

The Accuracy of Current Transformers Adjacent to High-Current Busses

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CURRENT transformers normally have excellent accuracy when applied correctly under conditions for which they were designed. While the common operating conditions affecting accuracy are usually recognized and properly considered, there is one factor, the importance of which is frequently underestimated or even overlooked entirely. This is the effect of stray flux produced by high-current busses adjacent to the transformer.

When current transformers are applied in the vicinity of busses carrying large currents, the transformer accuracy may be very seriously affected. This is particularly true on systems which normally carry over 2,000 amperes and may carry 10 or 20 times this current during faults. A current transformer designed to operate protective relays during such a fault may be rendered completely useless by improper installation. For instance, Figure 10 gives the accuracy of a certain current transformer with the primary return conductor at various spacings. If a burden of 0.190 ohm were used, and a return spacing of 12 inches, saturation would occur at about 10,000 amperes. However, with the same burden and a return spacing of 6 inches, saturation would occur at only 6,000 amperes. This could mean the difference between relays operating properly and relays not operating at all. Because such operating conditions are difficult to reproduce in test, it is usually necessary to rely on calculated rather than tested accuracy when such a situation is encountered.

There has been a certain amount of published information^{1,2} on the effect of stray fields on current transformer accuracy. However, in general, such information has been qualitative rather

than quantitative; and, as a result, designers have considerable difficulty in predicting performance if the conditions are somewhat unusual. An investigation has been undertaken of both the theoretical and practical aspects of this problem. This investigation is divided into three sections as follows:

1. Theoretical analysis (basis of tests).
2. Flux pick-up factors (empirical data).
3. Accuracy calculations and tests (use of data).

While the theoretical study is applicable to any type of current transformer, empirical pick-up factors were determined primarily for bushing transformers because other types take an almost prohibitive number of shapes and designs. However, with a little care, the data presented can be applied to almost any design with sufficient accuracy to permit reasonable estimates of performance.

Theoretical Analysis

In a perfect current transformer, the ampere-turns produced in the secondary would be exactly equal to the ampere-turns applied to the primary. Practically, this condition is never attained because the core material is not perfect and, therefore, uses some of the primary ampere-turns in maintaining magnetic flux in the core. Since these exciting ampere-turns are used in the core, they cannot be reproduced in the secondary, and an error results. The core flux may be considered to be made up of two components; first, the mutual or burden flux which links both the primary and secondary windings and is necessary to produce the required secondary voltage; and second, the leakage or stray flux

which may exist in some sections of the core due either to the leakage reactance of the primary and secondary coils themselves or to other current carrying conductors in the immediate vicinity.

The effect of stray flux produced by internal transformer leakage has been understood for many years. Since leakage flux (produced either by the coils themselves or by external conductors) will change the flux density in certain sections of the core, the errors will be affected. This fact is just as true in a bushing transformer with a uniformly distributed winding as in any other type of transformer, although this has not always been generally understood.

In the theoretical study of the effect of stray flux on current transformer accuracy, two conditions were analyzed as being representative of the general problem. The first considered an annular core transformer with uniformly distributed secondary winding, and the second an annular core transformer with the secondary concentrated on two small sections of the core. In both cases, a bar type (single turn) primary was assumed since nearly all high-current transformers have a single turn primary.

RING CORE WITH DISTRIBUTED SECONDARY WINDING AND SINGLE BAR PRIMARY

The following assumptions were made:

1. Transformer completely symmetrical above and below a plane taken through the primary conductor and the adjacent conductor.
2. Primary bar perfectly centered in core.
3. Secondary winding evenly distributed around core.
4. Stray flux enters at one point of core. In practice, other flux entering at other points would act in a similar manner since the point of entry is general.

Paper 51-308, recommended by the AIEE Instruments and Measurements Committee and approved by the AIEE Technical Program Committee for presentation at the AIEE Pacific General Meeting, Portland, Oreg., August 20-23, 1951. Manuscript submitted May 21, 1951, made available for printing July 9, 1951.

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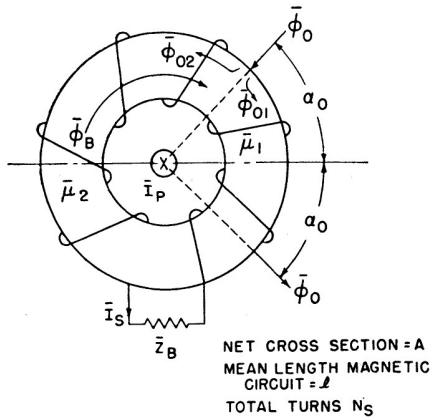


Figure 1 (left). Ring core with distributed winding

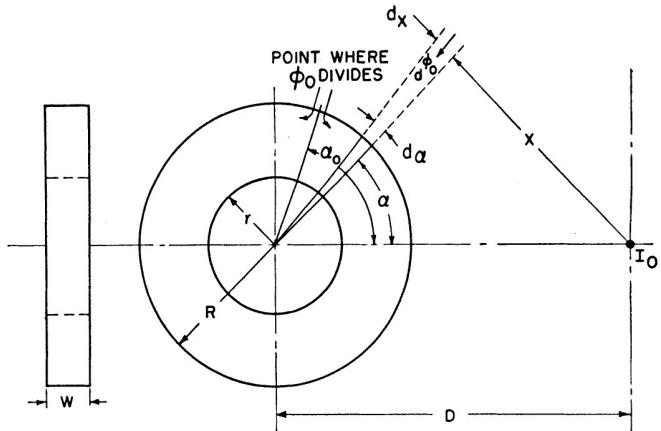


Figure 2 (right). Ring core with adjacent conductor

Referring to Figure 1, three equations may be written as follows:

The total voltage across the burden must equal the voltage induced by the flux linking the windings

$$I_s Z_B = \frac{\alpha_0}{\pi} N_s (\phi_B + \phi_{01}) + \left(1 - \frac{\alpha_0}{\pi}\right) N_s (\phi_B - \phi_{02}) \quad (1)$$

The total primary ampere turns minus the total secondary ampere turns must equal the magnetizing ampere turns of the core

$$I_p - I_s N_s = \frac{\alpha_0 l}{A \mu_1} (\phi_B + \phi_{01}) + \frac{\left(1 - \frac{\alpha_0}{\pi}\right) l}{A \mu_2} (\phi_B - \phi_{02}) \quad (2)$$

The stray flux in the core must equal the flux entering the core

$$\phi_{01} + \phi_{02} = \phi_0 \quad (3)$$

Eliminating ϕ_{01} , ϕ_{02} , ϕ_B from equations 1, 2, and 3 gives

$$I_p - I_s N_s = \frac{\alpha_0 l}{A \mu_1} \left[\frac{I_s Z_B}{N_s} + \left(1 - \frac{\alpha_0}{\pi}\right) \phi_0 \right] + \frac{\left(1 - \frac{\alpha_0}{\pi}\right) l}{A \mu_2} \left[\frac{I_s Z_B}{N_s} - \frac{\alpha_0}{\pi} \phi_0 \right] \quad (4)$$

While equation 4 may be solved for I_s , it is most conveniently used in its present form.

It should be noted that equation 4 is in exactly the same form as equation 2 and, therefore,

$$\phi_B = \frac{I_s Z_B}{N_s}, \quad \phi_{01} = \left(1 - \frac{\alpha_0}{\pi}\right) \phi_0, \quad \phi_{02} = \frac{\alpha_0}{\pi} \phi_0$$

Since, in the above equation, ϕ_B produces all the required burden voltage, no voltage is induced by ϕ_0 . The voltage induced by ϕ_{01} is exactly equal and opposite to that induced by ϕ_{02} . This flux condition is independent of the permeabilities in the two sides of the core.

However, if the primary current is reduced to zero, secondary current will flow due to the action of ϕ_0 unless the permeabilities are equal.

From a practical point of view, equation 4 indicates the basic method of test or calculation. From the above definitions, ϕ_B , ϕ_{01} , and ϕ_{02} may be calculated directly at any value of secondary current I_s if ϕ_0 and α_0 are known. Once the flux is determined, known data on the core material will give the permeabilities, μ_1 and μ_2 , and I_p can be calculated from equation 4.

To show that ϕ_0 can always be assumed to divide such that ϕ_{01} and ϕ_{02} induce equal and opposite voltages in the secondary winding, a nonuniformly wound secondary is considered.

RING CORE WITH TWO CONCENTRATED SECONDARY WINDING AND SINGLE BAR PRIMARY

The assumptions were the same as for the distributed winding calculation except that the secondary is wound in two sections, one on either side of the core at the central horizontal plane.

The analysis is the same except for equation 1 which becomes

$$I_s Z_B = \frac{N_s}{2} (\phi_B + \phi_{01}) + \frac{N_s}{2} (\phi_B - \phi_{02}) \quad (5)$$

The same solution as before gives the following result

$$I_p - I_s N_s = \frac{\frac{\alpha_0}{\pi} l}{A \mu_1} \left[\frac{I_s Z_B}{N_s} + \frac{\phi_0}{2} \right] + \frac{\left(1 - \frac{\alpha_0}{\pi}\right) l}{A \mu_2} \left[\frac{I_s Z_B}{N_s} - \frac{\phi_0}{2} \right] \quad (6)$$

This is basically the same as equation 4 for the distributed winding. The same discussion holds, and the only point to be made is that again the assumed ϕ_0 split should be made such as to induce no voltage in the winding. In this case, this means that $\phi_{01} = \phi_{02}$ regardless of the entry point of ϕ_0 if it is between the two coils.

In a practical case, the stray flux does not enter the core at one point but rather at all points on the core. However, the same assumption of no apparent induced

Nomenclature

- I_p = primary current (rms amperes)
- I_0 = adjacent conductor current (rms amperes)
- I_s = secondary current (rms amperes)
- E_s = secondary voltage (rms volts)
- ϕ_B = circulating flux in core (rms volts/turn)
- ϕ_0 = total stray flux entering core from adjacent conductor (rms volts/turn)
- ϕ_{01} = portion of ϕ_0 which follows core side 1 (rms volts/turn)
- ϕ_{02} = portion of ϕ_0 which follows core side 2 (rms volts/turn)
- ϕ_{0a} = stray flux at any point (a) in core (rms volts/turn)
- H = magnetic intensity at any point in core (rms ampere turns/inch)
- μ_1 = permeability of core side 1 (rms volts/turn square inch/rms ampere turns/inch)
- μ_2 = permeability of core side 2 (rms volts/turn square inch/rms ampere turns/inch)
- α_0 = angle around core to point where $d\phi_0$ enters core, radians
- α_0 = angle around core to point where ϕ_0 divides, radians
- R_s = secondary resistance, ohms
- Z_B = total secondary burden including R_s , ohms
- N_s = total secondary turns
- A = net cross section of core, square inches
- l = mean length of magnetic circuit, inches
- R = outside radius of core, inches
- r = inside radius of core, inches
- W = width of core, inches
- D = distance from center of primary conductor to center of adjacent conductor, inches
- x = distance from adjacent conductor to point where $d\phi_0$ enters core, inches
- k = core pick-up factor (ratio of the flux entering a magnetic core to the flux entering a nonmagnetic core of the same dimensions when placed in a magnetic field)

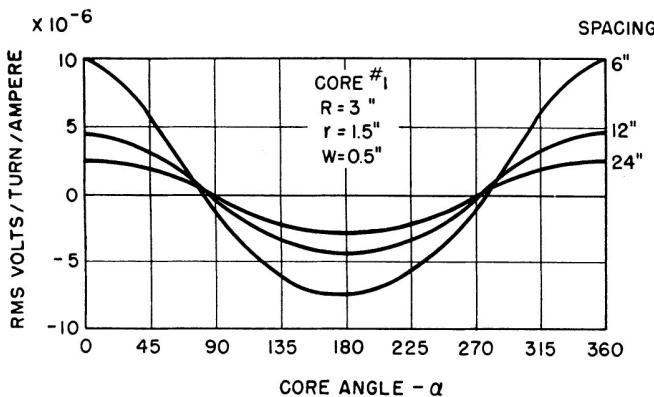


Figure 3. Pick-up flux from adjacent conductor core number 1

voltage may be used to determine how ϕ_0 divides.

FLUX PICK-UP FACTOR OF RING CORES

The other question to be given some theoretical consideration is involved in the amount of stray flux picked up by a magnetic core located in the field of a current-carrying conductor. The core will be considered to have a rectangular cross section but annular in shape, similar to the usual bushing transformer core.

The following assumptions were made:

- Core completely symmetrical above and below the plane through the axis of the core and the adjacent conductor.
- Adjacent conductor straight and infinite in length.
- Permeability of core large compared to air.
- Path of flux from adjacent conductor can be considered a complete circle around conductor. Empirical pickup factors include the effect of shortening of air path by magnetic core.
- Pick-up factor uniform over core (not strictly true, but gives sufficient accuracy for most purposes).

Referring to Figure 2, the flux $d\phi_0$ entering the core over an incremental distance dx at a distance x from the adjacent conductor (neglecting the effect of the core in shortening the air path) is

$$d\phi_0 = k \frac{3.2 \times 1.414 I_0}{2\pi x} W dx \frac{1}{4.44 \times 60 \times 10^{-8}} \\ = 1.92 \times 10^{-6} k WI_0 \frac{dx}{x} \text{ volts/turn at } 60 \text{ Cycles} \quad (7)$$

Table I

Transformer Number	R, Inches	r, Inches	W, Inches	N _s	R _s , Ohms
1.....3.....1.5....0.5....183....0.139					
2.....3.....1.5....1.5....165....0.174					
3.....3.....2.5....1.5....200....0.157					

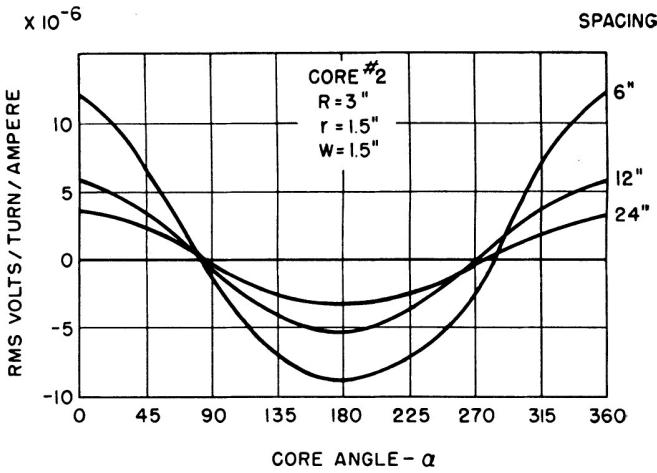


Figure 4. Pick-up flux from adjacent conductor core number 2

Writing x in terms of the angle α around the core

$$x = [(D-R \cos \alpha)^2 + (R \sin \alpha)^2]^{1/2} \quad (8)$$

$$x = [D^2 - 2RD \cos \alpha + R^2]^{1/2} \quad (9)$$

$$dx = \frac{RD \sin \alpha d\alpha}{[D^2 - 2RD \cos \alpha + R^2]^{1/2}} \quad (10)$$

Substituting equations 9 and 10 in equation 7

$$d\phi_0 = 1.92 \times 10^{-6} k W D R I_0 \frac{\sin \alpha d\alpha}{D^2 - 2RD \cos \alpha + R^2} \quad (11)$$

Equation 11 is an expression for the flux entering the core through the differential angle $d\alpha$ at any point on the core described by the angle α . The flux in any section of the core may be found by integrating equation 11 from the point where the flux divides, α_0 , to that section. However, equations 4 and 6 stated that α_0 must be taken such that no voltage is induced in the windings. This condition involves an integration which becomes very difficult to perform.

In order to avoid this integration, it is better to numerically integrate equation 11 for the particular core involved. By carrying on this integration from $\alpha=0$ toward α_0 and $\alpha=\pi$ toward α_0 simultaneously,

ouslly, α_0 may be found for the condition of zero induced voltage.

The total flux picked up by the core may be found directly by integrating equation 11 from $\alpha=0$ to $\alpha=\pi$.

$$\phi_0 = 1.92 \times 10^{-6} k W D R I_0 \times \int_0^\pi \frac{\sin \alpha d\alpha}{D^2 - 2RD \cos \alpha + R^2} \quad (12)$$

$$\phi_0 = 1.92 \times 10^{-6} k W D R I_0 \times \left[\frac{1}{2RD} \log_n (D^2 - 2RD \cos \alpha + R^2) \right]_0^\pi \quad (13)$$

$$\phi_0 = 1.92 \times 10^{-6} k W I_0 \log_n \frac{D+R}{D-R} \quad (14)$$

The only unknown factor involved in equation 14 is the pick-up factor k . Considerable data has been taken in order to evaluate k empirically.

Flux Pick-Up Test

For the purpose of evaluating k for ring cores, three experimental transformers were made up of mumetal. A 20-turn search coil was placed each $22\frac{1}{2}$ degrees around the cores before winding the evenly distributed secondary. Table I gives the dimensions, secondary turns, and secondary resistance of each.

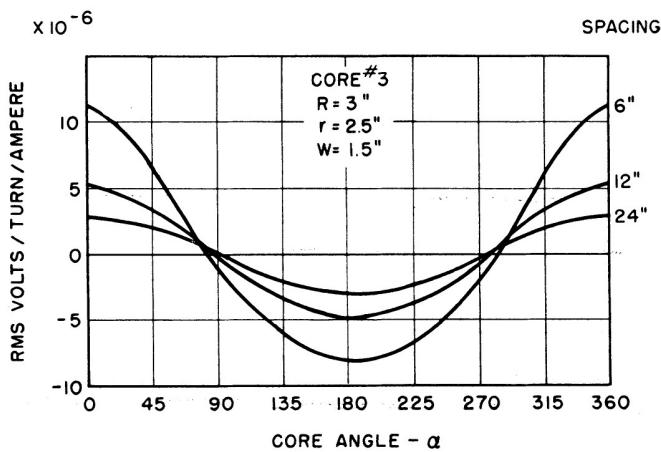


Figure 5. Pick-up flux from adjacent conductor core number 3

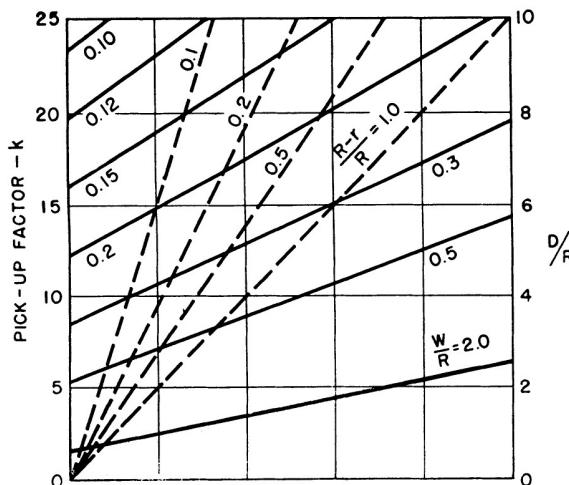


Figure 6 (left). Core pick-up factor—enter with D/R , horizontally to $(R-r)/R$, vertically to W/R , horizontally to k

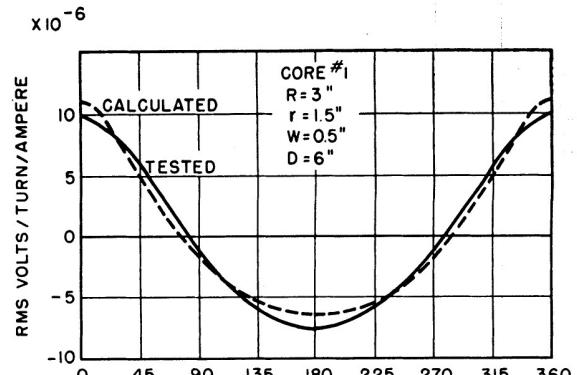


Figure 7 (right). Pick-up flux from adjacent conductor core number 1

These transformers were placed in a test fixture in such a manner that the bar primary passed through the core and returned at distances of 6, 12 or 24 inches, as desired. The secondary was short-circuited and search-coil voltages measured for known values of primary current. That component of search-coil voltage which was 90 degrees out-of-phase with the primary current, was assumed to be due to the adjacent conductor. The in-phase component was constant around the core and corresponded to the resistance load of the secondary winding. Figures 3, 4, and 5 give the results in terms of volts/turn/primary ampere at 60 cycles.

The total flux picked up by these three cores at the three spacings was substituted into equation 14 and the pick-up

factor, k , calculated in Table II. The dimensionless ratios D/R , W/R , and $(R-r)/R$, also are included in Table II.

Figure 6 is a plot of pick-up factor versus the dimensionless ratios given in Table II. While there was insufficient data taken to accurately determine the exact functions involved, the curves fit the ranges of dimensions covered by test and are sufficiently accurate for practical purposes.

While these curves were made for ring cores, sufficient checks have been made against rectangular cores to show that estimated radii of rectangular cores can be used to obtain the pick-up factor in the above range of dimensions.

The calculated flux distribution in the cores was found to agree quite well with the tested values. As might be expected,

the pick-up factor was smaller on the side of the core which was close to the return. This tended to make the calculated α_0 smaller than the tested α_0 , and the calculated flux in the side of the core next to the return larger than the tested value. Figure 7 is a comparison of the tested curve for a 6-inch spacing on core number 1 (see curve 3) against a numerically integrated curve (equation 11) for the same conditions. This integration is given in Appendix I.

Accuracy Calculations and Tests

A current transformer which is being subjected to interference flux from an adjacent conductor presents the same problem of accuracy calculation as a transformer with internal leakage. The general method is to divide the core into several sections, estimate the flux density in each section, find the exciting ampere turns necessary to produce this density in each section, and vectorially add these exciting ampere turns to give the total exciting ampere turns from which ratio and phase angle errors may be found. This type of calculation has been discussed in previous papers, so it will not be considered further here. Appendix II gives a sample calculation for one of the above transformers.

The three bushing-type transformers were tested at two burdens and three return conductor spacings. The accuracy curves are presented on Figure 8, 9, and 10. Several calculated points are included to give an idea of the comparison between calculated and tested results.

It should be noted that when calculations are made in the region where the core iron is saturating, very small errors in density produce very large changes in calculated accuracy. This means that calculated accuracy cannot be expected to check tested accuracy closely if the transformer errors are large. Usually the point where the errors start to increase rapidly can be calculated with reason-

Table II

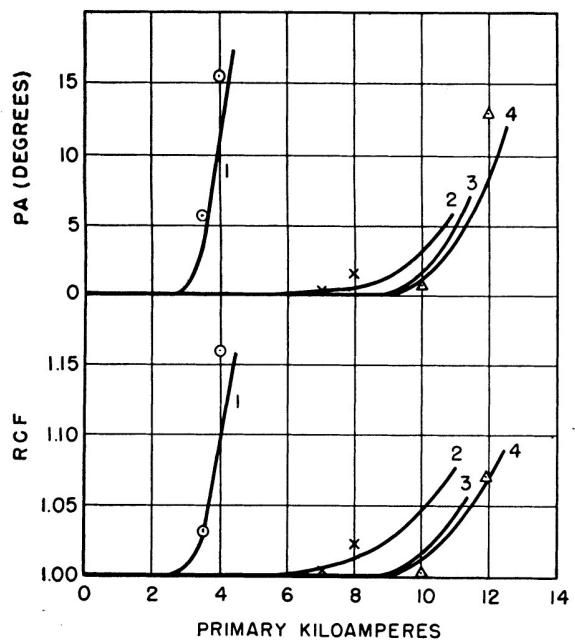
Transformer Number	$\frac{R-r}{R}$	$\frac{W}{R}$	$\frac{D=6 \text{ inch}}{D/R \ k}$	$\frac{D=12 \text{ inch}}{D/R \ k}$	$\frac{D=24 \text{ inch}}{D/R \ k}$
1.....	0.5.....	0.167.....	2.....16.6.....	4.....18.7.....	8.....22.8.....
2.....	0.5.....	0.5.....	2.....6.60.....	4.....7.59.....	8.....10.4.....
3.....	0.167.....	0.5.....	2.....6.15.....	4.....7.00.....	8.....8.16.....

Table III

α	$d\phi_0$	$Nd\phi_0$	E_s	$\alpha = \alpha - 5^\circ$ ϕ_{0a}	α	$d\phi_0$	$Nd\phi_0$	$-E_s$	$\alpha = \alpha + 5^\circ$ ϕ_{0a}
5.....	0.47.....	0.24.....	0.24.....	11.05.....	175.....	0.05.....	0.03.....	0.03.....	6.44.....
15.....	1.26.....	1.89.....	2.13.....	10.58.....	165.....	0.16.....	0.24.....	0.27.....	6.39.....
25.....	1.71.....	4.27.....	6.40.....	9.32.....	155.....	0.27.....	0.68.....	0.95.....	6.23.....
35.....	1.85.....	6.48.....	12.88.....	7.61.....	145.....	0.38.....	1.34.....	2.29.....	5.96.....
45.....	1.81.....	8.15.....	21.03.....	5.76.....	135.....	0.50.....	2.26.....	4.55.....	5.58.....
55.....	1.67.....	9.21.....	30.24.....	3.95.....	125.....	0.62.....	3.42.....	7.97.....	5.08.....
65.....	1.52.....	9.86.....	40.10.....	2.28.....	115.....	0.75.....	4.89.....	12.86.....	4.46.....
56% of 75.....	0.76.....	5.71.....	45.81.....	0.76.....	105.....	0.89.....	6.66.....	19.52.....	3.71.....
	11.05.....				95.....	1.03.....	8.75.....	28.27.....	2.82.....
					85.....	1.19.....	11.28.....	39.55.....	1.79.....
					44% of 75.....	0.60.....	6.25.....	45.80.....	0.60.....
						6.44.....			

$$d\phi_0 = 7.96 \frac{\sin \alpha d\alpha}{1.25 - \cos \alpha}$$

Figure 7 is a plot of ϕ_{0a} versus α
 $\alpha_0 = 75.6$ ($\Sigma E_s = 0$)



Tested Curve	Calculated Points	Connected Burden	Return Spacing, Inches
1	○	0.766+j0	6
2	x	0.166+j0	6
3	x	0.166+j0	12
4	△	0.166+j0	24

able accuracy, but no attempt is made to carry the curve further.

In the practical case, it is usually unnecessary to go through the complete accuracy calculation in order to determine whether or not a given installation will be satisfactory. Referring to Table III, Appendix II, it may be noted that nearly all of the error is caused by saturation in

Figure 9. Overcurrent accuracy of transformer number 2

Tested Curve	Calculated Points	Connected Burden	Return Spacing, Inches
1	○	0.750+j0	6
2	x	0.150+j0	6
3	x	0.150+j0	12

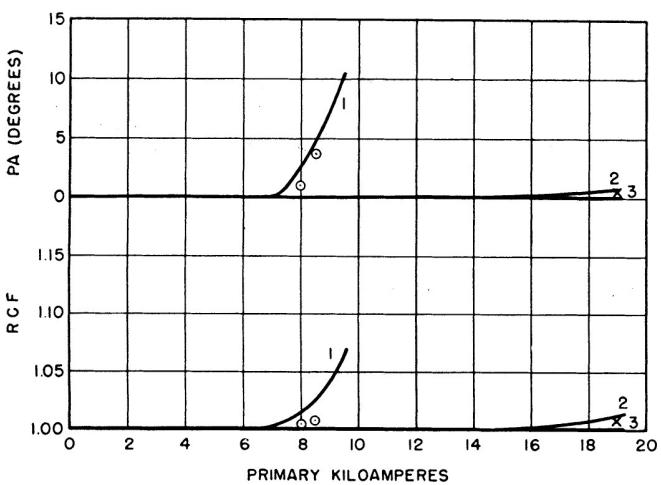


Figure 8 (left). Overcurrent accuracy of transformer number 1

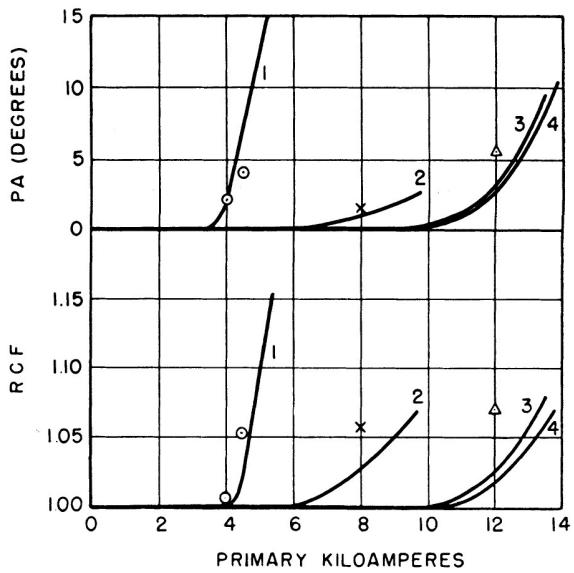


Figure 10 (right). Overcurrent accuracy of transformer number 3

section 1, which is the section of the core closest to the return conductor. This means that the transformer accuracy will be good for primary current up to the point where section 1 starts to saturate. By estimating the flux density in this section, the maximum safe operating current may be obtained. The burden flux ϕ_B , and the stray flux ϕ_{01} will be assumed to be in phase since this represents the worst possible condition. By definition, the burden flux, ϕ_B , is given by

$$\phi_B = \frac{I_s Z_B}{N_s} = \frac{I_p Z_B}{N_s^2} \text{ volts/turn} \quad (15)$$

for a transformer with a single-turn primary. If the only stray flux to be considered is that produced by an adjacent conductor (no internal leakage), equation 14 may be used to find the total flux picked up by the core.

$$\phi_0 = 1.92 \times 10^{-6} k W I_0 \log_n \frac{D+R}{D-R} \quad (14)$$

From the curves of Figure 3, 4, and 5,

Tested Curve	Calculated Points	Connected Burden	Return Spacing, Inches
1	○	0.790+j0	6
2	x	0.190+j0	6
3	△	0.190+j0	12
4	△	0.190+j0	24

it may be assumed that $\phi_{01} = 60$ per cent of ϕ_0 , that is 60 per cent of the flux picked up by the core passes through the section nearest the return conductor so

$$\phi_{01} = 1.15 \times 10^{-6} k W I_0 \log_n \frac{D+R}{D-R} \quad (16)$$

For the transformer calculated in Appendix II

$$\begin{aligned} \phi_B &= \frac{I_p \times 0.305}{(183)^2} = 9.1 \times 10^{-6} I_p \text{ volts/turn} \\ \phi_{01} &= 1.15 \times 10^{-6} \times 0.5 \times I_0 \log_n \frac{6+3}{6-3} \\ &= 10.5 \times 10^{-6} \text{ volts/turn} \end{aligned}$$

In this case, the adjacent conductor is the primary return so that $I_p = I_0$ and the total flux in section 1 is

$$\phi_B + \phi_{01} = (9.1 + 10.5) 10^{-6} I_p = 19.6 \times 10^{-6} I_p$$

Figure 11. Allegheny mumetal 0.025-inch thick ring punchings transformer numbers 1, 2, 3 at 60 cycles

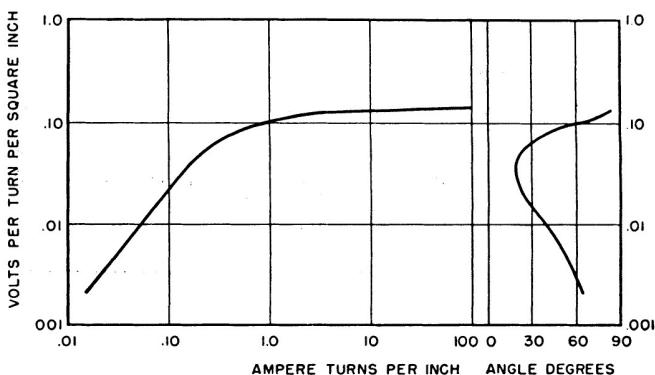


Table IV

Section	Section <i>l</i>	$\Phi_{B/A}$	$\Phi_{0a/A}$	$\frac{\Phi_B + \Phi_{0a}}{A}$	<i>H</i>	<i>Hl</i> section
1	0.88	0.103 + j0	0 + j0.109	0.150 + j46	180 - j87	120 - j104
2	0.88	0.103 + j0	+ j0.103	0.145 + j45	100 - j87	66 - j58
3	1.76	0.103 + j0	+ j0.084	0.133 + j39	18 - j83	23 - j22
4	1.76	0.103 + j0	+ j0.046	0.113 + j24	1 - j73	1 - j2
5	1.76	0.103 + j0	+ j0.006	0.103 + j3	1 - j63	1 - j2
6	1.76	0.103 + j0	- j0.026	0.106 - j15	1 - j67	0 - j2
7	1.76	0.103 + j0	- j0.056	0.117 - j29	2 - j76	- 1 - j4
8	1.76	0.103 + j0	- j0.074	0.127 - j36	6 - j81	- 5 - j10
9	1.76	0.103 + j0	- j0.082	0.132 - j39	16 - j83	- 15 - j24
$\Sigma l = 14.1$.190 - j228
$RCF = 1 + \frac{190}{8,000} = 1.024$				$PA = -57.3 - \frac{228}{8,000} = +1.6^\circ$		

The core cross section is 0.71 square inch, so the flux density is

$$\frac{\Phi_B + \Phi_{0a}}{A} = \frac{19.6 \times 10^{-6} I_p}{0.71} = 27.6 \times 10^{-6} I_p \text{ volts/turn/square inch}$$

From Figure 11, core saturation will occur at about 0.14 volt per turn per square inch, so the limiting primary current for good accuracy is approximately

$$I_p = \frac{0.14}{27.6 \times 10^{-6}} = 5,100 \text{ amperes}$$

From Figure 8, this is seen to be somewhat conservative, but nevertheless, a very good guide in determining whether or not a given installation is satisfactory.

Conclusions

1. A practical method of estimating the flux in a transformer core under the influence of an adjacent current-carrying bus has been investigated. It has been shown that by assuming a fixed secondary current, the stray flux in the core may be assumed to divide in such a manner as to induce no voltage in the secondary winding.

2. Empirical data is given for estimating the pick-up factor of a ring core in such a field. This may be applied with reasonable accuracy to rectangular cores by

using the estimated equivalent radii of the rectangular core.

3. The use of this data in calculating current transformer accuracy is discussed and verified by test results.

4. A simplified method of calculating the approximate maximum current at which good accuracy can be expected is demonstrated.

Appendix I

Numerical calculation of equation 11:

$$d\phi_0 = 1.92 \times 10^{-6} k W D R I_0 \frac{\sin \alpha d\alpha}{D^2 - 2RD \cos \alpha + R^2} \quad (11)$$

For transformer number 1 with 6 inch return:

$$W=0.5, D=6, R=3, I_0=1 \text{ ampere}$$

k from Figure 6 is found to be 16.6. Substituting the constants in equation 11 gives

$$d\phi_0 = 7.96 \frac{\sin \alpha d\alpha}{1.25 - \cos \alpha} \text{ microvolts/turn}$$

dα is taken as 10 degrees for the numerical integration. Referring to Table III, $d\phi_0$ is calculated for various values of *α* starting simultaneously from *α*=0 and *α*=180 degrees. $Nd\phi_0$ is taken as the number of sections through which $d\phi_0$ passes times $d\phi_0$ since the turns per section is a constant. E_s is the sum of the $Nd\phi_0$ terms up to *α*. This integration is carried on until the

Discussion

E. C. Wentz (Westinghouse Electric Corporation, Sharon, Pa.): Mr. Pfuntner is to be congratulated on a most workmanlike and practical solution of a most difficult yet important problem. The importance of stray fields from high current busses in affecting nearby current transformers is not generally realized.

Even when this effect has been understood to exist, no good method for calculation of the effect has been published. Any exact method of calculation is entirely too complicated for practical use, and any practical methods have been not much better than rules of thumb.

Stray fields due to current limiting reactors also are known to affect current transformers. I would like to know whether any simple adaptation of this method can be used to deal with this problem.

The usual installation has three current transformers side by side in a 3-phase bus. The outside transformers are exposed to the fields from the other two phases. Presumably the flux would have to be calculated for each bus and added vectorially.

The principal utility of this method is in number 4 of the author's conclusions—the calculation of the saturation point of the transformer. At densities below saturation the leakage fluxes flowing in opposite directions in the opposite parts of the core tend to

point, α_0 , is found where the voltage is equal from both sides. The actual flux in the core ϕ_{0a} is found by totaling $d\phi_0$ and subtracting the amount which entered at each section.

Appendix II. Accuracy Calculation of Transformer Number 1 Return Spaced Six Inches from Primary

Primary current ... $I_p = I_0 = 8,000$ amperes

Secondary turns ... $N_s = 183$

Secondary resistance ... $R_s = 0.139$ ohms

Net cross section of core ... $A = 0.71$ inch²

Mean length of magnetic circuit of core ... $l = 14.1$ inches

External burden ... $= 0.166 + j0$ ohms

Secondary burden ... $Z_B = 0.305 + j0$ ohms

Core material ... Mumetal

(See Figure 11 for data)

The core is divided into 18 sections; 14 sections are $22\frac{1}{2}$ degrees and the other 4 are $11\frac{1}{4}$ degrees. The smaller sections are taken next to the return conductor because the density changes rapidly in this area. Since the core is symmetrical above and below the plane of the primary and return, the top and bottom half may be combined to give seven sections of 45 degrees and two sections of $22\frac{1}{2}$ degrees.

The procedure is conventional in that the density, magnetic intensity from Figure 11, and exciting ampere-turns are found for each section and then added to give the total exciting ampere turns. The error in terms of ratio correction factor and phase angle are calculated from the in-phase and in-quadrature components of the ampere turns necessary to maintain the flux in the core. Table IV gives the calculations for this example.

References

- LEAKAGE PHENOMENA IN RING-TYPE CURRENT TRANSFORMERS, A. H. M. Arnold. *Journal, Institution of Electrical Engineers (London, England)*, volume 74, May 1934, pages 413-23.
- INFLUENCE OF LEAKAGE FLUX UNDER VARIOUS CONDITIONS, H. E. Forest. *Electrical Review (London, England)*, volume 143, July 9, 1948, pages 51-54.
- COMPUTATION OF ACCURACY OF CURRENT TRANSFORMERS, A. T. Sinks. *Electrical Engineering (AIEE Transactions)*, volume 59, December 1940, pages 663-68.

cancel their effects, but when one part of the core begins to saturate the transformer rapidly becomes useless. Therefore the method is not usually necessary for calculation of metering accuracy over the range of current up to 100 per cent current, but will often be of value in determining the operation of relays at high currents.

The method is of value in determining the performance of existing installations, or proposed installations with existing designs. It will possibly be of interest to describe how we have designed transformers for some 20 years to overcome the effect of return busses or stray fields so that it is not necessary to perform any special calculations to insure good accuracy. The construction we have

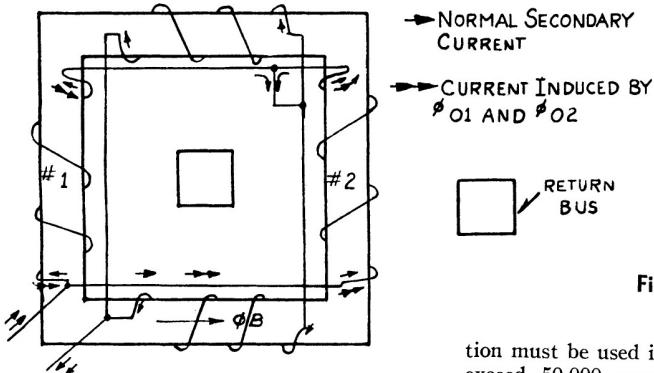


Figure 1

used is shown in Figure 1 of this discussion. Inspection of this figure will show that coils number 1-2 will supply approximately equal currents as they are effectively connected in parallel with respect to the flux ϕ_B , but they are effectively short circuited against each other with respect to the leakage fluxes ϕ_{01} and ϕ_{02} and current flows around the number 1-2 coil circuit which opposes the leakage flux. Inasmuch as there is relatively little leakage current magnetomotive force developed across the space occupied by the current transformer, the leakage flux is readily diverted from the transformer into the space around the transformer and the core is effectively free of leakage flux.

A rule of thumb which has been generally successful is that this series parallel connec-

tion must be used if the fault current is to exceed 50,000 amperes with 20-inch bus spacing. Supposing a typical transformer to have constants $W=2$, $R=5$, $r=4$ with 1.8 square inch core section, 50,000 amperes produces, by the author's method about 7,500 gauss, or more than one-third the effective "saturation" density, showing that the rule of thumb should be reasonably successful. It is always encouraging to get scientific confirmation of a rule of thumb. In this case, as in many others, the scientific confirmation shows where the rule of thumb may get us into trouble. It appears, however, that the series parallel connection must still be used when the calculation according to Mr. Pfuntner's method shows too high a leakage flux density.

A parallel connection scheme, used in wound-type current transformers to eliminate leakage flux, has been patented.

R. A. Pfuntner: I would like to express my appreciation for the interesting comments and discussion presented by Mr. Wentz. While, as Mr. Wentz says, it is always encouraging to get scientific confirmation of a rule of thumb, it is likewise very encouraging to have one's scientific data confirmed by a rule of thumb developed through years of operating experience.

With respect to the effect of current limiting reactors on current transformer accuracy, I have had no occasion to work on this problem, but I see no reason why the pick-up factors obtained in the paper could not be applied quite easily.

It is true that in a polyphase installation it is necessary to vectorially add the flux produced by each bus. While under normal operating conditions with balanced phase currents there is usually some cancellation effect, under fault conditions this may not be true. The interference problem is most serious during faults when currents are very high. These bus currents may be widely unbalanced depending on the type of fault encountered.

Generally the short calculation of the saturation point is all that is required. In most cases this will show that a given installation is either well within the safe operating range or seriously in trouble. Occasionally installations are found where it is desirable to make a more exact check in order to be sure that the operation will be satisfactory under all conditions.