

Lecture 5.

$$\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$$

$$\text{foldr } f \ k \ [] = k$$

$$\text{foldr } f \ k \ (x:xs) = x \cdot f \ (\text{foldr } f \ k \ xs) \\ = f \ x \ (\text{foldr } f \ k \ xs)$$

$$\text{foldr}^{f \ k} (+) \ 0 \ [1, 2, 3] =$$

$$= (+) \ 1 \ (\text{foldr } (+) \ 0 \ [2, 3])$$

$$= 1 + (\text{foldr } (+) \ 0 \ [2, 3])$$

$$= 1 + (2 + (\text{foldr } (+) \ 0 \ [3]))$$

$$= 1 + (2 + (3 + \text{foldr } (+) \ 0 \ []))$$

$$= 1 + (2 + (3 + 0))$$

=

6.

$$\boxed{\text{sum } xs = \text{foldr } (+) \ 0}$$

Function composition

$$f(x) = x^2 + 1$$

$$\text{square} :: \text{Int} \rightarrow \text{Int}$$

$$\text{square } x = x * x$$

$$\text{succ} :: \text{Int} \rightarrow \text{Int}$$

$$\text{succ } x = x + 1$$

$$\text{succSquare} :: \text{Int} \rightarrow \text{Int}$$

$$\text{succSquare} = \cancel{(x * x)} + 1$$

$$= \cancel{\text{succ}(\text{square } x)}$$

$$= (\text{succ} \cdot \text{square}) x$$

$$xs = [1, 2, 3]$$

$$\text{map } \cancel{\text{succSquare}} [2, 5, 10]$$

$\downarrow \text{succSquare}$
 succSquare

$$\text{map } (\text{succ} \cdot \text{square}) xs$$

$$c \xleftarrow{\quad} a$$

$$c \xleftarrow{g} b \quad b \xleftarrow{f} a$$

$$(\cdot) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$$

$$(g \cdot f) x = g (f x)$$

$\begin{array}{c} \uparrow \\ b \rightarrow c \end{array} \quad \begin{array}{c} \uparrow \\ a \rightarrow b \end{array} \quad \begin{array}{c} \uparrow \\ a \end{array}$

$$(g \cdot f) :: a \rightarrow c$$

$$h: c \rightarrow d$$

$$h \cdot g \cdot f$$

Monoids

$$\begin{array}{l} x + 0 = x \\ 0 + x = x \end{array} \quad \left. \vphantom{\begin{array}{l} x + 0 = x \\ 0 + x = x \end{array}} \right\} \text{unit laws}$$

$$x + (y + z) = (x + y) + z \quad \left. \vphantom{x + (y + z) = (x + y) + z} \right\} \text{associative law.}$$

$$+ \quad 0$$

$$* \quad 1$$

$$\&\& \wedge \quad \text{True}$$

$$\# \vee \quad \text{False}$$

$$\sqcup \quad -\infty$$

$$\sqcup = \text{max}$$

$$\sqcap \quad \infty$$

$$\cup \quad \emptyset$$

$$\cap \quad \text{PX} \quad \cup$$

$$\text{mplus} \quad \text{mzero}$$

$$\text{mplus} :: a \rightarrow a \rightarrow a$$

$$\text{mzero} :: a$$

$$\text{foldr mplus mzero}$$

Type classes

class Monoid ^a ~~no~~ where
 mplus :: a → a → a
 mzero :: a ~~no~~

instance Monoid Int where
 mplus = (+)
 mzero = 0

instance Monoid [a] where
 mplus =
 mzero =

mplus :: [a] → [a] → [a]
 mplus = (++)

mzero :: [a]
 mzero = []

$(==) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}$
 $5 == 3$

$(==) :: \text{Char} \rightarrow \text{Char} \rightarrow \text{Bool}$
 $'a' == 'a'$

$(==) :: \text{String} \rightarrow \text{String} \rightarrow \text{Bool}$
 $"hello" == "world"$

principal type: everything has just 1 type.

class Eq a where
 $(==) :: a \rightarrow a \rightarrow \text{Bool}$

instance Eq Int where ...

instance Eq Char where ...

instance Eq a \Rightarrow Eq [a] where

$[] == [] = \text{True}$

$(x:xs) == (y:ys) = x == y \ \&\& \ xs == ys$

$[] == (y:ys) = \text{False}$

$(x:xs) == [] = \text{False}$

$[a] \rightarrow [a] \rightarrow \text{Bool}$