Allmänt byte (om inre derivatan saknas)

$$\int \frac{1}{1+\sqrt{1-x}}\,dx = \left[t=\sqrt{1-x},\ x=1-t^2 \right]$$

$dx=-2t\,dt$ (derivera med hänsyn på t)

$$= \int \frac{1}{1+t}\cdot(-2t\,dt) = -2\int \frac{t}{1+t}\,dt$$

12.1

a) $x^5/5+C$ b) $\ln|x|+C$ c) e^x+C d) $\sin x+C$

e) $-\cos x + C$ f) $\tan x + C$ g) $\cot x + C$

h) $\arctan x + C$ i) $\arcsin x + C$ j) $\ln|x+\sqrt{x^2+\alpha}|+C$

12.2 a) $\frac{1}{4}\cdot x^4$ b) $\frac{x^{-2}}{-2}$ c) $\frac{x^{1.5}}{1.5}=\frac{3x\sqrt{x}}{2}$ d) $\frac{x^{2.5}}{2.5}$

e) $2\sqrt{x}$ f) $\frac{x^{-0.5}}{-0.5}$

12.3

d) $\int \frac{2}{x+1}\,dx = 2\int \frac{1}{x+1}\,dx = 2\ln|x+1|$

e) $\int \frac{1}{2x+1}\,dx = \int \frac{1}{2}\cdot\frac{1}{x+\frac{1}{2}}\,dx = \frac{1}{2}\ln|x+\frac{1}{2}|$

l) $\int \frac{2}{(1-3x)^2}\,dx = 2\int \frac{1}{(1-3x)^2}\,dx = \left[t=1-3x,\ dt=-3dx \right]$

$$= 2\int \frac{1}{t^2}\,\frac{dt}{-3} = \frac{2}{-3}\cdot\frac{t^{-1}}{-1}+C = \frac{2}{3t}+C = \frac{2}{3-9x}+C$$

12.5

a) $\int \sin 2x \, dx = \int \sin 2x \cdot 2 \cdot \frac{1}{2} dx$

$= \frac{1}{2} \cdot -\cos 2x = \underline{\frac{-\cos 2x}{2}}$

b) $\int \sin \frac{x}{3} dx = \int \sin \frac{x}{3} \cdot \frac{1}{3} \cdot 3 \, dx$

$= 3 \cdot -\cos \frac{x}{3} = -3\cos \frac{x}{3}$

c) $\int \sin 2x + \frac{\pi}{3} dx = \int \sin 2x + \frac{\pi}{3} \cdot \frac{1}{2} \cdot 2 \, dx$

$= \frac{1}{2} \cdot -\cos 2x + \frac{\pi}{3}$

12.6 $\frac{e^{x^2} \cdot 2x \cdot 2x - e^{x^2} \cdot 2}{4x^2} \quad \frac{e^{x^2}(4x^2 - 2)}{4x^2} \neq e^{x^2}$ Nej

12.7

a) $2x$ är inre derivatan så primitiv är e^{x^2}

b) $\int e^{x^2} \cdot 2x \, dx = \left[t = x^2, \; dt = 2x \, dx \right]$

$= \int e^t dt = e^t + C = e^{x^2} + C$

12.8

a) $\int e^{x^2} \cdot x \, dx = \int e^{x^2} \cdot 2x \cdot \frac{1}{2} dx = \frac{1}{2} \cdot e^{x^2}$

e) $\int x^2 \cos x^3 \, dx = \int 3x^2 \cos x^3 \cdot \frac{1}{3} = \frac{1}{3} \cdot \sin x^3$

12.8 h) $\int 2x(x^2+5)^8\,dx = \frac{(x^2+5)^9}{9}$

12.9

a) $\int \sin^2 x \cos x\,dx = [t = \sin x,\ dt = \cos x \cdot dx]$

$= \int t^2 \cdot dt = \frac{t^3}{3} + C = \frac{\sin^3 x}{3} + C$

b) $\int \cos x \sin^3 x\,dx = [t = \sin x,\ dt = \cos x \cdot dx]$

$= \int t^3 \cdot dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$

c) $\int \sin x \cdot \cos x\,dx = [t = \sin x,\ dt = \cos x \cdot dx]$

$= \int t\,dt = \frac{t^2}{2} + C = \frac{\sin^2 x}{2} + C$

d) $\int \cos x \cdot \frac{1}{\sin^2 x}\,dx = [t = \sin x,\ dt = \cos x \cdot dx]$

$= \int \frac{1}{t^2}\,dt = \frac{1}{-t} + C = \frac{1}{-\sin x} + C$

e) $\int \cos x \frac{1}{\sin x}\,dx = \int \frac{1}{t}\,dt = \ln|t| + C$

$= \ln|\sin x| + C$

f) $\ln|\sin x| + C$ ser uppgift e)

g) $\int \frac{\sin x}{\cos x}\,dx = [t = \cos x,\ dt = -\sin x\,dx] = -\int \frac{1}{t}\,dt$

$= -\ln|t| + C = -\ln|\cos x| + C$

12.9

h) $\int \tan x = \int \frac{\sin x}{\cos x} = -\ln|\cos x| + C$ ser uppgift g)

12.10

a) $\int \frac{1}{x^2+1} \cdot 2x \, dx = \left[t = x^2+1, \; dt = 2x\,dx \right]$

$= \int \frac{1}{t} \cdot dt = \ln|t| + C = \ln|x^2+1| + C$

b) ser uppgift a) $\ln|x^2+1| + C$

c) $\int \frac{x}{x^2+1} dx = \int \frac{1}{x^2+1} \cdot 2x \cdot \frac{1}{2} dx = \frac{1}{2} \ln|x^2+1| + C$

d) $\int \frac{3x^2}{x^3+1} dx = \left[t = x^3+1, \; dt = 3x^2 dx \right]$

$= \int \frac{1}{t} dt = \ln|t| + C = \ln|x^3+1| + C$

e) $\int \frac{x^2}{x^3+1} dx = \int \frac{1}{x^3+1} \cdot \frac{1}{3} \cdot 3x^2 dx = \frac{1}{3} \int \frac{3x^2}{x^3+1} dx$

$= \frac{1}{3} \ln|x^3+1| \, dx$

f) $\int \frac{e^x}{e^x+1} dx = \left[t = e^x+1, \; dt = e^x \cdot dx \right]$

$= \int \frac{1}{t} dt = \ln|e^x+1| + C$

g) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \left[t = e^x + e^{-x}, \; dt = e^x - e^{-x} dx \right]$

$= \int \frac{1}{t} dt = \ln|e^x + e^{-x}| + C$

12.10 h) $\int \frac{e^x+1}{e^x}dx = \int 1 + \frac{1}{e^x}dx$

$= x - e^{-x}$

12.11 a) $\int \frac{1}{x}(\ln x)^2 dx = [t=\ln x, dt = \frac{1}{x}dx]$

$= \int t^2 dt = \frac{t^3}{3} = \frac{(\ln x)^3}{3}$

b) $\int \frac{1}{x}\ln x dx =$ ser a) $= \frac{(\ln x)^2}{2}$

c) $\int \frac{(\ln x)^2}{x}dx = \int \frac{1}{x}\cdot \ln x^2 dx =$ ser a) $\frac{(\ln x)^3}{3} + C$

d) $\int \frac{\ln x}{x}dx$ ser b) $= \frac{(\ln x)^2}{2}$

e) $\int \frac{1}{x}\sin(\ln x)dx = [t=\ln x, dt=\frac{1}{x}dx]$

$= \int \sin t\, dt = -\cos t + C = -\cos(\ln x) + C$

f) $\int \frac{1}{x}\cdot\frac{1}{\ln x}dx = [t=\ln x, dt=\frac{1}{x}dx]$

$= \int \frac{1}{t}dt = \ln t + C = \ln|(\ln x)|$ frågar abs.

g) $\int \frac{1}{x\ln x}dx = \int \frac{1}{x}\cdot\frac{1}{\ln x}dx$ ser f)

h) $\int \frac{\sin(\ln x)}{x}dx = [t=\ln x, dt=\frac{1}{x}dx]$

$= \int \sin t\, dt = -\cos t + C = -\cos(\ln x) + C$

12,13

$$\int e^x(e^x+5)^8 dx = [t = e^x+5,\ dt = e^x dx]$$

$$= \int t^8 dt = \frac{t^9}{9} + C = \frac{(e^x+5)^9}{9} + C$$

12,14

a) se 12,11 f)

b) $\int \sin x \cos^{-4/3} = [t = \cos x,\ dt = -\sin x\, dx]$

$$= -\int t^{-4/3} dt = -\frac{t^{-1/3}}{-\frac{1}{3}} + C = \frac{3}{t^{1/3}} + C = \frac{3}{(\cos x)^{1/3}} + C$$

12,15

c) $\int x\sqrt{7x^2+5}\, dx = [t = 7x^2+5,\ dt = 14x\, dx]$

$$= \frac{1}{14}\int \sqrt{t}\, dt = \frac{1}{14} \cdot \frac{t^{3/2}}{3/2} + C = \frac{t^{3/2}}{21} + C$$

$$= \frac{(7x^2+5)^{3/2}}{21} + C$$

d) $\int \frac{x}{\sqrt{x^2+5}} dx = [t = x^2+5,\ dt = 2x\, dx]$

$$= \frac{1}{2}\int \frac{1}{\sqrt{t}} dt = \frac{1}{2} \cdot \frac{\sqrt{t}}{1/2} + C = \sqrt{t} + C = \sqrt{x^2+5} + C$$

e) Ja

12.16

a) $\int \frac{1}{e^x+e^{-x}}dx = \int \frac{1}{\frac{e^{2x}+1}{e^x}}dx$ $\left[t=e^x,\ dt=e^x \cdot dx,\ \frac{dt}{e^x}=\frac{dt}{t}=dx\right]$

$=\int \frac{1}{t^2+1}dt = \arctan t + C \quad = \arctan e^x + C$

b) $\int x\sqrt{x+1}\,dx = \left[t=\sqrt{x+1},\ x=t^2-1,\ dx=2t\,dt\right]$

$=\int (t^2-1)\cdot t\cdot 2t\,dt = \int 2t^4-2t^2\,dt$

$=\frac{2t^5}{5}-\frac{2t^3}{3}+C = \frac{6t^5-10t^3}{15}+C$

$=\frac{6\sqrt{x+1}^5-10\sqrt{x+1}^3}{15}+C$

c) $\int \frac{x}{\sqrt{2x+5}}dx = \left[t=\sqrt{2x+5},\ x=\frac{t^2-5}{2},\ dx=\frac{2t}{2}dt = t\,dt\right]$

$=\int \frac{\frac{t^2-5}{2}}{t}t\,dt = \int \frac{t^2-5}{2}dt = \frac{t^3}{6}-\frac{5t}{2}+C$

$=\frac{t^3-15t}{6}+C = \frac{\sqrt{2x+5}^3-15\sqrt{2x+5}}{6}+C$

d) $\int \frac{1}{x+x^{1/3}}dx = \left[t=x^{1/3},\ x=t^3,\ dx=3t^2dt\right]$

$\int \frac{1}{t^3+t}\cdot 3t^2dt = 3\int \frac{t}{t^2+1}dt = 3\int \frac{2t}{t^2+1}\cdot\frac{1}{2}dt$

$=\frac{3}{2}\ln|t^2+1|+C = \frac{3\ln|x^{2/3}+1|}{2}+C$