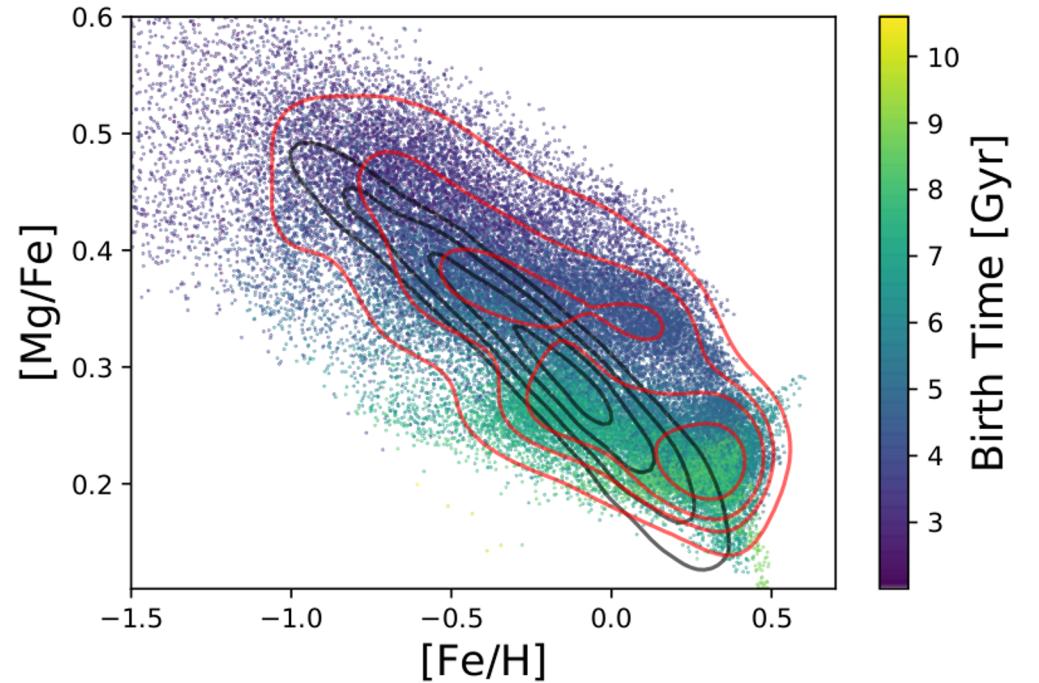
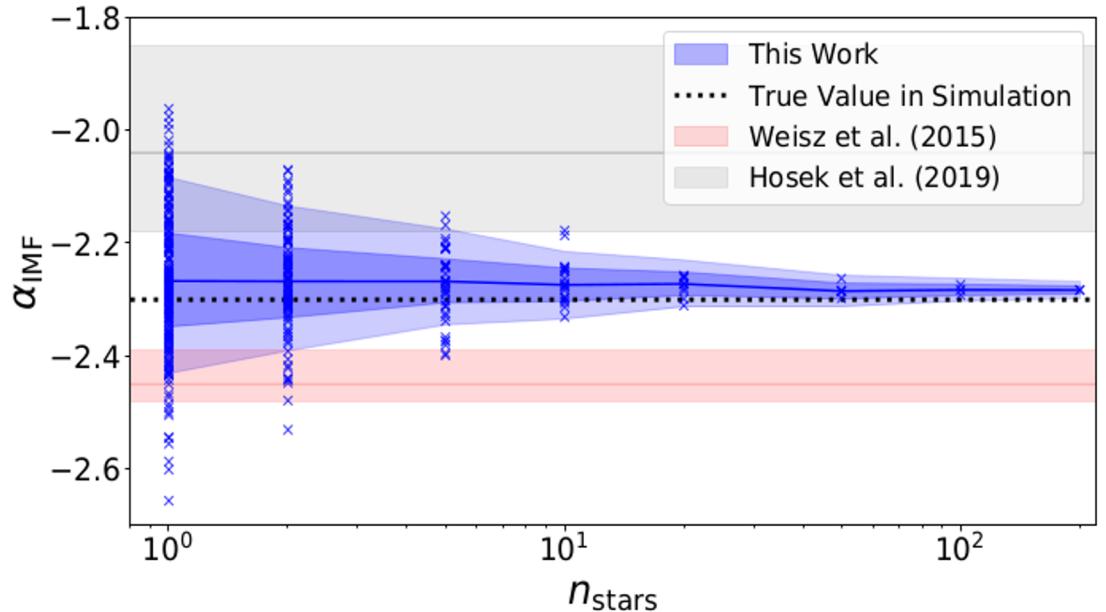


CHEMISTRY



PHYSICS



Inferring Galactic Parameters from Chemical Abundances

OLIVER PHILCOX (PRINCETON)

with:

Jan Rybizki (MPIA, Heidelberg)

JOINT STATISTICAL MEETING

Aug 5th 2020

arXiv: 1909.00812



Mock Galaxy
(Auriga Simulations)

*Are these
consistent?*

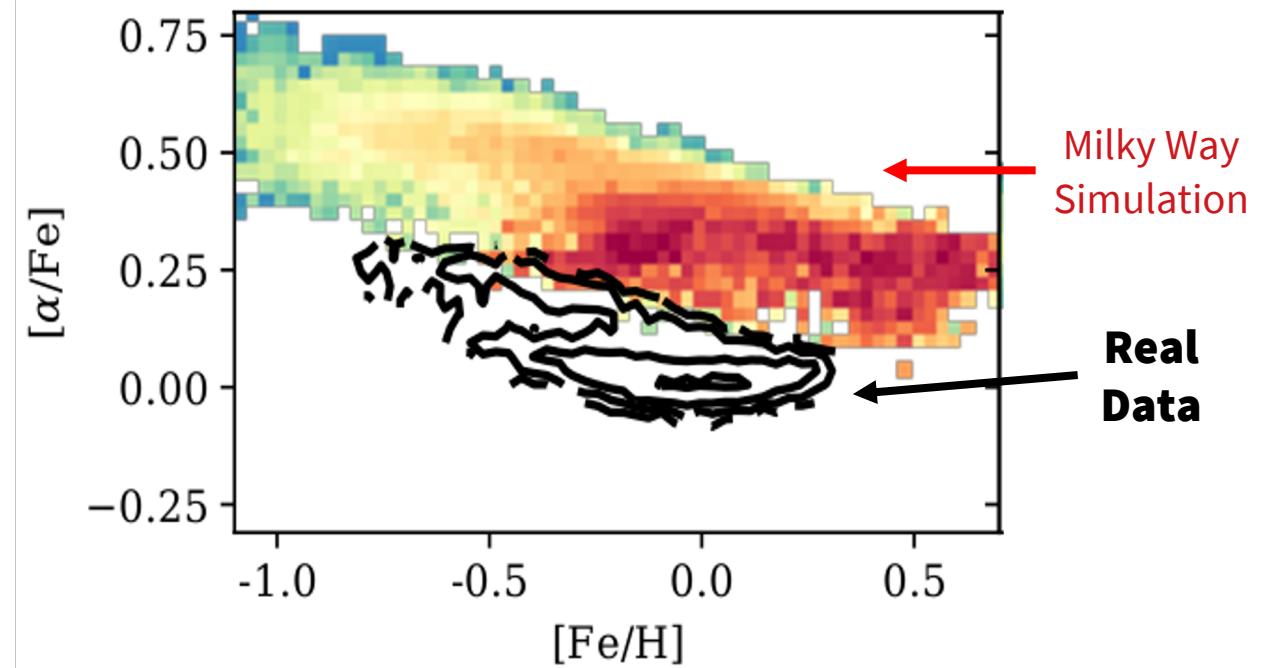
Simulating Galaxies

Real Galaxy
(M101)



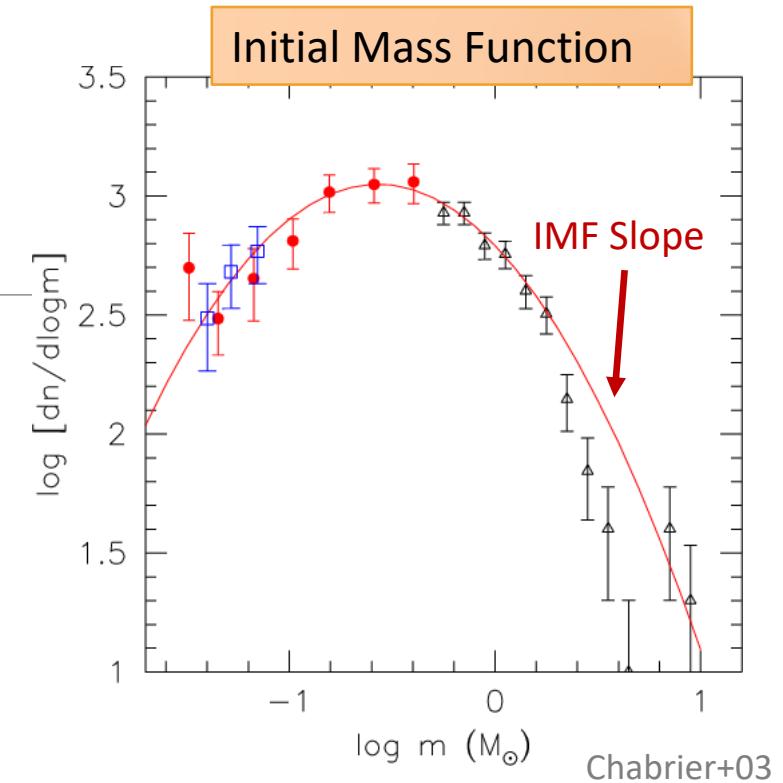
Are our Simulations Accurate?

- Simulations of galaxies do **not** match the Milky Way
- **Chemical evolution** is wrong
- They depend on **poorly constrained** parameters

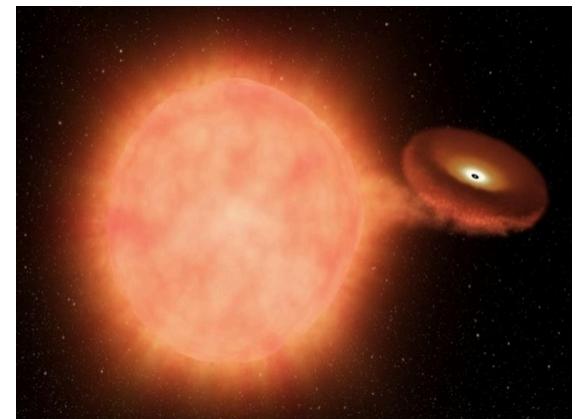


Parametrizing the Galaxy

- What are these parameters?
- The slope of the **Initial Mass Function (IMF)**
 - Controls stellar mass distribution
- The number of **Type Ia Supernovae**
 - Removes gas from galaxies and spreads metals
- And many more...

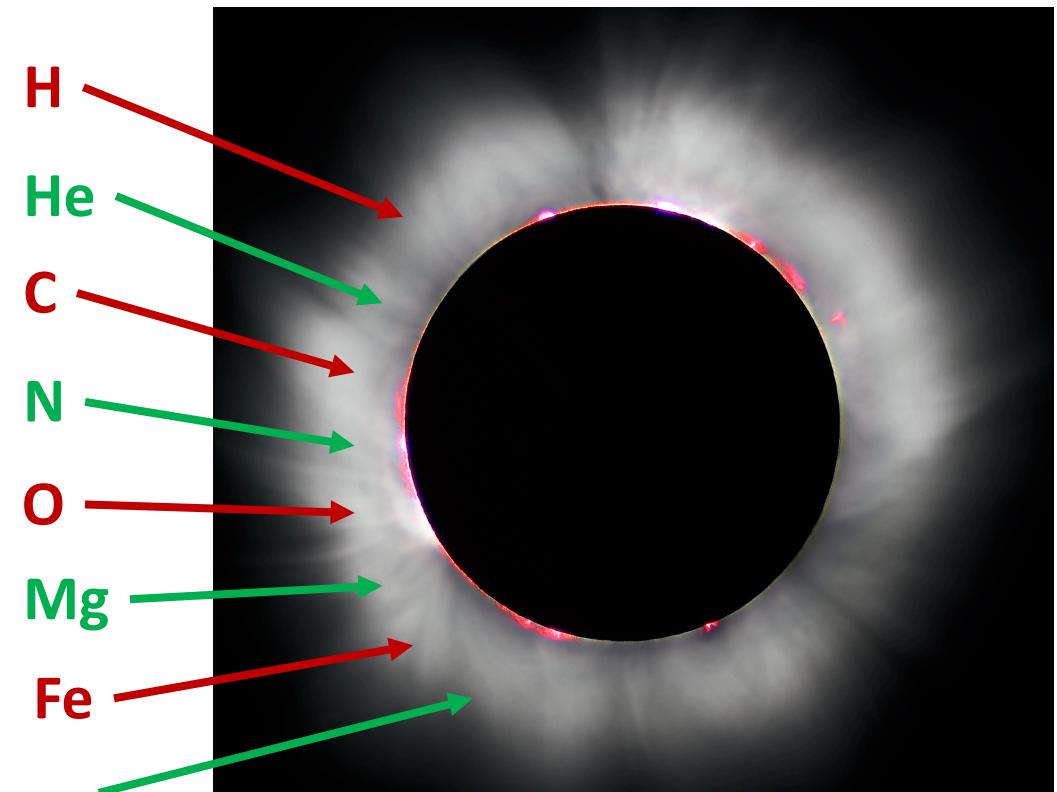
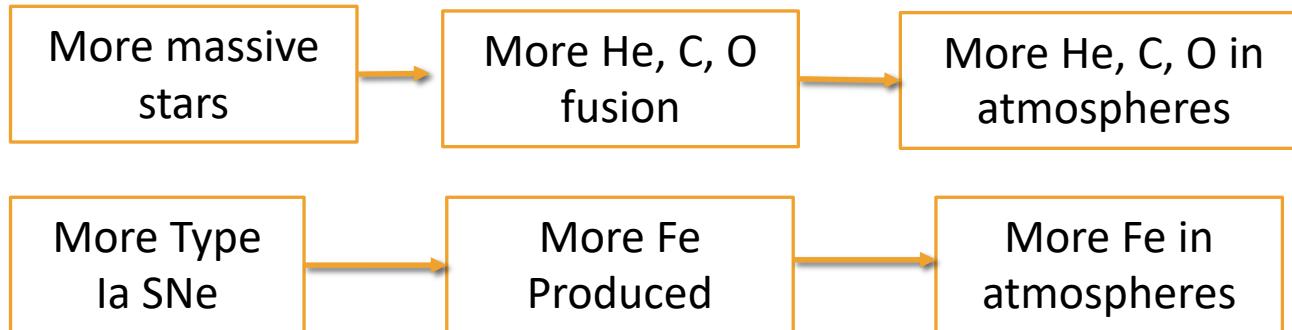


Type Ia Supernovae



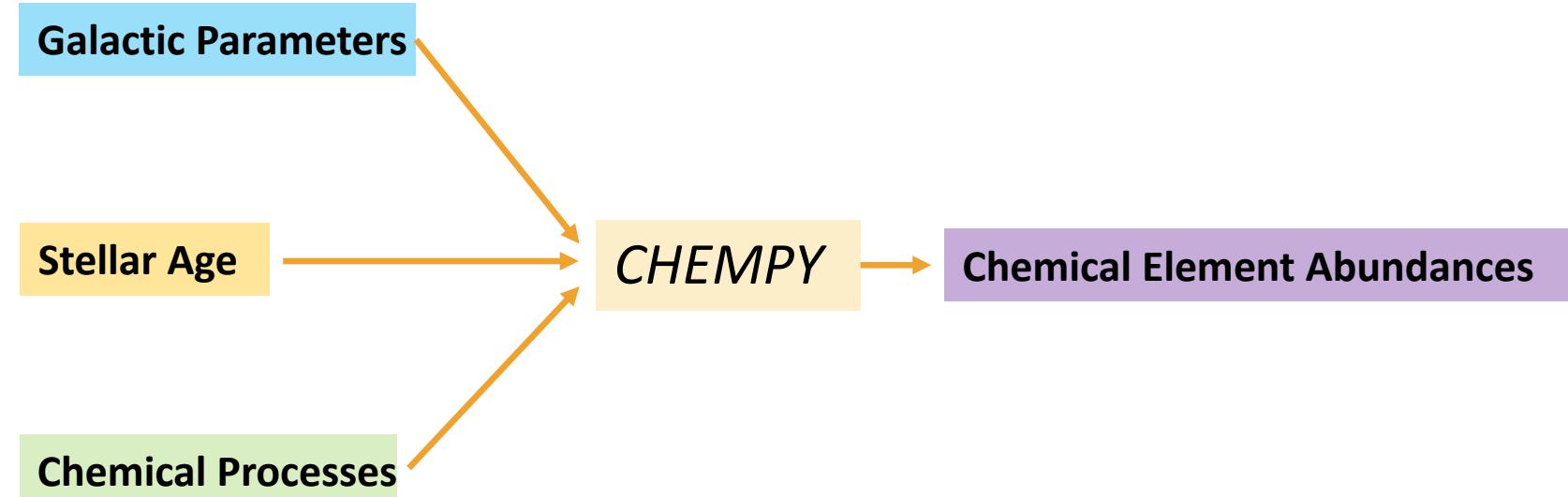
Chemistry from Stellar Atmospheres

- Stars **explode** and enrich the **interstellar medium**
- Stellar atmospheres encode chemical **abundances** of the interstellar medium
- These depend on **galactic parameters**, e.g.,



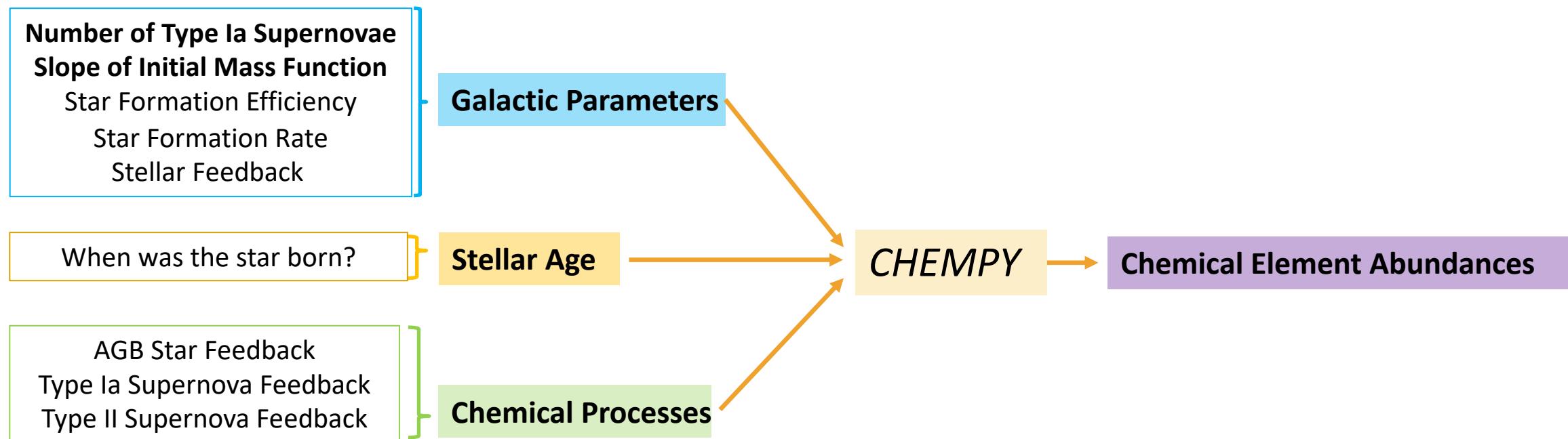
Modeling Chemical Evolution

- Given **galactic parameters** we can model the **element abundances** in stellar atmospheres
- We need a **fast** and **flexible** model: *Chempy*



Modeling Chemical Evolution

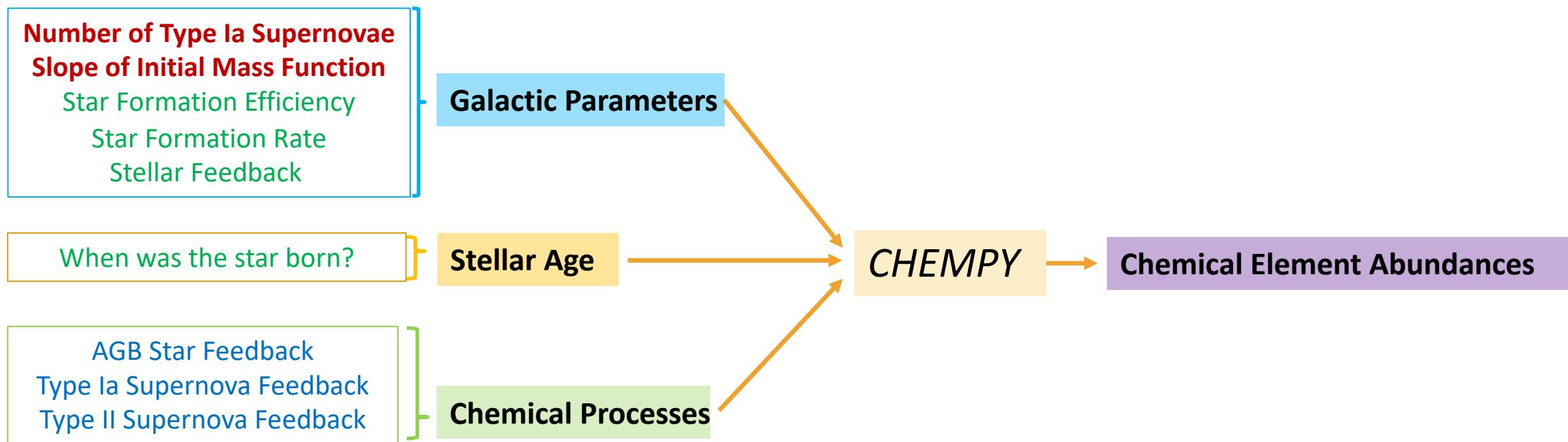
- Given **galactic parameters** we can model the **element abundances** in stellar atmospheres
- We need a **fast** and **flexible** model: *Chempy*



Modeling Chemical Evolution

- Given **galactic parameters** we can model the **element abundances** in stellar atmospheres
- We need a **fast** and **flexible** model: *Chempy*

EXTERNAL LOCAL GLOBAL



A Bayesian Framework

- From a set of **stellar chemical abundances** we can infer **galactic parameters**, Λ
- Via Bayes' theorem:

$$P(\Lambda | \text{Data}) \sim \int d\Theta dT \ P(\text{Data} | \Lambda, \Theta, T) p(\Lambda) p(\Theta) p(T)$$

Likelihood *Priors*

integrating over local parameters Θ and the age of the star, T with priors p .

- We can extend this to **multiple stars**

$$P(\Lambda | \text{Data}) \sim p(\Lambda) \prod_{i=1}^{n_{\text{stars}}} \int d\Theta_i dT_i \ P(\text{Data}_i | \Lambda, \Theta_i, T_i) p(\Theta_i) p_i(T_i)$$

ith star likelihood *Priors*

A Bayesian Framework

- We can extend this to **multiple stars**

$$P(\boldsymbol{\Lambda}|\text{Data}) \sim p(\boldsymbol{\Lambda}) \prod_{i=1}^{n_{\text{stars}}} \int d\boldsymbol{\Theta}_i dT_i \ P(\text{Data}_i|\boldsymbol{\Lambda}, \boldsymbol{\Theta}_i, T_i) p(\boldsymbol{\Theta}_i) p_i(T_i)$$

ith star likelihood *Priors*

- What about inadequacies in our model?

- Add free **model error** parameters, σ_{model} (one per chemical element)

- These will **downweight** constraints from poorly modeled elements

$$P(\boldsymbol{\Lambda}|\text{Data}) \sim p(\boldsymbol{\Lambda}) \int d\sigma_{\text{model}} p(\sigma_{\text{model}}) \prod_{i=1}^{n_{\text{stars}}} \int d\boldsymbol{\Theta}_i dT_i P(\text{Data}_i|\boldsymbol{\Lambda}, \boldsymbol{\Theta}_i, T_i, \sigma_{\text{model}}) p(\boldsymbol{\Theta}_i) p_i(T_i)$$

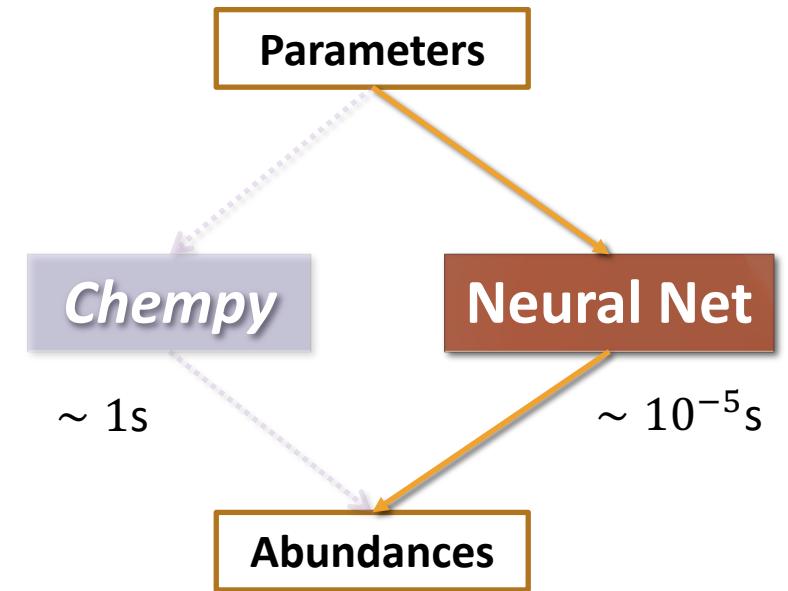
21st Century Statistics

- Our model for the two **global** galactic parameters depends on:
 - 3 local **star formation** parameters per star
 - 1 **age** parameter per star
 - 1 **error** parameter per element.

For 100 stars with 8 elements, this gives **408** free parameters!

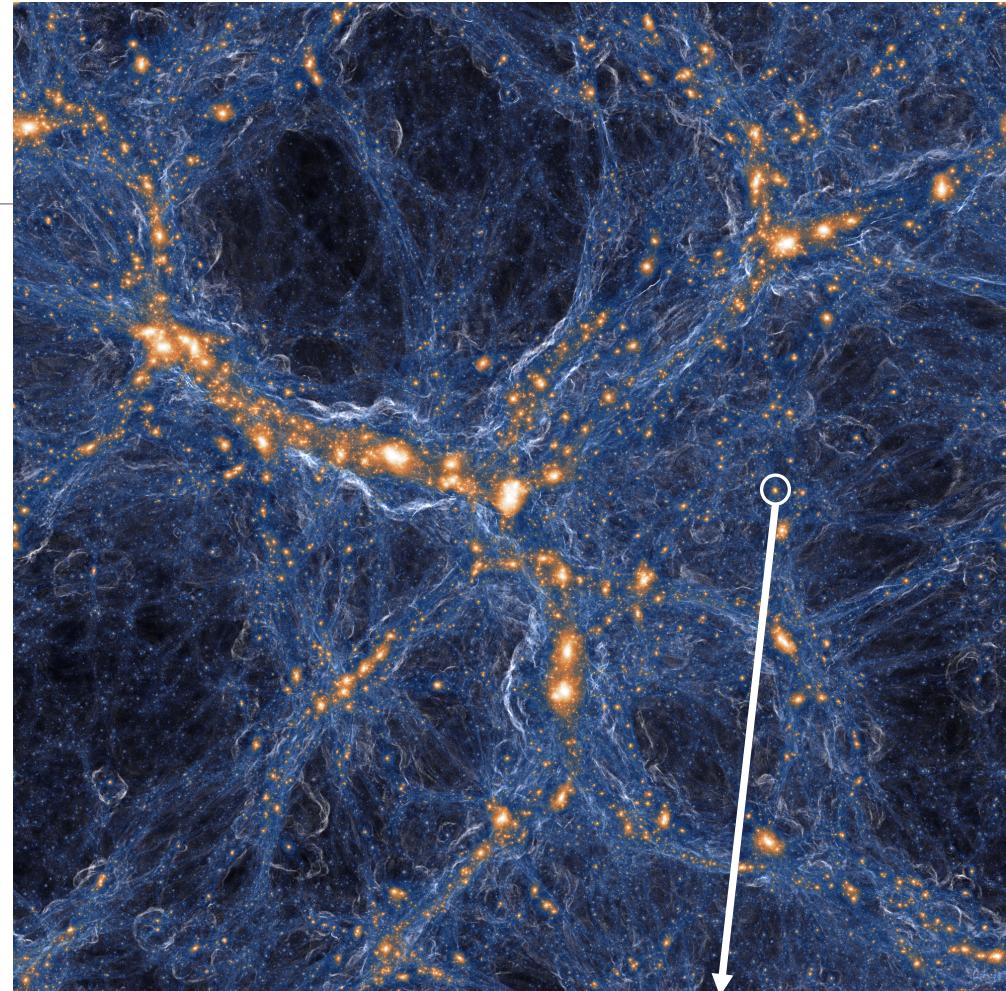
- We need **modern methods** for efficient sampling:

- 1. Replace the **slow** *Chempy* model with a (differentiable) trained **neural network**
- 2. Use **Hamiltonian Monte Carlo** with **No U-Turn Sampling** (NUTS)



Mock Data

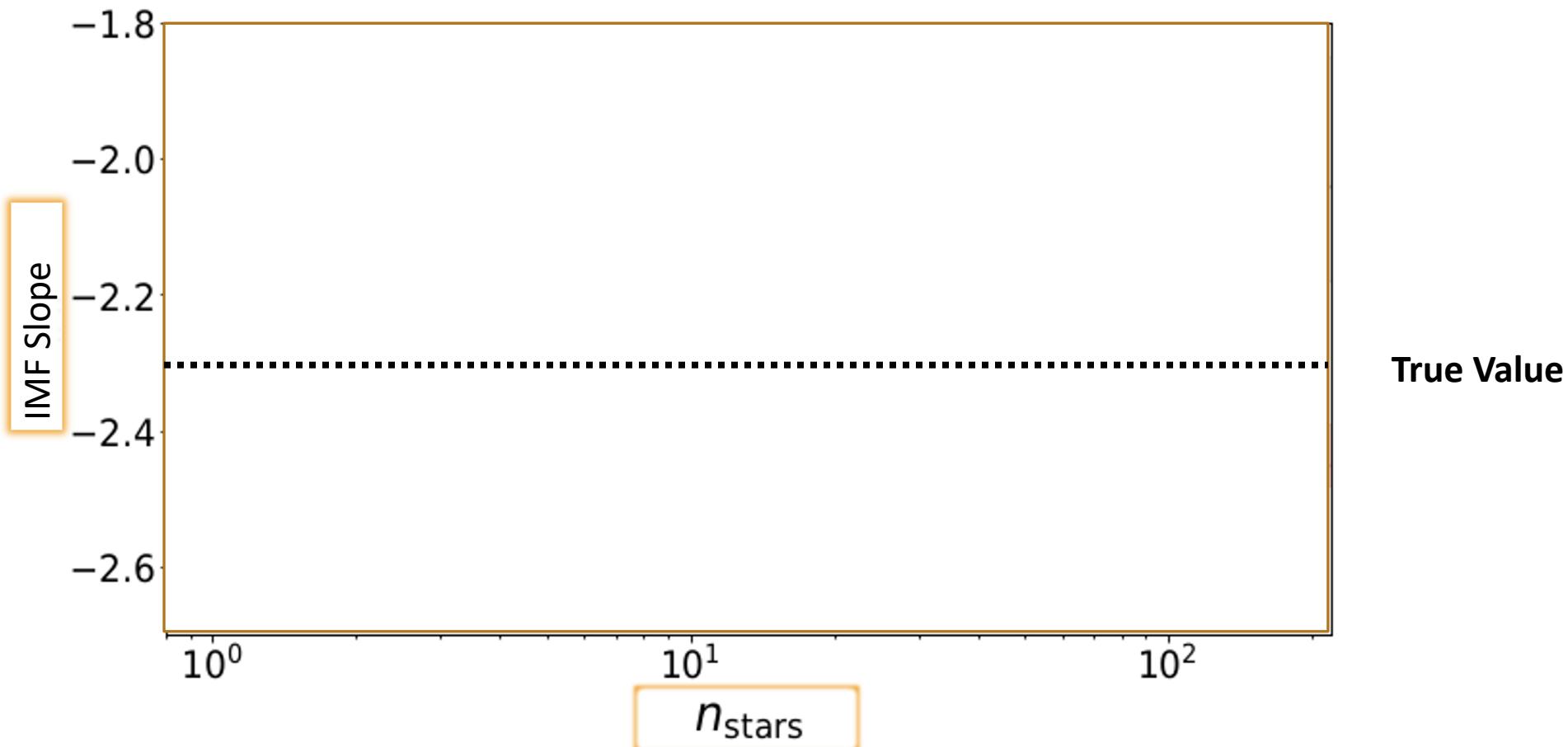
- Use the **IllustrisTNG-100** simulation
 - Has **much** more complex physics than *Chempy!*
- Extract ~200 ‘**stellar particles**’ from a **Milky Way-like** galaxy each with:
 - **Stellar ages**
 - **Abundances** of H, He, C, N, O, Mg, Si, Ne, Fe
- Supplement these with realistic observational **errors**



Milky Way v2.0

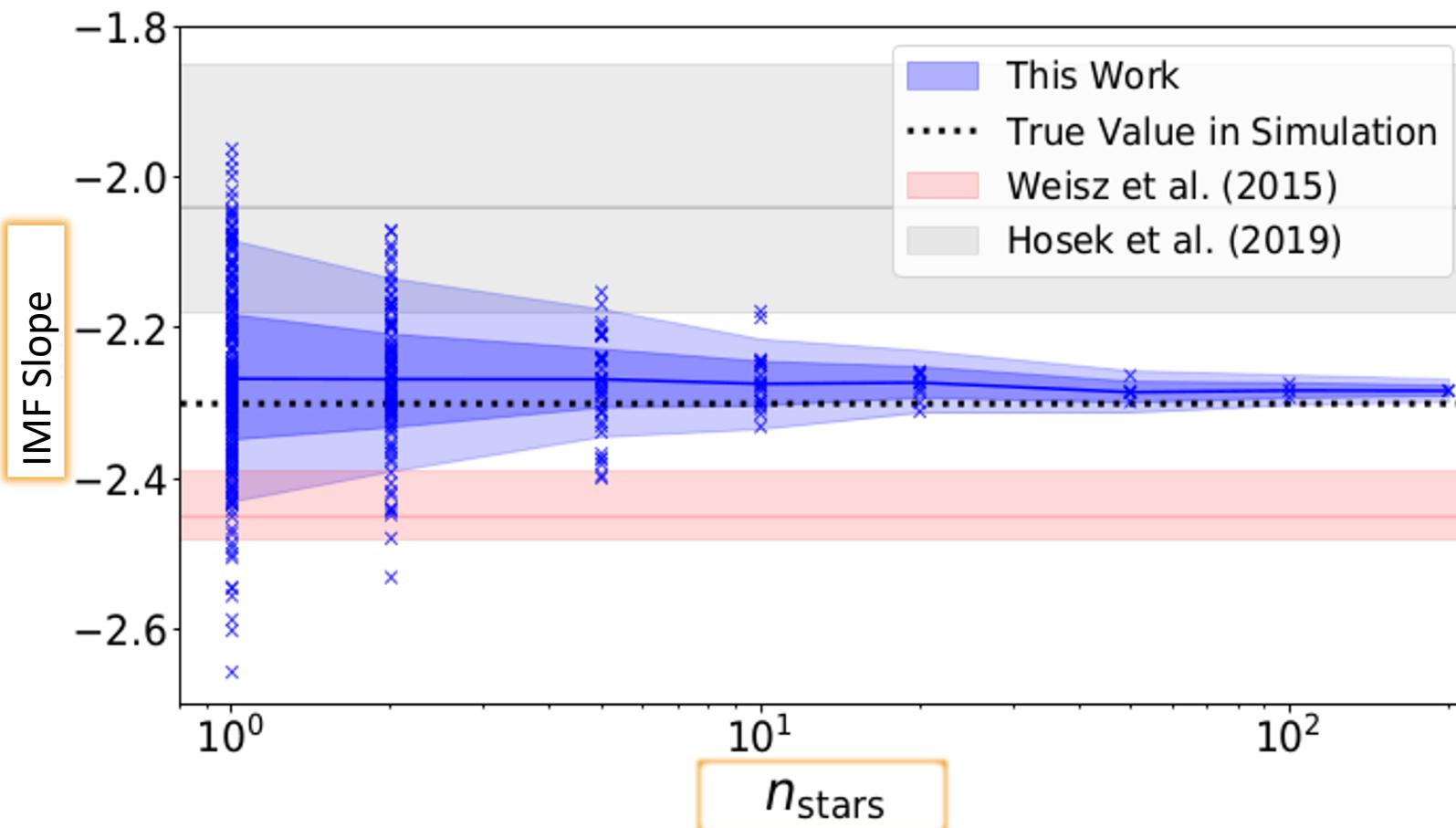
How well do we do?

- ☐ Compare our parameter inference with the true values in the simulation



How well do we do?

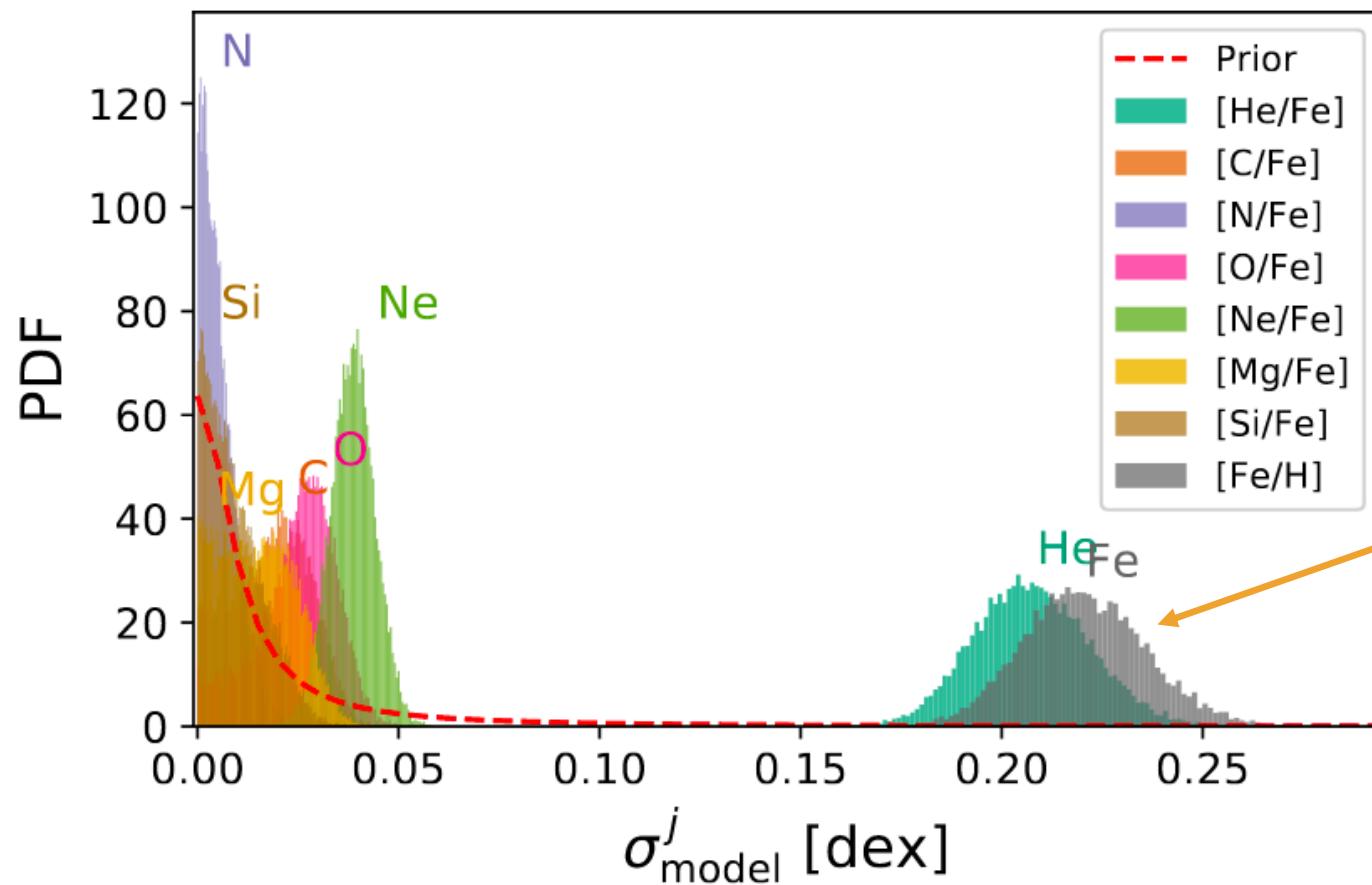
- Compare our parameter inference with the true values in the simulation



- A strong measurement of galactic parameters
- Tightens with more stars

How well do we do?

Model error
posteriors



Absolute metallicity
is **poorly predicted**
but metal line
ratios are useful

Summary

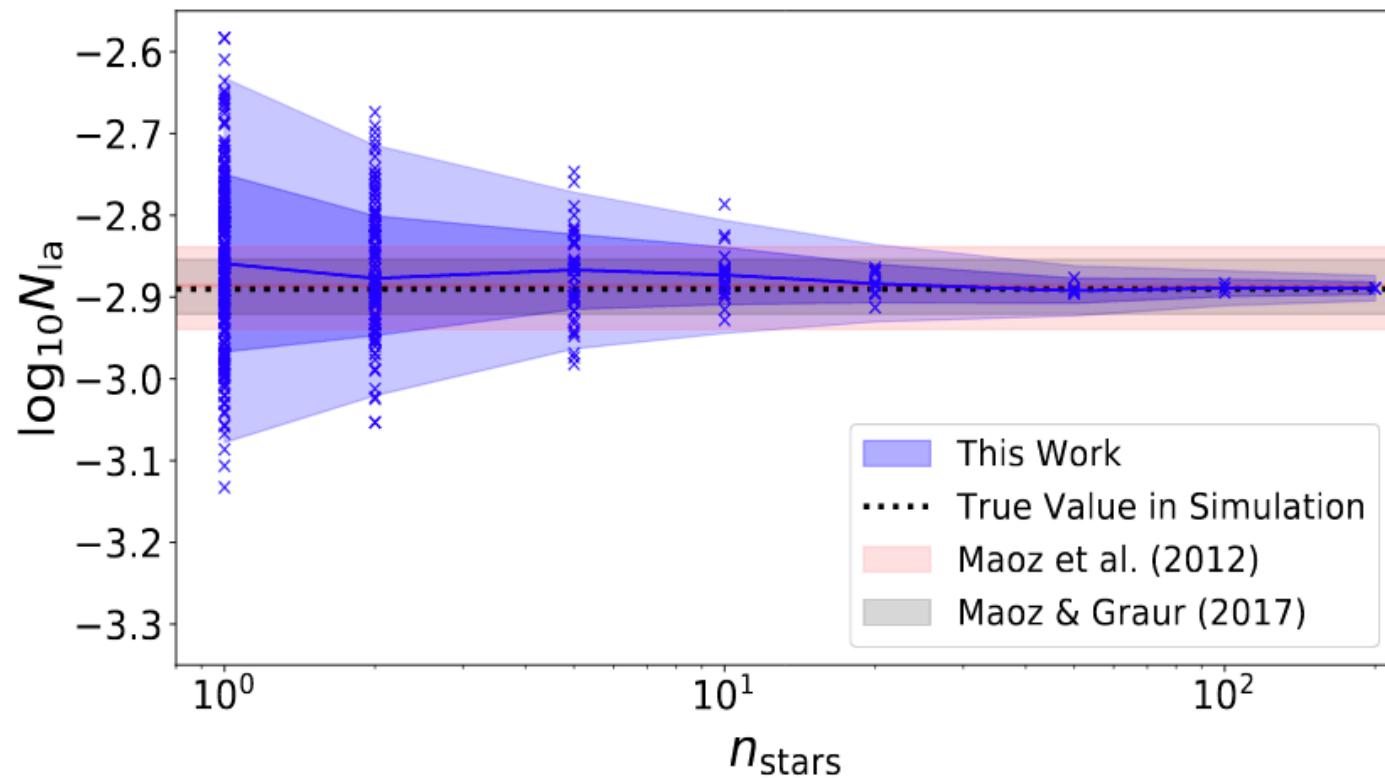
- We can use measure **galactic evolution** parameters from **stellar atmospheres** using:
 - A **simple** chemical evolution model (*Chempy*)
 - **Modern** statistical methods
- This has other applications e.g.:
 - **Application** to real data
 - **Optimizing** hydrodynamical simulation parameters
 - **Constraining** stellar nucleosynthesis

Further Questions?

ophilcox@princeton.edu

Type Ia Supernova Constraints

- Compare our parameter inference with the true values in the simulation



Neural Networks

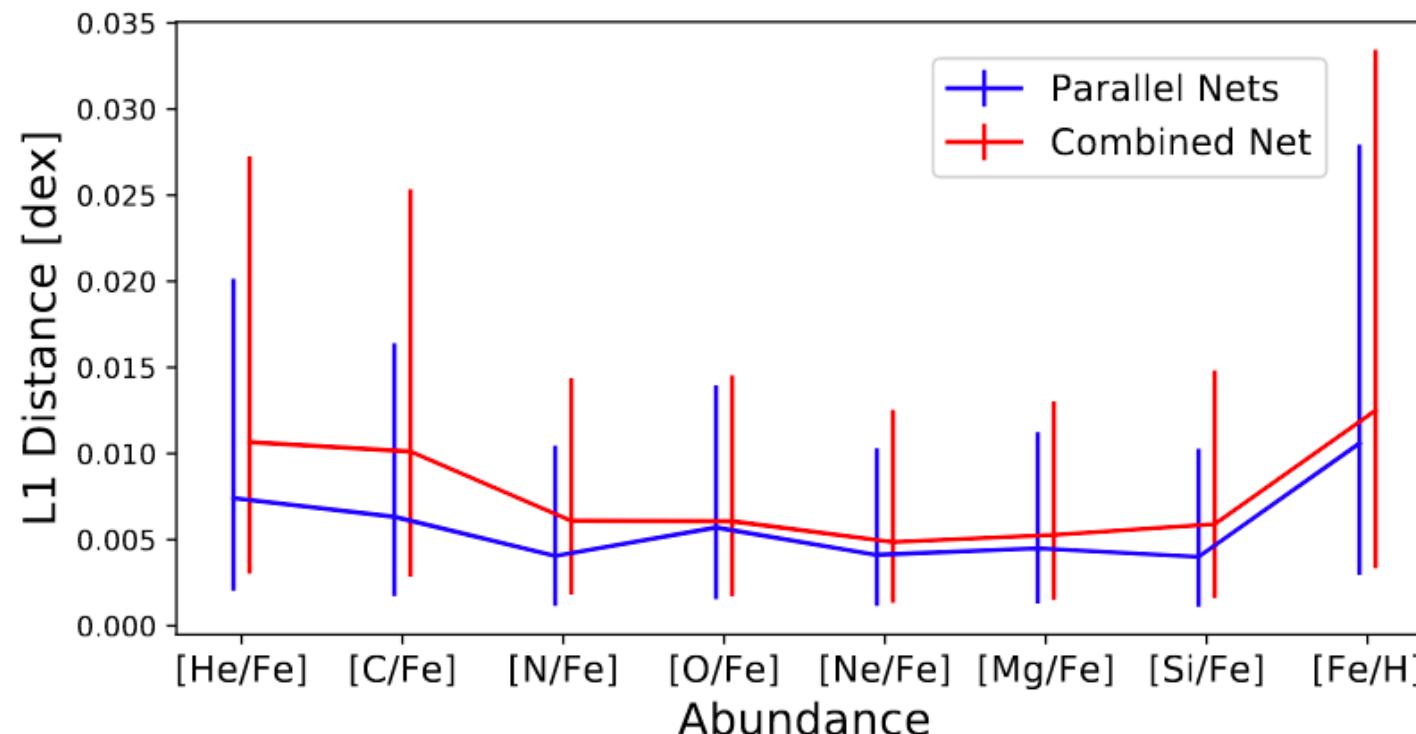
- Evaluation of the *Chempy* function is slow ($\sim 1\text{s}$)
- Use neural networks to **predict** output chemical elements from input parameters.
- This acts as a **fast, non-linear** interpolator, which is **differentiable**.

The diagram illustrates a neural network architecture. It starts with an **Input** arrow pointing to the equation for the **Hidden Layer**: $\mathbf{h} = \mathbf{W}_0 \cdot \mathbf{x} + \mathbf{b}_0$. Below this, another arrow points to the **Output** equation: $\mathbf{y} = \mathbf{W}_1 \cdot f(\mathbf{h}) + \mathbf{b}_1$. A vertical orange arrow points from the **Hidden Layer** equation up to the **Output** equation, labeled **Non-Linearity**, indicating that the hidden layer's output is passed through a non-linear activation function before becoming the final output.

$$\text{Hidden Layer} \longrightarrow \mathbf{h} = \mathbf{W}_0 \cdot \mathbf{x} + \mathbf{b}_0$$
$$\text{Output} \longrightarrow \mathbf{y} = \mathbf{W}_1 \cdot f(\mathbf{h}) + \mathbf{b}_1$$

Neural Networks

- This is trained by running *Chempy* on $\sim 10^4$ points in parameter space.



Errors are much
below observational
error ~ 0.05 dex

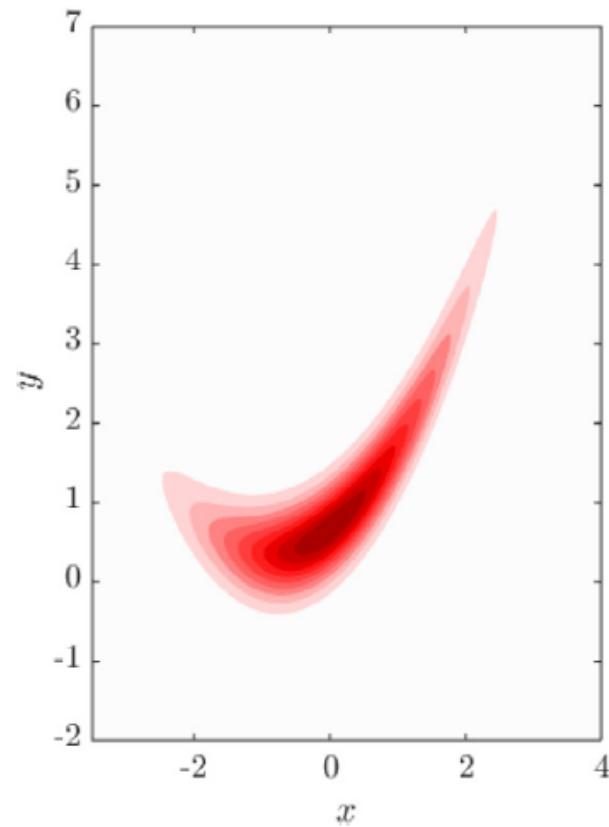
Hamiltonian Monte Carlo (HMC)

- Markov Chain Monte Carlo (MCMC) is **slow** and **unsuitable** for high-dimensional problems.
- MCMC works by jumping between points in parameter space at random.
- HMC preferentially samples where the posterior is **large**.
- It's **much more efficient** but requires a **differentiable** model.

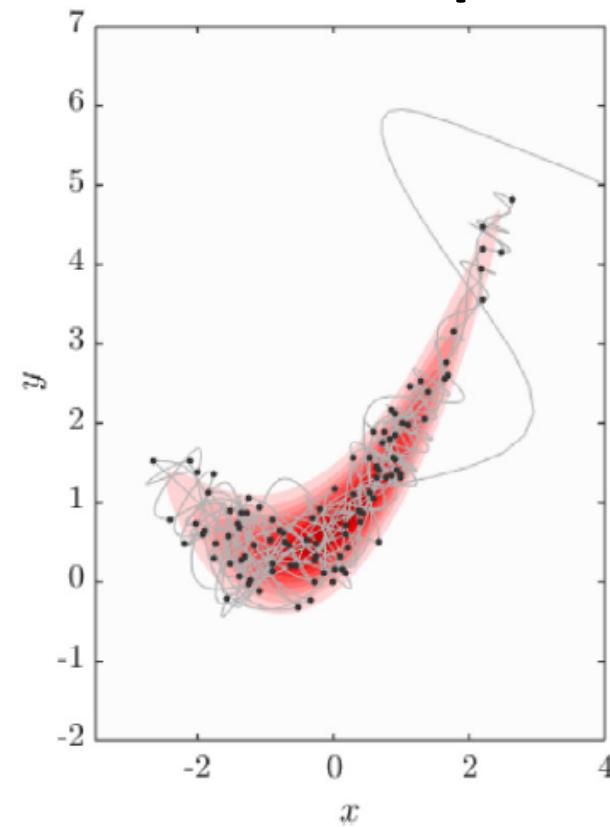
Hamiltonian Monte Carlo (HMC)

- HMC preferentially samples where the posterior is **large**.

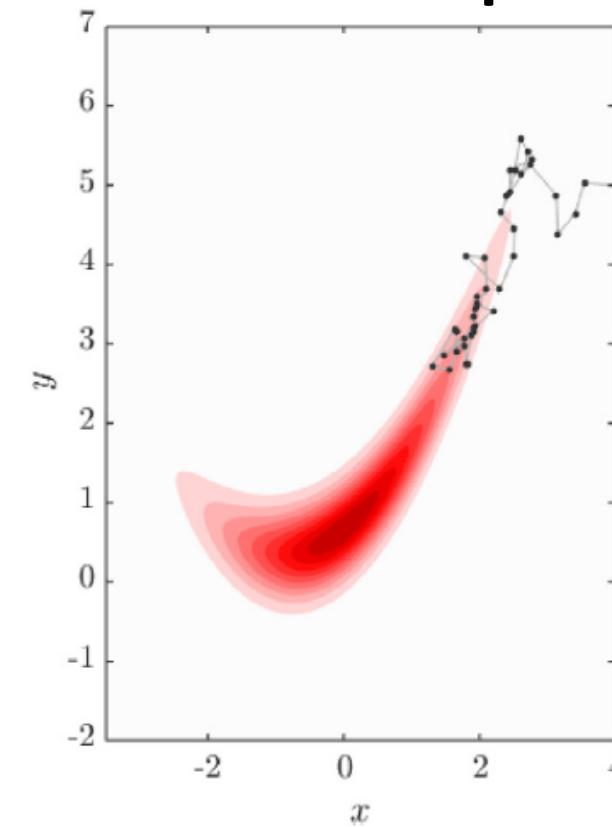
Posterior



HMC Samples

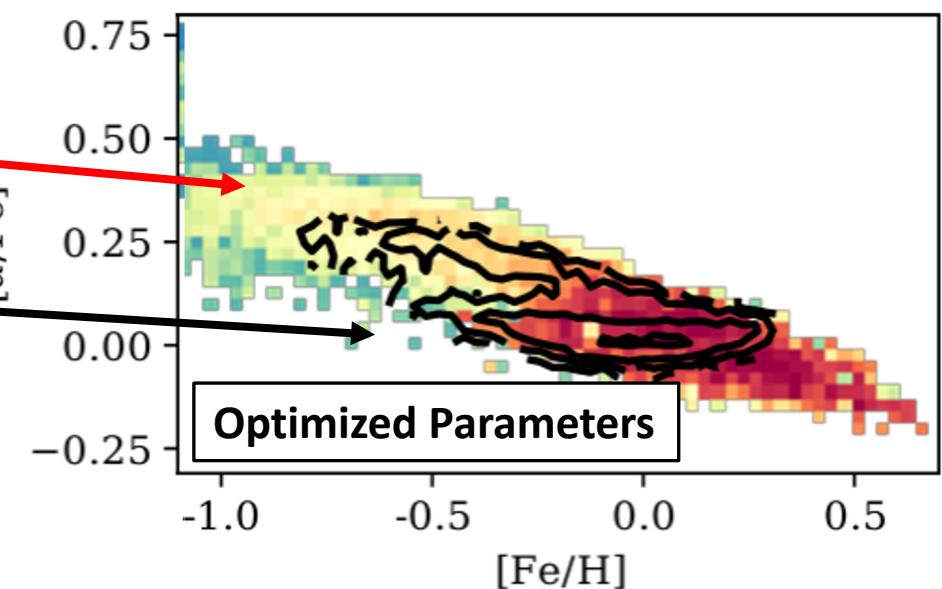
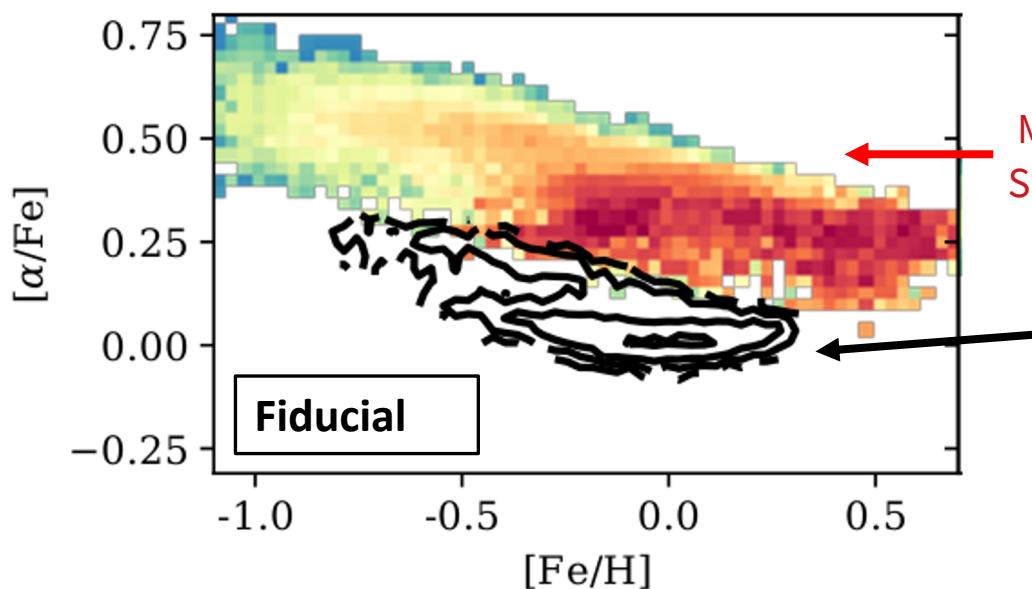


MCMC Samples



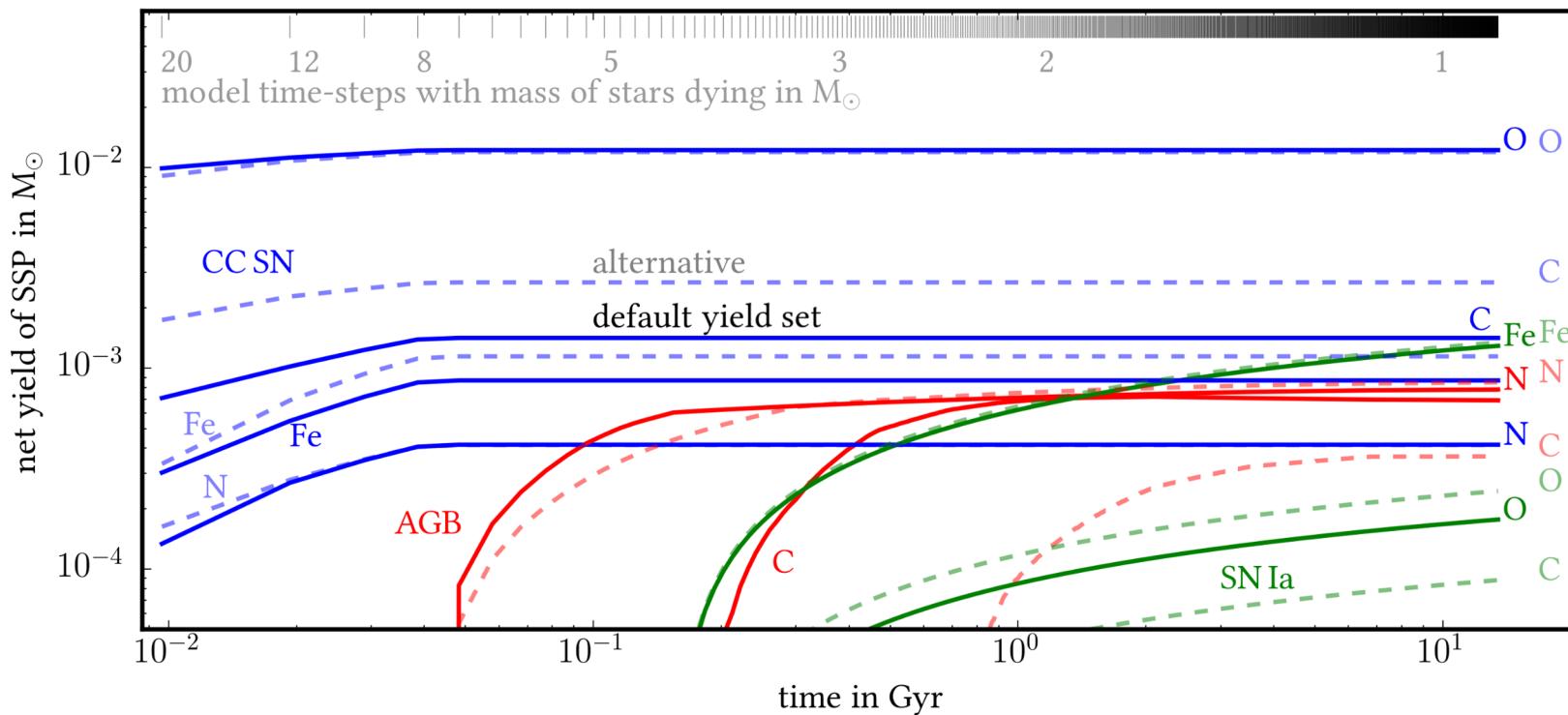
Optimizing Simulations

- ❑ *Chempy* can be combined with **solar abundances** to measure parameters.
- ❑ This is done via **MCMC** or more advanced methods
- ❑ To test, we can put our **best-fit parameters** into a cosmological simulation
 - ❑ Do we get a more realistic galaxy simulation?



Building a Fast and Flexible GCE: *Chempy*

- ❑ *Chempy* (including **chemical yields**, and **SSP** parameters):
 - ❑ **IMF** integrated, metallicity-dependent yield over time



SSP = Simple Stellar Population
(A group of stars born at the same time in the same environment)

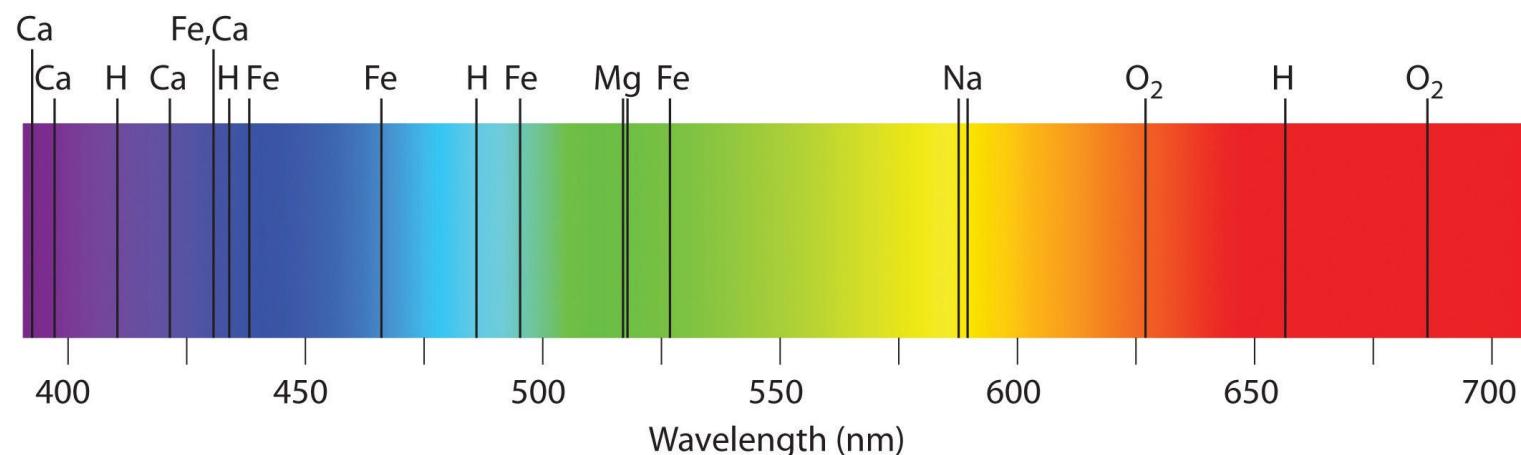
IMF = Initial Mass Function
(Contribution of stars born in a certain mass range to total mass)

How do we measure ISM Abundances?

- Option 1:** Spectroscopy of the ISM
- Difficult
- Depends on the ISM temperature and density

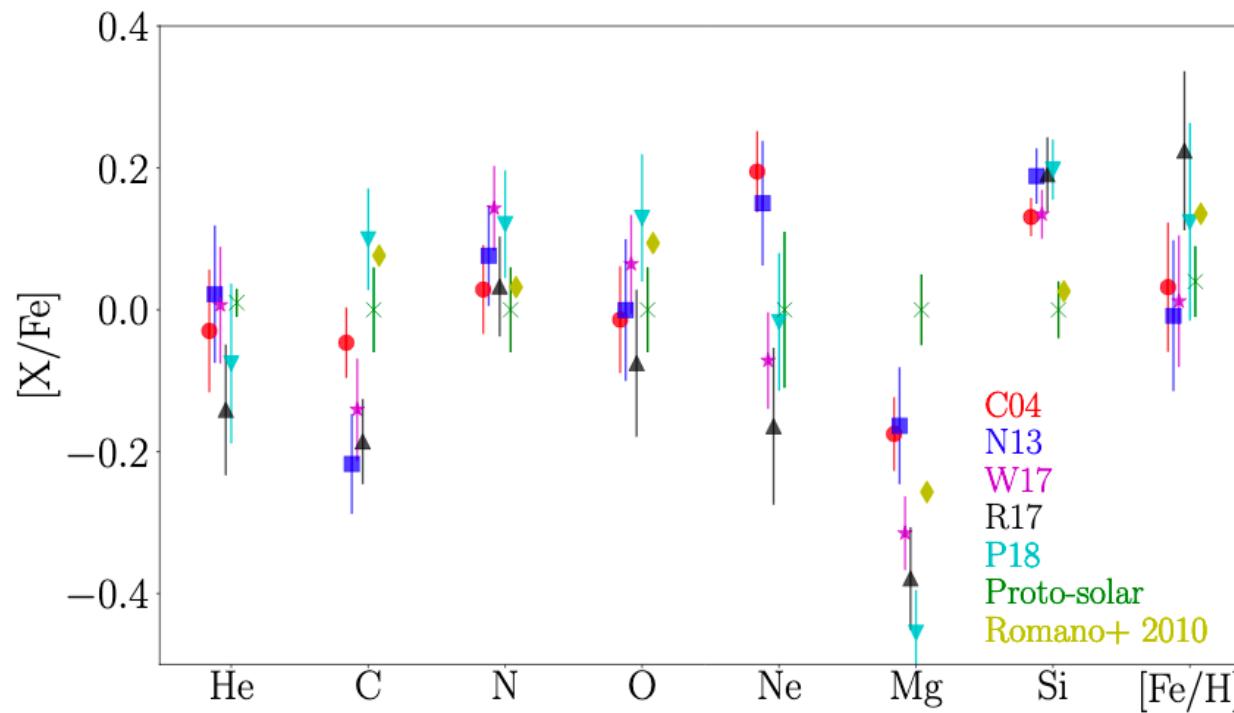
- Option 2:** Stellar Atmospheres
- Can observe stellar chemistry in **absorption lines**
- Many data catalogs exist, e.g. APOGEE
- Metal lines have little contamination

To Begin: Use a *single* star – the **Sun** with 28 elements



Yield Table Scoring

- Different Yield Tables predict **very different** proto-Solar abundances



Yield Table Scoring

❑ Method:

- ❑ Compare *Chempy* observations to **proto-Solar** abundances
- ❑ **Marginalize** over SSP and ISM parameters
- ❑ Include **model error** to account for modeling inaccuracies

❑ Model Comparison Statistics:

- ❑ Bayes Factor
 - ❑ $B \sim \int d\Theta \text{Posterior}(\Theta | Data)$

❑ Cross-Validation

- ❑ Use $n - 1$ elements to predict n -th element

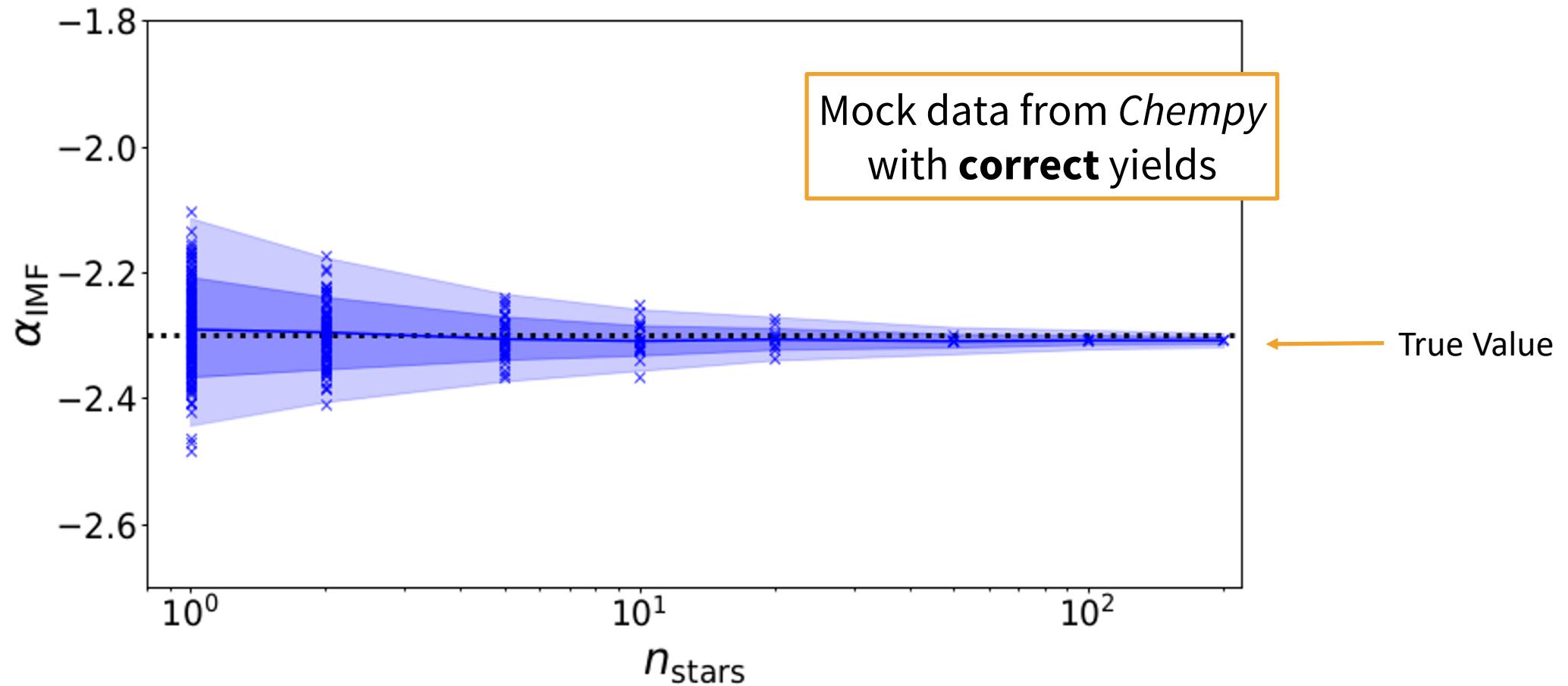
Yield Table Scoring

Comparing Core-Collapse Supernovae Yield Tables using 28 Elements

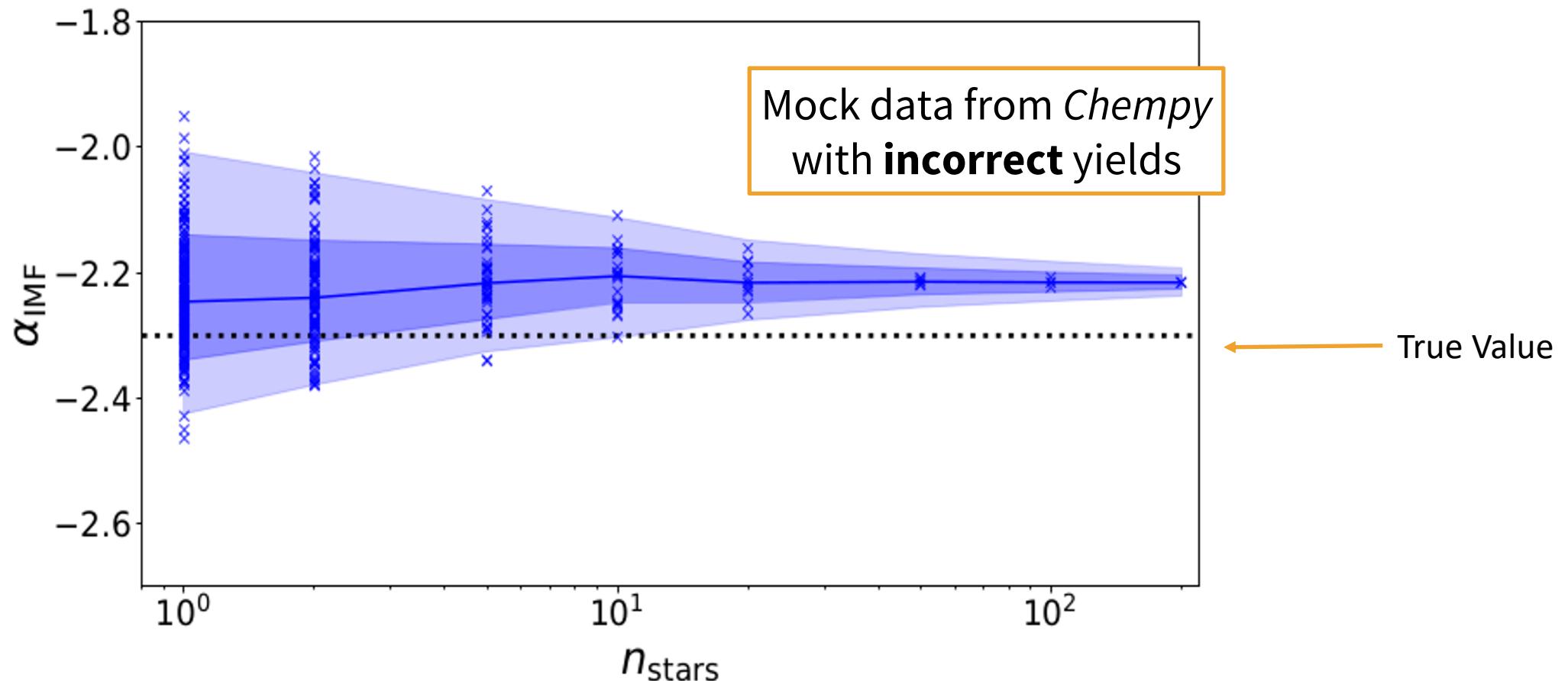
Yield Set	Bayes Score
C04	-1.21
N13	-5.69
W17	-0.78
R17	-6.11
P18	0.86

Nikos Prantzos'
yields perform
best here!

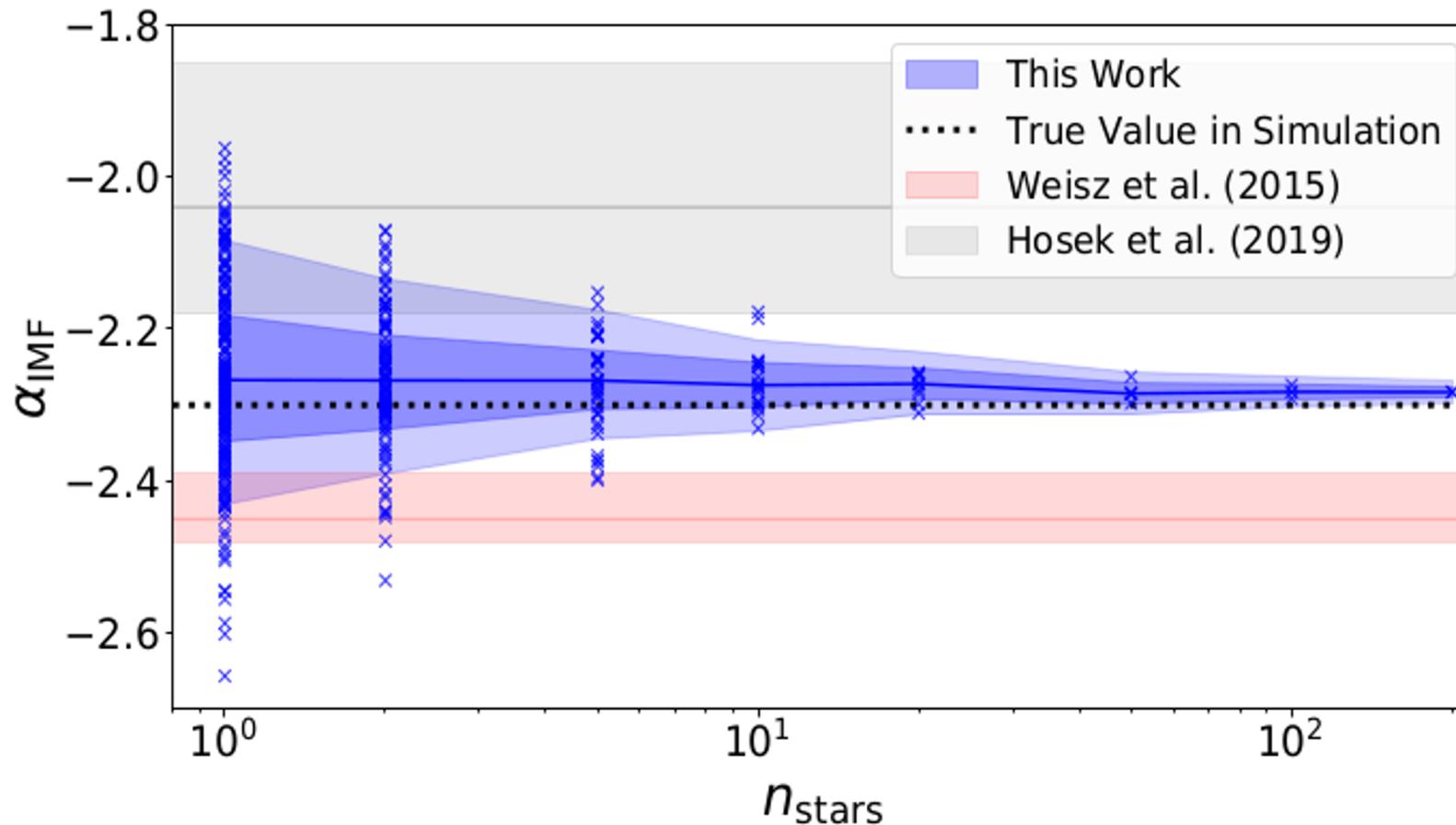
Multi-Star Inference with *Chempy*



Multi-Star Inference with *Chempy*



Multi-Star Inference with Chempy

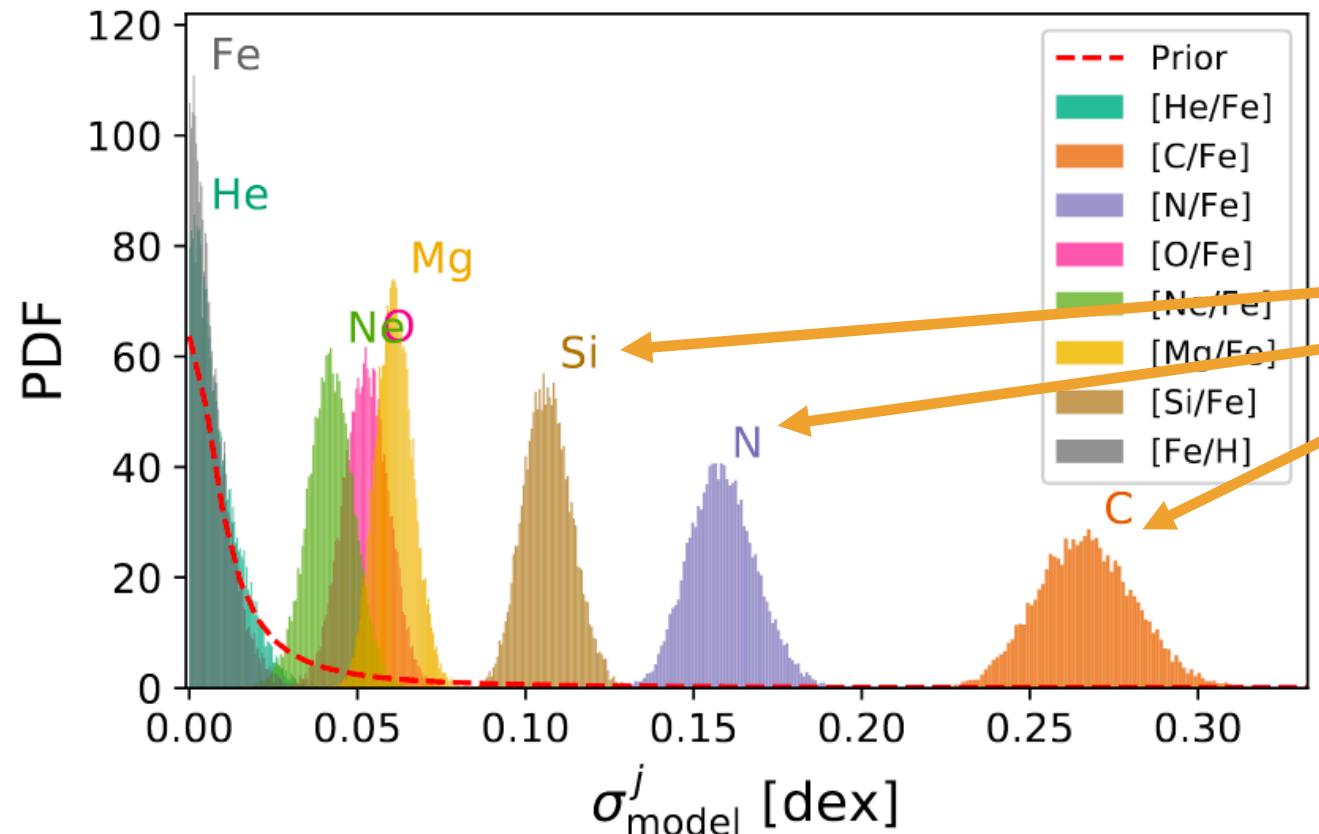


Mock data from
IllustrisTNG with
correct yields

Multi-Star Inference with Chempy

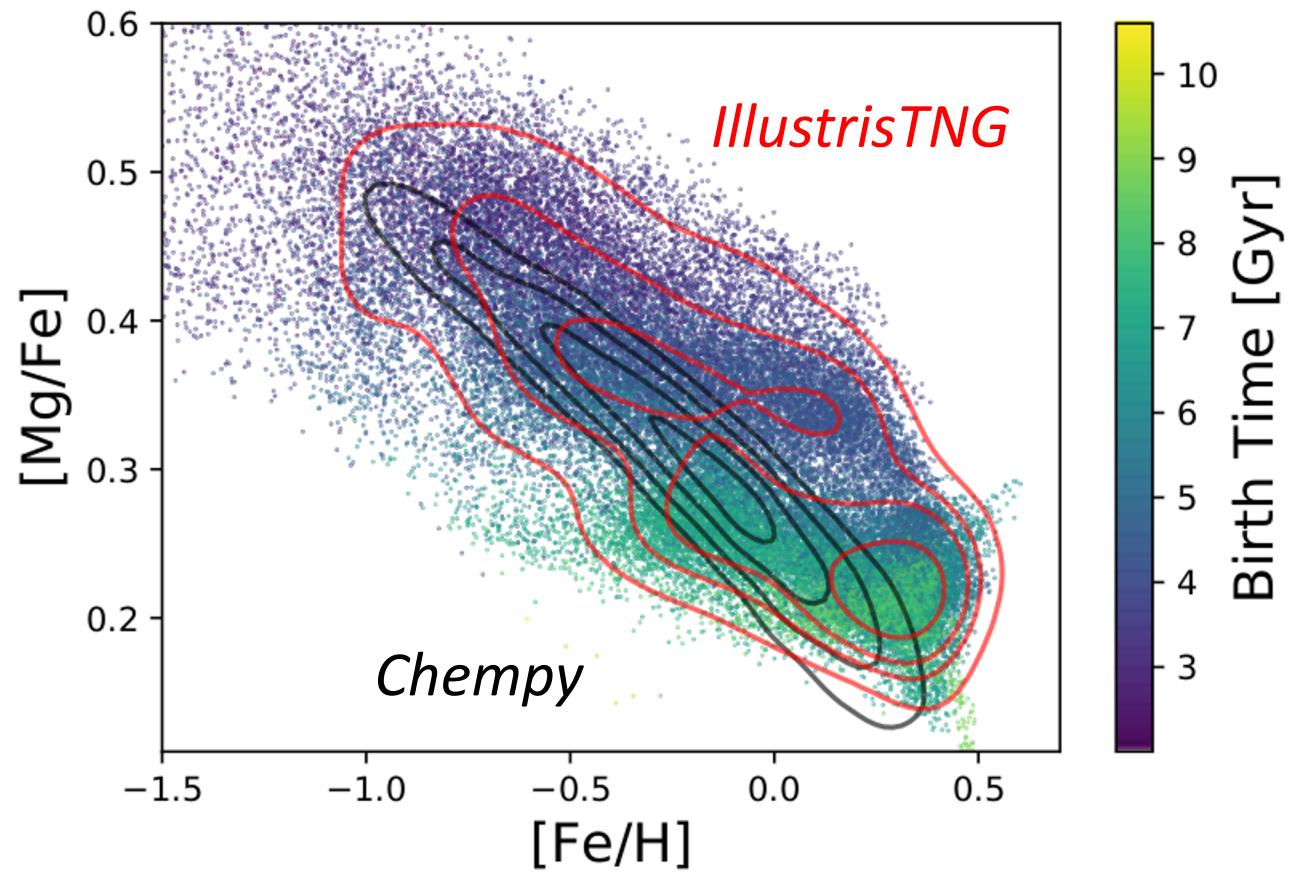
- **Model errors** indicate errors in our yield tables.

Model Error
Distributions
for analysis
with **incorrect**
yield set

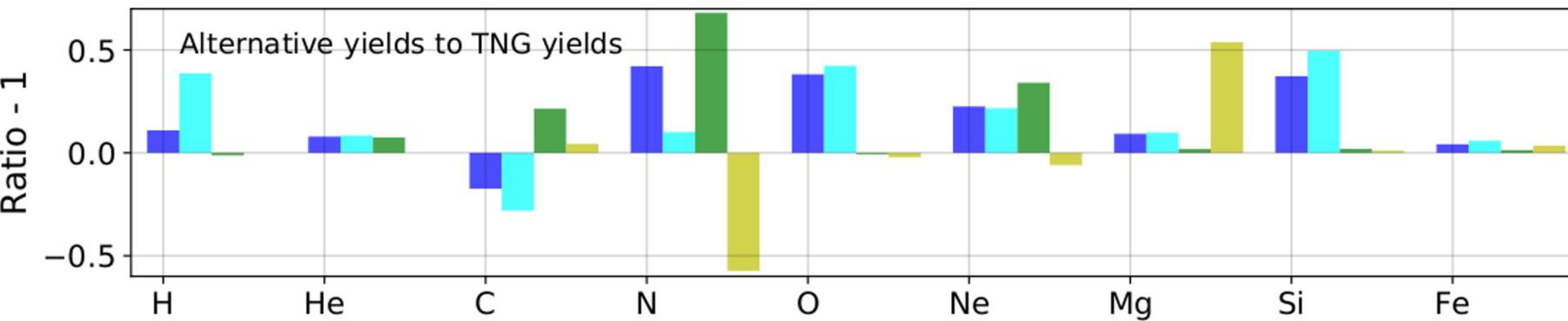
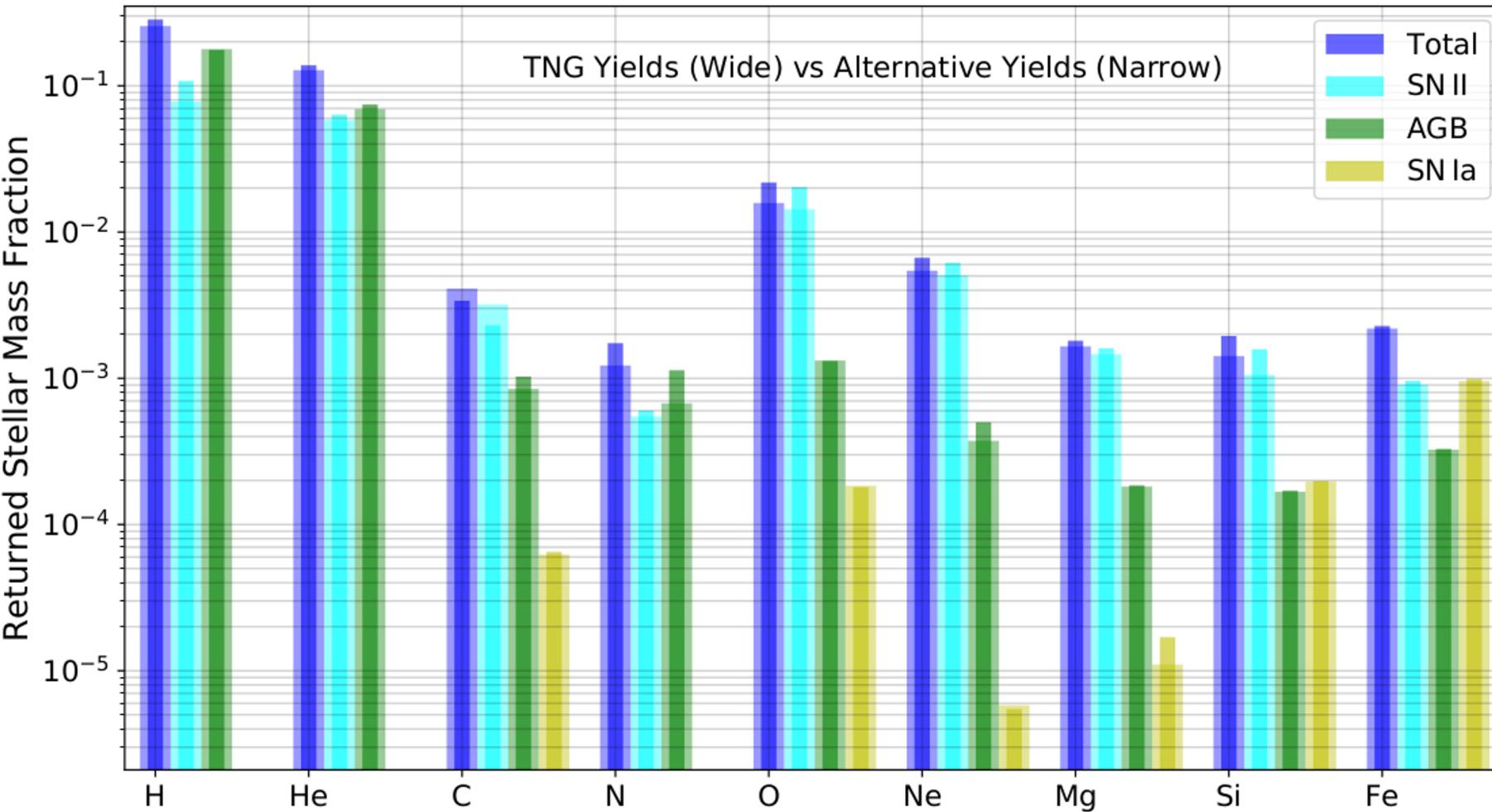


Si, N and C are the most discrepant elements between our yield sets!

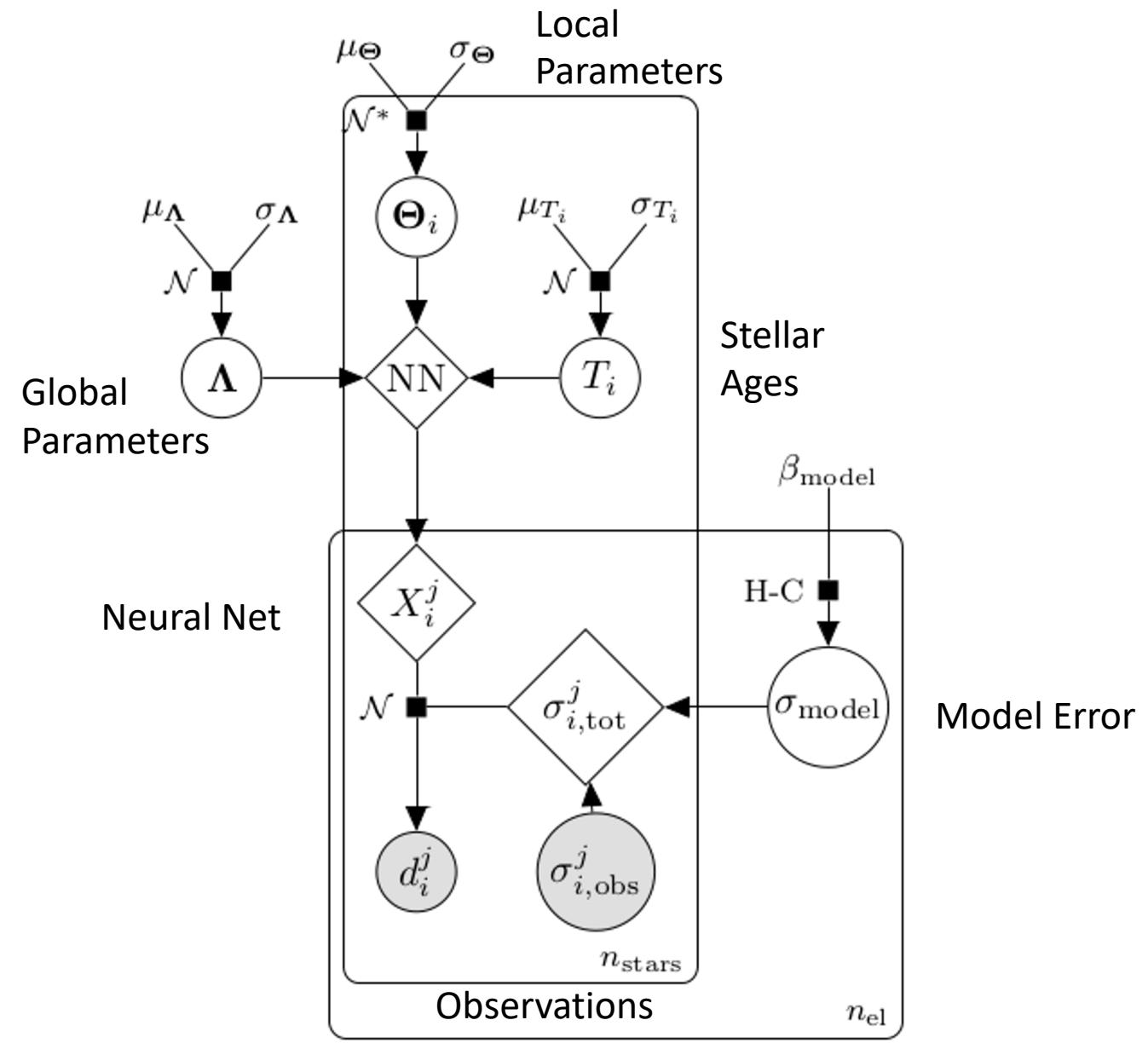
Abundance Diagrams



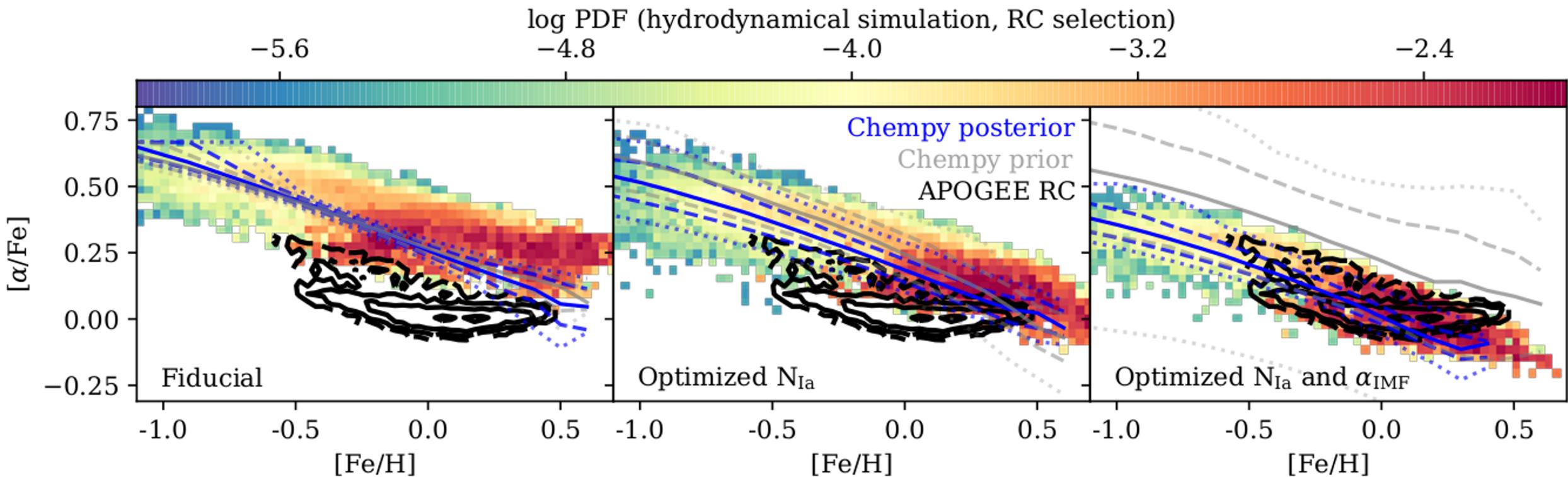
Default and Alternative Yields



ChempyMulti Architecture



Full Hydrodynamical Simulation Optimization



Full Corner Plot

