$So(4)$ 1th boundary map is transpose of 6
2th boundary map is transpose of 5
3th boundary map is not transpose of 4
4th boundary map is not transpose of 3
5th boundary map is transpose of 2
6th boundary map is transpose of 1
1 th homology generators are

$$(1, 1, 1, 1, 1, 1, 1, 1)$$

2 th homology generators are

$$(0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0)$$

3 th homology generators are

$$(0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0)$$

4 th homology generators are

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0)$$

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0)$$

5 th homology generators are

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0)$$

6 th homology generators are

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1)$$

7 th homology generators are

$$(0, 0, 0, 0, 0, 0, 0, 1)$$

$$H_1 = \mathbb{Z}_2^1$$
$$H_2 = \mathbb{Z}_2^1$$
$$H_3 = \mathbb{Z}_2^1$$
$$H_4 = \mathbb{Z}_2^2$$
$$H_5 = \mathbb{Z}_2^1$$
$$H_6 = \mathbb{Z}_2^1$$
$$H_7 = \mathbb{Z}_2^1$$