Assignment 1
Introduction to System Identification

Deadline: Monday September 30th, 2014, 23:59h

Subjects:

Correlation functions	Lec. 2
Covariance	Lec. 2
Spectral Densities	Lec. 2
Impulse response function (IRF)	Lec. 3
Discrete time models	Lec. 4

Important notes

- Reports handed in after the deadline will not be marked, resulting in a zero.
- Working in teams of (not more than) two students is strongly encouraged.

How to get help

- Always check the *Assignment Guide* on Blackboard for support.
- The functions *HELP* or *DOC* <*FUNCTIONNAME*> can be used for instant support to MATLAB functions.
- Assistance hours (see schedule on Blackboard).
- Help each other using the *Discussion Board* on Blackboard.

Report requirements

- Reports **must** be written in English.
- Each student hands in a report using Blackboard. Always provide the name of your team partner in the report.
- Always provide explanations in your answers and be precise and concise (providing irrelevant information can negatively influence your mark). Formulate your answer in a way that shows that you understood the question.
- **Use figures** to illustrate your findings. Do not forget to add a caption and the units on the axes. When plotting more than one curve in a figure, use different colors and markers. Do not forget to use the correct legends. Choose proper axes scales. Unclear Figures and Tables will not be marked.
- Do not submit MATLAB code, unless it is clearly stated that you should do it.
- Put your name (and that of your team partner, if applicable), page number, and the assignment number on each page of your report.

Goal

The goal of this assignment is to understand how the relationships between signals can be estimated, what the requirement are for proper estimators, and how the estimates can be used to identify an unknown system. You will also identify issues involving time domain models.

Accessories: *ASS1.M*
ASS1_SYSTEM.MDL

Question 1: Signal Analysis in the Time Domain

(Covariance functions, Correlation coefficients, and impulse response function)

Open the MATLAB script *ASS1.M* and the Simulink model *ASS1_SYSTEM.MDL*. This model can generate the signals *u(t)* (input) and *y(t)* (output). Run the second cell in the *ASS1.M* script. Take a look into the model to see its structure and the additional output noise *n(t)*.

a) Use the MATLAB function *XCOV* to estimate the covariance functions $C_{uu}(\tau)$, $C_{yy}(\tau)$, $C_{uy}(\tau)$, and $C_{yu}(\tau)$. Plot all functions in one figure using four subplots. Don't forget to label the axes.
b) Analyze and discuss the results. What are the variables on both figure axes? Is there a difference between $C_{uy}(\tau)$ and $C_{yu}(\tau)$? If so, elaborate on the differences (explain why).
c) What is the theoretical value of the auto-covariance function $C_{uu}(\tau)$ for all τ? What would be the theoretical value of the cross-covariance function between the input *u(t)* and the noise *n(t)* at the output: $C_{nu}(\tau)$? Note that both signals are designed as white noise.
d) Calculate the unbiased covariance function $C_{yu}(\tau)$. Explain the difference between biased and unbiased estimations of the covariance function. Plot both functions on the same graph. Explain why it is necessary to bias the covariance estimator?
e) Write **your own MATLAB function** for estimation of the covariance function. Provide the biased/unbiased option, and a *MAXLAG* option (use *HELP XCOV* to get information). <u>It is necessary to submit the code your function.</u> Include comments in your code, explaining what is being done.
f) Calculate the cross-correlation coefficients $K_{uu}(\tau)$ and $K_{yu}(\tau)$ from $C_{uu}(\tau)$ and $C_{yu}(\tau)$. Indicate the value of τ that corresponds to the maximum correlation coefficient?
g) Compare the biased and unbiased estimations of the covariance functions obtained using *XCOV* and your own function. Show that the estimations coincide.

The cross-covariance of the input and output signals of a system is the same as the impulse response of that system, because white noise and an impulse theoretically have the same frequency content.

h) Plot the true impulse response of the system using the MATLAB function *IMPULSE*. Plot a graph of $C_{yu}(\tau)$ in the same axis. Explain the difference.
i) Run the model *ASS1_SYSTEM.MDL* for 100 seconds. Plot $C_{yu100}(\tau)$ in the same figure. Explain the differences with $C_{yu}(\tau)$. Adjust the time axis for proper view.

Question 2: Fourier transform and spectral densities

(obtained using covariance functions or using direct Fourier transformation of signals)

In question 1 you analyzed the signal correlations using covariance estimators in the time domain. Relationships between signals can also be determined in the frequency domain using estimators of the spectral density. The spectral densities will be used for deriving a transfer function of the (unknown) system. This function is called the Frequency Response Function (FRF).

There are two ways to obtain the spectral density functions:

- Direct approach: Transform the signals into frequency domain and then derive the spectral densities.
- Indirect approach: Derive the covariance functions and then transform to frequency domain.

This part of the assignment will familiarize you with spectral analysis and the relationship between covariance functions and spectral densities.

Generate a signal *u(t)* as the summation of a 8 Hz sine with amplitude 2 and a random signal (*RANDN*) with zero mean and unitary variance. Make the signal *u(t)* lasts five seconds with a sample frequency f_s = 500 Hz. Apply a second order Butterworth filter (*BUTTER* & *FILTER*) to *u(t)* with a cutoff frequency of 20 Hz. Call the resulting signal *y*(*t*). Fourier transforms are denoted by upper cases, e.g. *Y(ω)*. Accordingly, discrete time signals are denoted *y(k)* and its Fourier transform as *Y(n)*.

a) Use the MATLAB function *FFT* to perform a discrete Fourier transform (DFT) on *y(k)*. Plot the magnitude and phase of *Y(n)*. Take proper axes scales. What is the meaning of the coefficients? From a system identification point of view, describe what you see in |*Y(n)*|.

Run the model *ASS1_SYSTEM.MDL* once more to generate *u(t)* and *y(t)* again.

b) Use the **direct** approach to obtain $S_{uu}(\omega)$ and $S_{yu}(\omega)$, and plot your results.
c) Apply the **indirect** approach to obtain $S_{uu}(\omega)$ and $S_{yu}(\omega)$. First rearrange the covariance functions from $[-\tau_{max},\ldots, \tau_{max}]$ into $[0, \ldots, 2\tau_{max}]$ as lag axis. Use the biased and unbiased estimators for covariance. Plot the spectral densities in logarithmic scales (*LOGLOG*).
d) Compare the results from a and b (biased and unbiased estimators) and explain the differences. Remember to construct the correct frequency axis for each approach!

Question 3: Time Domain Models

Use the open loop model *ASS1_SYSTEM.MDL* and white noise as input *u(t)* to generate an output *y(t)* with total time of 50 s and sampling rate of 100 Hz.

a) Store the data as an *IDDATA* object (see the *Assignment Guide*) and use the MATLAB function *ARX* to estimate a model on this dataset. Use a model order na=nb=12. Plot a Bode diagram of the ARX model (use *FREQRESP*) and compare it with the true system described by the model *ASS1_SYSTEM.MDL*. Discuss the results.
b) Derive the correct model structure (OE, ARMAX, BJ, etc) for this SISO open loop system.
c) Estimate the correct model structure onto the dataset (functions: *OE*, *ARMAX*, *BJ*). Use a model order of 3 (for all models). Also estimate an 3rd order ARX model. Include both models into the Bode diagram. Compare them using the function *COMPARE*.
d) Propose a method for defining the model order if this is unknown.