tail of the arrow, and each vertex operation to a Z gate acting on the vertex.

Straight away this can be seen to satisfy the intuitive conditions i), iii), iv) and v). Also as X and Y anti-commute, it is clear that every pair of edges that meet head-to-tail anti-commute, whereas pairs of edges that meet head-to-head or tail-to-tail commute. We can correct this behaviour by mapping every pair of these edges, those which currently commute but should anti-commute, to act upon a common ancillary qubit with X and Y gates respectively. Though this procedure is performed on a directed graph it still holds for edges in both directions as $E_{ij} = -E_{ji}$, so if E_{ij} anti-commutes with something so will E_{ji}.

On a square lattice this common ancillary qubit can be the face qubit adjacent to the edges, as every pair of head-to-head or tail-to-tail edges share a common face qubit (see Fig **??**). It can be seen that using an ancillary "face" qubit to introduce two anti-commutation relations does not break the restriction iv) as long as opposite edges of black squares act in commutable ways upon the "face" qubit (i.e. with same gate).

Finally, in order to satisfy the loop condition applying any given loop of edges needs to correspond to an identity in the fermionic Fock space. In other words this mapping is only valid for fermionic state representations that lie in the space stabilised by cycles of edges. It can be shown that the stabiliser code space of cycles is isomorphic to the group of even Majorana operators:

> To summarize, given a connected fermionic graph G, corresponding to some fermionic system, to construct a local fermionic encoding it suffices to specify 5 a local mapping of the edges and vertices of the graph to multi-qubit Pauli operators, satisfying the relations 7, such that no element of CG is mapped to 1.

need to prove this then done

First can show that the group of even majorana operators is isomorphic to the quotient group M_G/C_G, and so if we have a mapping to qubit operators such that all members of $\sigma(C_G)$ 1 have a common +1 eigenspace U then by projecting on it. We have a mapping σ_U that takes all members of C_G to 1, and so C_G is in the kernel of σ_U. Therefore, σ_U maps from M_G/C_G to $L(\mathcal{H}$ and so it maps from the even majorana operators to the qubit operators. As C_G all commute, and as long as the cycles never map to -1 they form a stabilizer of a $+1$ eigenspace that can be taken to be U (nieslen chuang 455). Therefore the even majorana states map into the code space of the stabilisers (the cycles). In this case if there wasn't anywhere we could inject majarona operators then the only states possible would be those that where themselves under application of an even face cycle so ones that satsify $\prod_i V_i = 1$ so the even fermonic algebra. In order to discuss how the code space actually represents the odd algebra we need to insert marjoranas at corners.
As every cycle can be decomposed in to a combination of cycles around individual faces (**proof**), we must simply consider the impact of cycling around a black or a white face.

As shown in Fig **??**, looping a black face produces $-I$ meaning it cannot be in a stabilizer, however this can be fixed by simply flipping the sign of one of the four edges. Then we get $X_1Y_5Y_2 \otimes -Y_2X_5X_3 \otimes X_3Y_5Y_4 \otimes Y_4X_5X_1 = -X_1^2Y_2^2X_3^2Y_4^2(Y_5X_5)^2 = -(iZ_5)^2 = I$.

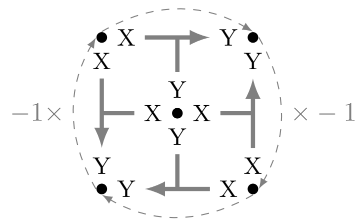

Fig. 4. Diagram of looping around black face. This gives $X^2 = Y^2 = I$ at every "vertex" qubit, but $(-X)Y(-X)Y = (XY)^2 = (iZ)^2 = -I$ at the "face" qubit. The negative signs are introduced by having to flip the orientation of the vertical edges, however they cancel out as they can both be consider to act upon the "face" qubit.

It can be seen in Fig **??**, that looping a white face produces a non-trivial stabilizer.

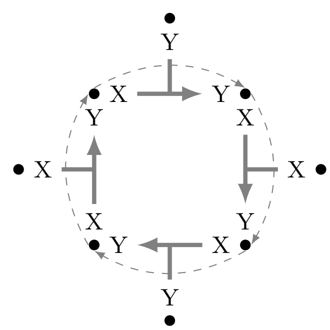

Fig. 5. Diagram of looping around white face.

Therefore, we can map the edges in the direction of their assigned orientation as:

$$E_{ij} = \begin{cases} X_iY_jX_{f(i,j)} & (i,j) \text{ oriented downwards} \\ -X_iY_jX_{f(i,j)} & (i,j) \text{ oriented upwards} \\ X_iY_jY_{f(i,j)} & (i,j) \text{ horizontal} \end{cases} \tag{40}$$

with the last term ignored if the edge lies on a boundary with no adjacent face qubit. Traversing edges in the opposite direction to their orientation just involves applying a sign change.

We also map the vertex operators to:

$$V_j = Z_j \tag{41}$$

These satisfy the anti-commutation relations as shown below:

$$\{E_{jk}, V_j\} = \begin{cases} \{X_jY_kX_{f(j,k)}, Z_j\} \\ -\{X_jY_kX_{f(j,k)}, Z_j\} \\ \{X_jY_kY_{f(j,k)}, Z_j\} \end{cases} = \begin{cases} \{X_j, Z_j\}Y_kX_{f(j,k)} \\ -\{X_j, Z_j\}Y_kX_{f(j,k)} \\ \{X_j, Z_j\}Y_kY_{f(j,k)} \end{cases} = 0 \tag{42}$$

$$\{E_{ij},E_{jk}\} = \begin{cases} \{X_iY_jX_{f(i,j)}, X_jY_kX_{f(j,k)}\} & (i,j) \text{ and } (j,k) \text{ oriented downwards } (1) \\ \{-X_iY_jX_{f(i,j)}, X_jY_kX_{f(j,k)}\} & (i,j) \text{ oriented upwards and } (j,k) \text{ oriented downwards } (2) \\ \{X_iY_jY_{f(i,j)}, X_jY_kX_{f(j,k)}\} & (i,j) \text{ oriented horizontal and } (j,k) \text{ oriented downwards } (3) \\ \{X_iY_jX_{f(i,j)}, -X_jY_kX_{f(j,k)}\} & (i,j) \text{ oriented downwards and } (j,k) \text{ oriented upwards } (4) \\ \{-X_iY_jX_{f(i,j)}, -X_jY_kX_{f(j,k)}\} & (i,j) \text{ and } (j,k) \text{ oriented upwards } (5) \\ \{X_iY_jY_{f(i,j)}, -X_jY_kX_{f(j,k)}\} & (i,j) \text{ oriented horizontal and } (j,k) \text{ oriented upwards } (6) \\ \{X_iY_jX_{f(i,j)}, X_jY_kY_{f(j,k)}\} & (i,j) \text{ oriented downwards and } (j,k) \text{ oriented horizontal } (7) \\ \{-X_iY_jX_{f(i,j)}, X_jY_kY_{f(j,k)}\} & (i,j) \text{ oriented upwards and } (j,k) \text{ oriented horizontal } (8) \\ \{X_iY_jY_{f(i,j)}, X_jY_kY_{f(j,k)}\} & (i,j) \text{ and } (j,k) \text{ oriented horizontal } (9) \end{cases} \tag{43}$$

$$\{E_{ij},E_{jk}\} = \begin{cases} \{Y_j, X_j\}Y_jX_{f(i,j)}Y_kX_{f(j,k)} & (1) \text{ or } (5) \\ \{Y_jY_{f(i,j)}, X_jX_{f(i,j)}\}X_iY_k & (3) \text{ and } f(i,j)=f(j,k) \\ \{Y_jX_{f(i,j)}, X_jY_{f(i,j)}\}X_iY_k & (7) \text{ and } f(i,j)=f(j,k) \\ \{Y_j, X_j\}X_iY_kY_{f(i,j)}X_{f(j,k)} & (3) \text{ or } (7) \text{ and } f(i,j)\neq f(j,k) \\ -\{Y_jY_{f(i,j)}, X_jX_{f(i,j)}\}X_iY_k & (6) \text{ and } f(i,j)=f(j,k) \\ -\{Y_jX_{f(i,j)}, X_jY_{f(i,j)}\}X_iY_k & (8) \text{ and } f(i,j)=f(j,k) \\ -\{Y_j, X_j\}X_iY_kY_{f(i,j)}X_{f(j,k)} & (6) \text{ or } (8) \text{ and } f(i,j)\neq f(j,k) \\ \{X_iY_jY_{f(i,j)}, X_jY_iY_{f(i,j)}\} & (9) \text{ and } f(i,j)=f(j,k) \\ \{Y_j, X_j\}X_iY_{f(i,j)}Y_kY_{f(j,k} & (9) \text{ and } f(i,j)\neq f(j,k) \end{cases} \tag{44}$$

Therefore, as $\{Y_j, X_j\} = 0$, we have $\{E_{ij}, E_{jk}\} = 0$ for $f(i,j) \neq f(j,k)$. So any directed edges that meet head to tail anti-commute.

It is necessary to introduce a sign change into the definition of the vertical edge operators depending on orientation so the loops around the black faces equal 1 (and so satisfy the loop condition).

Derby-Klassen have developed a design technique for compact fermionic encoding. First you take an undirected graph, then start assigning arrows to it. You then assign qubit operators to every arrow with a X operator at the head of every arrow and a Y operator at the tail. These will automatically anticommute with the Z vertex operator at every mode, and they will anticommute with each other if they are head to tail. However, they will commute with each other if they meet tial to tail or head to head and we want them to anticommute so we add an ancilla qubit to produce this behaviour. For every pair of qubits that commute and we want to anticommute assign a common ancilla and have them act with X and Y respectively. In some instances we can use this qubits for multiple pairs of edges as on the square lattice without breaking the existing commutation and anticommutation rules. Got all this insight from the video. If every e_{ij} anticommutes with e_{rs} then it will anticommute with e_{sr} as $e_{rs} = e_{sr}$, so we only need to consider a directed graph.

8 Fermionic Enumeration

9 Relative performance

Talk about how the long strings of Z increase the depth as it makes it hard to decompose in parallel

10 Introduction

The journal of *Quantum Information and Computation*, for both on-line and in-print editions, will be produced by using the latex files of manuscripts provided by the authors. It is therefore essential that the manuscript be in its final form, and in the format designed for the journal because there will be no futher editing. The authors are strongly encouraged to use Rinton latex template to prepare their manuscript. Or, the authors should please follow the instructions given here if they prefer to use other software. In the latter case, the authors ought to provide a postscript file of their paper for publication.

11 Text

Contributions are to be in English. Authors are encouraged to have their contribution checked for grammar. Abbreviations are allowed but should be spelt out in full when first used.

The text is to be typeset in 10 pt Times Roman, single spaced with baselineskip of 13 pt. Text area (excluding running title) is 5.6 inches across and 8.0 inches deep. Final pagination and insertion of running titles will be done by the editorial. Number each page of the manuscript lightly at the bottom with a blue pencil. Reading copies of the paper can be numbered using any legible means (typewritten or handwritten).

12 Headings

Major headings should be typeset in boldface with the first letter of important words capitalized.

12.1 Sub-headings

Sub-headings should be typeset in boldface italic and capitalize the first letter of the first word only. Section number to be in boldface roman.

12.1.1 Sub-subheadings

Typeset sub-subheadings in medium face italic and capitalize the first letter of the first word only. Section number to be in roman.

12.2 Numbering and Spacing

Sections, sub-sections and sub-subsections are numbered in Arabic. Use double spacing before all section headings, and single spacing after section headings. Flush left all paragraphs that follow after section headings.

12.3 Lists of items

Lists may be laid out with each item marked by a dot:

- item one,
- item two.

Items may also be numbered in lowercase roman numerals:

(i) item one

(ii) item two

  (a) Lists within lists can be numbered with lowercase roman letters,

(b) second item.

13 Equations

Displayed equations should be numbered consecutively in each section, with the number set flush right and enclosed in parentheses.

$$\mu(n,t) = \frac{\sum_{i=1}^{\infty} 1(d_i < t, N(d_i) = n)}{\int_{\sigma=0}^{t} 1(N(\sigma) = n) d\sigma} \, . \tag{45}$$

Equations should be referred to in abbreviated form, e.g. "Eq. (**??**)" or "(2)". In multiple-line equations, the number should be given on the last line.

Displayed equations are to be centered on the page width. Standard English letters like x are to appear as x (italicized) in the text if they are used as mathematical symbols. Punctuation marks are used at the end of equations as if they appeared directly in the text.

Theorem 1: Theorems, lemmas, etc. are to be numbered consecutively in the paper. Use double spacing before and after theorems, lemmas, etc.

Proof: Proofs should end with $\square$.

14 Illustrations and Photographs

Figures are to be inserted in the text nearest their first reference. The postscript files of figures can be imported by using the commends used in the examples here.

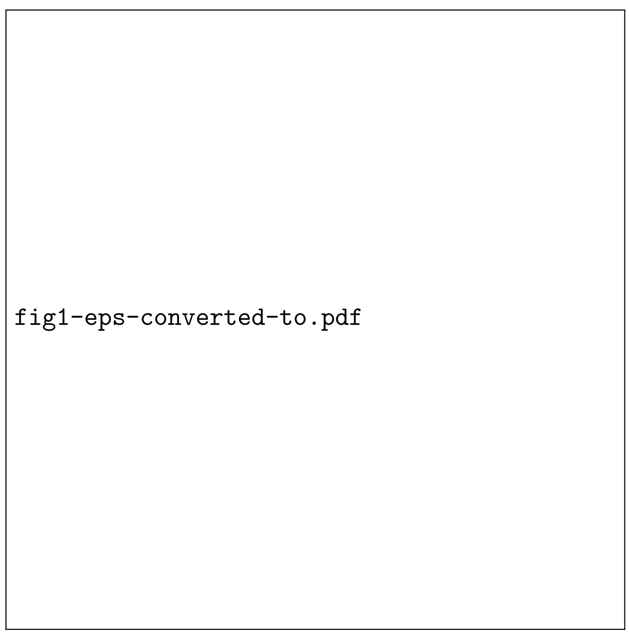

Fig. 6. figure caption goes here.

Figures are to be sequentially numbered in Arabic numerals. The caption must be placed below the figure. Typeset in 8 pt Times Roman with baselineskip of 10 pt. Use double spacing between a caption and the text that follows immediately.

Previously published material must be accompanied by written permission from the author and publisher.

15 Tables

Tables should be inserted in the text as close to the point of reference as possible. Some space should be left above and below the table.

Tables should be numbered sequentially in the text in Arabic numerals. Captions are to be centralized above the tables. Typeset tables and captions in 8 pt Times Roman with baselineskip of 10 pt.

Table 2. Number of tests for WFF triple NA = 5, or NA = 8.

		NP			
		3	4	8	10
NC	3	1200	2000	2500	3000
	5	2000	2200	2700	3400
	8	2500	2700	16000	22000
	10	3000	3400	22000	28000

If tables need to extend over to a second page, the continuation of the table should be preceded by a caption, e.g. "(*Table 2. Continued*)."

16 References Cross-citation

References cross-cited in the text are to be numbered consecutively in Arabic numerals, in the order of first appearance. They are to be typed in brackets such as [**?**] and [**?**, **?**, **?**].

17 Sections Cross-citation

Sections and subsctions can be cross-cited in the text by using the latex command shown here. In Section **??**, we discuss ....

18 Footnotes

Footnotes should be numbered sequentially in superscript lowercase Roman letters.[a]

Acknowledgements

We would thank ...

References

References are to be listed in the order cited in the text. For each cited work, include all the authors' names, year of the work, title, place where the work appears. Use the style shown in the following examples. For journal names, use the standard abbreviations. Typeset references in 9 pt Times Roman. pt!

[a]Footnotes should be typeset in 8 pt Times Roman at the bottom of the page.

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Appendix A

Appendices should be used only when absolutely necessary. They should come after the References. If there is more than one appendix, number them alphabetically. Number displayed equations occurring in the Appendix in this way, e.g. (**??**), (A.2), etc.

$$\langle \hat{O} \rangle = \int \psi^*(x) O(x) \psi(x) d^3x \ . \tag{A.1}$$