

Functional Programming Lecture II.

$x :: \text{Integer}$

$x = \underline{x} + 1$

$(\underline{x+1}) + 1$

$((\underline{x+1}) + 1) + 1$

\perp -- bottom.

$\text{infinity} :: \text{Integer}$

$\text{infinity} = \text{infinity} + 1$

↙ domain ↘ codomain / range.

$\text{two} :: \text{Int} \rightarrow \text{Int}$ ↻ Signature.

$\text{two } n = 2$

$\text{two } (\text{infinity}) :: \text{Int}$
 $= \text{two } (\text{infinity} + 1)$
 $= \text{two } (\text{infinity} + \dots)$

} applicative / eager

$\underline{\text{two}} (\text{infinity})$
 $= 2$

} normal / lazy

double :: Int \rightarrow Int

double x = x + x

eager

double (3+7)

= double 10

= 10 + 10

= 20

lazy

double (3+7)

= (3+7) + (3+7)

= 10 + (3+7)

= 10 + 10

= 20

let y = (3+7) in double y.

plus :: Int \rightarrow Int \rightarrow Int

plus x y = x + y

add :: (Int, Int) \rightarrow Int

add (x, y) = x + y

Int \rightarrow Int \rightarrow Int

=

Int \rightarrow (Int \rightarrow Int)

plus7 :: Int \rightarrow Int

plus7 = plus 7

\uparrow partially evaluated

Recursion. $\text{fac} :: \text{Int} \rightarrow \text{Int}$
 $\text{fac } 0 = 1$
 $\text{fac } n = \text{fac } (n-1) * n$

Fibonacci numbers.

	1	1	2	3	5	8	...
fib	0	1	2	3	4	5	

$\text{fib} :: \text{Int} \rightarrow \text{Int}$

$\text{fib } 0 = 1$

$\text{fib } 1 = 1$

$\text{fib } n = \text{fib } (n-1) + \text{fib } (n-2)$
 $\begin{cases} n > 0 & = \\ \text{otherwise} & = \text{error "fib exploded!"} \end{cases}$

$\text{min} :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$\text{min } x \ y = \text{if } x < y \text{ then } x \text{ else } y$

* Pattern guards.

$\text{min} :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$\text{min } x \ y$

$\mid x < y = x$

$\mid \text{otherwise} = y$
 $\mid x \geq y$

$\text{otherwise} = \text{True}$

$\text{error} :: \text{String} \rightarrow \text{Int}$

~~Int~~
a

polymorphic
type.

Lists

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[1, 1, 2, 3, 5, 8, ...]

[] empty list
(x : xs)
↑ ↑
element list of elements.

1 : 1 : 2 : ...

= ['h', 'e', 'l', 'l', 'o']

'h' : 'e' : 'l' : 'l' : 'o' : []

[] :: [a]
x : xs
↑
(:) :: Char → [Char] → [Char]
:: a → [a] → [a]
too specific!

head :: [~~a~~] → ~~a~~ head [5, 3, 7] = 5

head [] = error "empty list".

head (x : xs) = x



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 $\text{tail} :: [a] \rightarrow [a]$ $\text{tail } [] = \text{error "empty list"}$ $\text{tail } (x:xs) = xs$ $\text{head } (\text{tail } [5, 4, 3])$  $\text{nats} :: [\text{Int}]$ $\text{nats} = [1..]$ $\text{take} :: \text{Int} \rightarrow [a] \rightarrow [a]$ $\text{take } n [] = []$ $\text{take } n (x:xs) = [x, \dots, x_n]$ $\quad | n == 0 = []$ $\quad | \text{otherwise} = x : \text{take } (n-1) xs$

↓ pattern matching.

 $\text{take } 0 xs = []$ $\text{take } n (x:xs) = x : \text{take } (n-1) xs.$ $\text{filter} :: (a \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [a]$

