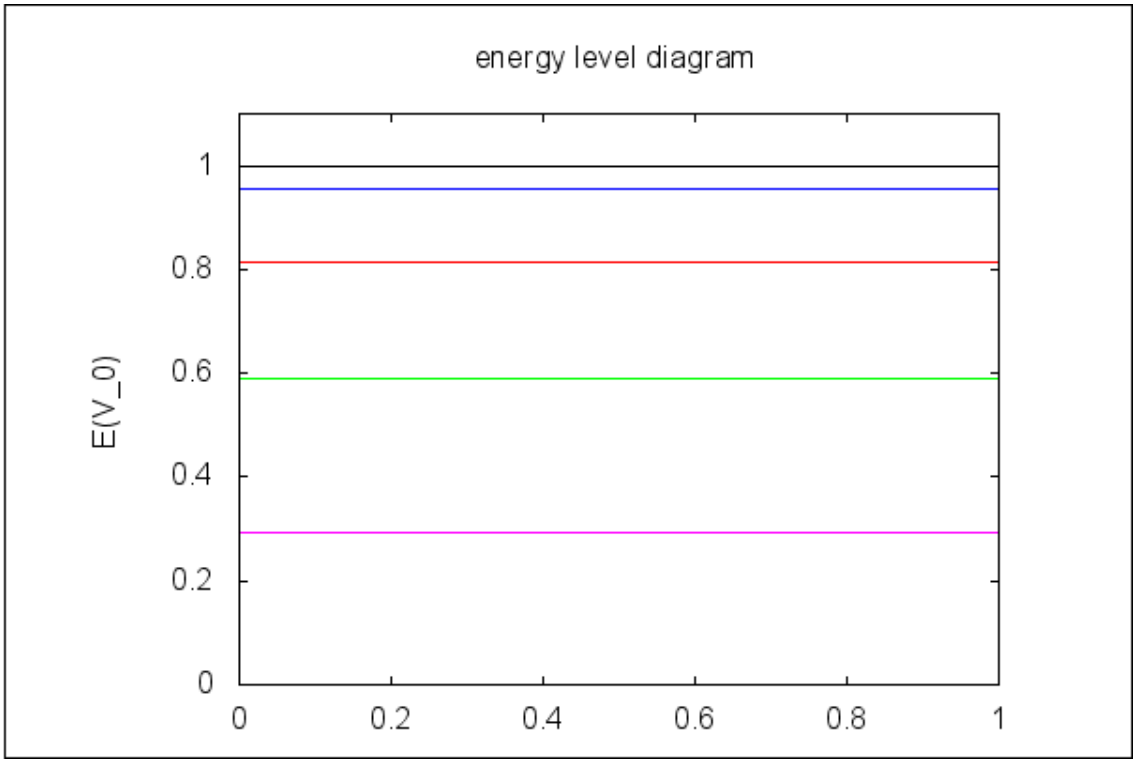
```
(%i6) find_root(-cot(x)-((2*%pi)^2/x^2-1)^(1/2), x, 5.1, 5.5);
(%o6) 5.284079702656433
```

The energy levels in terms of V_0 to 4-place accuracy are given by
E_n=-(1-(theta_n^2/theta_0^2))V_0
theta_0=2pi. Hence,

E_1 = -0.9536 V_0
E_2 = -0.8156 V_0
E_3 = -0.5910 V_0
E_4 = -0.2927 V_0
E_5 = 0.0 V_0

```
(%i13) wxplot2d([0.9536, 0.8156, 0.5910, 0.2927, 1], [x,0,1], [y,0,1.1],
       [title, "energy level diagram"], [legend, false],
       [xlabel, false], [ylabel,"E(V_0)"])$
```

(%t13)



Created with wxMaxima.