Simulation of goods receipt session

Summary of approach:

This task seeks to evaluate the suitability of a 10-minute truck offloading schedule for an inbound warehouse shipment operator. This is tackled by building model of the warehouse's goods receipt process over a 3-hour goods receipt session, then performing a Monte Carlo simulation with the model by feeding it with randomly generated offload times and arrival times in line with the provided probability distributions.

The stepwise approach is as follows (note this is in line with the accompanying Python Jupyter notebook, see that for details):

1. Create functions to generate random numbers according to the provided probability distributions
2. Create function to simulate a single goods receipt session
3. Run a Monte Carlo simulation for a set of goods receipt sessions using random numbers generated by the functions in (2)
4. Output simulation results and plots
5. Evaluation of 10-minute offloading schedule adequacy

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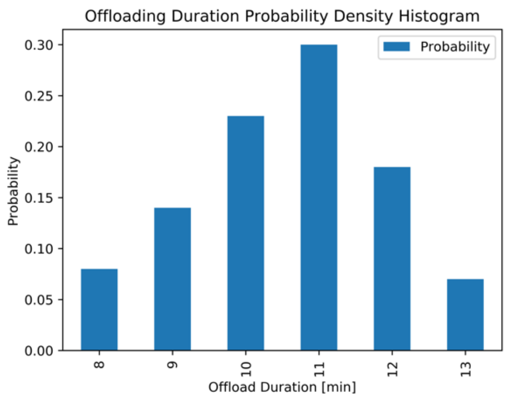
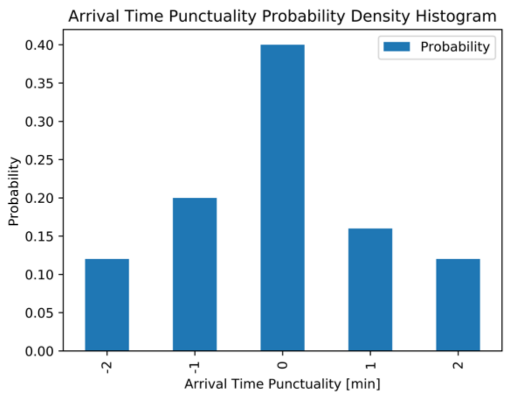
1. Functions to generate random numbers according to provided probability distributions

The provided probability distributions for truck offloading times and truck arrival time punctuality are shown in the figures below:



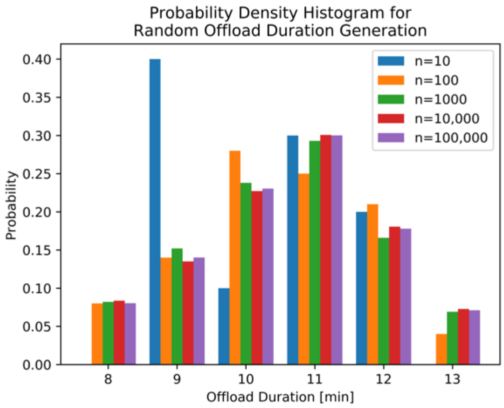


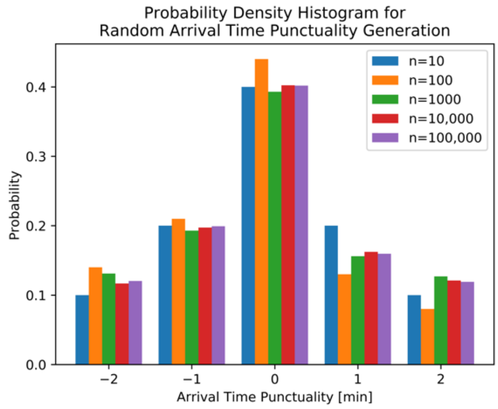
From the histogram on the left it is immediately clear that the mean offloading duration is between 10- and 11-minutes (10m34s).

From the histogram on the right it is clear that the mean arrival time punctuality is just slightly less than 0 (–0m02s).

These values immediately raise suspicion around the feasibility of a 10-minute scheduling interval, as the mean duration to offload exceeds this (even if trucks arrived earlier the benefits would not compensate for this as they do not compound because it is always relative to the initial schedule). In a 180-minute window, the **theoretical maximum number of trucks unloaded is therefore 17.0** (180 min/10.57 min).

Functions were then written to generate random numbers in-line with these distributions, the histograms below show the resulting distributions of generated random numbers for varying quantities of generated numbers (n):





The histograms above show that the distribution of generated random numbers approaches the input distribution once n>1000. As a result, n=10,000 is chosen for the number of Monte Carlo simulations to perform: (1) to ensure the distribution of generated numbers accurately reflects the input distribution; and (2) because compute time is not of great concern.

2. Function to simulate a single goods receipt session

The simulation function written generates a table of results for the goods receipt session, an example is shown below:

truck_ no	arrival_ scheduled [min]	offload_ duration [min]	arrival_ actual [min]	start_ time [min]	start_ delta [min]	finish_ time [min]	finish_ delta [min]	wait_time_ labour [min]	wait_time_ truck [min]	truck_ completion [%]
0	0	11	1	1	1	12	2	0	0	100%
1	10	9	10	12	2	21	1	0	2	100%
2	20	12	19	21	1	33	3	0	2	100%
3	30	9	32	33	3	42	2	0	1	100%
4	40	10	38	42	2	52	2	0	4	100%
5	50	11	52	52	2	63	3	0	0	100%
6	60	10	60	63	3	73	3	0	3	100%
7	70	10	68	73	3	83	3	0	5	100%
8	80	10	80	83	3	93	3	0	3	100%
9	90	10	90	93	3	103	3	0	3	100%
10	100	11	100	103	3	114	4	0	3	100%
11	110	9	110	114	4	123	3	0	4	100%
12	120	11	120	123	3	134	4	0	3	100%
13	130	11	131	134	4	145	5	0	3	100%
14	140	10	140	145	5	155	5	0	5	100%
15	150	9	149	155	5	164	4	0	6	100%
16	160	10	161	164	4	174	4	0	3	100%
17	170	10	170	174	4	184	4	0	4	60%
18	180	12	178	184	4	196	6	0	6	0%

The most important note to make here is that this function simulates each truck individually, not each 10-minute interval. Each row of the table is a truck scheduled to be offloaded and all other data follows, the columns of importance are explained below:

Column name	*Description*	*Formula*
arrival_scheduled	Scheduled arrival time for truck	$truck\ no \cdot scheduled\ interval$
offload_duration	Randomly generated offload duration using provided probability distribution	[randomly generated offload duration]
arrival_actual	Randomly generated actual arrival using provided probability distribution and scheduled interval	$arrival\ scheduled$ +[randomly generated arrival time punctuality]
start_time	Actual time truck begins to be unloaded	$\max(arrival\ actual_i; finish\ time_{i-1})$
start_delta	Difference between scheduled and actual start time	$start\ time - arrival\ scheduled$
finish_time	Actual time truck unloading is completed	$start\ time + offload\ duration$
finish_delta	Difference between scheduled and actual finish time	$finish\ time_i - arrival\ scheduled_{i+1}$
wait_time_labour	Duration warehouse labour is idle between completing the previous truck and beginning this truck	$start\ time_i - finish\ time_{i-1}$
wait_time_truck	Duration truck is waiting to be unloaded once arrived	$start\ time - arrival\ actual$
truck_completion	Percentage of the truck contents that were unloaded within the 3-hour simulation window	N/A (see detail in Python Jupyter notebook)

3. Monte Carlo simulation of goods receipt sessions

The mechanics of running the Monte Carlo simulation are quite simple. As mentioned in section 1, 10,000 simulated goods receipt sessions are performed in which the total simulation period is 3-hours and the scheduling interval is to be kept constant (initially at 10-minutes, other intervals explored are discussed in section 5).

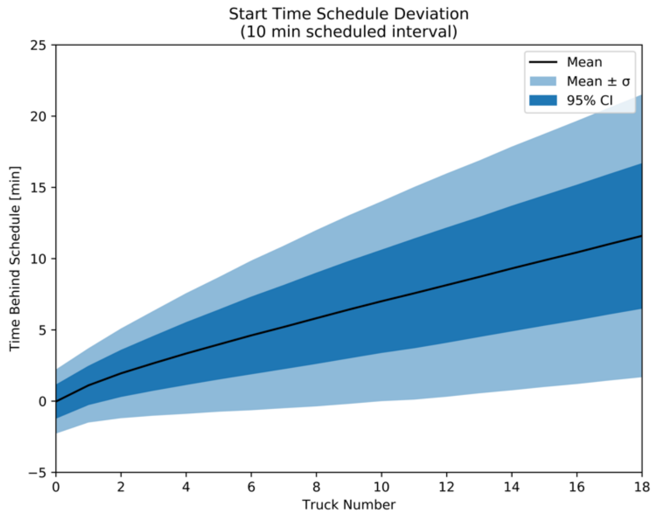
For each simulation, the following summary data of the result table is stored to be used for result analysis:

- Total number of trucks unloaded in the 3-hour window (including the fraction of the final truck unloaded within the 3-hour window)
- Total time labour spent idle during the 3-hour window
- Total time trucks spent waiting before being unloaded during the 3-hour window
- The time behind (or ahead) of schedule each truck in the session began to be unloaded.

4. Monte Carlo simulation results

For a 10-minute schedule interval over a simulation period of 3-hours, the following results were obtained:

4.a. Over time, deviation from the schedule continues to increase

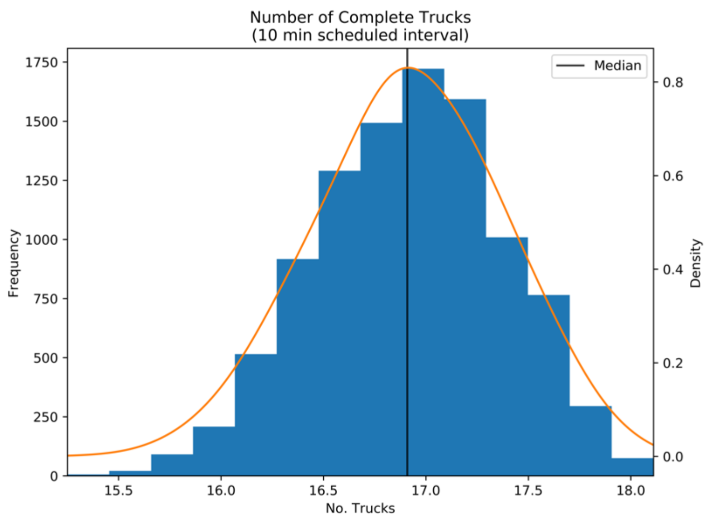


The plot above illustrates the mean (and surrounding confidence intervals) deviation from the scheduled offloading start time and the actual offloading start time. This demonstrates that the scheduling interval is ineffective as it does not reach stability (at least within a 3-hour window). This is to be expected, as the mean offloading time is greater than 10-minutes, so it is impossible for the two to synchronise and for deviation to plateau.

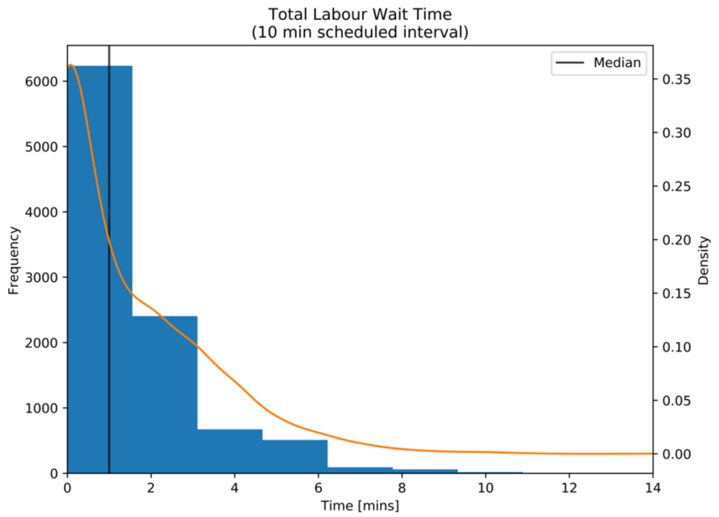
4.b. Other performance metrics

Number of trucks unloaded

The average (mean and median) number of trucks unloaded is 16.9. This is below the scheduled 18, but very close to the theoretical maximum of 17.0. See the histogram below for distribution.

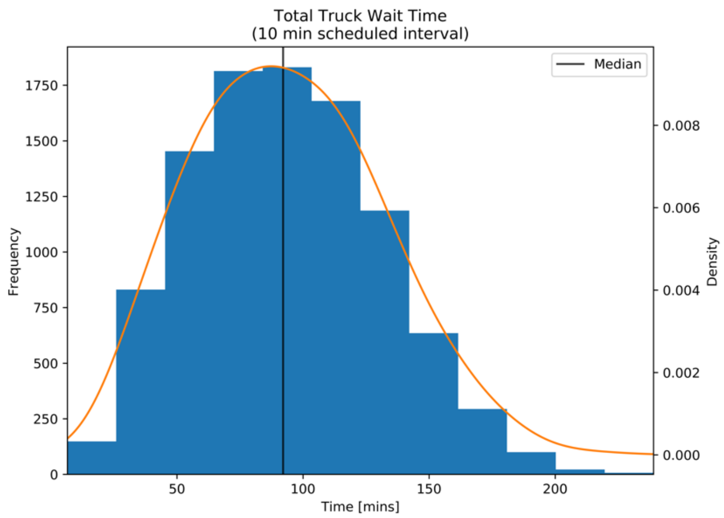


Labour idle time



Total labour wait time over the 3-hour window is very low, with a median of 1-minute (mean is not appropriate for this highly skew distribution). This indicates that the workers are operating at effectively maximum capacity.

Truck wait time



Conversely, truck wait time is very high with a mean total of 1h33m time spent waiting in the 3-hour window. Due to the constantly increasing deviation from schedule, more and more trucks will build up on the lot over time. This may be a significant issue as: (1) the trucks could get in the way and create further logistical challenges; (2) the logistics companies may take issue with the warehouse repeatedly wasting driver and truck time.

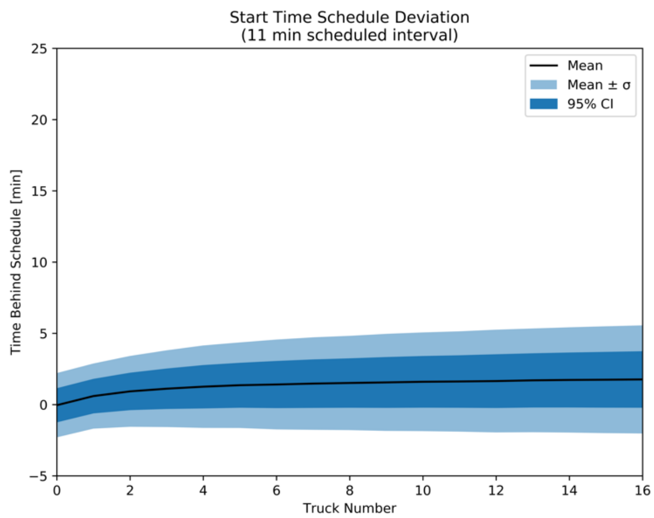
More statistical data relating to the above histograms is available in the table below:

	Total Labour Wait Time	*Total Truck Wait Time*	*No. Trucks Unloaded*
count	10,000	10,000	10,000
mean	1.43	93.75	16.91
std	1.82	37.35	0.46
min	0.00	6.55	15.25
Q1	0.00	65.45	16.60
median	1.00	92.09	16.91
Q3	2.00	119.80	17.23
max	14.00	239.00	18.11

5. Evaluation of scheduling interval adequacy

A 10-minute scheduling interval is clearly inadequate. As demonstrated in the discussions above, the schedule never reaches equilibrium and trucks become progressively more and more behind schedule. Even though labour is operating at effectively maximum capacity and unloading almost the theoretical maximum number of trucks they are capable of unloading in 3-hours, it is not a sustainable approach as the delay constantly increases.

As an alternative, the Monte Carlo simulation was run again for scheduling intervals of 11- and 12-minutes. (and 9-minutes, for curiosity). For 11-minute intervals the system reaches a steady-state, meaning deviation from the schedule at some point ceases to increase (see the figure below).



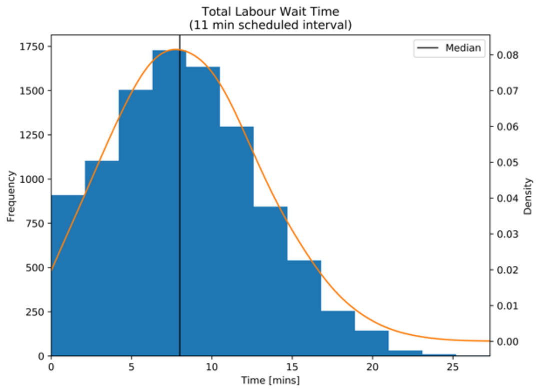
The trade-off for this stability is a slight decrease in the total truck offloading capacity of the warehouse during the 3-hour window (from 16.9 to 16.2 trucks per 3-hours) due to an increase in worker idle time. However, this scheduling window means there are only ever a maximum of two trucks on the lot at a time, whereas the build-up of trucks caused by a 10-minute interval would inevitably cause other challenges and reduce the long-term offloading efficiency.

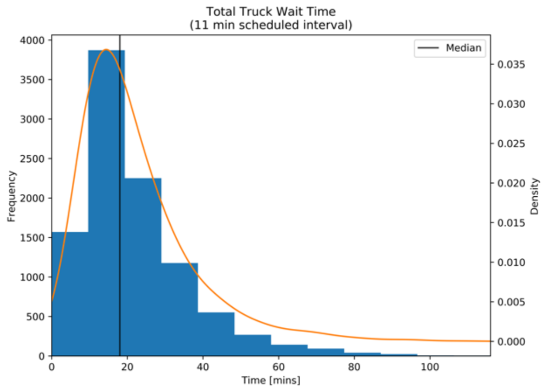
Further increasing the scheduled interval is not of much practical use. A 12-minute window results in a tighter deviation from scheduled time, but 11-minutes is already sufficient. Furthermore, a 12-minute window decreases the total number of trucks offloaded in the 3-hour substantially (down to 15.0).

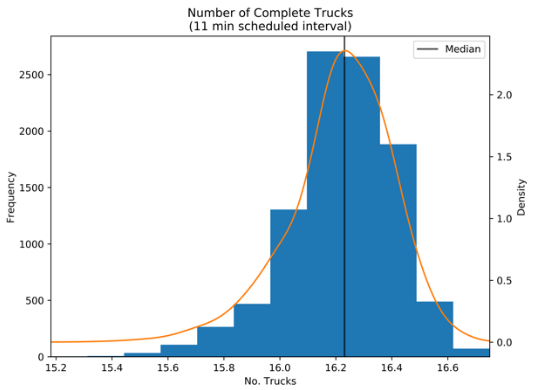
Histograms and plots for all these simulations can be seen in the appendix that follows.

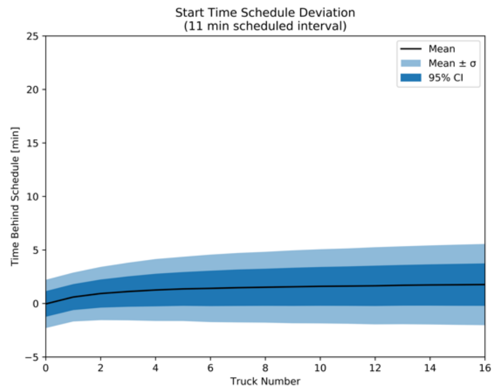
Appendix: Summary plots and tables for various scheduling intervals

A.1. 11-minute scheduling interval



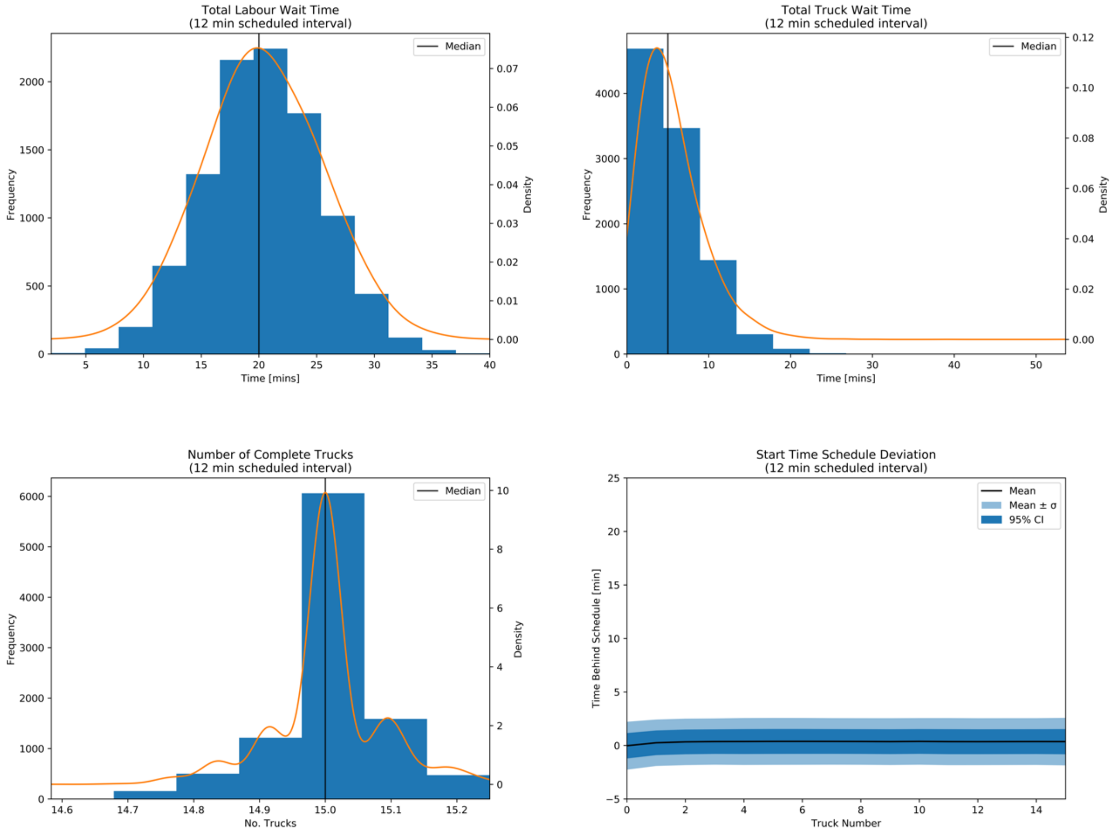






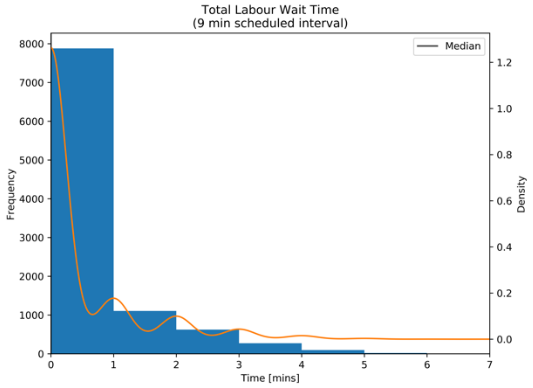
	total_wait_ time_labour	total_wait_ time_truck	no_trucks_ complete
count	10000	10000	10000
mean	8.44	21.89	16.22
std	4.54	14.48	0.19
min	0.00	0.00	15.18
Q1	5.00	12.00	16.11
median	8.00	18.00	16.23
Q3	11.33	27.60	16.33
max	27.31	116.00	16.75

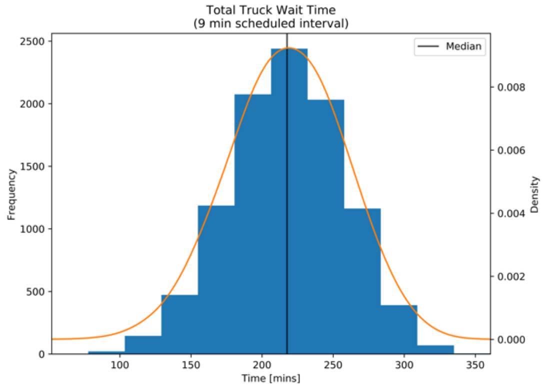
A.2. 12-minute scheduling interval

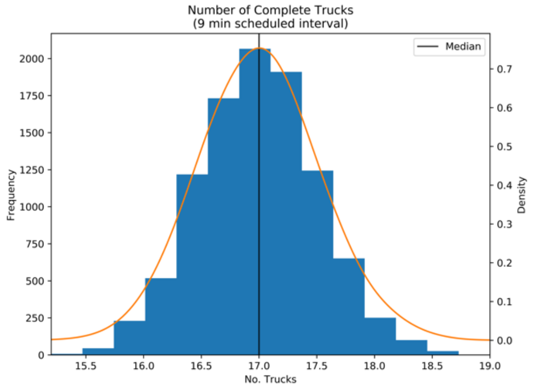


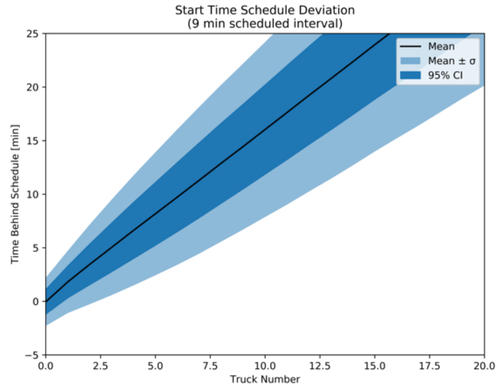
	total_wait_ time_labour	total_wait_ time_truck	no_trucks_ complete
count	10000	10000	10000
mean	20.39	5.50	15.00
std	5.12	3.87	0.08
min	2.00	0.00	14.58
Q1	17.00	3.00	15.00
median	20.00	5.00	15.00
Q3	24.00	7.08	15.00
max	40.00	53.60	15.25

A.3. 9-minute scheduling interval









	total_wait_ time_labour	total_wait_ time_truck	no_trucks_ complete
count	10000	10000	10000
mean	0.37	217.49	17.00
std	0.83	40.69	0.51
min	0.00	58.64	15.18
Q1	0.00	190.50	16.64
median	0.00	218.69	17.00
Q3	0.00	245.50	17.33
max	5.00	352.00	19.20