Donald A. Glaser Advanced Lab

OPT Pre-Lab

PDF version

INCLUDE THIS SHEET AS THE FIRST PAGE OF YOUR REPORT.

Student's Name: Oliver Gorton

Partner's Name: David Zepeda

Before the 1st Day of Lab

Pre-lab Questions and Sign Off Sheet

It is your responsibility to discuss this lab with an instructor on the first day of your scheduled laboratory period. This signed sheet must be included as the first page of your report. Without it you will lose 1/3 of a letter grade. You should think about and be prepared to discuss at least the following questions before you come to lab:

1. What is the general principle of optical pumping? Go over your derivation of the Breit-Rabi formula and the values of the Lande g-factors of the hyperfine energy levels of 85Rb and 87Rb. Draw qualitative energy-level diagrams for 85Rb and 87Rb showing the fine, hyperfine, and Zeeman splittings. How do the Lande g-factors affect the ordering of the Zeeman levels? Show the transitions between these levels that are important to this experiment. Include these drawings in your write-up. For our rubidium system, what is the pumping process? Where is the pumped level? Where is the RF transition?
2. Why do we modulate (vary sinusoidally) the external magnetic field? How would we take data if the magnetic field were not modulated?
3. In this experiment, how will you determine the resonance frequency? How can you best estimate the error? Will the modulation amplitude affect your result? What data will you take, and what plots will you make?

Staff Signature ______ Date 3/11/2016

Completed *before the first* day of lab? (circle) Yes / No

Mid-lab Questions and Sign Off Sheet

On day 2 of this lab, you should have successfully produced a plot of frequency versus current for at least one rubidium isotope, and have made an estimate of the earth's magnetic field. Show them to an instructor and ask for a signature.

Staff Signature ______ Date 4/26/16

Completed on the *second* day of lab? (circle) Yes / No

Please also fill out the Student Evaluation of Experiment.

Optical Pumping

Oliver Gorton
Partner: Daniel Zepeda

May 2016

Abstract

In this experiment atoms of rubidium are placed in a weak magnetic field and filtered RF oscillations applied to the sample cause particular energy levels to become highly populated, inducing a non-Boltzmann distribution of quantum states. The resulting observations in conjunction with the Breit-Rabi equation allow for experimental measurement of the nuclear spins of the two most common isotopes of rubidium and an accurate measurement of the Earth's magnetic field. [Major results: $I_{Rb85} = 5/2$, $I_{Rb87} = 3/2$, $B_{earth} = 0.45 \pm 0.1$Gauss.]

1 Introduction

Optical pumping is an important experiment that brings together nuclear physics, electrodynamics, and quantum mechanics. The atomic energy levels of atoms of rubidium (held in their gaseous form) as described by quantum mechanics are manipulated using static magnetic fields and rapidly oscillating electromagnetic fields. An important part of experimental physics is measuring the energy levels of various particle systems. Optical pumping is used here to measure the quantum energy levels of rubidium at the level of the fine structure, the hyperfine structure, and at the level of Zeeman splitting. This experiment is also closely related to the methods of nuclear magnetic resonance. Both of these experiments reveal information about the fundamental structure of the atoms under study, and both have extensive applications to other fields such as medical physics.

In this experiment we will use optical pumping to determine experimentally the nuclear spins of rubidium 85 and rubidium 87 - the two most common isotopes of rubidium. Once the nuclear spins are know, measurements of the resonant frequencies of these atoms can be used to accurately measure the magnetic field of the Earth.

2 Theory

2.1 Rubidium and energy levels

In this experiment we deal with the effects of the energy level splittings of isotopes of rubidium, Rb85 and Rb87. In the presence of a weak magnetic field, the hyperfine structures loses degeneracy as the magnetic moments of the electrons and nucleons interact with the magnetic field. Specifically, we are dealing with the valence electrons of Rb85 and Rb87 in the 5S and 5P states. At the fine-structure level, the 5P state split into $^2P_{3/2}$ and $^2P_{1/2}$ levels (calculated from electron spin and relativistic corrections). The 5S state remains degenerate at the fine-structure level and is labeled ${}^2S_{1/2}$. Transitions between ${}^2P_{1/2}$ and ${}^2S_{1/2}$ occur will be called D1 transitions and transitions between ${}^2P_{3/2}$ and ${}^2S_{1/2}$ will be called D2 transitions. In order to simply our system we filter out all light at D2 frequencies to that only ${}^2P_{1/2}$ - ${}^2S_{1/2}$ transitions occur. After the fine-structure we see the hyperfine splitting due to interaction between the magnetic dipole moment of the nucleus and the magnetic field it sees from the electron. Such transitions demand that the quantum number $F = I + J$ satisfy $\Delta F = 0, or \pm 1$.

If we place our atoms in a magnetic field then we introduce another term in the Hamiltonian: interaction between the magnetic field and the magnetic moments of our particles. This interaction is called the Zeeman effect. Zeeman splitting is the smallest at on the order of MHz and are magnetic transitions satisfying $\Delta m_f = 0, or \pm 1$.

By circularly polarizing our light we can further restrict to $\Delta m_f = +1$ absorptions, since circularly polarized light can only increase the momentum of the system by one quanta. Transitions will also occur in the other direction both due to stimulated emission and random decays. In these cases $\Delta m_f = 0, or \pm 1$, but notice (see energy level diagram) that once an electron is in the ${}^2S_{1/2}, m_f = +2$ state for Rb87 or the ${}^2S_{1/2}, m_f = +3$ state for Rb85, circularly polarized light cannot excite it into any higher energy state. We are now ready to induce the process known as Optical Pumping. Statistical mechanics tells us that the atoms' quantum states will be distributed according to the Boltzmann distribution. But by exiting the sample using circularly polarized D1 light, all of the atoms will eventually be 'pumped' into the most positive ${}^2S_{1/2}, m_f$ states where they become optically isolated. When the system has been pumped, it can absorb no more D1 light, and is therefore transparent to that frequency.

2.2 Resonant frequency

If once the atoms have been optically pumped we apply an oscillating electromagnetic wave at the larmour frequency, then we will stimulate transitions to lower energy level and the atoms can again absorb the polarized light. This can be measured as a spike intensity of light coming from the sample when we apply the correct (radio frequency) signal. We therefore need to determine the relationship between the applied resonant frequency and the applied magnetic field using results from quantum mechanics. The Hamiltonian is

$$H = -\vec{\mu_I} \cdot \left(\vec{B_J} + \vec{B_{ext}}\right) - \vec{\mu_J} \cdot \vec{B_{ext}} \qquad (1)$$

where the I indices refer to nuclear spin and J indices refer to electron nuclear spins. By assuming a weak magnetic field (for weak Zeeman splitting) and that the interaction of the nucleus with the field is small compared to that of the electron (since $m_e << m_p$) we can arrive at the Breit-Rabi equation

$$\frac{f_{resonance}}{B_{ext}} = \frac{g_s \mu_b}{(2I+1)h} = \frac{2.799}{2I+1}\frac{MHz}{Gauss} \qquad (2)$$

(see appendix for full derivation) which is the primary relation used in this experiment.

Our sample is placed in the center of Helmholtz coils which are aligned with the Earth's magnetic field and which supply the magnetic field to induced Zeeman splitting. Therefore under operating circumstances $B_{ext} = B_{earth} + B_{coils}$. The magnetic field at the center of the coils can be derived from basic magneto statics (see appendix for derivation) and is approximately

$$B_{coils} = \frac{8\mu_0 Ni}{5\sqrt{5}r} = 9 * 10^{-3}\frac{Ni}{r}\frac{Gauss * meter}{ampere} \qquad (3)$$

where N is the number of coils, i is the current in the coils, and r is the radius of the coils. For our experimental setup N = 135 and r =

0.275 m. In total

$$B_{coils} = 4.42 * i\frac{Gauss * meter}{ampere}. \tag{4}$$

Lastly we can rearrange our results for experimental application. To determine the nuclear spin of an isotope for which we have observational resonant frequencies and known current in the Helmholtz coils, we plot $f_{resonance}$ against the current i to find the ratio $df_{resonance}/di = Slope$. Then observing $df/di = 4.42 * df/dB_{coils}$,

$$I = \frac{1}{2}\left(\frac{12.37}{Slope} - 1\right). \tag{5}$$

The Earth's magnetic field can be found by finding the Y-intercept of the same plot. When $i = 0$, $B_{ext} = B_{earth}$ and

$$B_{earth} = \frac{(2I+1)}{2.799} f_0 \tag{6}$$

where f_0 is the resonance frequency when $i = 0$: the Y-intercept. Yet another method for finding the value of Earth's magnetic field is to adjust the resonance parameters until the resonance signal disappears. For such values, the field from the Helmholtz coils exactly cancels the field from the Earth, thus causing the Zeeman splitting to disappear.

3 Apparatus and Procedure

3.1 Apparatus

The optical pumping equipment setup can be seen in a block diagram in the appendix. The basic elements are these:

1. A rubidium sample containing Rb85 and Rb87
2. A heat source and method for controlling the sample's temperature
3. Helmholtz coils with AC modulation control unit
4. RF oscillator with circular polarizer and D1-pass filter
5. Additional magnetic coils near the sample to apply MHz-range signals to the sample, perpendicular to the Helmholtz field
6. Photodiode detector placed oppositely to RF oscillator

3.2 RF Modulation vs Coil Modulation

There are two methods for applying equation (5) to practice. Both methods attempt to identify a resonance signal where for a given value of either the magnetic field or the applied RF frequency, the unique value of the other parameter is located by probing nearby values.

The first method involves holding the magnetic field from the Helmholtz coils fixed and varying the RF oscillator signal. This is referred to as the RF Modulation method. Data is taken by observing peaks in the photodetector output in the time-domain, corresponding to resonances at particular RF frequencies.

The second method involves holding the RF oscillator fixed near a known resonance frequency and then varying the magnetic field from the Helmholtz coils around a given offset. This is referred to as the Coil Modulation method. Data is taken by observing resonance signals in the phase-domain of the magnetic field modulation, which oscillates sinusoidally about some constant offset.

4 Analysis

4.1 Nuclear spins and Earth's magnetic field

Here are the results summarized from the graphs:

RF Modulation:

Isotope	Y-intercept	Slope
Rb85	0.20 ± 0.01	2.14 ± 0.01
Rb87	0.31 ± 0.01	3.19 ± 0.01

Coil Modulation:

Isotope	Y-intercept	Slope
Rb85	0.17 ± 0.01	2.05 ± 0.02
Rb87	0.23 ± 0.02	3.07 ± 0.02

From these values and the Breit-Rabi equation we can determine both the magnetic field of the earth and the nuclear spins of both isotopes of rubidium. Above we showed that

$$I = \frac{1}{2}\left(\frac{12.37}{Slope} - 1\right) \tag{7}$$

and that I must be a half-integer value. Propagating the errors appropriately we arrive at the following results.

RF Modulation:
$I_{Rb85} = 2.39 = \frac{5}{2}$
$I_{Rb87} = 1.43 = \frac{3}{2}$
$B_{earth} = 0.41 \pm 0.02$ Gauss (from rb85)
$B_{earth} = 0.43 \pm 0.01$ Gauss (from rb87)

Coil Modulation:
$I_{Rb85} = 2.52 = \frac{5}{2}$
$I_{Rb87} = 1.51 = \frac{3}{2}$
$B_{earth} = 0.37 \pm 0.02$ Gauss (from rb85)
$B_{earth} = 0.33 \pm 0.03$ Gauss (from rb87)

We see that the values from the RF modulation method are incompatible with the values from the Coil modulation method in that their errors exclude the other's values. This is despite the fact that within a given method the value do agree. We use another method here to verify the discrepancy: Take the difference between resonant frequencies for positive and negative currents and calculate earth's magnetic field from these values, taking the mean and standard deviation of the set as the value and error, respectively. (See appendix for code.)

From frequency modulation method:
From Rb85: $B_{earth} = 0.35 \pm 0.09$ Gauss
From Rb87: $B_{earth} = 0.55 \pm 0.04$ Gauss
Average: $B_{earth} = 0.45 \pm 0.10$ Gauss.

From coil modulation method:
From Rb85: $B_{earth} = 0.29 \pm 0.08$ Gauss
From Rb87: $B_{earth} = 0.36 \pm 0.17$ Gauss
Average: $B_{earth} = 0.33 \pm 0.19$ Gauss.

Average of coil modulation average and frequency modulation average:
$B_{earth} = 0.39 \pm 0.21$ Gauss.

(Errors on the average were calculated correctly from the results of the Error Analysis Assignment, using error propagation techniques.)

From this value we see that we have very little certainty about what the earth's magnetic field is, with a standard deviation nearly two thirds of the mean. We expect the earth's magnetic field in Berkeley to be around 0.48 Gauss based on our coordinates (See NOAA reference). There is reason to suspect, however, that the data set collected during the coil modulation method may have been influenced by the proximity of a compass needle. It is reasonable to suspect that the needle's field has sufficient strength to have such a noticeable effect since it was observed during setup of the coil modulation method that spinning the needle nearby the coils would cause the resonance signal to oscillate around nearby resonance frequencies. If the compass used to test the orientation of the Helmholtz coils were left too close to the apparatus then the alignment of the compass needle with the coils would have had the effect of partially canceling out part of the earth's magnetic field, leading to an apparent weaker field from the earth.

If the second set of results were influenced by an unaccounted-for external magnetic field, then the results from the first set of data yielding $B_{earth} = 0.45 \pm 0.10$ is the more accurate and precise value.

4.2 Zero-field

By setting the coils to cancel out the magnetic field of the Earth (identified by zero resonance signal) we can calculate the Earth's magnetic field. Using this method:

$B_{earth} = 0.44 \pm 0.22$ Gauss.

This is consistent with the value found above for RF Modulation.

4.3 Optical Signal vs Temperature

The rubidium sample is heated in order to maintain optimal signal strength. Data was collected before the main experiment in order to determine the optimal sample temperature. According to the results displayed in figure 1, the optimal signal occurs around 41 degrees Celsius. For this reason the sample was continuously heated to 42 degrees Celsius, allowed to cool while measurements were taken, then, once the temperature reached 39 degrees Celsius, heated again.

5 Conclusion

We successfully measured the nuclear spins of Rb85 and Rb87 and measured Earth's magnetic field strength with some uncertainty do to possible experimental negligence.

6 Acknowledgments

The author would like to thank Daniel Zepeda for his partnership during this experiment and he would also like to thank Professor Hartmut Haeffner and T.A. William T. who both guided us during this experiment.

7 References

1. Bevington, P.R., *Data Reduction and Error Analysis.* Third Edition. McGrawHill.

2. Bloom, A. L., *Optical Pumping.* Scientific American, Oct. 1960, p.72.

3. De Zafra, R. L., *Optical Pumping.* Amer. Journ. of Phys. 28, 646 (1960).

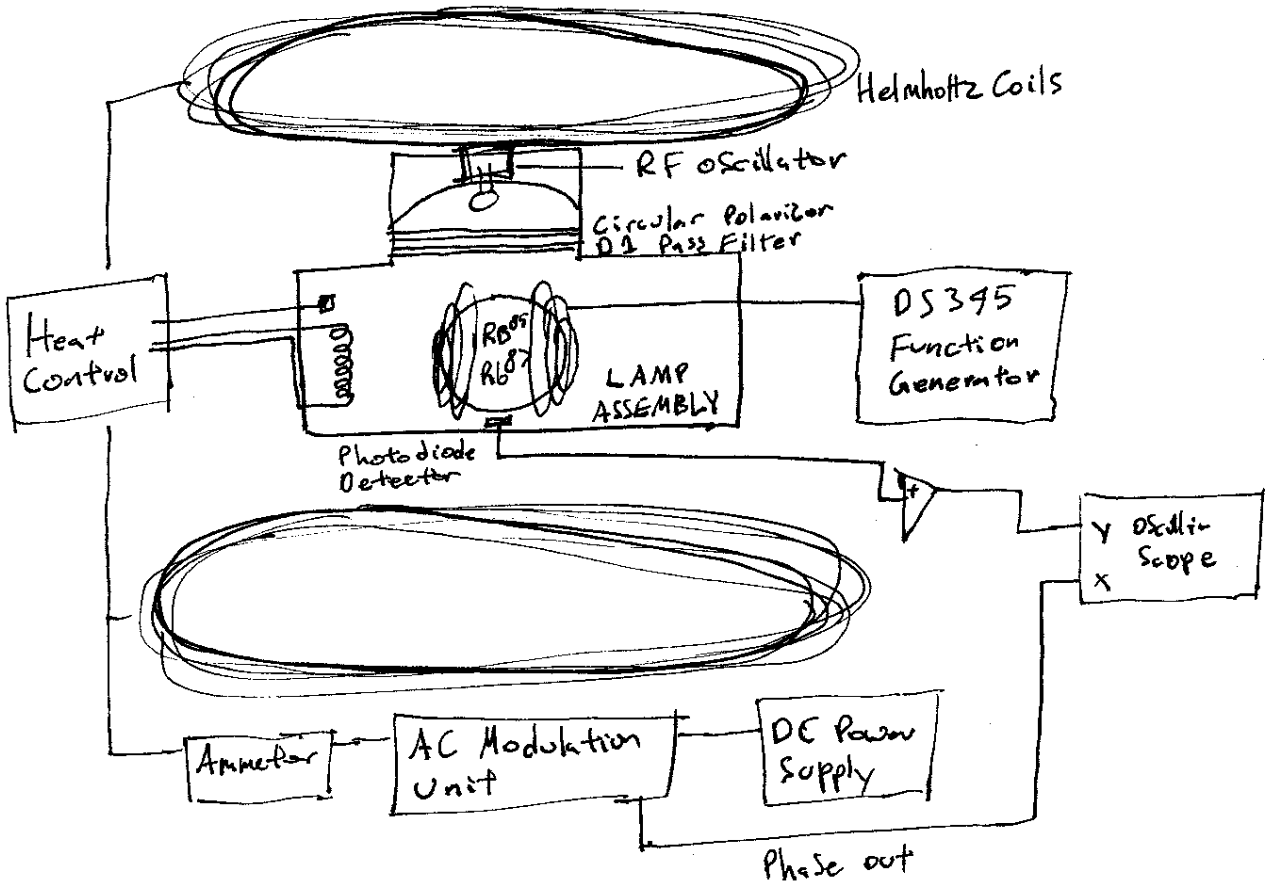
4. Griffiths, D. J., *Introduction to Electrodynamics.* Third Edition, p.259.

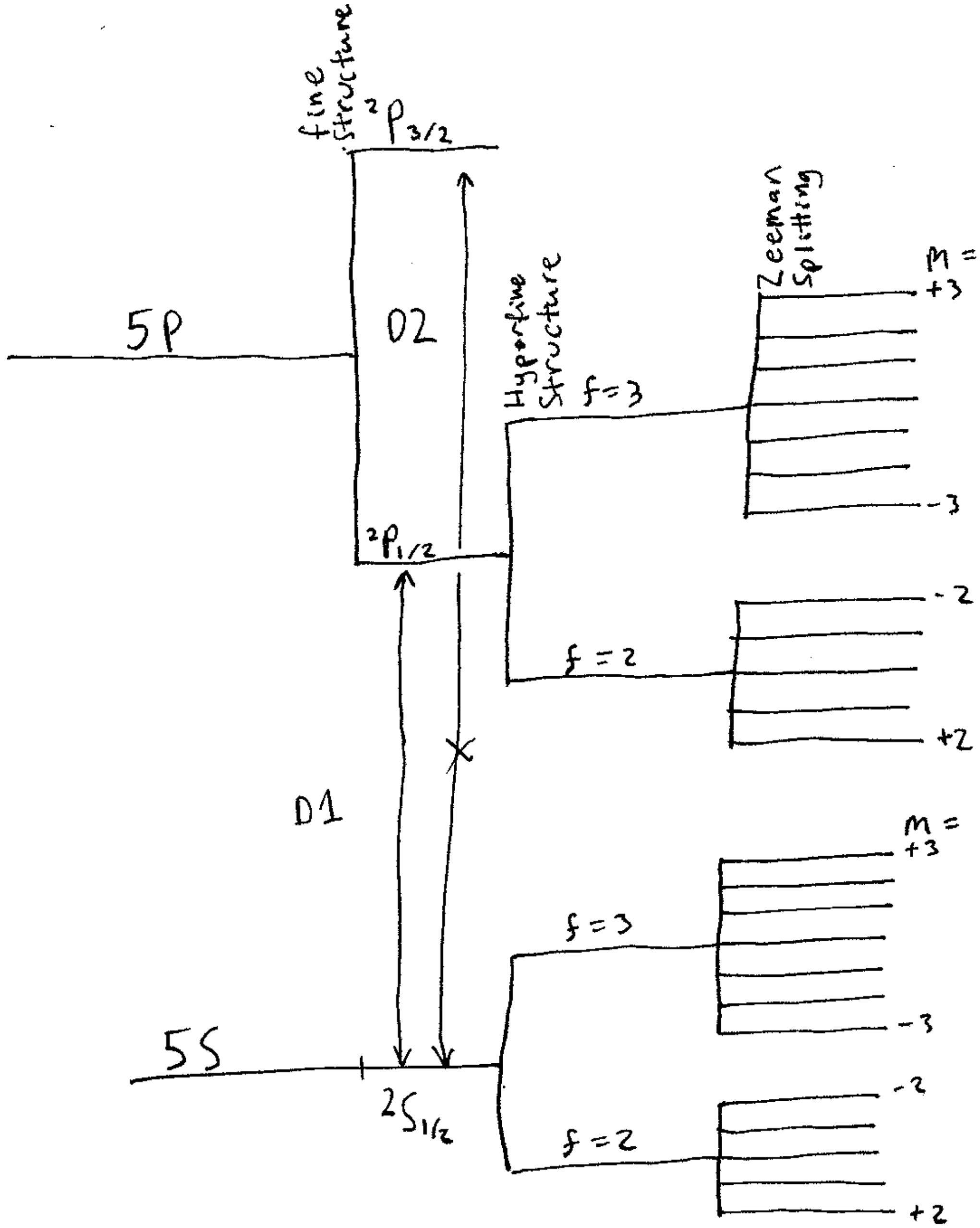
5. Lyons, L., *A Practical Guide to Data Analysis for Physical Science Students.* Cambridge, 1991.

6. NOAA, *Magnetic Field Calculators.* Online resource. Accessed May 1, 2016.

7. OPT Ref. 6, *Experiment 7; Electron Spin Resonance by Optical Pumping.* Origin Unknown. Physics 111-Lab Library Reprints.

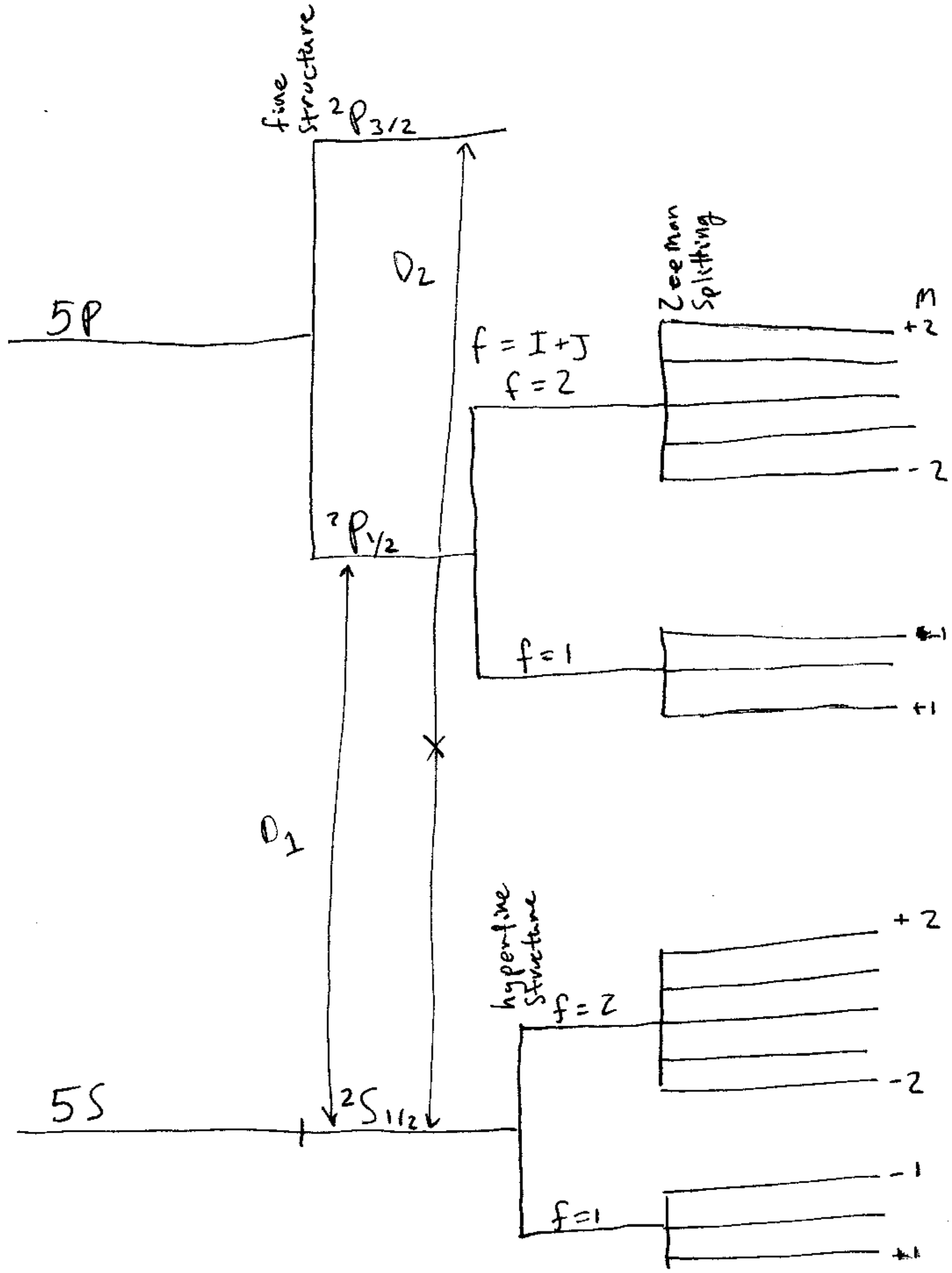
Optical Pumping Equipment Setup





Rb-85 Energy Level Diagram

(I= 5/2)



Rb-87 Energy Level Diagram
(I = 3/2)

Figure 1: Optical signal as a function of sample temperature.

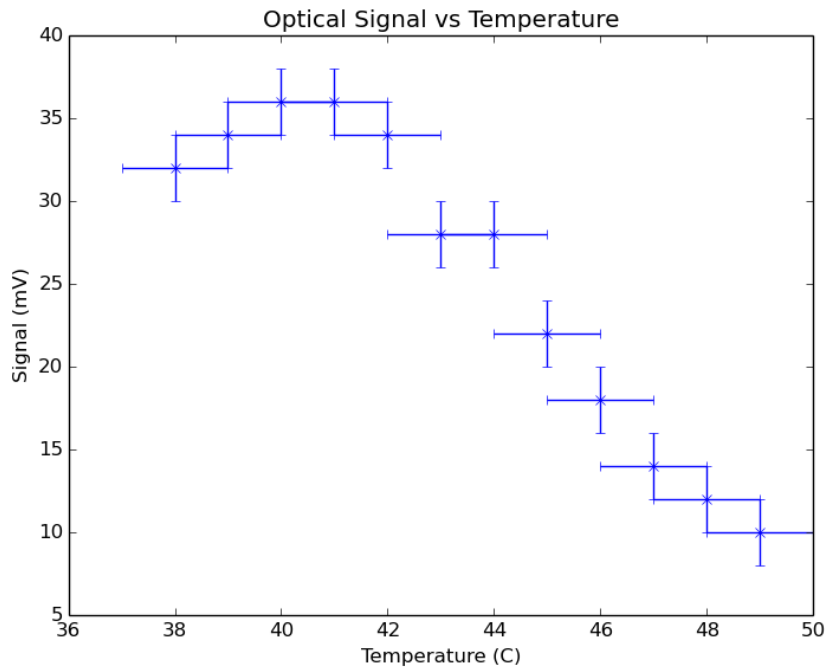


Figure 2: Rb85 Resonant frequency versus coil current using frequency modulation. Contains combined data from forward and reverse current. Reduced Chi2 = 0.08, 99 percent probability that model is good. See appendix for code. [Slope = 2.14 ± 0.01, Y-intercept = 0.20 ± 0.01]

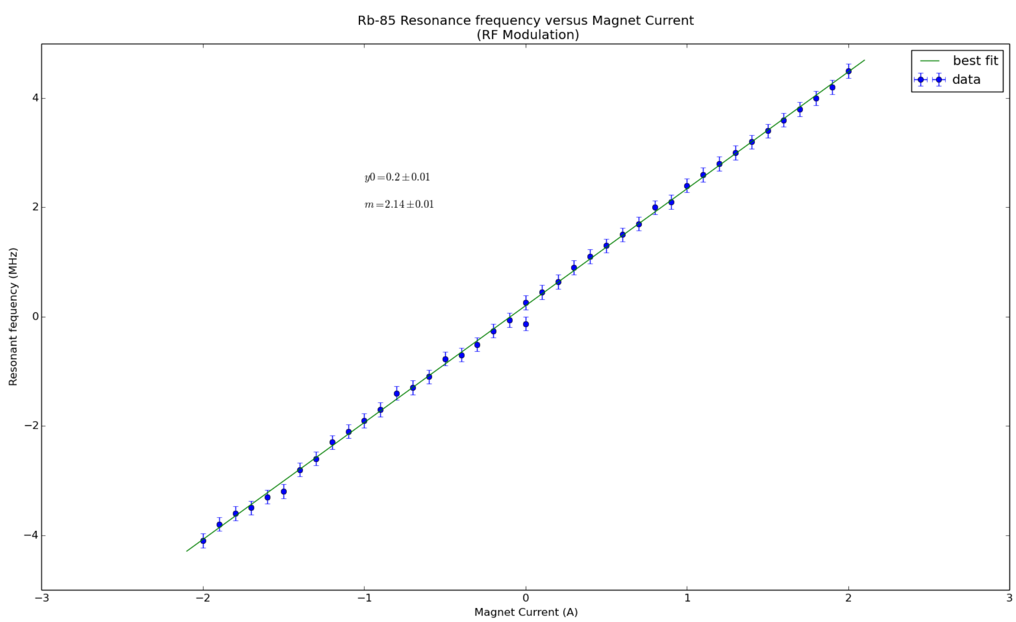


Figure 3: Rb87 Resonant frequency versus coil current using frequency modulation method. Contains combined data from forward and reverse current. Notice that values for small current for which no signal could be distinguished from noise. Reduced Chi2 = 0.13, 99 percent probability model is good. See appendix for code. [Slope = 3.19 ± 0.01, Y-intercept = 0.31 ± 0.01]

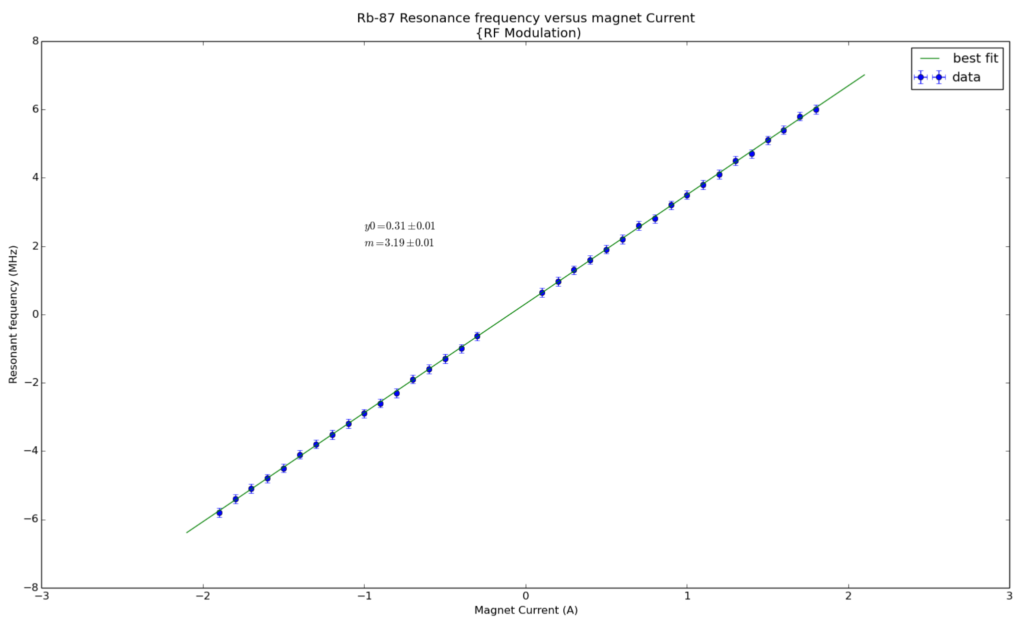


Figure 4: Rb85 Resonant frequency versus coil current using coil modulation technique. Contains combined data from forward and reverse current. Reduced Chi2 = 0.58, 92 percent probability model is good. See appendix for code. [Y-intercept = 0.17 ± 0.01, Slope = 2.05 ± 0.02]

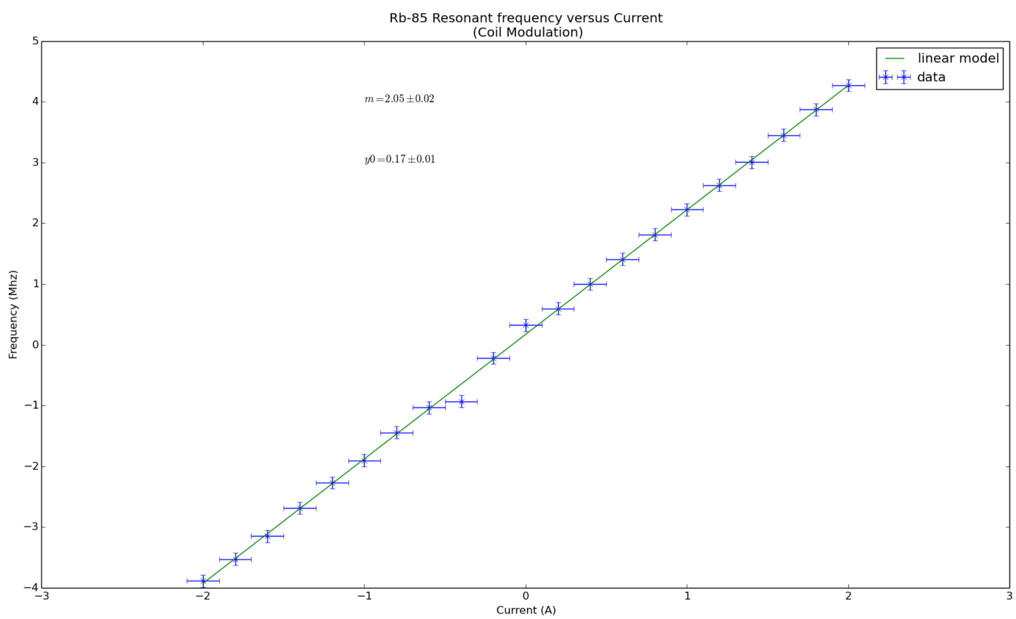
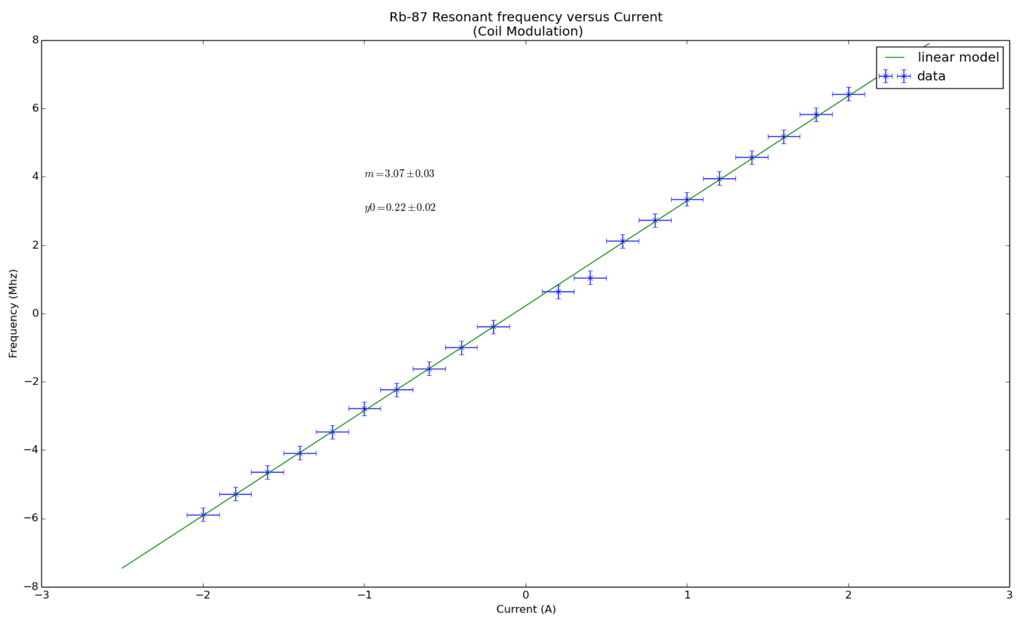


Figure 5: Rb87 Resonant frequency versus coil current using coil modulation technique. Contains combined data from forward and reverse current. Reduced Chi2 = 0.32, 99 percent probability model is good. See appendix for code. [Y-intercept = 0.23 ± 0.02, Slope = 3.07 ± 0.02]



Python Scripts for Optical Pumping Lab
Physics 111B Spring 2016
Oliver Gorton

```
#earth_field.py
#This program calculates Earth's magnetic field using the frequency-difference
#method
import numpy as np
def B(df):
      return 0.5*(df/2.799)*(2*I+1)
#Rb85:
I = 5/2
i, ff = np.loadtxt('Rb85_resonance_v_current_forward.txt', unpack = True,
skiprows=1)
i, fr = np.loadtxt('Rb85_resonance_v_current_reverse.txt',unpack=True, skiprows=1)
df=np.zeros(len(ff))
b=np.zeros(len(ff))
for i in range(0,len(ff)):
      df[i] = ff[i]-fr[-i-1]
      b[i] = B(df[i])
print round(np.mean(b),2), '+/-', round(np.std(b),2)
#Rb87:
I=3/2
I = 5/2
i, ff = np.loadtxt('Rb87_resonance_v_current_forward.txt', unpack = True,
skiprows=1)
i, fr = np.loadtxt('Rb87_resonance_v_current_reverse.txt',unpack=True, skiprows=1)
df=np.zeros(len(ff))
b=np.zeros(len(ff))
for i in range(0,len(ff)):
      df[i] = ff[i]-fr[i]
      b[i] = B(df[i])
print round(np.mean(b),2), '+/-', round(np.std(b),2)
'''
Print output:
0.35 +/- 0.09
0.55 +/- 0.04
'''
############################################################
#Rb85:
I = 5/2
i, ff = np.loadtxt('coilmod_rb85_forward.txt', unpack = True, skiprows=1)
i, fr = np.loadtxt('coilmod_rb85_reverse.txt',unpack=True, skiprows=1)
ff /= 1000.
fr /= 1000.
df=np.zeros(len(ff))
b=np.zeros(len(ff))
for i in range(0,len(ff)):
      df[i] = ff[i]-fr[i]
      b[i] = B(df[i])
print round(np.mean(b),2), '+/-', round(np.std(b),2)
#Rb87:
I=3/2
I = 5/2
i, ff = np.loadtxt('coilmod_rb87_forward.txt', unpack = True, skiprows=1)
i, fr = np.loadtxt('coilmod_rb87_reverse.txt',unpack=True, skiprows=1)
ff /= 1000.
fr /= 1000.


df=np.zeros(len(ff))
b=np.zeros(len(ff))
for i in range(0,len(ff)):
      df[i] = ff[i]-fr[i]
      b[i] = B(df[i])
print round(np.mean(b),2), '+/-', round(np.std(b),2)
```

```
'''
Print output:
0.29 +/- 0.08
0.36 +/- 0.17
'''
```

```
#optical_signal_v_temp.py
#opt
#This program plots the optical signal versus temperature data

import numpy as np
import matplotlib.pyplot as plt

temp, sign = np.loadtxt('optical_signal_v_temp.txt', unpack = True, skiprows = 2)

temp_err = 1 #deg celcius
sign_err = 2 #mV

plt.figure()
plt.title('Optical Signal vs Temperature')
plt.ylabel('Signal (mV)')
plt.xlabel('Temperature (C)')
plt.errorbar(temp, sign, xerr=temp_err, yerr=sign_err, fmt='x')

plt.show()
```

```
#Rb85_resonance_v_current.py
#OPT
#This program plots the data collected for Rb85 and calculates all of the relevant
#statistical quantities as per the error analysis section of the lab report.

import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize as fitter
from scipy.stats import chi2

current, resonance = np.loadtxt('Rb85_resonance_v_current.txt', unpack = True,
skiprows = 1)
x = np.linspace(-2.1,2.1,100)
def linear_model(x, m, y0):
      return y0+m*x
plt.figure()
plt.title('Rb-85 Resonance frequency versus Magnet Current \n (RF Modulation)')
plt.xlabel('Magnet Current (A)')
plt.ylabel('Resonant fequency (MHz)')
plt.errorbar(current, resonance, fmt='o', xerr = 0.01, yerr = 0.2*0.64,
label='data')
par, cov = fitter.curve_fit( linear_model, current, resonance)
y0 = par[1]
y0 = round(y0,2)
err_y0 = np.sqrt(cov[0,0])
err_y0 = round(err_y0,2)
y0_result = r'$y0 = %s \pm %s$'%(y0,err_y0)
plt.text(-1,2.5, y0_result)
m = par[0]
m = round(m, 2)
err_m = np.sqrt(cov[1,1])
err_m = round(err_m, 2)
m_result = '$m = %s \pm %s$'%(m, err_m)
plt.text(-1,2, m_result)
plt.plot(x, linear_model(x,m,y0),label='best fit')
sig=0.2*0.64
print 'sigma: ', sig
sigma = np.ones(len(current))*sig
chi_squared = 0
for i in range(0,len(current)):
```

```
      chi_squared+=(((linear_model(float(current[i]), float(par[0]),float(par[1]))-
resonance[i])/sigma[i])**2)
nu = len(current)-len(par)
reduced_chi_squared = (chi_squared)/(nu)
print 'nu = ', nu
print 'chi^2 = {0:5.2f}'.format(chi_squared)
print 'chi^2/d.f.={0:5.2f}'.format(reduced_chi_squared)
p=1-chi2.cdf(chi_squared,nu)#probability that a randon chi2 would be greater than
our chi
print 'probability that the straight line we found is an adequate description:', p
perr=np.sqrt(np.diag(cov))
print r'y0-uncertainty: ', round(perr[0],3)
'''
Output:
sigma:  0.128
nu =  33
chi^2 =  2.57
chi^2/d.f.= 0.08
probability that the straight line we found is an adequate description: 1.0
y0-uncertainty:  0.006
'''
plt.legend()
plt.show()
```

Programs for other resonance versus current data (for other isotopes and collection methods) is of the same form as Rb85_resonance_v_current.py and so it is not included here.

T(°C)	Peak (div)	T(°C)	Peak
50	[illegible]	39	
49	3.2	38	
48		37	
47		36	
46		35	
45			
44			
43			
42			
41			
40			

Earth's field

$$.375\,MHz = H\left(\frac{2.799}{6}\right)$$

T(°C)	Peak(div)
49	1
48	1.2
47	1.4
46	1.8
45	2.2
44	2.4
43	2.8
42	3.4
41	3.6
40	3.6
39	3.4
38	3.2

10 mV/div

± 2mV

• Coil current changes moves resonant frequency

09.26 Day 2 of OPT

Field current versus resonance frequency and voltage.

0.5 V/div

I(A) ±0.01	Peak 1 (div) 10 ±0.2 div	Peak 2 (div)		~~Peak~~ 2 (div)
0.0	0.4	—		
0.1	0.7	1.0	-0.1	0.1	—
2	~~0.7~~ 1.0	1.5	-0.2	0.4	—
3	1.4	2.0	-0.3	0.8	1.0
4	1.7	2.5	-0.4	1.1	1.6
5	2.0	3.0	-0.5	1.2	2.0
6	2.3	3.5	-0.6	1.7	2.5
7	2.6	4.0	-0.7	2.0	3.0
8	3.0	4.4	-0.8	2.2	3.6
9	3.3	5.0	-0.9	2.7	4.0
1.0	3.7	5.5	-1.0	3.0	4.5
1.1	4.0	6.0	-1.1	3.3	5.0
1.2	4.3	6.4	-1.2	3.6	5.5
1.3	4.7	7.0	-1.3	4.0	6.0
1.4	5.0	7.4	-1.4	4.4	6.5
1.5	5.3	8.0	-1.5	4.7	7.1
1.6	5.6	8.2	-1.6	5.0	7.5
1.7	6.0	9.0	-1.7	5.4	8.0

• 90 - 91 °C

• 9.4 div ⟺ ± phase

⇒ 0.64 MHz / div

Part 2 #6

215.2 KHz (Rb85) f

321 KHz

320.1 KHz K

215. KHz

I(A)	(KHz) RB 85	(KHz) RB 87	(A) I	(KHz) 85	(KHz) 87
0	215	321	0	219	320
0.2	598	640	-0.2	220	393
0.4	1000	1045	-0.4	932	998
0.6	1411	2118	-0.6	1035	1617
0.8	1815	2727	-0.8	1446	2233
1.0	2225	3343	-1.0	1904	2788
1.2	2629	3951	-1.2	2222	3465
1.4	3004	4571	-1.4	2688	4081
1.6	3449	5182	-1.6	3150	4650
1.8	3820	5820	-1.8	3530	5280
2.0	4270	6420	-2.0	3890	5890

#7 Max res. Signal @ 0.1 A ± .1 A

Derivation of the Breit-Rabi equation from Quantum Mechanics

From OPT Source #6 "Experiment 7"

In a low magnetic field the hyperfine interaction Couples ~~both~~ the electron and nuclear angular momentum:

$$\underset{\text{total}}{\vec{F}} = \underset{\text{elect.}}{\vec{J}} + \underset{\text{nuc.}}{\vec{I}} \quad ; \quad (\underset{\text{elect.}}{\vec{J}} = \underset{\text{orb.}}{\vec{L}} + \underset{\text{spin}}{\vec{S}}) \; ; \; \vec{F}\cdot\vec{F} = f(f+1)\hbar^2 .$$

The energy of a magnetic dipole moment $\vec{\mu}$ is

$$E = \langle -\vec{\mu}\cdot\vec{B} \rangle , \quad \text{but} \quad \vec{\mu} = \vec{\mu}_J + \vec{\mu}_I :$$

$$E = -\langle \vec{\mu}_J \cdot \vec{B} \rangle - \langle \vec{\mu}_I \cdot \vec{B} \rangle$$

$$= \mu_b g_F \langle \vec{F}\cdot\vec{B} \rangle \quad \text{since} \quad \vec{\mu} = g_F \mu_b \vec{F}$$

$$\text{where} \quad \mu_b = \frac{e\hbar}{2m_e c} = 9.27\cdot 10^{-24}\, J/T.$$

$$0 = \langle (g_F\vec{F} - g_J\vec{J} - g_I\vec{I})\cdot\vec{B} \rangle \quad \forall \; \vec{B} \quad \text{so}$$

$$0 = \langle (g_F\vec{F} - g_J\vec{J} - g_I\vec{I})\cdot\vec{F} \rangle \quad \text{also.}$$

$$0 = g_F f(f+1) - g_J \langle \vec{J}\cdot\vec{F} \rangle - g_I \langle \vec{I}\cdot\vec{F} \rangle$$

Quantum mechanics tells us that

$$f(f+1) = I(I+1) + J(J+1) + 2\langle \vec{I}\cdot\vec{J} \rangle$$

$$\Rightarrow \langle \vec{I}\cdot\vec{J} \rangle = \frac{1}{2}\left[f(f+1) - I(I+1) - J(J+1) \right]$$

Similar arguments yield $\langle \vec{J}\cdot\vec{F}\rangle$ and $\langle \vec{I}\cdot\vec{F}\rangle$.

Rearranging

$$g_F = \frac{f(f+1) + J(J+1) - I(I+1)}{2f(f+1)} g_J$$

$$+ \frac{f(f+1) + I(I+1) - J(J+1)}{2f(f+1)} g_I$$

Since $g_I / g_J \approx m_e/m_p \ll 1$, the second term can be ignored. But in the same manner

$$g_J = \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)} g_L$$

$$+ \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} g_S$$

$$g_J \underset{\substack{L=0 \\ J=S=\frac{1}{2}}}{=} g_S .$$

For $J = 1/2$, $F = I \pm \frac{1}{2}$:

$$g_F \approx \frac{(I \pm 1/2)(I \pm 1/2 + 1) + 3/4 - I(I+1)}{2(I \pm 1/2)(I \pm 1/2 + 1)} g_S = \frac{\pm g_S}{2I+1}$$

$$E = -g_F \mu_0 \langle \vec{F}\cdot\vec{B}\rangle \equiv -g_F \mu_0 M_F B$$

$$\Delta E = |E_{M_F+1} - E_{M_F}| = g_F \mu_0 B$$

$$\frac{\nu}{B} = \frac{\Delta E}{hB} = \frac{g_F \mu_0}{h} = \frac{\pm g_S \mu_0}{(2I+1)h} \approx \boxed{\frac{2.799}{2I+1} \frac{\text{MHz}}{\text{gauss}}}$$