3GTMDMa/4GT – Game Theory
Extra Task: Graphical methods and their extensions

1. Consider the matrix game

$$A = \begin{pmatrix} -1 & 1 & 1/2 \\ 1 & -1 & -1/2 \end{pmatrix}$$

   (i) Use the graphical method to compute the value of this game and the optimal strategy of player I.

   (ii) Consider **all three** 2×2 matrices resulting from deleting a column in A, and for each of them find the value of the matrix game and an optimal strategy of each player.

   (iii) Among the three strategies of player II that you found in (ii), which of them can be extended (by setting the remaining third component to 0) to optimal strategies in the initial game defined by A, and which of them cannot be extended to optimal strategies in the initial game defined by A? Prove all claims that you make.

   (iv) Based on your experience with this problem, describe a rule that can be used to characterise the 2×2 matrix games to which a general game with 2 rows can be reduced.

   Note: This problem is based on P. Morris, Exercise 6 in Section 2.3.

2. Consider the following parametric matrix game:

$$A(t_1, t_2) = \begin{pmatrix} 2 & 6 & 9 \\ t_1 & t_2 & 5 \\ 3 & 4 & 7 \\ 1 & 2 & 3 \end{pmatrix}.$$

   and let $v(A(t_1, t_2))$ be the value of that game (understood as the value of the game in pure strategies if the game has solution in pure strategies, and as the value of the game in mixed strategies otherwise).

   For each $t_1 \in \mathbb{R}$, find the infimum (that is, the greatest lower bound) of the set of values $v(A(t_1, t_2))$ over $t_2 \in \mathbb{R}$, that is, $\inf_{t_2 \in \mathbb{R}} v(A(t_1, t_2))$. Furthermore, find out for which values of t_1 this infimum is attained at some values of t_2, and for which values of t_1 it is not attained by any value of t_2. Give your arguments.

   Hint: use the row and column domination and a graphical method.

3. By extending the graphical method for matrix game with 2 rows, formulate a graphical method for finding the value of the game and an optimal strategy of player I for a matrix game with 3 rows. Explain why the method should work and give an example of its application.

   Hint: When giving an example, you can use any software to draw surfaces in three-dimensional space.