KAPITEL 12

12.1

a) $\int x^4 = \boxed{\frac{x^5}{5} + C}$ | b) $\int \frac{1}{x} = \boxed{\ln|x| + C}$

c) $\int e^x = \boxed{e^x + C}$ | d) $\int \cos x = \boxed{\sin x + C}$

e) $\int \sin x = \boxed{-\cos x + C}$ | f) $\int \frac{1}{\cos^2 x} = \boxed{\tan x + C}$

g) $\int \frac{1}{\sin^2 x} = \boxed{\cot x + C}$ | h) $\int \frac{1}{1+x^2} = \boxed{\arctan x + C}$

i) $\int \frac{1}{\sqrt{1-x^2}} = \boxed{\arcsin x + C}$

j) $\int \frac{1}{\sqrt{x^2+a}} = \boxed{\ln\left|x + \sqrt{x^2+a}\right| + C}$

12.2

a) $\boxed{\frac{x^4}{4}}$ b) $\frac{1}{x^2} \cdot \frac{1}{2} = \boxed{-\frac{1}{2x^2}}$ c) $\boxed{\frac{2x^{3/2}}{3}}$

d) $\int x\sqrt{x} = \frac{x \cdot 2x^{3/2}}{3} - \int \frac{2x^{3/2}}{3} \cdot 1 = \frac{2 x^{5/3}}{3} - \frac{4 x^{5/3}}{15} = \boxed{\frac{3x^{5/3}}{5}}$

$= \boxed{\frac{3}{5} x^2 \cdot \sqrt{x}}$

e) $$\int \frac{1}{\sqrt{x}} = \int x^{-1/2} = x^{1/2} \cdot 2 = \boxed{2\sqrt{x}}$$

f) $$\cancel{\int \frac{1}{x\sqrt{x}} = \left[\begin{array}{l} \sqrt{x} = t \Rightarrow t^2 = x \\ \frac{dt}{dx} = 2t \Rightarrow dx = \frac{dt}{2t} \end{array}\right] = \int \frac{1}{t^3}\,dx = \int \frac{dt}{2t^4} =}$$

$$= \int \frac{1}{x\sqrt{x}}\,dx = \left[\begin{array}{l} t = \sqrt{x} \Rightarrow t^2 = x \\ \frac{dx}{dt} = 2t \Rightarrow dx = 2t\,dt \end{array}\right] = \int \frac{1}{t^3} \cdot 2t\,dt =$$

$$= \int \frac{2}{t^2}\,dt = \int 2t^{-2}\,dt = -2t^{-1} = -\frac{2}{t} = \boxed{-\frac{2}{\sqrt{x}}}$$

$\boxed{12.3}$

a) $$\int \frac{1}{x}\,dx = \boxed{\ln|x|}$$

b) $$\int \frac{1}{x-2}\,dx = \int (x-2)^{-1}\,dx = \boxed{\ln|x-2|}$$

c) $$\int \frac{1}{1-x}\,dx = -\int \frac{1}{x-1} = \boxed{-\ln|x-1|}$$

12.5

a) $\int \sin 2x \, dx = \boxed{\dfrac{\cos 2x}{-2}}$

b) $\int \sin \frac{x}{3} \, dx = \boxed{-3\cos\frac{x}{3}}$

c) $\int \sin\left(2x+\frac{\pi}{3}\right) dx = \boxed{\dfrac{-\cos\left(2x+\frac{\pi}{3}\right)}{2}}$

12.6

Är $\boxed{\int e^{x^2} dx = \dfrac{e^{x^2}}{2x}}$?

$$D\left(\frac{e^{x^2}}{2x}\right) = \frac{(2x)^2 e^{x^2} - 2e^{x^2}}{4x^2} = \frac{4x^2e^{x^2} - 2e^{x^2}}{4x^2} = \frac{e^{x^2}(2x^2-1)}{4x^2}$$

Svar: Nej

12.7 a) $\int e^{x^2} \cdot 2x \, dx = \left[t = x^2 = \frac{dt}{dx} = 2x\right] = \int e^t dt = e^t = \boxed{e^{x^2}+C}$

$\boxed{12.8}$

a) $\int e^{x^2} \cdot x\,dx = \left[t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow x \cdot dx = \frac{dt}{2} \right] =$

$= \int e^t \cdot \frac{dt}{2} = \frac{1}{2} \int e^t dt = \frac{1}{2} e^t = \boxed{\frac{e^{x^2}}{2} + c}$

e) $\int x^2 \cdot \cos(x^3) dx = \left[\begin{array}{l} t = x^3 = \frac{dt}{dx} = 3x^2 \\ \Rightarrow x^2 dx = \frac{dt}{3} \end{array} \right] =$

$= \int \frac{\cos t}{3} dt = -\frac{\sin t}{3} = \boxed{-\frac{\sin x^3}{3}}$

h) $\int 2x(x^2+5)^8 dx = \left[\begin{array}{l} t = x^2 \Rightarrow \frac{dt}{dx} = 2x \\ \Rightarrow 2x\,dx = dt \end{array} \right] = \int (t+5)^8 dt =$

$= \frac{(t+5)^9}{9} = \boxed{\frac{(x^2+5)^9}{9}}$

12.9

a) $\int \sin^2 x \cdot \cos x \, dx = \left[\sin x = t \Rightarrow \frac{dt}{dx} = \cos x \right] =$

$= \int t^2 \cdot dt = \frac{1}{3} t^3 = \boxed{\frac{1}{3} \sin^3 x}$

e) $\int \cos x \cdot \frac{dx}{\sin x} = \left[\begin{array}{l} t = \sin x \Rightarrow \frac{dt}{dx} = \cos x \\ \Rightarrow \cos x \cdot dx = dt \end{array} \right] =$

$= \int \frac{dt}{t} = \ln|t| = \boxed{\ln|\sin x|}$

h) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \left[\begin{array}{l} \cos x = t \\ dt = -\sin x \, dx \end{array} \right] =$

$= -\int \frac{dt}{t} = -\ln|t| = \boxed{-\ln|\cos x|}$

12.10

a) $\int \frac{1}{x^2+1} \cdot 2x\,dx = \begin{bmatrix} x^2 = t \\ dt = 2x\,dx \end{bmatrix} =$

$= \int \frac{1}{t+1}\,dt = \ln|t+1| = \boxed{\ln|x^2+1|}$

e) $\int \frac{x^2}{x^3+1}\,dx = \begin{bmatrix} t = x^3 \\ \frac{dt}{3} = \cancel{3}x^2\,dx \end{bmatrix} = \int \frac{dt}{3(t+1)} =$

$= \boxed{\frac{1}{3}\ln|x^3+1|}$

h) $\int \frac{e^x+1}{e^x}\,dx = \begin{bmatrix} e^x = t \\ dt = e^x \cdot dx \end{bmatrix} = \int 1 + \frac{1}{e^x} =$

$= \boxed{x + \ln|e^x|}$

h) $\int \frac{e^x+1}{e^x}dx = \int \frac{e^x}{e^x} + \frac{1}{e^x}dx = \int 1dx + \int e^{-x}dx = \boxed{x - e^{-x}}$

$\boxed{12.11}$

a) $\int \frac{1}{x}(\ln x)^2 dx = \left[\begin{matrix} t = \ln x \\ \frac{dt}{dx} = \frac{1}{x} \end{matrix}\right] = \int t^2 dt = \frac{t^3}{3} = \boxed{\frac{\ln^3 x}{3}}$

e) $\int \frac{1}{x}\sin(\ln x)dx = \left[\begin{matrix} t = \ln x \\ dt = \frac{dx}{x} \end{matrix}\right] = \int \sin t\, dt = \cos t =$

$= \boxed{\cos(\ln x)}$

h) $\int \frac{\sin(\ln x)}{x}dx = \left[\begin{matrix} t = \ln x \\ dt = \frac{dx}{x} \end{matrix}\right] = \int \sin t\, dt =$

$= -\cos t = \boxed{-\cos(\ln x)}$

$\boxed{12.13}$

a) $$\int e^x (e^x+5)^8 dx = \begin{bmatrix} t = e^x \\ dt = e^x dx \end{bmatrix} = \int (t+5)^8 dt =$$

$$= \frac{(t+5)^9}{9} + c = \boxed{\frac{(e^x+5)^9 + c}{9}}$$

$\boxed{12.14}$

a) $$\int \frac{1}{x \ln x} dx = \begin{bmatrix} t = \ln x \\ dt = \frac{dx}{x} \end{bmatrix} = \int \frac{1}{t} dt =$$

$$= \ln|t| + c = \boxed{\ln|\ln x| + c}$$

b) $$\int \sin x \cos^{-4/3} x \, dx = \begin{bmatrix} t = \cos x \\ dt = -\sin x \, dx \end{bmatrix} = -\int t^{-4/3} dt =$$

$$= - -3t^{-1/3} + c = \boxed{3\cos^{-1/3} x + c}$$

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20, 22a, 23b, 24ad, 25ab, 26bd 27ab 30a 31b

$\frac{dy}{dx}$

$\boxed{12.16}$

a) $\int \frac{1}{e^x + e^{-x}} dx = \begin{bmatrix} t = e^x \\ \frac{dt}{dx} = e^x = t \end{bmatrix} = \int \frac{1}{t + \frac{1}{t}} \cdot \frac{dt}{t} = \int \frac{1}{t^2 + 1} dt =$

$= \arctan(t) = \boxed{\arctan(e^x)}$

b) $\int x\sqrt{x+1}\, dx = \begin{bmatrix} t = \sqrt{x+1} \\ x = t^2 - 1 \end{bmatrix} = \int (t^2 - 1) \cdot t \cdot 2t\, dt =$
$dx = 2t\, dt$

$= \int 2t^4 - 2t^2 dt = \boxed{\frac{2t^5}{5} - \frac{2t^3}{3} + C}$

c) $\int \frac{x}{\sqrt{2x+5}} dx = \begin{bmatrix} t = \sqrt{2x+5} \\ x = \frac{t^2 - 5}{2} \\ dx = t\, dt \end{bmatrix} = \int \frac{\frac{t^2 - 5}{2}}{t} t\, dt = \int \frac{t^2 - 5}{2} dt =$

$= \frac{t^3 - 15t}{6} = \boxed{\frac{\sqrt{2x+5}^3 - 15\sqrt{2x+5}}{6}}$

d) $\displaystyle\int \frac{1}{x+x^{1/3}}\,dx = \begin{bmatrix} t=x^{1/3} \\ x=t^3 \\ dx=3t^2dt \end{bmatrix} = \int \frac{1}{t^3+t}\,3t^2dt = \int \frac{3t}{t^2+1}\,dt =$

$\displaystyle = 3\int t \cdot \frac{1}{t^2+1}\,dt = \begin{bmatrix} g=t^2 \\ \frac{dg}{dt}=2t \end{bmatrix} = 3\int t \cdot \frac{1}{g+1}\,\frac{dg}{2t} =$

$\displaystyle = \frac{3}{2}\int \frac{1}{g+1}\,dg = \frac{3}{2}\int (g+1)^{-1}dg = \ln(g+1) =$

$\displaystyle = \boxed{\ln(x^{2/3}+1)\cdot\frac{3}{2}+C}$

$\boxed{12.17}$

a) $\displaystyle\int (x^2\ln x)\,dx = \frac{x^3}{3}\cdot\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx =$

$\displaystyle = \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} = \boxed{\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right)+C}$

b) $\displaystyle\int x\cdot e^{-x}\,dx = -xe^{-x} + \int e^{-x} = -xe^{-x} - e^{-x} = \boxed{-e^{-x}(x+1)} + C$

d) $\int \arctan x = \boxed{\frac{1}{x^2+1} + c}$

e) $\int x \cdot \arctan x \, dx = \frac{x^2}{2} \arctan x - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} \, dx$

$\triangleright \frac{1}{2} \int \frac{x^2}{1+x^2} dx = \frac{1}{2} \int 1 - \frac{1}{1+x^2} = \frac{1}{2}(x - \arctan x)$

$$\begin{array}{r|l} 1+x^2 & \overline{\;x^2\;} \quad 1 \\ & -x^2-1 \\ \hline & -1 \end{array} \Rightarrow \boxed{\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}}$$

REMEMBER

<u>så</u>

$\int (x \cdot \arctan x) dx = \boxed{\frac{1}{2} x^2 \cdot \arctan x - \frac{1}{2} x - \arctan x + c}$

f) $\int (\ln(x+1))dx = \begin{bmatrix} t = x+1 \\ \frac{dt}{dx} = 1 \end{bmatrix} = \int \ln(t)\,dt = t(\ln t - 1) + c$

$\ln 1$

$= \boxed{(x+1)(\ln(x+1) - 1) + c}$

g) $\int \ln^2 x\,dx = x(\ln x - 1) \cdot \ln x - \int \frac{\cancel{x}(\ln x - 1)}{\cancel{x}} =$

$= x\ln^2 x - x\ln x - x(\ln x - 1) + x + c =$

$= \boxed{x\ln^2 x - 2x\ln x + 2x + c}$

h) $\int x^2 \cdot \sin x\,dx = -\cos x \cdot x^2 - \int -\cos x\,2x =$

$= -x^2 \cdot \cos x + \sin x\,2x - \int 2\sin x =$

$= \boxed{-x^2\cos x + 2x\sin x + 2\cos x}$

$= \boxed{(2 - x^2)\cos x + 2x\sin x}$

$\boxed{12.18}$

a) $\int e^x \cdot \sin x \, dx = e^x \cdot \sin x + \int e^x \cos x \, dx =$

$\cancel{= e^x \cdot \sin x + e^x \sin x - \int e^x \sin x \, dx}$

$\cancel{= 2e^x \cdot \sin x - \int e^x \cdot \sin x \, dx}$

$\boxed{\text{GÅ IGENOM NOGA}}$

$\boxed{12.20}$

$$\int e^{\sqrt{x}} dx = \begin{bmatrix} t = \sqrt{x} \\ t^2 = x \\ dx = 2t\,dt \end{bmatrix} = \int e^t \cdot 2t \, dt =$$

$$= e^t \cdot 2t - \int 2e^t \, dt = e^t \cdot 2t - 2e^t = 2e^t(t-1) =$$

$$= \boxed{2e^{\sqrt{x}}(\sqrt{x}-1) + C}$$

$\boxed{12.22}$

a) $\displaystyle\int \frac{x^2+4}{x-1}\,dx = \int x+1\,dx + \int \frac{5}{x-1}\,dx =$

$$\begin{array}{r} x+1 \\ x-1\,\big)\overline{\,x^2+4\,} \\ -x^2+x \\ \hline x+4 \\ -(x-1) \\ \hline 5 \end{array}$$

$= \boxed{\frac{1}{2}x^2 + x + 5\ln(x-1)}$

$\boxed{12.23}$

b) $\displaystyle \frac{x+13}{x^2-4x-5} = \frac{x+13}{(x+5)(x-5)} = \frac{A}{x+5} + \frac{B}{x+1} = \frac{A(x-5)+B(x+1)}{x^2-4x-5}$

$\left(2 \pm \sqrt{4+5} = 2 \pm 3\right)$

$x+13 = A(x-5) + B(x+1)$

$x=5 \Rightarrow 18 = 6B \Leftrightarrow B=3$

$x=-1 \Rightarrow 12 = -6A \Leftrightarrow A=-2$

$= \boxed{-2\ln|x+5| + 3\ln(x+1)}$

$\boxed{12.24}$

a) $$\frac{1}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{Cx+D}{(x-3)^2}$$

$$= \frac{A(x-3)^2 + B(x-3)x + (Cx+D)x}{x(x-3)^2}$$

$$x = 3 \Rightarrow 1 = 9C + 3D$$

$$x = 0 \Rightarrow 1 = 9A \Leftrightarrow \boxed{A = \frac{1}{9}}$$

$$1 = (x^2 - 6x + 9)\frac{1}{9} + Bx^2 - 3Bx + Cx^2 + D$$

$$\begin{cases} \frac{x^2}{9} + Bx^2 + Cx^2 = 0 \\ -\frac{6x}{9} - 3Bx = 0 \end{cases} = \begin{cases} \frac{1}{9} + B + C = 0 \Leftrightarrow \boxed{C = \frac{1}{9}} \\ \frac{2}{9} + B = 0 \Leftrightarrow \boxed{B = -\frac{2}{9}} \end{cases}$$

$$1 + D = 1 \Rightarrow \boxed{D = 0}$$

$$= \frac{1}{9}\int \frac{1}{x} - \frac{2}{x-3} + \frac{x}{(x-3)^2}\,dx = \frac{1}{9}\left(\ln|x| - 2\ln|x-3|\right.$$

$\boxed{12.24}$

$\frac{1}{4} \quad -1 \pm \sqrt{1-1}$

d) $\dfrac{1}{x^3+2x^2+x} = \dfrac{1}{x(x^2+2x+1)} = \dfrac{1}{x(x+1)^2} = \dfrac{A}{x} + \dfrac{Bx+C}{(x+1)^2}$

$\Leftrightarrow 1 = A(x+1)^2 + Bx^2 + Cx$

$\Leftrightarrow 1 = Ax^2 + 2Ax + A + Bx^2 + Cx$

$$\begin{cases} A+B=0 \\ 2A+C=0 \\ A \quad = 1 \end{cases} \Leftrightarrow \begin{cases} B=-1 \\ C=-2 \\ A=1 \end{cases}$$

$$\int x^{-1} - \int \frac{x+2}{(x+1)^2} = \ln x + \int \frac{x}{(x+1)^2} + \int \frac{2}{(x+1)^2} =$$

$= \ln|x|$

$\boxed{12.25}$

a) $\displaystyle\int \frac{dx}{x^2+1} = \boxed{\arctan x + C}$

$\boxed{C \atop +}$

b) $\displaystyle\int \frac{dx}{(2x)^2+1} = \begin{bmatrix} t = 2x \\ dx = \frac{dt}{2} \end{bmatrix} = \frac{1}{2}\int \frac{1}{t^2+1}\,dt = \boxed{\tfrac{1}{2}\arctan 2x}$

$\boxed{12.26}$

b) $\displaystyle\int \frac{dx}{\frac{x^2}{9}+1} = \int \frac{dx}{(\frac{1}{3}x)^2+1} = \begin{bmatrix} t = \frac{1}{3}x \\ dx = 3dt \end{bmatrix} = 3\int \frac{1}{t^2+1}\,dt =$

$= \boxed{3\arctan(\tfrac{1}{3}x) + C}$

d) $\displaystyle\int \frac{dx}{x^2+9} = \int \frac{1}{(\frac{1}{3}x)^2+1}\,dx = \begin{bmatrix} t = \frac{1}{3}x \Leftrightarrow 3t = x \\ dx = 3dt \end{bmatrix} =$

$\displaystyle = 9 \cdot 3\int \frac{1}{t^2+1}\,dt = \boxed{27\arctan \tfrac{1}{3}x + C}$

$\boxed{12.27}$

a) $\dfrac{1}{x^2-2x+2} = \dfrac{1}{x^2-2x+1+2-1} \quad \displaystyle\int \frac{1}{(x-1)^2+1}\,dx = \left[\begin{array}{l} t = x-1 \\ \frac{dt}{dx} = 1 \end{array}\right] =$

$$= \int \frac{1}{t^2+1}\,dt = \boxed{\arctan(x-1) + C}$$

b) $\dfrac{1}{x^2+4x+5} = \dfrac{1}{(x+1)^2-4+5} = \left[\begin{array}{l} t = x+2 \\ \frac{dt}{dx} = 1 \end{array}\right.$

$$\int \frac{dt}{t^2+1} = \boxed{\arctan(x+2)}$$

$\boxed{12.28}$

a) $\dfrac{x+1}{x^2-2x+2} = \dfrac{x+1}{\underbrace{x^2-2x+1}-1+2} = \dfrac{x+1}{(x-1)^2+1} = \dfrac{x+1}{(x-1)^2+1}$

$\left[x-1 = t \Leftrightarrow x+1 = t+2\right]$
$\left[dt = dx\right]$

$$\int \frac{t}{t^2+1} + \int \frac{1}{t^2+1} = \boxed{\frac{1}{2}\ln(t^2+1) + \arctan t + C}$$

12.28

b) $\frac{x+1}{x^2+4x+5} = \frac{x+1}{(x+2)^2+1} = \left[\begin{matrix} t = x+2 \Leftrightarrow x+1 = t-1 \\ \frac{dt}{dx} = 1 \Leftrightarrow dt = dx \end{matrix}\right]$

$= \int \frac{t-1}{t^2+1} dt = \int \frac{t}{t^2+1} - \frac{1}{t^2+1} dt =$

$= \frac{1}{2} \ln|t^2+1| - \arctan t + C$

$= \boxed{\frac{1}{2} \ln|(x+2)^2+1| - \arctan(x+2) + C}$

12.30

a) $\frac{2}{(x^2+1)(x-1)} = \frac{Bx+C}{(x^2+1)} + \frac{A}{x-1} = \frac{(Bx+C)(x-1)+A(x^2+1)}{(\ \ )(\ \ )}$

$Bx^2 - Bx + Cx - C + Ax^2 + A = 2$

$\begin{cases} A+B = 0 \\ -B+C = 0 \\ A - C = 2 \end{cases} = \begin{cases} A+B = 0 \\ -B+C = 0 \\ A - B = 2 \end{cases}$ $\boxed{\begin{matrix} A = 1 \\ B = -1 \\ C = -1 \end{matrix}}$

$$\int \frac{1}{x-1} - \frac{x-1}{x^2+1}\,dx = \boxed{\ln(x-1) - \frac{1}{2}\ln|x^2+1| - \arctan x} + C$$

$\boxed{12.31}$

b)

$$\frac{x^2+8x+4}{x^2+4x+8} = 1 + \frac{4x-4}{\underset{+4-4}{x^2+4x+8}} = 1 + \frac{4x-4}{(x+2)^2+4} =$$

$$\begin{array}{r} 1 \\ x^2+4x+8 \overline{\big) x^2+8x+4} \\ -x^2-4x-8 \\ \hline 4x-4 \end{array}$$

$$1 + \frac{x-4}{\left(\frac{x+2}{2}\right)^2+1} = \begin{bmatrix} \frac{x+2}{2} = t \iff x = 2t-2 \\ dx = dt \end{bmatrix} = 1 + \frac{2t-6}{t^2+1}$$

$$2\int \frac{t}{t^2+1}\,dt - 6\int \frac{1}{t^2+1}\,dt + \int 1\,dx =$$

$$= \boxed{2\ln|t^2+1| - 6\arctan t + x}$$

$\boxed{12.32}$

a) $t = \tan\frac{x}{2}$, $|x| < \pi$

Visa att:

$$\boxed{x} = 2\arctan\left(\tan\frac{x}{2}\right) = 2 \cdot \frac{x}{2} = \boxed{x}$$

och:

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\sin x + \frac{\sin^2 x/2}{\cos^2 x/2}\sin x = 2\tan\frac{x}{2} \quad ?$$

b)

$$\int \frac{3}{4+5\sin x}\,dx =$$

$\boxed{12.33}$

a) $$\frac{1}{2+\sin x} = \begin{bmatrix} t = \tan\frac{x}{2} \\ dt = \frac{1}{1+\frac{x^2}{4}}\,dx \end{bmatrix}$$

$\boxed{12.34}$

b) $$\frac{\sin 2x}{\cos^3 x} = \frac{2\sin x}{\cos^2 x} = \begin{bmatrix} t = \cos x \\ \frac{dt}{dx} = -\sin x \end{bmatrix}$$

$$-2\int \frac{\frac{dt}{\cancel{dx}}}{t^2}\,\cancel{dx} = -2\int t^{-2}\,dt = \frac{2}{t} + C = \boxed{\frac{2}{\cos x} + C}$$

d) $$\int \cos x \cdot \sin^9 x\,dx = \begin{bmatrix} t = \sin x \\ dt = \cos x\,dx \end{bmatrix} = \int t^9\,dt = \boxed{\frac{\sin^{10} x}{10} + C}$$

12.35

a) $\int \sin^5 x\, dx = \int (\sin^2 x)^2 \cdot \sin x\, dx =$

$$\int (1-\cos^2 x)^2 \sin x\, dx = \begin{bmatrix} \cos x = t \\ dt = \sin x\, dx \end{bmatrix} =$$

$$= \int (1-t^2)^2 dt = \int 1 - 2t^2 + t^4\, dt =$$

$$= t + \frac{2t^3}{3} + \frac{t^5}{5} = \boxed{\cos x + \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x}$$

b) $\int \sin^4 x\, dx = \int (1+\cos^2 x)^2 dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$$= \frac{x^2}{4} + \frac{1}{2}\int \cos^2 2x\, dx = \frac{x^2}{4} + \frac{1}{2}\int \frac{1+\cos 4x}{2} dx =$$

$$\frac{x^2}{4} + \frac{x^2}{8} + \frac{1}{4}\sin 4x \cdot \frac{1}{4} + C = \frac{3x^2}{8} + \frac{1}{16}\sin 4x + C$$

$\boxed{12.35}$

b) $\int \sin^4 x \, dx = \int (1-\cos^2 x)^2 dx =$

$\int (1 - 2\cos^2 x + \cos^4 x) dx = x - 2\int \cos^2 x \, dx + \int \cos^4 x \, dx$

$= \triangle \int \frac{1+\cos 2x}{2} dx = \frac{x}{2} + \frac{1}{4}\sin 2x$

$\triangle \int \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int 1 + 2\cos 2x + \frac{1+\cos 4x}{2} \, dx =$

$= \frac{x}{4} + \sin 2x + \frac{x}{8} + \frac{1}{8}\sin 4x$

så

$\int \sin^4 x = -\frac{1}{2}\sin 2x + \frac{3x}{8} + \sin 2x + \frac{1}{8}\sin 4x$

$= \frac{3x}{8} + \frac{1}{2}\sin x + \frac{1}{8}\sin 4x$

d) $\int \cos^2 x = \int \left(\frac{1+\cos 2x}{2}\right) dx = \boxed{\frac{x}{2} + \frac{\sin 2x}{4} + C}$

$\boxed{12.36}$

$$\int \frac{1+\sqrt{x+1}}{1-\sqrt{x+1}}\,dx = \left[\begin{array}{l} t=\sqrt{x+1} \Leftrightarrow x = t^2-1 \\ dx = 2t\,dt \end{array}\right] =$$

$$\int \frac{1+t}{1-t}\,2t \cdot dt =$$