

zip [1,2,3] ['a','b','c']

1

Lecture 4.

= [(1,'a'), (2,'b'), (3,'c')]

zip :: [a] → [b] → [(a,b)]

zip [] ys = []

zip xs [] = []

zip (x:xs) (y:ys) = (x,y) : zip xs ys

zip [1,2,3,4,5] ['a','b','c']

= (1, 'a') : zip [2,3,4,5] ['b','c']

= (1, 'a') : (2, 'b') : zip [3,4,5] ['c']

zipWith (add) [1,2,3] [4,5,6]

= [~~1+4~~, ~~2+5~~, ~~3+6~~]  
add 14    add 25    add 36

zipWith :: (a → b → c) → [a] → [b] → [c]

zipWith f [] ys = []

zipWith f xs [] = []

zipWith f (x:xs) (y:ys) = f x y : zipWith f xs ys  
 $\underbrace{\underbrace{(x:xs)}_{[a]} \underbrace{(y:ys)}_{[b]}}_{[a] \ [b]} \quad \underbrace{f \quad \underbrace{x \ y}_c}_{[c]}$

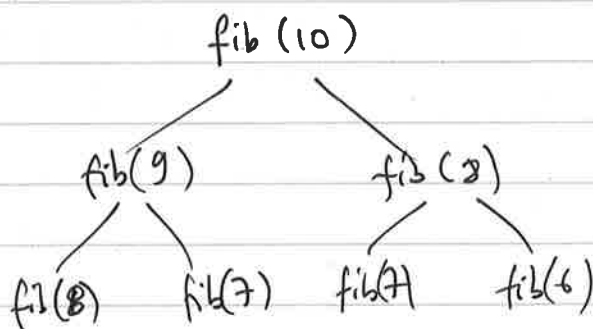
map :: (a → b) → [a] → [b]

$\text{fib} :: \text{Int} \rightarrow \text{Int}$

$\text{fib } 0 = 1$

$\text{fib } 1 = 1$

$\text{fib } n = \text{fib}(n-1) + \text{fib}(n-2)$



$\text{fibs} :: [\text{Int}]$        $\text{tail fibs}$

$\text{fibs} = 1 : 1 : 2 : 3 : 5 : \dots$        $\text{fibs}$

$\begin{array}{c} 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad \dots \\ \hline 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad \dots \end{array}$

$\begin{array}{c} 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \quad \dots \\ \hline 1 \quad 1 \quad 2 \quad 3 \quad 5 \quad 8 \quad \dots \end{array}$

$\text{fibs} = 1 : 1 : \text{zipwith } \text{add} \text{ fibs } (\text{tail fibs})$

$\text{fib} :: \text{Int} \rightarrow \text{Int}$

$\text{fib } n = \text{fibs} !! n$

$(+)$        $:: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$\text{add} = (+)$

## Higher-order functions.

A higher-order function takes a function as a parameter.

$\text{map} :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$

$\text{zipWith} :: (a \rightarrow b \rightarrow c) \rightarrow [a] \rightarrow [b] \rightarrow [c]$

$\text{filter} :: (a \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [a]$

$[1, 2, 3, 4]$

$= 1 : 2 : 3 : 4 : []$

$\downarrow^+ \quad \downarrow^+ \quad \downarrow^+ \quad \downarrow^+ \quad \downarrow^0$

$1 + (2 + (3 + (4 + 0)))$

$\text{sum } xs = \text{foldr } (+) 0 \text{ } xs :: \text{Int}$

$[1, 2, 3, 4]$

$= 1 : 2 : 3 : 4 : []$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$1 * 2 * 3 * 4 * 1$

$\text{fac } n = \text{foldr } (*) 1 [1..n]$

$\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$

$\text{foldr } f \ k \ [] = k$

$\text{foldr } f \ k \ (x:xs) = f \ x (\text{foldr } f \ k \ xs)$

$f :: a \rightarrow b \rightarrow b$

$(+) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$\text{foo} :: \text{Char} \rightarrow \text{Int} \rightarrow \text{Int}$       $a :: \text{Char}$

$\text{bar} :: \text{Int} \rightarrow \text{Char} \rightarrow \text{Int}$       $b :: \text{Int}$