Conservation of Mechanical Energy (Draft)

Introduction

We'll use the rollercoaster track and marble to measure the marble's speed at certain points on the track. If we know the height and speed of the marble at a point on the track, we can calculate its total energy.

The diameter of the marble is about $d = 19\text{mm}$. If the marble breaks the beam on a photogate for a duration of time Δt, then the speed of the marble as it pass through the photogate is $v = d/\Delta t$.

The *kinetic* energy of the marble is given by the expression $\frac{1}{2}mv^2$ and the potential energy of the marble is mgh where m is the marble's mass, $g = 9.81$ m/s^2 is the acceleration due to gravity, and h is the marble's height above the table surface.

Part 1

- Measure the marble's velocity at 12 locations on the track and fill out the provided spreadsheet; your data will look something like the figure on the right

	A	B	C	D
1	s (meters)	h (meters)	dt (seconds)	v (meters/second)
2	0	0.362		0.000
3	0.05	0.352	0.0517	0.368
4	0.43	0.145	0.0109	1.743
5	0.15	0.308	0.0198	0.960
6	0.5	0.156	0.0116	1.638
7	0.2	0.270	0.0156	1.218
8	0.6	0.219	0.0141	1.348

- Add a new column to the spreadsheet titled `k.e. (joules)` and use the formula for kinetic energy (above) to fill in the values of that column
- Add another column titled `p.e. (joules)` and use the formula for potential energy to fill in the values
- The total energy of the marble is just the sum *kinetic energy* + *potential energy*. Add another column title `total (joules)` and fill in the values by adding `k.e. + p.e.`; Does is appear that the total energy is conserved?
- To investigate this a little further, add on more column titled `Ediff (joules)`. If energy is conserved then the total energy should always be equal to $mg\,h_0$ where h_0 is the height of the marble at the start of the track (i.e. where its speed is zero); Fill in this final column with the values `mgh - total`. This is difference between the expected value of energy and what you actually measured. What do you notice?

Part 2

- Make a plot of `k.e.` versus `Ediff1`. You should see that your data is distributed around a line
- Now, use the trendline spreadsheet feature to plot the line that best fits the data
- The slope of this line corresponds to the quantity called the *moment of inertia*; we could do all the math to calculate, from first principles, the moment of inertia of a solid sphere (like the marble) and we would find it to be ⅖ = 0.4. Is your value close?
- Let us test the conservation of energy now using the theoretical value of the moment of inertia. Including rotation, the total energy at a point of the track is given by $\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mv^2 + mgh = \frac{7}{10}mv^2 + mgh$. Use this formula to fill in a new column title `total2`.