

# Lecture 7

$qsort' :: \text{Ord } a \Rightarrow [a] \rightarrow [a]$

$qsort' [] = []$

$qsort' (x:xs) = qsort' ys \# [x] \# qsort' zs$

where

$(ys, zs) = \text{partition } (\leq x) \ xs$

$\text{partition} :: (a \rightarrow \text{Bool}) \rightarrow [a] \rightarrow ([a], [a])$

$\text{partition } p \ xs = (\text{filter } p \ xs, \text{filter } (\text{not} \cdot p) \ xs)$

$(\leq x) :: \text{Ord } a \Rightarrow a \rightarrow \text{Bool}$

$(\leq x) \ y = \text{not } y \leq x$

data Maybe a = Nothing  
| Just a

—  $\#$  LANGUAGE  
EADs  $\#$  - 3

data Maybe a where

Nothing :: Maybe a

Just :: a  $\rightarrow$  Maybe a

fromMaybe :: Maybe a  $\rightarrow$  [a]

fromMaybe Nothing = []

fromMaybe (Just x) = [x]

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data Tree a = Tip

| Node (Tree a) a (Tree a)

data Tree a where

Tip :: Tree a

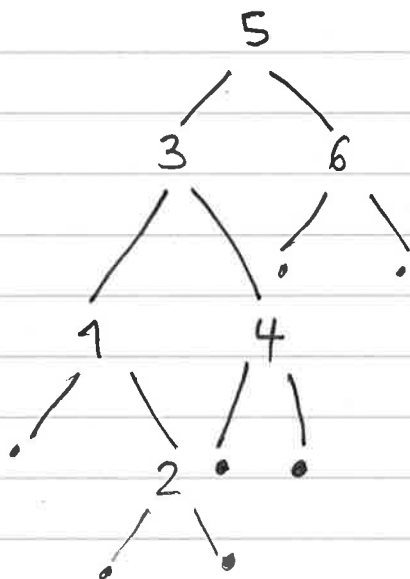
Node :: Tree a  $\rightarrow$  a  $\rightarrow$  Tree a  $\rightarrow$  Tree a

$qSort [5, 3, 1, 4, 2, 6]$   
 $=$   
 $flatten (mkTree [5, 3, 1, 4, 2, 6])$   
 $=$   
 $flatten (Node (mkTree [3, 1, 4, 2])$   
 $\quad 5$   
 $\quad (mkTree [6]))$

$mkTree [3, 1, 4, 2]$   
 $= Node (mkTree [1, 2])$   
 $\quad 3$   
 $\quad (mkTree [4])$

$mkTree [1, 2]$   
 $= Node (mkTree [])$   
 $\quad 1$   
 $\quad (mkTree [2])$

$mkTree [] = Tip$   
 $mkTree [2] = Node (mkTree [])$   
 $\quad 2$   
 $\quad (mkTree [])$



## Functors.

class Functor f where

$fmap :: (a \rightarrow b) \rightarrow f a \rightarrow f b$

Maybe a

Tree a

$[a] = [] a$   
List a

instance Functor [] where

$fmap :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$

$fmap = map$

instance Functor Maybe where

$fmap :: (a \rightarrow b) \rightarrow Maybe a \rightarrow Maybe b$

$fmap f Nothing = Nothing$

$fmap \underbrace{f}_{a \rightarrow b} (\underbrace{Just x}_a) = Just (f x)$

### Example.

$Nothing :: Maybe Int$

$alpha :: Int \rightarrow Char$

$alpha \ 1 = 'a'$

$alpha \ 26 = 'z'$

$fmap alpha Nothing = Nothing$

$fmap alpha (Just 2) = Just 'b'$

$$\text{fmap } \alpha \text{pha } [1, 2, 3] = ['a', 'b', 'c']$$

instance Functor Tree where

~~$$\text{fmap} :: (a \rightarrow b) \rightarrow f a \rightarrow f b$$~~

$$\text{fmap} :: (a \rightarrow b) \rightarrow \text{Tree } a \rightarrow \text{Tree } b$$

$$\text{fmap } f \text{ Tip} = \text{Tip}$$

$$\text{fmap } \underset{\substack{\downarrow \\ a \rightarrow b}}{f} (\text{Node } \underset{\substack{\downarrow \\ \text{Tree } a}}{l} \underset{\substack{\downarrow \\ a}}{x} \underset{\substack{\downarrow \\ \text{Tree } a}}{r}) =$$

$$\begin{array}{l} \text{Node } (\text{fmap } f \ l) \\ (f \ x) \leftarrow b \\ (\text{fmap } f \ r) \end{array}$$

Tree b

$$\text{fmap } \alpha \text{pha } \left( \begin{array}{c} 1 \\ / \quad \backslash \\ 2 \quad 3 \\ / \backslash \quad / \backslash \\ \bullet \quad \bullet \quad \bullet \quad \bullet \end{array} \right) = \begin{array}{c} 'a' \\ / \quad \backslash \\ 'b' \quad 'c' \\ / \backslash \quad / \backslash \\ \bullet \quad \bullet \quad \bullet \quad \bullet \end{array}$$